Surface Parameter Estimation: Basics and Advanced Concepts

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Motivation of Radar Remote Sensing

- Independent from Weather
  - Data acquisition during cloud cover, rain, ...
- Radar is complementarily to optics

What does the Radar measure?

Radar reflectivity (backscattered signal) of the target as a function of position.

radar transmits a pulse
(travelling velocity is equal to velocity of light)

some of the energy in the radar pulse is reflected back towards the radar.

This is what the radar measures.
It is known as radar backscatter $\sigma_0$
(sigma nought or sigma zero).
Normalized radar cross-section (backscattering coefficient) is given by:

\[ \sigma_0 \text{ (dB)} = 10 \cdot \log_{10} \left( \frac{\text{energy ratio}}{\text{energy reflected in an isotropic way}} \right) \]

whereby

\[ \text{energy ratio} = \frac{\text{received energy by the sensor}}{\text{energy reflected in an isotropic way}} \]

i.e.

The backscattered coefficient can be a positive number if there is a focusing of backscattered energy towards the radar

or

The backscattered coefficient can be a negative number if there is a focusing of backscattered energy way from the radar (e.g. smooth surface)

<table>
<thead>
<tr>
<th>Levels of Radar backscatter</th>
<th>Typical scenario</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very high backscatter (above -5 dB)</td>
<td>( \Rightarrow ) man-made objects (urban) ( \Rightarrow ) terrain slopes towards radar ( \Rightarrow ) very rough surface ( \Rightarrow ) radar looking very steep</td>
</tr>
<tr>
<td>High backscatter (-10 dB to 0 dB)</td>
<td>( \Rightarrow ) rough surface ( \Rightarrow ) dense vegetation (forest)</td>
</tr>
<tr>
<td>Moderate backscatter (-20 to -10 dB)</td>
<td>( \Rightarrow ) medium level of vegetation ( \Rightarrow ) agricultural crops ( \Rightarrow ) moderately rough surfaces</td>
</tr>
<tr>
<td>Low backscatter (below -20 dB)</td>
<td>( \Rightarrow ) smooth surface ( \Rightarrow ) calm water ( \Rightarrow ) road ( \Rightarrow ) very dry terrain (sand)</td>
</tr>
</tbody>
</table>

**Commonly Used Frequency Bands**

<table>
<thead>
<tr>
<th>Frequency band</th>
<th>Frequency range</th>
<th>Application examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>VHF</td>
<td>300 KHz - 300 MHz</td>
<td>foliage/ground penetration, biomass</td>
</tr>
<tr>
<td>P-Band</td>
<td>300 MHz - 1 GHz</td>
<td>soil moisture, biomass, penetration</td>
</tr>
<tr>
<td>L-Band</td>
<td>1 GHz - 2 GHz</td>
<td>agriculture, forestry, soil moisture</td>
</tr>
<tr>
<td>C-Band</td>
<td>4 GHz - 8 GHz</td>
<td>ocean, agriculture</td>
</tr>
<tr>
<td>X-Band</td>
<td>8 GHz - 12 GHz</td>
<td>agriculture, ocean, high resolution radar</td>
</tr>
<tr>
<td>Ku-Band</td>
<td>14 GHz - 18 GHz</td>
<td>glaciology (snow cover mapping)</td>
</tr>
<tr>
<td>Ka-Band</td>
<td>27 GHz - 47 GHz</td>
<td>high resolution radar</td>
</tr>
</tbody>
</table>

**Environmental Sensing from Air and Space**

**Airborne measurements**

- Highly flexible operation
- Coverage of dedicated areas
- Experimental configuration
- Sensor specific data formats
- Short re-visit times

**Spaceborne measurements**

- Highly regular observation
- Wide area coverage
- Highly operational & reliable
- Standard product delivery
- Long term observations
The Polarimetric Scattering Problem

Incident (Plane) Wave

\[ \mathbf{E}_i(\mathbf{r}) \begin{bmatrix} E_{ih} \\ E_{iv} \end{bmatrix} \]

Scattered Field

In the far zone region

\[ \mathbf{E}_s(\mathbf{r}) \begin{bmatrix} E_{sh} \\ E_{sv} \end{bmatrix} \]

Scatterer

Transforms the incident into the scattered wave

2x2 Complex Scattering Matrix

\[ \begin{bmatrix} S_{hh} & S_{hv} \\ S_{vh} & S_{vv} \end{bmatrix} \]

Mapping of the 2-dim incident vector \( \mathbf{E}_i(\mathbf{r}) \) into the 2-dim scattered vector \( \mathbf{E}_s(\mathbf{r}) \)

First Order Parameters:
- Norm. Backscattering Cross Section \( \sigma_0 \)
- Intensity (Cross Section \( \sigma_0 \)) ratios
- Polarimetric phase differences

Second Order Parameters:
- Polarimetric (inter channel) coherences

Applications
- Soil moisture/roughness estimation
- Wetland/Veg. characterisation/mapping
- Snow & ice mapping (type classification)
- Ship & Oil spill detection
- Classification/Segmentation (Pol based)
The Polarimetric Scattering Problem

Incident (Plane) Wave

\[
\begin{bmatrix}
E_i^v(t) \\
E_i^h(t)
\end{bmatrix} = \begin{bmatrix}
K_i^v(t) \\
K_i^h(t)
\end{bmatrix}
\]

(Jones Vector Representation)

Scattered Field

In the far zone region

\[
\begin{bmatrix}
E_s^v(t) \\
E_s^h(t)
\end{bmatrix} = \begin{bmatrix}
K_s^v(t) \\
K_s^h(t)
\end{bmatrix}
\]

(Jones Vector Representation)

Scatterer

Transforms the incident wave into the scattered wave

1: Changes the polarization state of the incident wave
2: Changes the degree of polarization of the incident wave

