ADM Aeolus L2A aerosol and cloud products.

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Section I) L2A algorithms
High Spectral Resolution Lidar: Rayleigh and Mie signals

\[
s_{ray}(r) = K_{ray} N_p E_0 (C_1(P,T,\nu)X(r) + C_2(\nu)Y(r))
\]

\[
s_{mie}(r) = K_{mie} N_p E_0 (C_4(P,T,\nu)X(r) + C_3(\nu)Y(r))
\]
High Spectral Resolution Lidar: Rayleigh and Mie signals

\[
s_{ray}(r) = K_{ray} N_p E_0 (C_1(P,T,\nu) X(r) + C_2(\nu) Y(r))
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\[
s_{mie}(r) = K_{mie} N_p E_0 (C_4(P,T,\nu) X(r) + C_3(\nu) Y(r))
\]

\[X(r) : \text{Molecular backscattered signal} \ (m^{-3} \text{sr}^{-1})\]

\[Y(r) : \text{Particle backscattered signal} \ (m^{-3} \text{sr}^{-1})\]

\[
X(r) = \frac{\beta_m(r)}{r^2} \exp \left( -2 \int_0^r (\alpha_m(u) + \alpha_p(u)) \, du \right)
\]

\[
Y(r) = \frac{\beta_p(r)}{r^2} \exp \left( -2 \int_0^r (\alpha_m(u) + \alpha_p(u)) \, du \right)
\]

C1, C2, C3, C4 = coefficients computed from the calibration processors (A.Dabas presentation on Wednesday)
Signals are integrated in height bins with a variable vertical resolution:
- 250m/500m for $z < 2$ km
- 1 km for $2$ km < $z$ < 16 km
- 2 km for $z > 16$ km

Retrieval of $\alpha_{aer}$ and $\beta_{aer}$ are representative of height-bin averages.
Cross-talk correction

System of 2 equations with 4 unknown variables

\[
S_{\text{ray},i} = K_{\text{ray}} N_p E_0 (C_{1,\text{ray},i} X_{\text{ray},i} + C_{2,\text{ray},i} Y_{\text{ray},i})
\]

\[
S_{\text{mie},i} = K_{\text{mie}} N_p E_0 (C_{4,\text{mie},i} X_{\text{mie},i} + C_{3,\text{mie},i} Y_{\text{mie},i})
\]

C1 >> C2
C4 << C3
Cross-talk correction

If there is a matching between the bins of Mie and Rayleigh scales, the system is reduced to 2 equations and 2 unknowns and we can separate the molecular signal $X_i$ from the particle signal $Y_i$.

\[
S_{ray,i} = K_{ray} N_p E_0 (C_{1,ray,i} X_{ray,i} + C_{2,ray,i} Y_{ray,i})
\]
\[
S_{mie,i} = K_{mie} N_p E_0 (C_{4,mie,i} X_{ray,i} + C_{3,mie,i} Y_{ray,i})
\]

\[
\begin{align*}
X_i &= \frac{K_{mie} C_{3,i} S_{ray,i} - K_{ray} C_{2,i} S_{mie,i}}{N_p E_0 K_{ray} K_{mie} (C_{1,i} C_{3,i} - C_{2,i} C_{4,i})} \\
Y_i &= \frac{K_{mie} C_{4,i} S_{ray,i} - K_{ray} C_{1,i} S_{mie,i}}{N_p E_0 K_{ray} K_{mie} (C_{1,i} C_{3,i} - C_{2,i} C_{4,i})}
\end{align*}
\]

C1 $>>$ C2
C4 $<<$ C3
L2A processor: Aerosol extinction coefficient retrieval

1) Observed molecular signal $X_{i, obs}$: after cross-talk correction

$$X_i = \int_{R_{i-1}}^{R_i} \frac{\beta_m(r)}{r^2} \exp \left( -2 \int_0^r \left( \alpha_m(u) + \alpha_p(u) \right) du \right) dr$$
1) **Observed molecular signal $X_{i,\text{obs}}$:** after cross-talk correction

\[
X_i = \int_{R_{i-1}}^{R_i} \frac{\beta_m(r)}{r^2} \exp \left( -2 \int_0^r \left( \alpha_m(u) + \alpha_p(u) \right) du \right) dr
\]

