Terrain Motion 2: Volcano Deformation, Models and Examples

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Volcano Deformation – Models and Examples

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A simple volcanic system
1. Charging the system

A simple volcanic system
1. Discharging the system (a. eruptions)
A simple volcanic system
1. Discharging the system (b. intrusions)

Outline

• Simple volcanic deformation sources
  – Point Pressure (‘Mogi’ Model for magma chambers)
  – Elastic Dislocations (‘Okada’ model for Dykes and Sills)
  – Penny-shaped crack (for sills)
  – Other models
• Limitations of Volcanic Geodesy
• How to interpret volcanic deformation
• Comparison of InSAR and GPS
• Science vs Monitoring
Simple Sources: 1. The Mogi Model

Assumptions
1. Isotropic elastic half space (Poisson’s ratio $\nu$; Shear modulus $\mu$)
2. $\alpha << d$ (i.e. spherical point source)
3. Incompressible magma

\[
\begin{bmatrix}
  u \\
v \\
w
\end{bmatrix} = \alpha^3 \Delta P \frac{(1-\nu)}{\mu} \begin{bmatrix}
x / R^3 \\
y / R^3 \\
d / R^3
\end{bmatrix}
\]

But $\Delta V \approx \frac{\Delta P}{\mu} \pi \alpha^3 \Rightarrow$

\[
\begin{bmatrix}
  u \\
v \\
w
\end{bmatrix} = \Delta V \frac{(1-\nu)}{\pi} \begin{bmatrix}
x / R^3 \\
y / R^3 \\
z / R^3
\end{bmatrix}
\]

Simple Sources: 1. The Mogi Model

3D Surface Displacements

Source Depth = 5 km; Volume = $10^7$ m$^3$
Maximum uplift = 95 mm


Simple Sources: 2. Tensile Elastic Dislocations (Okada model – sills or dykes)

Assumptions
1. Isotropic elastic half space
2. Uniform opening (for dykes/sills) on rectangular dislocation

Parameters
8 parameters needed to define dislocation:

- **Strike, Dip** [2]
- Location $(x_c,y_c)$ or $(x_p,y_p)$ [2]
- $d_c$ and Width (or $d_{top}$, $d_{bot}$) [2]
- Length
- Opening $(u_⊥)$ [1]

Can simplify

e.g. **Length = Width** for sills;
**Dip = 0** for sills, **90°** for dykes

Simple Sources: 2. Tensile Elastic Dislocations (Okada model – *sills* or dykes)

Sill 1: Dip = 0, Strike = 0, x = y = d_z = 5 km
Length = Width = 1 km
Opening = 10 m [Volume = $10^7$ m$^3$]


Simple Sources: 2. Tensile Elastic Dislocations (Okada model – *sills* or dykes)

Sill 2: Dip = 0, Strike = 0, x = y = d_z = 5 km
Length = Width = 5 km
Opening = 0.4 m [Volume = $10^7$ m$^3$]

Simple Sources: 2. Tensile Elastic Dislocations (Okada model – sills or dykes)

Sill 3: Dip = 0, Strike = 0, x=y=0, \( d_z = 5 \) km
Length = Width = 10 km
Opening = 0.1 m  [Volume = \( 10^7 \text{ m}^3 \)]


Simple Sources: 2. Tensile Elastic Dislocations (Okada model – sills or dykes)

\[ \text{Vertical deformation normalised by Mogi uplift} \]
\[ \text{Distance normalised by depth of source} \]
Simple Sources: 2. Tensile Elastic Dislocations (Okada model – sills or dykes)


- Vertical deformation normalised by Mogi uplift
- Distance normalised by depth of source
Simple Sources: 2. Tensile Elastic Dislocations (Okada model – sills or dykes)

- Vertical deformation normalised by Mogi uplift
- Distance normalised by depth of source

Source volume vs uplift volume

Delaney and McTigue (1994) showed:

1. For sills, source volume change = surface uplift volume
2. For Mogi, source volume change = surface uplift volume / 2(1-ν)
   e.g. For ν = 0.25, source volume = (2/3) * surface uplift volume

These relationships are only true if the Magma is incompressible. See Johnson et al (2000), Rivalta and Segall, (2008) for details
Simple Sources: 2. Tensile Elastic Dislocations (Okada model – sills or dykes)

Dyke 1: Dip = 90, Strike = 0, x = y = 0
\(d_{top} = 2\,\text{km}, \, d_{bot} = 8\,\text{km}\)
Length = 10 km
Opening = 1 m [Volume = \(6 \times 10^7\,\text{m}^3\)]

Simple Sources: 2. Tensile Elastic Dislocations (Okada model – sills or dykes)

Dyke 2: Dip = 90, Strike = 0, x=y=0
\( d_{top} = 1 \text{ km}, d_{bot} = 8 \text{ km} \)
Length = 10 km
Opening = 1 m [Volume = 7 \times 10^7 \text{ m}^3]

Simple Sources: 2. Tensile Elastic Dislocations (Okada model – sills or dykes)