2x2 Complex Scattering Matrix

\[
\begin{bmatrix}
E_s^v(t) \\
E_s^h(t)
\end{bmatrix} = \begin{bmatrix}
S_{sv} & S_{sh} \\
S_{hv} & S_{hh}
\end{bmatrix} \begin{bmatrix}
E_i^v(t) \\
E_i^h(t)
\end{bmatrix}
\]

Coherent Scattering Matrix

\[
\begin{bmatrix}
E_i^v(t) \\
E_i^h(t)
\end{bmatrix} \exp(i \phi_i) = \begin{bmatrix}
S_{sv} & S_{sh} \\
S_{hv} & S_{hh}
\end{bmatrix} \begin{bmatrix}
E_s^v(t) \\
E_s^h(t)
\end{bmatrix}
\]

Total Scattered Power:

\[
TP = 2 \left( |S_{sv}|^2 + |S_{sh}|^2 + |S_{hv}|^2 + |S_{hh}|^2 \right)
\]

Scattering Amplitude Images

Bi- & Mono-Static Measurement of the Scattering Matrix

\[
T = \frac{1}{PRF}
\]

Bi-SAR / Test Site: Oberpfafenhoffen

Scattering Amplitude Images
Backscattering (FSA & BSA)

In the case of monostatic backscattering from reciprocal scatterers:

Reciprocity Theorem

\[ S_{\text{BSA}} = S_{\text{FSA}} = S_{\text{BSA}}^{*} \quad \text{and} \quad (S_{\text{FSA}}^{*} = S_{\text{BSA}}^{*} = S_{\text{BSA}}^{\text{adj}}) \]

\[ S = \begin{bmatrix} S_{\text{XX}} & S_{\text{XY}} & S_{\text{XZ}} \\ S_{\text{YX}} & S_{\text{YY}} & S_{\text{YZ}} \\ S_{\text{ZX}} & S_{\text{ZY}} & S_{\text{ZZ}} \end{bmatrix} \]

\[ \exp(\phi_{\text{XX}} - \phi_{\text{XY}}) \begin{bmatrix} |S_{\text{XX}}| & |S_{\text{XY}}| & |S_{\text{XZ}}| \\ |S_{\text{YX}}| & |S_{\text{YY}}| & |S_{\text{YZ}}| \\ |S_{\text{ZX}}| & |S_{\text{ZY}}| & |S_{\text{ZZ}}| \end{bmatrix} \]

Absolute Phase Factor

Five Parameters: 3 Amplitudes & 2 Phases

The scattering problem can be addressed in the 3-dim complex space:

3-dim Lexicographic (L) and Pauli (P) Scattering Vectors:

\[ \vec{k}_{\text{L}} = \begin{bmatrix} S_{\text{XX}} \\ S_{\text{XY}} \\ S_{\text{XZ}} \end{bmatrix} \quad \vec{k}_{\text{P}} = \frac{1}{\sqrt{2}} \begin{bmatrix} S_{\text{XX}} + S_{\text{YY}} \\ S_{\text{XX}} - S_{\text{YY}} \\ 0 \end{bmatrix} \]

Note: The factor \( \sqrt{2} \) is required to keep the vector norm of \( \vec{k}_{\text{L}} \) invariant.

Covariance & Coherency Matrices in Backscattering

Lexicographic Scattering Vector:

\[ \vec{k}_{\text{L}} = \begin{bmatrix} S_{\text{XX}} \\ S_{\text{XY}} \\ S_{\text{XZ}} \end{bmatrix} \]

Covariance Matrix \([C]:\]

\[ [C]_L := \langle \vec{k}_{\text{L}} \cdot \vec{k}_{\text{L}}^* \rangle \]

\[ [C]_P := \begin{bmatrix} \langle S_{\text{XX}} \rangle & \langle S_{\text{XY}} \rangle & \langle S_{\text{XZ}} \rangle \\ \langle S_{\text{YX}} \rangle & \langle S_{\text{YY}} \rangle & \langle S_{\text{YZ}} \rangle \\ \langle S_{\text{ZX}} \rangle & \langle S_{\text{ZY}} \rangle & \langle S_{\text{ZZ}} \rangle \end{bmatrix} \]

Pauli Scattering Vector:

\[ \vec{k}_{\text{P}} = \begin{bmatrix} \frac{1}{\sqrt{2}} (S_{\text{XX}} + S_{\text{YY}}) \\ \frac{1}{\sqrt{2}} (S_{\text{XX}} - S_{\text{YY}}) \\ 0 \end{bmatrix} \]

Coherency Matrix \([T]:\]

\[ [T]_L := \langle \vec{k}_{\text{L}} \cdot \vec{k}_{\text{L}}^* \rangle \]

\[ [T]_P := \begin{bmatrix} \langle (S_{\text{XX}} + S_{\text{YY}})^2 \rangle & \langle (S_{\text{XX}} + S_{\text{YY}})(S_{\text{XX}} - S_{\text{YY}}) \rangle & \langle (S_{\text{XX}} + S_{\text{YY}})S_{\text{XZ}} \rangle \\ \langle (S_{\text{XX}} - S_{\text{YY}})(S_{\text{XX}} + S_{\text{YY}}) \rangle & \langle (S_{\text{XX}} - S_{\text{YY}})^2 \rangle & \langle (S_{\text{XX}} - S_{\text{YY}})S_{\text{XZ}} \rangle \\ 2 \langle S_{\text{XX}}(S_{\text{XX}} - S_{\text{YY}}) \rangle & \langle 2S_{\text{XX}}S_{\text{XZ}} \rangle & \langle 2S_{\text{XX}}S_{\text{XZ}} \rangle \end{bmatrix} \]

\([C]_L \text{ and } [T]_L \text{ are by definition 3x3 hermitian positive semi-definite matrices & contain in general 9 independent parameters.}\]

Partial Scatterers

Deterministic Scatterers

Partial Scatterers

Point Scatters

- Change the polarisation state of the wave
- Do not change the degree of polarisation