2) Computation of the **synthetic molecular signal $X_{i,\text{sim}}$:** only molecules are considered (no particles):

\[
X_{i,\text{sim}} = \int_{R_{i-1}}^{R_i} \frac{\beta_m(r)}{r^2} \exp \left( -2 \int_0^r \alpha_m(u) du \right) dr
\]

Molecular coefficients $\beta_m$ (m$^{-1}$ sr$^{-1}$) and $\alpha_m$ (m$^{-1}$) are a function of the air density and are computed from ECMWF profiles of pressure and temperature (*Collis and Russel* 1976)
**L2A processor : Aerosol extinction coefficient retrieval**

Definition of the **Normalized Integrated Two Way Transmission Ratio**:

\[ \text{NITWT}(i) = \frac{X_{\text{obs}}(i)}{X_{\text{sim}}(i)} = T_{p,\text{sat}}^2(i-1) \left(1 - e^{-2 \alpha_p(i) \Delta R_i}\right) / \left(2 \alpha_p(i) \Delta R_i\right) \]

\[ \alpha_p(i) = f(\Delta R_i, \text{NITWT}_i, T_{p,\text{sat}}^2(i-1)) \]

Only one unknown: the extinction coefficient
L2A processor: Aerosol extinction coefficient retrieval

Definition of the Normalized Integrated Two Way Transmission Ratio:

$$NITWT(i) = \frac{X_{\text{obs}}(i)}{X_{\text{sim}}(i)} = T_{p,\text{sat}}^2(i-1) \left(1-e^{-2\alpha_p(i)\Delta R_i}\right) / \left(2\alpha_p(i)\Delta R_i\right)$$

$$\alpha_p(i) = f(\Delta R_i, NITWT_i, T_{p,\text{sat}}^2(i-1))$$

**Extinction coefficient $\alpha$**

- Derived by recursion:
  - If $T_{p,\text{sat},i-1}^2$ is known, $X_{\text{obs}}(i)$ and $\Delta R_i$ are known from bin data
  - $\alpha_p(i)$ is computed
  - The transmission from the satellite to bin $i$ is computed:
    $$T_{p,\text{sat},i}^2 = T_{p,\text{sat},i-1}^2 e^{-\alpha_p(i)\Delta R_i}$$

**Assumptions:**

- Small variation of molecular terms and range squared terms over the range bin
- Homogeneous particle filling of the range bin
- Bin matching between Rayleigh and Mie scales.


L2A processor: Backscatter coefficient retrieval

\[ X_i = \int_{R_{i-1}}^{R_i} \frac{\beta_m(r)}{r^2} \exp \left( -2 \int_0^r \left( \alpha_m(u) + \alpha_p(u) \right) du \right) dr \]

\[ Y_i = \int_{R_{i-1}}^{R_i} \frac{\beta_p(r)}{r^2} \exp \left( -2 \int_0^r \left( \alpha_m(u) + \alpha_p(u) \right) du \right) dr \]

\[ \hat{\beta}_{p,i} = \frac{Y_i}{X_i} \times \beta_{m,i,sim} \]

- Computed independently without recursion

Yi : Observed Particulate Signal (after cross-talk)
Xi : Observed Molecular Signal (after cross-talk)
\( \beta_{m,i,sim} \): Synthetic Molecular backscatter coefficient

Section II) L2A aerosol products
- Extinction coefficient retrieval: because of the recursion algorithm, propagation of the errors from bin i-1 to bin i. It will result in an oscillation between underestimation and overestimation of the extinction.

Stabilisation of the oscillation through an **averaging over 2 sequent bins** and definition of a new retrieval grid (**Mid Rayleigh Grid**).
L2A aerosol products

- Extinction coefficient retrieval: because of the recursion algorithm, propagation of the errors from bin i-1 to bin i. It will result in an oscillation between underestimation and overestimation of the extinction.

Stabilisation of the oscillation through an **averaging over 2 sequent bins** and definition of a new retrieval grid (**Mid Rayleigh Grid**).