Dyke 3: Dip = 90, Strike = 0, x=y=0
\(d_{\text{top}} = 4 \text{ km}, d_{\text{bot}} = 8 \text{ km}\)
Length = 10 km
Opening = 1 m [Volume = 4 x 10^7 m^3]

Simple Sources: 2. Tensile Elastic Dislocations (Okada model – sills or dykes)

Dyke 1: Dip = 90, Strike =0, x=y=0
\[ d_{top} = 2 \text{ km}, d_{bot} = 8 \text{ km} \]
Length= 10 km
Opening = 1 m [Volume = 6 x 10^7 m^3]


Simple Sources: 2. Tensile Elastic Dislocations (Okada model – sills or dykes)

Dyke 4: Dip = 90, Strike =0, x=y=0
\[ d_{top} = 2 \text{ km}, d_{bot} = 12 \text{ km} \]
Length= 10 km
Opening = 1 m [Volume = 10 x 10^7 m^3]

Simple Sources: 2. Tensile Elastic Dislocations (Okada model – sills or dykes)

Dyke 4: Dip = 90, Strike =0, x=y=0
\( d_{\text{top}} = 2 \text{ km}, d_{\text{bot}} = 12 \text{ km} \)
Length = 10 km
Opening = 1 m  \( \text{[Volume} = 10 \times 10^7 \text{ m}^3 \text{]} \)


Ascending vs Descending
Simple Sources: 3. Penny-shaped crack
(‘Fialko’ model for sills)

Assumptions
1. Isotropic elastic half space
   (Poisson’s ratio $\nu$; Shear modulus $\mu$)
2. Uniformly pressurized horizontal crack

Parameters
1. Depth
2. Radius of source
3. Pressure

For full equations see Fialko et al., 2001.

Fialko, Y. et al. (2001), Geophys. J. Int., 146, 181-190

Simple Sources: 3. Penny-shaped crack
(‘Fialko’ model for sills)

Penny Sill: Source Depth = 5 km; Radius = 5 km;
Pressure = 0.77 MPa

Fialko, Y. et al. (2001), Geophys. J. Int., 146, 181-190
Simple Sources: 3. Penny-shaped crack ('Fialko' model for sills)

Penny Sill: Source Depth = 5 km; Radius = 5 km;
Pressure = 0.77 MPa

Okada Sill: Source Depth = 5 km; Length = Width = 8.86 km (= $\sqrt{\pi}$);
Opening = 0.13 m (= 10/($\pi$)); [Volume = $10^7$ m$^3$]

Fialko, Y. et al. (2001), Geophys. J. Int., 146, 181-190

Simple Sources: 3. Penny-shaped crack ('Fialko' model for sills)

Penny Sill: Source Depth = 5 km; Radius = 5 km;
Pressure = 0.77 MPa

Okada Sill: Source Depth = 5 km; Length = Width = 8.86 km (= $\sqrt{\pi}$);
Opening = 0.13 m (= 10/($\pi$)); [Volume = $10^7$ m$^3$]

Mogi Point: Source Depth = 5 km; Volume Change = $10^7$ m$^3$

Fialko, Y. et al. (2001), Geophys. J. Int., 146, 181-190
Mt Peulik. Dormant Stratovolcano: Last eruption: 1852

Mount Peulik volcano inflated $0.051 \pm 0.005 \text{km}^3$ between October 1996 and September 1998.

Zhong Lu, Chuck Wicks, USGS

Limitations of Simple Models used in Volcano Geodesy

- Ambiguity of source geometries
- Isotropic, Elastic Half-Space Assumption
- Volumes/Pressures dependent on assumptions about geometries or magma/rock properties
- Models are purely kinematic
- Magma is hot $\rightarrow$ visco-elastic effects?
- How to interpret signals?

Fialko, Y. et al. (2001), Geophys. J. Int., 146, 181-190
InSAR vs GPS for geohazards

<table>
<thead>
<tr>
<th></th>
<th>InSAR</th>
<th>GPS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency of Observation</td>
<td>Every 10-40 days for each satellite.</td>
<td>Up to 50 Hz. Typically get daily positions for continuous sites.</td>
</tr>
<tr>
<td>Spatial coverage</td>
<td>Continuous (in coherent areas)</td>
<td>Points</td>
</tr>
<tr>
<td>Dimensionality</td>
<td>1D (Line of Sight) Displacement</td>
<td>3D Displacements</td>
</tr>
<tr>
<td>Cost</td>
<td>Data is cheap/free for most satellites</td>
<td>Cost of instruments vary</td>
</tr>
<tr>
<td>Field requirements</td>
<td>None</td>
<td>Installation and servicing on the ground</td>
</tr>
<tr>
<td>Accuracy</td>
<td>~1 cm for single interferogram. Better for time-series/stacks</td>
<td>~1mm/yr for continuous sites</td>
</tr>
</tbody>
</table>

Fialko, Y. et al. (2001), Geophys. J. Int., 146, 181-190