A short course on Altimetry

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\textit{with contributions by Peter Challenor, Ian Robinson, R. Keith Raney + some other friends…}
Outline

• Rationale
  • why we need altimetry

• A1 – Principles of altimetry
  • how it works in principle
  • New techniques

• A2 – Altimeter Data Processing
  • From satellite height to surface height: corrections
  • (or how it is made accurate)

• A3 – Altimetry and Oceanography

• A4 – Geophysical parameters and applications
  • what quantities we measure
  • how we use them!
Rationale for Radar Altimetry over the oceans

- Climate change
  - oceans are a very important component of the climate system

- Altimeters monitor **currents / ocean circulation**...

- …that can be used to estimate **heat** storage and transport

- … and to assess the interaction between **ocean and atmosphere**

- We also get interesting by-products: **wind/waves, rain**
The sea is not flat…

Surface dynamical features of height = tens of cm over lengths = hundreds of kms
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Altimetry 1 – principles & instruments
Basic Principles

- The altimeter is a radar at vertical incidence.
- The signal returning to the satellite is from quasi-specular reflection.
- Measure distance between satellite and sea (range).
- Determine position of satellite (precise orbit).
- Hence determine height of sea surface.
- Oceanographers require height relative to geoid.

DT = Orbit - Range - Geoid
• Measure travel time, $2T$, from emit to return

$$h = T \times c$$  \(c \approx 3 \times 10^8 \text{ m/s}\)

• **Resolution to ~1cm** would need a pulse of 3x$10^{-10}$s (0.3 nanoseconds)

• 0.3ns… That would be a pulse bandwidth of >3 GHz… Impossible!
Chirp, chirp….

• So we have to use tricks: **chirp** pulse compression

• …and average ~1000 pulses

• It is also necessary to apply a number of corrections for atmospheric and surface effects
• In principle here are two types of altimeter:
  • beam-limited
  • pulse-limited
Beam-Limited Altimeter

• Return pulse is dictated by the width of the beam
Beam-Limited altimeter

- A plot of return power versus time for a beam-limited altimeter looks like the *heights* of the specular points, i.e. the probability density function (pdf) of the specular scatterers.

The **tracking point** (point taken for the range measurement) is the maximum of the curve.
Beam-Limited: technological problems

- Narrow beams require very large antennae and are impractical in space
  - For a 5 km footprint a beam width of about 0.3° is required.
  - For a 13.6 GHz altimeter this would imply a 5 m antenna.
- Even more important: highly sensitivity to mispointing, which affects both amplitude and measured range
- New missions like ESA’s CryoSat (launched 8 Apr 2010) and Sentinel-3 use synthetic aperture techniques (delay-Doppler Altimeter) that “can be seen as” a beam-limited instrument in the along-track direction.
In a pulse-limited altimeter the shape of the return is dictated by the length (width) of the pulse.
The “pulse-limited” footprint

- Full illumination when rear of pulse reaches the sea – then area illuminated stays constant
- Area illuminated has radius \( r = \sqrt{2hc} \)
- Measure interval between mid-pulse emission and time to reach half full height

\[ t = T \]
\[ t = T + \frac{1}{2} \]
\[ t = T + 3 \]

Emitted pulse

2h/c

position of pulse at time:

Area illuminated at time:
- A plot of return power versus time for a pulse-limited altimeter looks like the integral of the heights of the specular points, i.e., the cumulative distribution function (cdf) of the specular scatterers.

![Diagram showing the tracking point as the half power point of the curve](image)
Pulse- vs Beam-Limited

• All the microwave altimeters flown in space to date, including very successful TOPEX/Poseidon, ERS-1 & 2 RA & Envisat RA-2, are pulse-limited except….

• … laser altimeters (like GLAS on ICESAT) are beam-limited

• …and a Delay-Doppler Altimeter “can be seen” as beam-limited in the along-track direction

• To understand the basics of altimetry we will focus on the pulse limited design
Basics of pulse-limited altimeter theory

• We send out a thin shell of radar energy which is reflected back from the sea surface
• The power in the returned signal is detected by a number of gates (bins) each at a slightly different time
Shell of energy from the pulse

Sea Surface

Power
If we add waves ...