<table>
<thead>
<tr>
<th>Rayleigh grid</th>
<th>Mid Rayleigh grid</th>
</tr>
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<tbody>
<tr>
<td>1</td>
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</table>

<table>
<thead>
<tr>
<th>Product on Rayleigh grid</th>
<th>Product on Mid Rayleigh grid</th>
</tr>
</thead>
<tbody>
<tr>
<td>Extinction $\alpha$</td>
<td>Extinction $\text{mid}_\alpha$</td>
</tr>
<tr>
<td>Backscatter $\beta$</td>
<td>Backscatter $\text{mid}_\beta$</td>
</tr>
<tr>
<td>Local Optical Depth LOD</td>
<td>Local optical depth $\text{mid}_{\text{LOD}}$</td>
</tr>
<tr>
<td>Scattering ratio SR</td>
<td>Backscatter-to-extinction ratio (BER)</td>
</tr>
</tbody>
</table>
Example of product: standard aerosol profile + cirrus cloud (optical thickness 0.2)

« True » aerosol profile used to simulate ALADIN data (E2S simulator)

- Good localisation of the cloud layer and the aerosol profile
- Oscillations observed in the extinction coefficient retrieval due to the iterative algorithme
- Good retrieval of the backscatter coefficient in the aerosol layer.
Impact of Depolarizing target on the backscatter retrieval

- Only the **co-polar component** of the backscattered signal is received

  **Significant underestimation** (/overestimation) of the backscatter coefficient (/BER) in case of highly **depolarizing** layer: cirrus clouds and desert dust.

  L2A retrieval if we would measure both the **co-polar and cross-polar** components

  L2A retrieval if we only measure the **co-polar** component

  Co-polar LR = 97
  Total LR = 58
Impact of Depolarizing target on the backscatter retrieval

- Only the **co-polar component** of the backscattered signal is received

**Significant underestimation** (/overestimation) of the backscatter coefficient (BER) in case of highly **depolarizing** layer: cirrus clouds and desert dust.

- L2A retrieval if we would measure both the **co-polar and cross-polar** components
- L2A retrieval if we only measure the **co-polar** component (ADM-Aeolus)

**Backscatter coefficient (BRC5)**

- Co-polar LR = 97
- Total LR = 58

**Underestimation** of the backscatter coefficient up to 50 % compared to an instrument that would measure both the co-polar and the cross-polar component.
Complex LITE Scenes, backscatter coefficient.

Observations sampled during the NASA's LITE experiment, ECMWF profiles for input wind, pressure and temperature.

- Good retrieval of aerosol/cloud properties
- Overestimation of $\beta$ for low aerosol layers
Complex LITE Scenes, mid extinction coefficient.

Observations sampled during the NASA's LITE experiment, ECMWF profiles for input wind, pressure and temperature.

Mid extinction L2A

Mid extinction E2S

Good retrieval of aerosol/cloud properties

No SCA retrieval due to: negative Rayleigh signal after cross-talk, large OD (~2, strong attenuation of the signal) or negative value after the inversion of the ration $\frac{NITWT}{T^{2}}_{p,1,i-1}$.
Other L2A products (heterogeneous scene, simulated)

**BER** \((\beta_p / \alpha_p)\) on **Mid** Rayleigh grids

- Potential in aerosol/cloud layers;
- BER retrieval affected by the large uncertainties in the mid \(\alpha\) coefficient.

**SR** \((1+\beta_p / \beta_m)\) on Rayleigh grids

- Good retrieval of the scattering ratio in aerosol layers due to the good accuracy of the \(\beta\) retrieval.
Conclusion

1. The Aeolus L2A products will be made available to users off-line.

2. The current evaluation of the products has shown the potential of ADM-Aeolus to retrieve aerosol properties.

3. Good quality of the backscatter coefficient.

4. For the extinction coefficient, more uncertainties due to the iterative algorithm: the use of the retrieval on Mid Rayleigh grid is highly recommended.

5. A data quality flag will be provided to users to invalidate bins with large estimated errors and/or low signal to noise ratio.