Monochromatic Incident Wave

Scattered Wave

Depolarisation described by second order statistics

Completely described by \([S]\)

Cannot be described by a single \([S]\)

Scattering Polarimetry

INTERPRETATION OF SCATTERING MECHANISMS

- Directly from the Scattering Matrix
- Model based
Interpretation of Scattering Mechanisms

Scattering Matrix:

\[
[S] = \begin{bmatrix}
S_{HH} & S_{HV} \\
S_{VH} & S_{VV}
\end{bmatrix}
\]

Unitary Representation:

\[\tilde{e} = \frac{1}{\sqrt{2}} \begin{bmatrix}
\cos \alpha \\
\sin \alpha \\
\cos \beta \\
\sin \beta
\end{bmatrix}
\]

\[\begin{bmatrix}
\cos \alpha \\
\sin \alpha \\
\cos \beta \\
\sin \beta
\end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix}
S_{HH} + S_{VV} \\
S_{HV} - S_{VH} \\
S_{VH} + S_{HH} \\
S_{VV} - S_{HH}
\end{bmatrix}
\]

Parameterisation of \(\tilde{e}\) in terms of five angles: \(\alpha, \beta, \phi_1, \phi_2, \phi_3\).

Change of Scattering Mechanism:

\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0
\end{bmatrix}
\]

\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]


Interpretation of Scattering Mechanisms

Point Reduction Theorem:

\[
\begin{bmatrix}
\exp(i \phi_1) \\
\exp(i \phi_2) \\
\exp(i \phi_1) \\
\exp(i \phi_2)
\end{bmatrix} = \begin{bmatrix}
0 & 0 & 0 & 1 \\
0 & \cos \beta & -\sin \beta & 0 \\
\sin \beta & \cos \beta & 0 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

Scattering Processes: Fresnel Scattering

Scattering Matrix:

\[
[S] = \begin{bmatrix}
R_{HH} & 0 \\
0 & R_{VV}
\end{bmatrix}
\]

Fresnel Reflection Coefficients:

\[R_{HH} = \frac{\cos \theta - \sin \theta}{\cos \theta + \sin \theta}\]

\[R_{VV} = \frac{\cos \theta - \sin \theta}{\cos \theta + \sin \theta}\]

\[R_{HV} = R_{VH} = 0\]

\[\text{where } \epsilon \text{ is the dielectric constant of the surface}\]
Scattering Processes: Bragg Scattering

\[
\begin{align*}
R_v &= \frac{\cos\theta - \sqrt{\varepsilon - \sin^2\theta}}{\cos\theta + \sqrt{\varepsilon - \sin^2\theta}} \\
R_s &= \frac{\varepsilon \left[\sin\theta \cos\phi \left(\sin\theta - \sin\theta \sin\phi\right) - \left(\sin\theta + \sin\theta \sin\phi\right)\right]}{\varepsilon \left[\sin\theta \cos\phi \left(\sin\theta - \sin\theta \sin\phi\right) + \left(\sin\theta + \sin\theta \sin\phi\right)\right]} \\
\end{align*}
\]

where \( \varepsilon \) is the dielectric constant of the surface.

Dihedral vs Volume Scattering (L- /P-Band E-SAR – Kryklan/Sweden)

Scattering Processes: Dihedral Scattering

\[
\begin{align*}
[S] &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \\
R_{dd} &= 0 \\
R_{dv} &= 0 \\
R_{vd} &= 0 \\
R_{vv} &= 0 \\
Fresnel Coefficients:\nR_v &= \frac{\varepsilon \cos\theta - \sqrt{\varepsilon - \sin^2\theta}}{\varepsilon \cos\theta + \sqrt{\varepsilon - \sin^2\theta}} \\
R_s &= \frac{\varepsilon \left[\sin\theta \cos\phi \left(\sin\theta - \sin\theta \sin\phi\right) - \left(\sin\theta + \sin\theta \sin\phi\right)\right]}{\varepsilon \left[\sin\theta \cos\phi \left(\sin\theta - \sin\theta \sin\phi\right) + \left(\sin\theta + \sin\theta \sin\phi\right)\right]} \\
\end{align*}
\]

Scattering Processes: Volume Scattering

\[
\begin{align*}
K(\alpha, \beta, \gamma) &= [K(\alpha) \ [K(\beta) \ [K(\gamma)]] \\
[S] &= \begin{bmatrix} a & b \\ b & d \end{bmatrix} \Rightarrow \vec{E} = \frac{a + b}{2} \\
\end{align*}
\]

Coherency Matrix:

\[
[T] = \begin{bmatrix} \cos\beta & 0 & -\sin\beta \\ -\sin\beta & 0 & \cos\beta \\ 0 & 0 & 1 \end{bmatrix}
\]

Random Volume
Scattering Processes: Volume Scattering

Principal Polarisability Matrix:

\[
[P] = \begin{bmatrix}
\rho_x & 0 & 0 \\
0 & \rho_y & 0 \\
0 & 0 & \rho_z
\end{bmatrix} \quad \text{with} \quad \rho_i = \frac{V}{4\pi L(L_i - 1)}
\]

where \( V \) is the particle volume and \( L_i = \int \frac{(l_i l_i / 8)}{(s + l_i / 2)^2} (s + l_i / 2)^2 (s + l_i / 2)^2 \mathrm{d}s \)

is the particle volume and \( L_1, L_2, L_3 = 1 \) is the particle volume and

\[
L = \int \frac{(l_1 l_2 l_3 / 8)}{(s + l_1 / 2)^2} (s + l_2 / 2)^2 (s + l_3 / 2)^2 \mathrm{d}s
\]

is the particle volume and

Particle Anisotropy:

\[
A_m = \frac{L_m (c - 1) + 1}{L_m (c - 1) + 1} \quad 0 \leq m \leq \infty
\]