Sea Surface

Power

Gate Number

Time
The area illuminated or ‘effective footprint’

• The total area illuminated is related to the significant wave height noted as SWH [or Hs] (SWH ≈ 4 × std of the height distribution)

• The formula is

\[
\frac{R_0 \left( c + 2H_s \right)}{1 + R_0 / R_E}
\]

Where
- \(c\) is the speed of light
- \(t\) is the pulse length
- \(H_s\) significant wave height
- \(R_0\) the altitude of the satellite
- \(R_E\) the radius of the Earth
## Diameters of the effective footprint

<table>
<thead>
<tr>
<th>$H_s$ (m)</th>
<th>ERS-2/1, ENVISAT Effective footprint (km) (800 km altitude)</th>
<th>TOPEX, Jason-1/2 Effective footprint (km) (1335 km altitude)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.6</td>
<td>2.0</td>
</tr>
<tr>
<td>1</td>
<td>2.9</td>
<td>3.6</td>
</tr>
<tr>
<td>3</td>
<td>4.4</td>
<td>5.5</td>
</tr>
<tr>
<td>5</td>
<td>5.6</td>
<td>6.9</td>
</tr>
<tr>
<td>10</td>
<td>7.7</td>
<td>9.6</td>
</tr>
<tr>
<td>15</td>
<td>9.4</td>
<td>11.7</td>
</tr>
<tr>
<td>20</td>
<td>10.8</td>
<td>13.4</td>
</tr>
</tbody>
</table>

From Chelton et al (1989)
The Brown Model

• Assume that the sea surface is a perfectly conducting rough mirror which reflects only at specular points, i.e. those points where the radar beam is reflected directly back to the satellite.
• Under these assumptions the return power is given by a three fold convolution

\[ P_r(t) = P_{FS}(t) \ast P_{PT}(t) \ast P_H(z) \]

Where

\( P_r(t) \) is the returned power
\( P_{FS}(t) \) is the flat surface response
\( P_{PT}(t) \) is the point target response
\( P_H(z) \) is the pdf of specular points on the sea surface
The Flat Surface Response Function

• The Flat surface response function is the response you would get from reflecting the radar pulse from a flat surface.

• It looks like

\[ P_{FS}(t) = U(t - t_0) \cdot G(t) \]

Where

- \( U(t) \) is the Heaviside function
  \[ U(t) = 0 \text{ for } t < 0; \quad U(t) = 1 \text{ otherwise} \]
- \( G(t) \) is the two way antenna gain pattern
• The point target response (PTR) function is the shape of the transmitted pulse

• Its true shape is given by

\[ P_{PT}(t) = \left( \frac{\sin\left(\frac{t}{t}\right)}{t} \right)^2 \]

• For the Brown model we approximate this with a Gaussian.
The Brown Model - III

\[ P_r(t) = P_{FS}(0) \quad P_T\sqrt{2} \quad \frac{p}{2} \quad 1 + \text{erf} \quad \frac{(t - t_0)}{\sqrt{2}c} \quad \text{for } t < t_0 \]

\[ P_r(t) = P_{FS}(t \quad t_0) \quad P_T\sqrt{2} \quad \frac{p}{2} \quad 1 + \text{erf} \quad \frac{(t - t_0)}{\sqrt{2}c} \quad \text{for } t \geq t_0 \]

\[ c = \sqrt{\frac{2 + \frac{4s^2}{c^2}}{p}} \]

\[ s \quad \frac{SWH}{4} \]

\[ P_{FS}(t) = \frac{G_0^2}{4(4)^2L_Ph^3}c^0 \exp \quad \frac{4\sin^2}{4(\frac{4ct}{h}\cos2}) \quad I_0 \quad \frac{4\sqrt{ct}}{h}\sin2 \quad \div \]

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where

$$erf(t) = \frac{2}{\sqrt{\pi}} \int_{0}^{t} e^{-x^2} \, dx$$

(compare this with the Normal cumulative distribution function)

$$I_0(t) = \frac{1}{\sqrt{2}} \int_{0}^{t} e^{\frac{x^2}{2}} \, dx$$

$$I(x) = \frac{1}{2} \left(1 + erf\left(x\frac{1}{\sqrt{2}}\right)\right)$$

$I_0()$ is a modified Bessel function of the first kind
What are we measuring?