6. Improvement of the feature finder and scene classification in order to produce aerosol and cloud products at a better resolution. To that end, the next step would be to use the information from NWP models (ECMWF) and forecasts from the Copernicus/MACC project.
Conclusion

ECMWF output
- Liquid water content
- Ice water content
- Cloud base
- Cloud coverage

MACII forecast 20150201 00+036

Identification of aerosol and cloud scenes
L2A retrievals at a better resolution. (< 90 km)
Thanks for your attention
Lidar ratio

![Lidar ratio graph with data points for different particle types: Ash, Dust, Dust+Marine, Dust+BBA, Marine, Pollution, and BBA. The x-axis represents the lidar ratio (blue) / co-polar lidar ratio (red), in sr, while the y-axis represents the particle linear depolarization ratio (blue) and circular depolarization ratio (red), in %.]
Complex LITE Scenes, backscatter coefficient.

Observations sampled during the NASA's LITE experiment, ECMWF profiles for input wind, pressure and temperature.

Good retrieval of aerosol/cloud properties

Overestimation of $\beta$ for low aerosol layers
Molecular backscatter coefficient $\beta_{\text{mol}} \text{ (m}^{-1} \text{ str}^{-1})$ and extinction coefficient $\alpha_{\text{mol}} \text{ (m}^{-1})$ are needed in the L2A.

The coefficients are computed using pressure and temperature information from the NWP model:

\[
\beta_m(z) \approx 1.38 \left( \frac{550}{355} \right)^{4.09} \frac{p(z)}{1013} \frac{288}{T(z)} \times 10^{-6} \\
\alpha_m(z) \approx 1.16 \left( \frac{550}{355} \right)^{4.09} \frac{p(z)}{1013} \frac{288}{T(z)} \times 10^{-5}
\]

The coefficients 1.38 and 1.16 have been determined experimentally, Collis and Russel 1976.
Why negative values are possible when retrieving $\alpha_p$

\[ \alpha_{p,i} = \frac{1}{2\Delta R_i} H^{-1}\left( \frac{1}{T_{p,1,i-1}^2} \text{NITWT}_i \right) \]

For $H(x) > 1$, $x < 0$

If $\text{NITWT}_i < T_{p,1,i-1}^2$, $H^{-1}(\text{NITWT}_i/T_{p,1,i-1}^2 > 1)$

We look for $x$ so that $H(x) > 1$ ($H^{-1}(H(x)) = x$)

This implies that $x < 0$
Why $\beta$ error bars larger in aerosol layers?

Backscatter coefficient (Mid Rayleigh grid)

$$\langle (\delta \beta_{p,i})^2 \rangle = \beta_{p,i}^2 \left[ \langle e_{X_i}^2 \rangle + \langle e_{Y_i}^2 \rangle - 2 \langle e_{X_i} e_{Y_i} \rangle \right]$$

<table>
<thead>
<tr>
<th>$\beta$ E2S</th>
<th>$\beta$ L2A</th>
<th>$F$</th>
<th>$\text{var}_\beta$ $\beta$ L2A</th>
<th>$\text{var}_\beta$ $\beta$ E2S</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1.4 \times 10^{-6}$</td>
<td>$5 \times 10^{-6}$</td>
<td>0.0458</td>
<td>$1.17 \times 10^{-12}$</td>
<td>$4.7 \times 10^{-14}$</td>
</tr>
<tr>
<td>$3.6 \times 10^{-6}$</td>
<td>$1 \times 10^{-5}$</td>
<td>0.0240</td>
<td>$2.7 \times 10^{-12}$</td>
<td>$2.8 \times 10^{-14}$</td>
</tr>
</tbody>
</table>
Example of product: standard aerosol profile + opaque cumulus cloud (optical thickness 2)

« True » aerosol profile used to simulate ALADIN data (E2S simulator)

SCA retrieval

- Strong attenuation of the lidar signal in the cloud layer
- Aerosol properties retrieved only above the cloud and at the cloud top
Example: **Mid Extinction** opaque cumulus cloud and standard RMA

- **Mid Extinction retrieval**
  
  \( z(\text{mid} \alpha_p) \) of BRC 2.

- **Data quality flag** (new estimation 1)
  
  \( z(\text{mid} \alpha_p) \) of BRC 2.

- Cumulus cloud: invalid retrieval on the lowest part of the cloud because of a large attenuation of the signal, only the cloud top value is retrieved.