Particle Shape Ratio:

\[
m > \frac{L_2}{L_3}, \quad m \leq 1 \quad \text{oblate spheroids}
\]

\[
m < \frac{L_2}{L_3}, \quad m \geq 1 \quad \text{prolate spheroids}
\]

Scattering Processes: Volume Scattering

Decomposition Theorems

\[
[S] \quad \text{COHERENT DECOMPOSITION}
\]

E. Krogager (1990)

W.L. Cameron (1990)

\[
[K] \quad \text{TARGET DICHOTOMY}
\]

J.R. Huyten (1970)

R.M. Barnes (1988)

\[
[T] \quad \text{AZIMUTHAL SYMMETRY}
\]

S.R. Cloude (1985)

A.J. Freeman (1992)

W.A. Holm (1988)

J.J. Van Zyl (1992)

\[
[C] \quad \text{EIGENVECTORS / EIGENVALUES ANALYSIS & MODEL BASED DECOMPOSITION}
\]

Decomposition Theorems

- Pauli Matrix Decomp.
- Model Based Decomp.
- Eigenvector Decomp.

Pauli Matrices Decomposition

\[
[S] = \begin{bmatrix}
S_{HH} & S_{HV} \\
S_{HV} & S_{VV}
\end{bmatrix} = \begin{bmatrix}
a+b & c-id \\
c+id & a-b
\end{bmatrix} = \begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix} + \begin{bmatrix}
1 & 0 \\
0 & -1
\end{bmatrix} + \begin{bmatrix}
0 & 1 \\
0 & 0
\end{bmatrix} + \begin{bmatrix}
0 & -i \\
i & 0
\end{bmatrix}
\]

- \([S]_1 = \begin{bmatrix}
1 & 0 \\
0 & i
\end{bmatrix}
\]
  Single Scattering: \(S_{HH} = S_{VV}\)

- \([S]_2 = \begin{bmatrix}
1 & 0 \\
0 & -1
\end{bmatrix}
\]
  Dihedral Scattering: \(S_{HH} = -S_{VV}\)

- \([S]_3 = \begin{bmatrix}
0 & 1 \\
1 & 0
\end{bmatrix}
\]
  Dihedral Scattering (... rotated by \(\pi/2\) about the LOS)

- \([S]_4 = \begin{bmatrix}
0 & -i \\
i & 0
\end{bmatrix}
\]
  Transforms all polarization states into their orthogonal states (disappears in backscattering)
**Decomposition Theorems**

- Pauli Matrice Decomp.
- Eigenvector Decomp.
- Model Based Decomp.

**Eigenvector Decomposition**

Coherence Matrix: $[\mathbf{T}] = \langle \mathbf{k} \cdot \bar{\mathbf{k}} \rangle$

Diagonalisation: $[\mathbf{T}] = \mathbf{U} \mathbf{\Lambda} \mathbf{U}^\dagger$

$[\mathbf{T}] = \sum_{i=0}^{2} \lambda_i (\mathbf{e}_i \cdot \mathbf{e}_i') \mathbf{e}_i \mathbf{e}_i' = \sum_{i=0}^{2} \lambda_i [\mathbf{e}_i \mathbf{e}_i']$

- 3 real positive eigenvalues
- 3 orthonormal eigenvectors $\lambda_0 \geq \lambda_1 \geq \lambda_2 \geq 0$

Where: $[\mathbf{\Lambda}] = \begin{bmatrix} \lambda_0 & 0 & 0 \\ 0 & \lambda_1 & 0 \\ 0 & 0 & \lambda_2 \end{bmatrix}$

$[\mathbf{U}] = \begin{bmatrix} e_0 & e_1 & e_2 \\ e_0' & e_1' & e_2' \end{bmatrix}$

$[\mathbf{U}]^\dagger = \begin{bmatrix} e_0'' & e_1'' & e_2'' \end{bmatrix}$

**Scattering Entropy / Anisotropy**

Coherency Matrix Diagonalisation:

$[\mathbf{T}] = \sum_{i=0}^{2} \lambda_i (\mathbf{e}_i \cdot \mathbf{e}_i') \mathbf{e}_i \mathbf{e}_i'$

- Scattering Entropy: $H \geq \sum_{i=0}^{2} P_i \log P_i$ with $P_i = \frac{\lambda_i}{\lambda_0 + \lambda_1 + \lambda_2}$

$0 \leq H \leq 1$

Where:

- $H = 0$ → Totally Polarised Scatterer
- $H = 1$ → Totally Unpolarised Scatterer

- Scattering Anisotropy: $A = \frac{\lambda_0 - \lambda_2}{\lambda_0 + \lambda_1 + \lambda_2}$

$0 \leq A \leq 1$

Where:

- $A = 0$ → 2 Equal Secondary Scattering Processes
- $A = 1$ → Only 1 Secondary Scattering Process
Scattering Entropy Images

C-band  L-band

Scattering Anisotropy Images

C-band  L-band

Mean Scattering Parameters: Eigenvector

Coherency Matrix Diagonalisation:

\[
\begin{bmatrix}
\cos(\alpha) \exp(i\gamma) \\
\sin(\alpha) \cos(\beta) \exp(i\delta) \\
\sin(\alpha) \sin(\beta) \exp(i\epsilon)
\end{bmatrix}
\]

Eigenvectors:

\[
\lambda_1, \lambda_2, \lambda_3
\]

Appearance Probabilities:

\[
P = \frac{\lambda_1}{\lambda_1 + \lambda_2 + \lambda_3}
\]

Mean \(\alpha\)-Angle:

\[
a = P_e, a_1 + P_e, a_2 + P_e, a_3
\]

Mean \(\beta\)-Angle:

\[
b = P_e, b_1 + P_e, b_2 + P_e, b_3
\]

Mean \(\gamma\)-Angle:

\[
c = P_e, c_1 + P_e, c_2 + P_e, c_3
\]

Mean \(\delta\)-Angle:

\[
d = P_e, d_1 + P_e, d_2 + P_e, d_3
\]

Mean \(\epsilon\)-Angle:

\[
e = P_e, e_1 + P_e, e_2 + P_e, e_3
\]

Mean Scattering Mechanism:

\[
\delta = \frac{\cos(\alpha) \exp(i\gamma)}{\sin(\alpha) \cos(\beta) \exp(i\delta) + \sin(\alpha) \sin(\beta) \exp(i\epsilon)}
\]
Decomposition Theorems

- Pauli Matrix Decom.
- Eigenvector Decomp.
- Model Based Decomp.