- **SWH** - significant wave height
- **t₀** - the time for the radar signal to reach the Earth and return to the satellite
  - we then convert into range and finally into height – see in the next slides
- **σ₀** - the radar backscatter coefficient
  - note this is set by the roughness at scales comparable with radar wavelength, i.e. cm, therefore it is (in some way) related to wind
- sometimes **mispointing angle** ξ can be also estimated from the waveforms
The Brown Model – measured parameters

\[ P_r(t) = P_{FS}(0) \quad P_T \sqrt{2} \quad \frac{p}{2} \quad 1 + \text{erf} \quad \frac{(t - t_0)}{\sqrt{c}} \quad \text{for } t < t_0 \]

\[ P_r(t) = P_{FS}(t \quad t_0) \quad P_T \sqrt{2} \quad \frac{p}{2} \quad 1 + \text{erf} \quad \frac{(t - t_0)}{\sqrt{c}} \quad \text{for } t > t_0 \]

\[ c = \sqrt{\frac{2 + \frac{4}{s}}{p} \quad \frac{s^2}{c^2}} \]

\[ P_{FS}(t) = \frac{G_0^2}{4(4)^2} \quad R \quad \frac{c}{L_p h^3} \quad \exp \quad \frac{4 \quad \sin^2}{\frac{4ct}{h} \quad \cos 2} \quad I_0 \quad \frac{4 \quad \sqrt{\frac{ct}{h} \quad \sin 2}}{\div} \]
What are the other parameters?

- $\lambda_R$ is the radar wavelength
- $L_p$ is the two way propagation loss
- $h$ is the satellite altitude (nominal)
- $G_0$ is the antenna gain
- $\gamma$ is the antenna beam width
- $\sigma_p$ is the pulse width
- $\eta$ is the pulse compression ratio
- $P_T$ is the peak power
- $\xi$ (as we said) is the mispointing angle
Looking at the slope of the leading edge of the return pulse we can measure wave height!
The effect of mispointing
Noise on the altimeter

• If we simply use the altimeter as a detector we will still have a signal - known as the thermal noise.
• The noise on the signal is known as fading noise
• It is sometimes assumed to be constant, sometimes its mean is measured
• For most altimeters the noise on the signal is independent in each gate and has a negative exponential distribution.
Exponential distribution

- pdf

\[ f(x) = \frac{1}{\theta} e^{-\frac{x}{\theta}} \quad \text{for} \quad 0 < x < \infty \]

- Mean = \theta
- Variance = \theta^2
Exponential pdf
Averaging the noise

• For a negative exponential distribution the variance is equal to the square of the mean. Thus **the individual pulses are very noisy!**

⇒ **We need a lot of averaging to achieve good Signal to Noise Ratio**

• The pulse repetition frequency is thousands per second
  • 1020 for ERS-1/2, 1800 for Jason & Envisat, 4500 for Topex

• Usually data are transmitted to the ground at ~20Hz and then averaged to ~1 Hz
A single pulse

![Graph showing received power over time](image)

- **Y-axis**: Received Power
- **X-axis**: Time (gate number)

Graph details:
- The graph illustrates the variation of received power over time, as indicated by the y-axis labeled 'Received Power' and the x-axis labeled 'Time (gate number)'.
- The data points show fluctuations, with peaks and troughs across the timeline.
- The graph provides a visual representation of how the signal strength changes with time, which is crucial in understanding the dynamics of the signal in the context of ocean remote sensing.
It is very difficult (if not impossible) to generate a single-frequency pulse of length 3 ns.

It is possible to do something very similar in the frequency domain using a chirp: modulating the frequency of the carrier wave in a linear way.

The equivalent pulse width = 1/chirp bandwidth.
A chirp is generated
Two copies are taken
The first is transmitted
The second is delayed so it can be matched with the reflected pulse
• The two chirps are mixed.
• A point above the sea surface gives returns at frequency lower than would be expected and vice versa
• So a ‘Brown’ return is received but with frequency rather than time along the x axis
A real waveform - from the RA-2 altimeter on ESA’s Envisat

How do we estimate the various parameters from this?

Ku band, 13.5 Ghz, 2.1 cm
“Retracking” of the waveforms

= fitting the waveforms with a waveform model, therefore estimating the parameters

- **Maximum amplitude**: related to wind speed
- **“Epoch”**: gives range (therefore height)
- **Slope of leading edge**: related to significant wave height