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METEO FRANCE

Toujours un temps d'avance
Data quality flag

- Creation of a Data Quality flag on the SCA retrievals at the BRC level
  Two flags: - one for the retrievals on Rayleigh grid: Profile(jProf).SCA.Info.QC
  - one for the retrievals on the mid Rayleigh grid: Profile(jProf).SCA.more.QC

For Rayleigh grid products: 8 bits
\( \alpha \) Validity | \( \beta \) validity | SNR Mie | SNR Ray | \( \alpha \) error bar | \( \beta \) error bar | total attenuation | 1 place holder

For mid Rayleigh grid products: 8 bits
\( \alpha \) Validity | \( \beta \) validity | BER validity | SNR Mie | SNR Ray | \( \alpha \) error bar | \( \beta \) error bar | total attenuation

\( \alpha \) and \( \beta \) valid if validity of the error bar threshold + signal not too attenuated (depends on the total optical depth) and reasonable values for Rayleigh/Mie SNR. BER valid if we obtain reasonable values.
Data quality flag

- These thresholds will be refined after the satellite launch.

```
<L2Ap>
  <Quality_flag>
    <Mie_snr_Threshold>6</Mie_snr_Threshold>
    <Rayleigh_snr_Threshold>70</Rayleigh_snr_Threshold>
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    <Mid_alpha_error_bar_Threshold unit="%">500</Mid_alpha_error_bar_Threshold>
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    <BER_max_Threshold unit="1/steradian">0.1</BER_max_Threshold>
    <BER_min_Threshold unit="1/steradian">0.01</BER_min_Threshold>
  </Quality_flag>
```
Error bars computation (backscatter)

- The error bar is computed as a function of the mean of the squared relative error on the Mie signal \(Y_i\) and the Rayleigh signal \(X_i\):

\[
\langle e_{X_i}^2 \rangle = \frac{x_{3,i} SNR_{Ray,i}^{-2} S_{Ray,i}^2 + x_{2,i} SNR_{Mie,i}^{-2} S_{Mie,i}^2}{X_i^2} \quad (1)
\]

\[
\langle e_{Y_i}^2 \rangle = \frac{x_{4,i} SNR_{Ray}^{-2} S_{Ray}^2 + x_{1,i} SNR_{Mie}^{-2} S_{Mie}^2}{Y_i^2} \quad (2)
\]

\[
\langle e_{X_i} e_{Y_i} \rangle = \frac{-x_{3,i} x_{4,i} SNR_{Ray,i}^{-2} S_{Ray,i}^2 + x_{2,i} x_{1,i} SNR_{Mie,i}^{-2} S_{Mie,i}^2}{X_i Y_i} \quad (3)
\]

- For the backscatter coefficient: The error bar = the root-mean-square-error between the backscatter estimation (retrieved) and the « true » backscatter coefficient. Thus, we plot an horizontal bar centered on the retrieved value +/- the square-root of the error estimation (4):

\[
((\delta\beta_{p,i})^2) = \beta_{p,i} [\langle e_{X_i}^2 \rangle + \langle e_{Y_i}^2 \rangle - 2 \langle e_{X_i} e_{Y_i} \rangle] \quad (4)
\]

- the error on the backscatter is thus directly retrieved from the Rayleigh and Mie signals and the Rayleigh and Mie SNR.
Error bars computation (extinction)

\[ \sigma_{L_{p,i}}^2 = \langle (\delta L_{p,i})^2 \rangle - \langle \delta L_{p,i} \rangle^2 \approx 4 \sum_{k=1}^{i} \langle e_{X_k}^2 \rangle - 3\langle e_{X_1}^2 \rangle - 3\langle e_{X_i}^2 \rangle \]

Depends only on the relative error of the molecular and particulate signal after cross-talk correction

\[ e_x = \frac{\delta X}{X} \]
\[ e_y = \frac{\delta Y}{Y} \]
E2S scenarios used in the presentation