Freeman 3 Component Decomposition

Vegetation Scattering: Bragg-Scattering + Dihedral Scattering + Random Volume of Dipoles

\[ \mathbf{T} = \mathbf{T}_1 + \mathbf{T}_2 + \mathbf{T}_3 \]

where

\[ \mathbf{T}_1 = \begin{bmatrix} \beta & 0 & 0 \\ \beta & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \mathbf{T}_2 = \begin{bmatrix} \alpha' & -\alpha & 0 \\ -\alpha & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \text{and} \quad \mathbf{T}_3 = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \]

\[ \mathbf{T} = \begin{bmatrix} f_1 \beta^2 + f_2 \alpha^2 + 2f_4 & f_1 \beta - f_2 \alpha & 0 \\ f_3 \beta - f_4 \alpha & f_3 + f_4 & 0 \\ 0 & 0 & f_5 \end{bmatrix} \]

4 Equations for 5 Unknowns

- Re(HH+VV) / 4 > 0 \quad \text{Single bounce is dominant} \quad \alpha = 1
- Re(HH+VV) / 4 < 0 \quad \text{Double bounce is dominant} \quad \beta = 1
Volume shape parameter \( \rho \) of a randomly oriented volume:

\[
\rho = \frac{1}{3} \left( |\alpha| < 1 \text{ dihedral} \right) \quad \rho = 1 \left( |\alpha| \geq 1 \text{ surface} \right)
\]

Scattering Power Components:

\[
P_\alpha = \ell_\alpha (|\alpha|^2) \quad P_\rho = \ell_\rho (3 - |\rho|^2)
\]

Summary: Polarimetric Parameters / Scattering Matrix

\[
[S] = \begin{bmatrix}
S_{HH} & S_{HV} & S_{VH} \\
S_{VH} & S_{VV} & S_{VH} \\
S_{VH} & S_{VH} & S_{VV}
\end{bmatrix}
\]

Absolute Phase Factor

\[
\begin{pmatrix}
\exp(\alpha_{HH}) & \exp(\alpha_{HV}) & \exp(\alpha_{VH}) \\
\exp(\alpha_{VH}) & 1 & \exp(\alpha_{VH}) \\
\exp(\alpha_{VH}) & \exp(\alpha_{VH}) & 1
\end{pmatrix}
\]

Five Parameters: 3 Amplitudes & 2 Phases

- Scattering Amplitudes
  
  \[
  \sigma_{HH}^0 = |S_{HH}| \quad \sigma_{HV}^0 = |S_{HV}| \quad \sigma_{VH}^0 = |S_{VH}|
  \]

- Total Power
  
  \[
  P^T = |S_{VV}|^2 + |S_{HH}|^2 + |S_{HV}|^2
  \]

- Amplitude Ratios
  
  \[
  \frac{\sigma_{HH}^0}{\sigma_{VV}^0} \quad \frac{\sigma_{VV}^0}{\sigma_{HH}^0} \quad \frac{\sigma_{HV}^0}{\sigma_{VH}^0} \quad \frac{\sigma_{VH}^0}{\sigma_{HV}^0} \quad \frac{\sigma_{HH}^0 + \sigma_{VV}^0}{\sigma_{HV}^0 + \sigma_{VH}^0}
  \]

- Pol Phase Differences
  
  \[
  \phi_{HVV} = \phi_{HHR} - \phi_{VHR}
  \]

- Helicity
  
  \[
  \text{Hel} = |S_{HH}| - |S_{VV}|
  \]

Applications of Radar Polarimetry

**Agriculture/Land-Use**
- Crop Classification/Moisture Content Estimation
- Urban Area mapping
- Urban Topography for Mobile comms

**Forestry**
- Biomass Estimation: (Saturation For High Biomass)
  - C-band saturation at 50 tons/hectare
  - L-band saturation at 100 tons/hectare
  - P-band saturation at 200 tons/hectare
- Deforestation
- Forest Canopy Height Estimation
- Tree Species Discrimination
- Forest Re-growth Monitoring

**Hydrology**
- Flood mapping/Forest Inundation
- Snow Hydrology
- Soil Moisture

**Sea Ice/Oceanography**
- Ice Roughness/Thickness Studies
- Polar Ice Cap Studies
- Extra-Terrestrial Ice/Water Studies

**Meteorology**
- Rain rate estimation
- Water/ice particle studies
- Severe Storm/Flood warning

**Topography/Cartography**
- Direct Surface Slope Estimation
- Accurate DEM Generation
- Difference of DEMs for Vegetation mapping

**Geology**
- Playas: Smooth Natural Surfaces (rms = 1cm)
- Alluvial fans, Sand Dunes, Moraines
- Sedimentary Rock formations
- Lava Flows (extreme in surface roughness)
- Weathering Erosion Studies
- Surface Roughness Estimates

**Humanitarian Demining**
- Surface Penetrating Radar (SPR)
- SAR for Mine Field Detection
Soil Moisture Estimation

- Surface characterisation
- \( m_v \) estimation over bare surface
- \( m_v \) estimation under the vegetation

Scattering @ Natural (Bare) Rough Surfaces

**Roughness Parameters**

- \( \text{rms (height)} \)
- \( \sigma \) [cm]
- \( \text{autocorrelation length} \) [cm]

**Volumetric Moisture Content**

\( m_v \) [vol. %]

The backscattered signal depends on the moisture, and roughness of the surface. Single channel SAR cannot resolve unambiguously the bare surface scattering problem.