Figure from J Gomez-Enri et al. (2009)
## Altimeters flown in space

<table>
<thead>
<tr>
<th>Height</th>
<th>inclination</th>
<th>accuracy</th>
<th>repeat period</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>GEOS-3 (04/75 – 12/78)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>845 km</td>
<td>115 deg</td>
<td>0.5 m</td>
<td>-</td>
</tr>
<tr>
<td><strong>Seasat (06/78 – 09/78)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>800 km</td>
<td>108 deg</td>
<td>0.10 m</td>
<td>3 days</td>
</tr>
<tr>
<td><strong>Geosat (03/85 – 09/89)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>785.5 km</td>
<td>108.1 deg</td>
<td>0.10 m</td>
<td>17.5 days</td>
</tr>
<tr>
<td><strong>ERS-1 (07/91 – 03/2000); ERS-2 (04/95 – 09/2011)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>785 km</td>
<td>98.5 deg</td>
<td>0.05 m</td>
<td>35 days</td>
</tr>
<tr>
<td><strong>TOPEX/Poseidon (09/92 – 10/2005); Jason-1 (12/01 – 06/2013); Jason-2 (06/08 – present)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1336 km</td>
<td>66 deg</td>
<td>0.02 m</td>
<td>9.92 days</td>
</tr>
<tr>
<td><strong>Geosat follow-on (GFO) (02/98 – 09/2008)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>800 km</td>
<td>108 deg</td>
<td>0.10 m</td>
<td>17.5 days</td>
</tr>
<tr>
<td><strong>Envisat (03/02 – 04/12)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>785 km</td>
<td>98.5 deg</td>
<td>0.03 m</td>
<td>35 days</td>
</tr>
<tr>
<td><strong>CryoSat-2 (04/10 – present) [delay-Doppler]</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>717 km</td>
<td>92 deg</td>
<td>0.05 m</td>
<td>369 days (30d sub-cycle)</td>
</tr>
<tr>
<td><strong>SARAL/AltiKa (02/13 – present) [Ka-band]</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>785 km</td>
<td>98.5 deg</td>
<td>0.02 m</td>
<td>35 days</td>
</tr>
</tbody>
</table>
1-D (along-track) measurement

TOPEXPOSEIDON ground tracks over a 10-day cycle
Example: Sea Surface Height along the ground track of a satellite altimeter
Radar Altimeters: Now and Then

High-inclination orbit

ERS-2 ESA
ENVISAT ESA
GFO
CRYOSAT-2 ESA
Saral/AltiKa India/France

HY-2A China
HY-2B, -2C, -2D

Sentinel-3A Europe
Sentinel-3B, -3C, -3D

Swath altimetry

GFO-FO US Navy

Swath altimetry

SWOT/WaTER-HM USA/Europe
TBD

High accuracy SSH (reference missions) from mid-inclination orbit

Jason-1 Fr./USA
Jason-2 Europe/USA
Jason-3 Europe/USA
Jason-CS/Jason-4 Europe/USA

Jason-CS successor Europe/USA

Adapted from CNES, 2009, with acknowledgement
Cryosat-2

- ESA mission; launched 8 April 2010
- LEO, non sun-synchronous
  - 369 days repeat (30d sub-cycle)
  - Mean altitude: 717 km
  - Inclination: 92°
- Prime payload: SIRAL
  - SAR/Interferometric Radar Altimeter (delay/Doppler)
  - Modes: Low-Res / SAR / SARIn
- Ku-band only; no radiometer
- Design life:
  - 6 months commissioning + 3 years
Conventional altimeter footprint scan

RA pulse-limited footprint in effect is dragged along the surface pulse by pulse as the satellite passes overhead.

Among other consequences, the effective footprint is expanded beyond the pulse-limited diameter.
Delay-Doppler Altimetry (aka SAR altimetry)


DDA spotlights each along-track resolved footprint as the satellite passes overhead

Improved along-track resolution, higher PRF, better S/N, less sensitivity to sea state,…
Tracker “reads” waveforms only from the center (1, 2, or 3) Doppler bins

Result? **Rejects all reflections from non-nadir sources**

Each surface location can be followed as it is traversed by Doppler bins
SARAL / AltiKa

- **Satellite:** Indian Space Research Organization (ISRO)
  - carrying **AltiKa** altimeter by CNES
  - Ka-band 0.84 cm (viz 2.2 cm at Ku-band)
  - Bandwidth (480 MHz) => 0.31 \( \rho \) (viz 0.47)
  - Otherwise “conventional” RA
  - PRF ~ 4 kHz (viz 2 kHz at Ku-band)
  - Full waveform mode

- Payload includes dual-frequency radiometer
- Sun-synchronous, 35-day repeat cycle (same as ERS/Envisat)
- Navigation and control: DEM and DORIS
- Launched February 2013
GNSS (GPS/Galileo) Reflectometry

HOW GNSSS-R WORKS