LITE SCENE C EXPERIMENT
SCA equations

\[ X_l = \int_{R_{l-1}}^{R_l} \frac{\beta_m(r)}{r^2} \exp \left( -2 \int_0^r \left( \alpha_m(u) + \alpha_p(u) \right) du \right) dr \quad \text{Eq. 6.31} \]

\[ X_l = T_{m,sat,i-1}^2 T_{p,sat,i-1}^2 \int_{R_{l-1}}^{R_l} \frac{\beta_m(r)}{r^2} \exp \left( -2 \int_{R_{l-1}}^r \left( \alpha_m(u) + \alpha_p(u) \right) du \right) dr \quad \text{Eq. 6.32} \]

Then, considering that molecular and squared range quantities are weakly varying over a range bin,

\[ X_l = \frac{T_{m,sat,i-1}^2 T_{p,sat,i-1}^2 \beta_{m,l}}{R_{\text{mean},l}^2} \int_{R_{l-1}}^{R_l} \exp \left( -2 \int_{R_{l-1}}^r \alpha_m(u) du \right) \exp \left( -2 \int_{R_{l-1}}^r \alpha_p(u) du \right) dr \quad \text{Eq. 6.33} \]

Inside the integral, variations of the molecular term \( \int_{R_{l-1}}^r \alpha_m(u) du \) are limited and can be approximated by its mean value over the bin, i.e.:

\[
\frac{1}{\Delta R_l} \int_{R_{l-1}}^{R_l} \int_{R_{l-1}}^r \alpha_m(u) du \ dr \approx \frac{1}{\Delta R_l} \int_{R_{l-1}}^{R_l} \alpha_{m,i}(r-R_{l-1}) \ dr
\]

\[
\approx \frac{1}{\Delta R_l} \alpha_{m,i} \left( \frac{(r-R_{l-1})^2}{2} \right)_{R_{l-1}}^{R_l}
\]

\[
\approx \alpha_{m,i} \frac{\Delta R_l}{2}
\]

\[
\approx \frac{L_{m,i}}{2}
\]

\[ \text{Eq. 6.34} \]
SCA equations

\[ X_{i,\text{sim}} = \int_{R_{i-1}}^{R_i} \frac{\beta_m(r)}{r^2} \exp \left( -2 \int_0^r \alpha_m(u) du \right) dr \]
\[ = \frac{T_{\text{m,sat},i-1}^2 \beta_{m,i}}{R_{\text{mean},i}^2} \int_{R_{i-1}}^{R_i} \exp \left( -2 \int_{R_{i-1}}^r \alpha_m(u) du \right) dr \] \quad \text{Eq. 6.37}

To stay consistent with previous developments, the same approximation (slow and small variations of molecular characteristics) is made for the molecular transmission:

\[ X_{i,\text{sim}} \approx \frac{T_{\text{m,sat},i-1}^2 \beta_{m,i}}{R_{\text{mean},i}^2} \int_{R_{i-1}}^{R_i} e^{-L_{m,i}} dr \]
\[ \approx \frac{T_{\text{m,sat},i-1}^2 \beta_{m,i}}{R_{\text{mean},i}^2} \Delta R_i e^{-L_{m,i}} \] \quad \text{Eq. 6.38}

\[
\begin{align*}
\frac{X_{i,\text{obs}}}{X_{i,\text{sim}}} & = \frac{T_{\text{m,sat},i-1}^2 T_{\text{p,sat},i-1}^2 \beta_{m,i}}{R_{\text{mean},i}^2} e^{-L_{m,i}} \left( \frac{1 - e^{-2L_{p,i}}}{2\alpha_{p,i}} \right) \\
& = \frac{T_{\text{m,sat},i-1}^2 \beta_{m,i}}{R_{\text{mean},i}^2} \Delta R_i e^{-L_{m,i}} \\
& = T_{\text{p,sat},i-1}^2 \left( \frac{1 - e^{-2L_{p,i}}}{2L_{p,i}} \right) \\
\frac{X_{i,\text{obs}}}{X_{i,\text{sim}}} & = T_{\text{p,sat},i-1}^2 H(2L_{p,i})
\end{align*}
\]
SCA equations

\[ X_i = \frac{T_{m, sat, i-1} T_{p, sat, i-1} \beta_{m,i}^2}{R_{\text{mean},i}^2} e^{-L_{m,i}} \int_{R_{i-1}}^{R_i} \exp \left( -2 \int_{R_{i-1}}^{r} \alpha_p(u) \, du \right) \, dr \]