Models for the Estimation of Bare Surface

**Theoretical Models**

- Geometrical-Optic 1963
- Physical-Optic 1963
- Small Perturbation 1988
- Integral Equation 1990

**Empirical Extensions**

- Oh Model 1992
- Dubois Model 1995
- Shi Model 1997

**Model Based Extensions**

- X-Bragg 1999

**Time**

- very complex
- inversion is restricted possible

Models for the Estimation of Vegetated Surface

**Theoretical Models**

- Vegetation Models (Stiles 2000)

**Scattering Decomposition Approaches**

- Eigen-Decomposition
- Model based-Decomposition

**Sem-I-Model Based Extensions**

- Optimisation (physical and numerical)
Small Perturbation Model

\[ \begin{bmatrix} S_{uu} & S_{uv} \\ S_{vu} & S_{vv} \end{bmatrix} = \left( 2k \cos \theta \right) \begin{bmatrix} R_u(\theta, \varepsilon) & 0 \\ 0 & R_v(\theta, \varepsilon) \end{bmatrix} \]

Exact solution of Maxwell Equation for \( s \to 0 \):

\[ R_u = \cos \theta \cos \varepsilon - \sin \theta \sin \varepsilon, \quad R_v = (\cos(\theta - \varepsilon) - \cos(\theta + \varepsilon)) / (2 \sin \varepsilon) \]

Roughness:
Depolarises the incident wave introducing (depolarised) HV and decreasing polarimetric coherence.

Ambiguity:
Terrain Slopes introduce also a HV component but this is co-related to HH and VV.

Moisture content:
Effects the scattering amplitude of all polarisations. For “smooth” (with respect to wavelength) surfaces the co-polarimetric ratio \( (HH/VV) \) becomes independent of roughness.

\[ \begin{bmatrix} S_{uu} & R_u \\ S_{vu} & R_v \end{bmatrix} \text{ is independent of } \begin{bmatrix} S_{uu} & S_{uv} \\ S_{vu} & S_{vv} \end{bmatrix} \]

Polarimetry provides an observation space that allows to separate roughness from moisture effects.

Small Perturbation Model (Bragg-Model)

Measured versus estimated volumetric moisture content \( m_v \) [vol. %] using SPM.

Semi-Empirical Models

Oh-Model


Dubois-Model


Quantitative Estimation of Volumetric Moisture \( m_v \) [vol %]

Validity range of the SPM for the ground measurements - surface roughness against volumetric soil moisture.
Quantitative Estimation of Surface Roughness $k_s$

**Dubois**

<table>
<thead>
<tr>
<th></th>
<th>Elbe RMS$_{\text{corr.}}$</th>
<th>Weih RMS$_{\text{corr.}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_s$</td>
<td>0.24</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>0.19</td>
<td>-0.7</td>
</tr>
</tbody>
</table>

**Oh**

Surface Parameter @ X-Bragg

Surface scattering model for the estimation of soil moisture content & surface roughness

**Advantage:**
- Simple model with direct relation to soil moisture content
- Not realistic enough for natural surfaces

**Disadvantage:**
- Very sensitive to vegetation cover

Prediction of the X-Bragg Model I

$$\begin{align*}
\text{HH} & \Rightarrow \text{VV} \\
(\text{HH+VV}) & \Rightarrow (\text{HH-VV})
\end{align*}$$

Cross polarised power versus $b_1$ for different local incidence angles

Prediction of the X-Bragg Model II

Variation of Anisotropy with Roughness (Model Parameter $\beta_1$)

Variation of Entropy/Alpha values with Dielectric Constant (45 degrees AOI)
Experimental Data from Anechoic Chamber (JRC - Ispra)
Collected in European Microwave Signature Laboratory (EMSL)
quad-pol scattering matrix (HH, VV, HV, VH); \( s = 0.4 \) cm; surface correlation length \( l = 6 \);
dielectric constant \( \varepsilon' = 8 \); frequency range = 1.5-18.5 GHz

Anisotropy versus frequency plot for the EMSL data
Entropy/alpha angle plot for the EMSL data

Inversion Results @ X-Bragg – Early Example on ELBE 2000

Volumetric Moisture

Soil Moisture Estimation
• Surface Characterisation
• \( m_v \) estimation over bare surface
• \( m_v \) estimation under the vegetation
  • 3 component model decomp.
  • improvement of the 3 component model decomp.
  • multi-angle approach combined with 3 comp. model decomp.
  • hybrid decomposition
3 Component Freeman Decomposition

\[
T_{\text{surf}} = -t_f \begin{bmatrix} 1 & \beta & 0 \\ \beta & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} R_u = \begin{bmatrix} \cos \theta - \sqrt{t_c \sin^2 \theta} \\ \cos \theta + \sqrt{t_c \sin^2 \theta} \\ t_c - \frac{1}{2} \sin^2 \theta \end{bmatrix}
\]

Surface

Dihedral

Volume

AGRISAR Campaign in Northern Germany 2006
1. Agricultural data base over a whole vegetation growth period - April - August 2006
2. 16 data acquisitions (DLR’s E-SAR) & ground measurements
3. Support by ESA for the space segment - Sentinel Program
3 Component Decomposition and Modifications

<table>
<thead>
<tr>
<th>Surface</th>
<th>Dihedral</th>
<th>Volume</th>
</tr>
</thead>
</table>

Decision rule: 
\[ \text{Decision rule } = \begin{cases} \text{Re} \left( S_{hh}S_{vv}^* \right) > 0 & \text{or} \quad \text{Re} \left( S_{hh}S_{vv}^* \right) < 0 \
\end{cases} \]