\[ = \frac{T_{m, sat, i-1} T_{p, sat, i-1} \beta_{m,i}^2}{R_{\text{mean},i}^2} e^{-L_{m,i}} \int_{R_{i-1}}^{R_i} \exp \left( -2 \alpha_{p,i}(r - R_{i-1}) \right) \, dr \]

\[ T_{m, sat, i-1} T_{p, sat, i-1} \beta_{m,i} - \int_{R_{i-1}}^{R_i} \left[ e^{-2\alpha_{p,i}(r-R_{i-1})} \right] \, dr \]

\[ X_{i, \text{sim}} = \int_{R_{i-1}}^{R_i} \frac{\beta_m(r)}{r^2} \exp \left( -2 \int_{0}^{r} \alpha_m(u) \, du \right) \, dr \]

\[ = \frac{T_{m, sat, i-1} \beta_{m,i}}{R_{\text{mean},i}^2} \int_{R_{i-1}}^{R_i} \exp \left( -2 \int_{R_{i-1}}^{r} \alpha_m(u) \, du \right) \, dr \quad \text{Eq. 6.37} \]

To stay consistent with previous developments, the same approximation (slow and small variations of molecular characteristics) is made for the molecular transmission:

\[ X_{i, \text{sim}} \approx \frac{T_{m, sat, i-1} \beta_{m,i}}{R_{\text{mean},i}^2} \int_{R_{i-1}}^{R_i} e^{-L_{m,i}} \, dr \]

\[ \approx \frac{T_{m, sat, i-1} \beta_{m,i}}{R_{\text{mean},i}^2} \Delta R_i e^{-L_{m,i}} \quad \text{Eq. 6.38} \]
<table>
<thead>
<tr>
<th>Mission</th>
<th>ADM-Aeolus</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lidar concept</td>
<td>ALADIN</td>
</tr>
<tr>
<td></td>
<td>One single wavelength laser. High Spectral Resolution receiver separates the laser light scattered by molecules and particles into two signals in two channels</td>
</tr>
<tr>
<td>Nd-YAG laser</td>
<td>355 nm</td>
</tr>
<tr>
<td>Operating wavelength(s)</td>
<td>110 mJ BOL / 100 mJ EOL</td>
</tr>
<tr>
<td>Transmitted energy per pulse</td>
<td>Circular</td>
</tr>
<tr>
<td>Laser polarization</td>
<td>It precludes polarization diversity</td>
</tr>
<tr>
<td>Pulse duration</td>
<td>26 ns</td>
</tr>
<tr>
<td>Pulse repetition frequency</td>
<td>50.5 Hz</td>
</tr>
<tr>
<td>Receiver telescope diameter</td>
<td>1.5 m</td>
</tr>
<tr>
<td>Receiver field-of-view (full angle)</td>
<td>18 μrad x 76 the telescope diffraction limit</td>
</tr>
<tr>
<td>Receiver</td>
<td>High Spectral Resolution</td>
</tr>
<tr>
<td></td>
<td>a) Rayleigh channel: Dual Fabry-Pérot interferometer for light scattered by air molecules</td>
</tr>
<tr>
<td></td>
<td>b) Mie channel: Fizeau interferometer for light scattered by particles</td>
</tr>
<tr>
<td>Receiver spectral bandwidth (measured)</td>
<td>a) 0.63 pm for Rayleigh channel</td>
</tr>
<tr>
<td></td>
<td>b) 0.067 pm for Mie channel</td>
</tr>
<tr>
<td>Vertical resolution (range bin)</td>
<td>250, 500, 1000, 2000 m</td>
</tr>
<tr>
<td>Horizontal resolution (along satellite track)</td>
<td>3 to 7.5 km (accumulation of $P = 20$ to $50$ shots)</td>
</tr>
<tr>
<td>Pointing of line-of-sight</td>
<td>35° off-nadir cross track</td>
</tr>
<tr>
<td>Orbit height</td>
<td>408 km (498 km lidar range)</td>
</tr>
<tr>
<td>Lidar footprint at surface</td>
<td>9 m</td>
</tr>
<tr>
<td>Footprint spacing</td>
<td>135 m</td>
</tr>
</tbody>
</table>