Surface modification (X-Bragg) \((\epsilon_s, \theta, 0)\): 
\[
\begin{pmatrix}
1 & \beta' \sin(2\phi) \\
\beta' \cos(2\phi) & \frac{3}{2} \beta' \left(1 - \cos(4\phi)\right) \\
0 & 0 \\
\end{pmatrix}
\]

Dihedral modification (modified Fresnel coefficients) \((\epsilon_s, \theta, \phi, \theta_s)\): 
\[
\begin{pmatrix}
\alpha' & \alpha' \\
\alpha' & 0 \\
\end{pmatrix}
\]

\[ L_S = \exp\left(-2 (4\pi)^2 \cos(\phi)^2\right) \]

\[ L_S = I - A \]

Time Series of Decompositions on Field Basis @ L-band

Wheat (No. 250) – far range

Wheat (No. 230) – near range

Rape (No. 140) – far range

Rape (No. 101) – far range
Inverted Soil Moisture @ L-band
Volume 2 (surface) and modified Dihedral (dihedral)

Time Series of Inverted Soil Moisture @ L-band
Wheat field (No. 250)  Wheat field (No. 230)

Time Series of Inverted Soil Moisture @ L-band
Rape field (No. 140)  Rape field (No. 101)

Soil Moisture Estimation
- Surface Characterisation
- \( m_v \) estimation over bare surface
- \( m_v \) estimation under the vegetation
  - 3 component model decomp.
  - improvement of the 3 component model decomp.
  - multi-angle approach combined with 3 comp. model decomp.
  - hybrid decomposition
**Polarimetric Decomposition Techniques & Inversion Scheme**

- Eigenvalue decomposition
- Calculation of entropy and alpha
- H2q criterion

**Soil Moisture Map**

- Vegetated soil
- Bare soil
- Soil moisture inversion from X-Bragg model

**Soil Moisture Map**

- Weisseritz Region

**Modeling of Volume Orientation**

Volume modelling of a cloud of uniformly shaped \( \psi \) particles with an orientation \( \varphi \) in azimuthal direction.

Selection of volume orientation by polarisation power ratio \( P_r \):

- \( P_r < -2 \text{dB} \)
- \( -2 \text{dB} < P_r < 2 \text{dB} \)
- \( P_r > 2 \text{dB} \)

**General Case**

- Vertical dipoles
- Random dipoles
- Horizontal dipoles

**Incorporation of a roughness and HV-decorrelation term**

\[ L_V = \exp(-\gamma (\mu_{\text{max}} - \mu_{\text{min}})) \]

\[ \mu_{\text{min}} = \frac{\alpha}{P_r} \]

**3 Component Decomposition and Modifications**

Surface

- Scattering dominance
  - \( \Re \left( S_{\text{HH-SV}} \right) > 0 \)
  - \( \Re \left( S_{\text{HH-SV}} \right) < 0 \)

Dihedral

- Surface model (X-Bragg) \( \{\psi_0, 0, 0\} \):
  - \( \left[ \begin{array}{c} a \sin \omega + b \cos \omega \\ a \cos \omega - b \sin \omega \end{array} \right] \)
  - \( \left[ \begin{array}{c} a \sin \omega - b \cos \omega \\ a \cos \omega + b \sin \omega \end{array} \right] \)

- Dihedral model (Fresnel with vegetation attenuation) \( \{\psi_0, 0, \varphi, \psi_L\} \):
  - \( \left[ \begin{array}{c} a \sin \omega + b \cos \omega \\ a \cos \omega - b \sin \omega \end{array} \right] \)

**Correction of Volume Intensity \( f_v \), with Eigen-based analysis**

- \( T_{11} \)
- \( T_{12} \)
- \( T_{21} \)
- \( T_{22} \)

- Eigenanalysis of Ground Components
  - Set Eigenvalues to zero (lower calculus limit)
  - Solutions for \( f_v \):
    - \( f_v = f_{v1} = 4T_{11} \)
    - \( f_{v2} = f_{v3} = T_{11} + 2T_{22} \)

- Decomposition with Corrected Volume Intensity \( f_v \)

\[ T_{\text{Surface}} = T_{\text{Dihedral}} \]

\[ f_v = \frac{1}{2} \text{Re}(S_{\text{HH-SV}}) \]

\[ f_v = \frac{1}{2} \text{Re}(S_{\text{HH-SV}}) \]

\[ f_v = \frac{1}{2} \text{Re}(S_{\text{HH-SV}}) \]

\[ f_v = \frac{1}{2} \text{Re}(S_{\text{HH-SV}}) \]
1. Soil moisture estimation under different land cover with measurement approaches at different scales
2. Radar data acquisition flights (single pol X-, dual-pol C- and quad-pol L- and P-band)
3. Ground measurements (soil moisture, soil roughness, biomass, ...)

**Goal:** Support flood forecasting by identification of critical catchment states

**OPAQUE – Campaign**
May 2007

- TDR and GPR
- Corner-reflector
- Weißeritz
- Flight strip
- Test site

- Winter triticale
- Winter wheat
- Summer barley
- Winter barley
- Grass land
- Winter rape
- Summer corn
- Bare soil

**Volume Orientation and Scattering Dominance @ L-band**

- Volume orientation: vertical, horizontal, random
- Surface dominant: grassland, winter wheat, etc.
- Dihedral dominant: X-Bragg only

**Model-based Decomposition @ L-band**

- Range:
  - 0°
  - 10°
  - 20°
  - 30°
  - 40°
  - 50°

**Inverted Soil Moisture @ L-band**

- + =+
Validation with TDR Measurements on Winter Wheat

Vegetation height: 55cm
Row distance: 10cm
Wet biomass: 2.85kg/m²

Validation with TDR Measurements on Summer Barley

Vegetation height: 45cm
Row distance: 23cm
Wet biomass: 0.93kg/m²

Validation with TDR Measurements on Winter Triticale

Vegetation height: 85cm
Row distance: 10cm
Wet biomass: 3.34kg/m²

Validation with TDR Measurements on Winter Barley

Vegetation height: 70cm
Row distance: 10cm
Wet biomass: 3.31kg/m²
**Soil Moisture Estimation**

- Surface Characterisation
- mv estimation over bare surface
- mv estimation under the vegetation
  - 3 component model decomps.
  - improvement of the 3 component model decomps.
  - multi-angle approach combined with 3 comp. model decomps.
  - hybrid decomposition

---

**Inversion Rate of Soil Moisture Estimation @ different AOI**

<table>
<thead>
<tr>
<th>Acquisition</th>
<th>Master</th>
<th>Opposite</th>
<th>Perpendicular</th>
</tr>
</thead>
<tbody>
<tr>
<td>One acquisition</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Single angular</td>
<td>40.32%</td>
<td>29.87%</td>
<td>48.53%</td>
</tr>
<tr>
<td>Two acquisitions</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Master-opposite</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Master-perpendicular</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Opposite-perpendicular</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bi-angular</td>
<td>55.87%</td>
<td>63.39%</td>
<td>60.71%</td>
</tr>
<tr>
<td>Three acquisitions</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Master-opposite-perpendicular</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tri-angular</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>70.89%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

---

**Validation of Soil Moistures @ different Incidence Angles**

- Sampling box: 13x13 pixels
- Box coverage: 70%
- Field mean values

---

**Multi-Angle Approach for Soil Moisture Inversion Rate Increase**

- Testsite: Weissertz Watershed, E-SAR L-band
**RMSE [vol.%] between Estimated and Measured Soil Moistures for Different Incidence Angle Constellations**

<table>
<thead>
<tr>
<th>Fields</th>
<th>m</th>
<th>o</th>
<th>p</th>
<th>m-o</th>
<th>m-p</th>
<th>o-p</th>
<th>m-o-p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Winter Triticale</td>
<td>6.34</td>
<td>6.34</td>
<td>10.33</td>
<td>5.42</td>
<td>6.27</td>
<td>9.48</td>
<td>5.72</td>
</tr>
<tr>
<td>Winter Barley</td>
<td>&lt;</td>
<td>&lt;</td>
<td>9.75</td>
<td>7.76</td>
<td>6.85</td>
<td>8.78</td>
<td>6.94</td>
</tr>
<tr>
<td>Winter Rye</td>
<td>5.25</td>
<td>6.11</td>
<td>5.86</td>
<td>5.81</td>
<td>4.89</td>
<td>6.42</td>
<td>5.27</td>
</tr>
<tr>
<td>Winter Wheat</td>
<td>4.52</td>
<td>&lt;</td>
<td>9.79</td>
<td>4.41</td>
<td>5.04</td>
<td>9.54</td>
<td>4.98</td>
</tr>
<tr>
<td>Summer Oat</td>
<td>8.55</td>
<td>6.43</td>
<td>-</td>
<td>6.35</td>
<td>8.55</td>
<td>6.43</td>
<td>6.35</td>
</tr>
<tr>
<td><strong>Mean</strong></td>
<td>6.17</td>
<td>6.29</td>
<td>8.93</td>
<td>5.95</td>
<td>6.32</td>
<td>8.13</td>
<td>5.85</td>
</tr>
</tbody>
</table>

* Out of scene
* Too less values for a valid analysis

**Single angular**

**Bi-angular**

**Tri-angular**

---

**Soil Moisture Estimation**

- Surface Characterisation
- mv estimation over bare surface
- mv estimation under the vegetation
  - 3 component model decomp.
  - improvement of the 3 component model decomp.
  - multi-angle approach combined with 3 comp. model decomp.
  - hybrid decomposition
Retrieval of the Ground Scattering Components

Hybrid Polarimetric Decomposition

Polarimetric SAR data

Surface scattering model
Bragg scatter modeling with $\theta_{loc}$ and a variety of soil dielectric constants $\varepsilon$,

$\beta_c = \frac{R_{HH} - R_{VV}}{R_{HH} + R_{VV}}$

$R_{HH}, R_{VV} = f(\varepsilon_s, \theta_{loc})$

Minimization

$\min \| \beta - \beta_c \|$

Pedo-Transfer Function of:
Topp et al. ($\varepsilon_s < 40$)
Roth et al. ($\varepsilon_s > 40$)

Soil moisture [vol.%]

Soil Moisture Inversion from Surface Scattering Component

Soil Moisture Inversion along Vegetation Growth Cycle

Validation of Moisture Inversion with Field Measurements (TDR)
Summary Soil Moisture Estimation

- Soil moisture inversion is still object of scientific research:
  - Main research focuses on the estimation of mv under the vegetation
  - One attempt is to use SAR polarimetry for scattering mechanism decomposition in order to characterise and subtract volume from a ground component (surface/dihedral)
  - PolSARPro provides the possibility to invert soil moisture over bare fields and allows a decomposition of scattering mechanisms
  - Outlook: Inversion procedures for vegetated areas are still missing

References (Part I)

Basic Text Books for SAR-Polarimetry and Pol-InSAR:

Surface Parameter Inversion:
Part II: Exercises with L-band Airborne Data

- Read the airborne SAR data
- Speckle Filtering (refined Lee)
- Oh, Dubois and X-Bragg Inversion

Test Date Used for the Exercise

- **Testsite: Demmin**
  - Location: Northern Germany
  - Acquisition Date: May 2012
  - Frequency: L-band
  - Data size: az: 2.75km rg: 2.2 km
  - Polarisation: 4 SLC
  - Resolution: az: 60cm x rg: 3.8m
  - Rows and columns: 7981 x 1837

First Steps in PolSARPro

- Please open PolSARPro
- Define your environment
- Open the DLR’s acquired test Radar data