GOCE gravity models

Roland Pail
Institute of Astronomical and Physical Geodesy
TU München
Contents

- Overview and comparison of models and methodologies
- Performance evaluation: internal and external validation
- Future prospects
- Global models vs. gradients
Spherical harmonic series

\[
V(r, \theta, \lambda) = \frac{G M}{R} \sum_{n=0}^{\infty} \left( \frac{R}{r} \right)^{n+1} \sum_{m=0}^{n} \bar{P}_{nm} \cos(\theta) \cdot \left[ \bar{C}_{nm} \cos(m \lambda) + \bar{S}_{nm} \sin(m \lambda) \right]
\]
Global gravity field determination

\[
V(r, \theta, \lambda) = \frac{G M}{R} \sum_{n=0}^{n_{\text{max}}} \left( \frac{R}{r} \right)^{n+1} \sum_{m=0}^{n} P_{nm}(\cos \theta) \cdot \left[ C_{nm} \cos(m \lambda) + S_{nm} \sin(m \lambda) \right]
\]

Satellite observations

Gravity field parameters

Satellite orbit

\[
V(r, \theta, \lambda) \approx \sum_{i=1}^{n_{\text{obs}}} A_{nm}(r, \theta, \lambda) \cdot x_{nm}
\]

\[
\approx a_i = \frac{\partial V}{\partial x_i}
\]

Gradiometry

\[
V_{ij} = \frac{\partial^2 V}{\partial x_i \partial x_j}
\]

Spherical harmonic coeff.

User:
- Given: coefficients → comp. of gravity field quantities
- Forward task: summation of terms of the series

Gravity field modelling:
- Given: grav. observations → comp. of coefficients
- Inverse problem
Global Gravity Models

http://icgem.gfz-potsdam.de/ICGEM/ICGEM.html

<table>
<thead>
<tr>
<th>Model</th>
<th>Year</th>
<th>Degree</th>
<th>Data</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>GO_CONS_GCF_2_DIR_R3</td>
<td>2011</td>
<td>240</td>
<td>S(Goce, Grace, Lageos)</td>
<td>Bruinsma et al, 2010</td>
</tr>
<tr>
<td>GO_CONS_GCF_2_TIM_R3</td>
<td>2011</td>
<td>250</td>
<td>S(Goce)</td>
<td>Pail et al, 2011</td>
</tr>
<tr>
<td>GIF48</td>
<td>2011</td>
<td>360</td>
<td>S(Grace), G, A</td>
<td>Ries et al, 2011</td>
</tr>
<tr>
<td>EIGEN-6C</td>
<td>2011</td>
<td>1420</td>
<td>S(Goce, Grace, Lageos), G, A</td>
<td>Förste et al, 2011</td>
</tr>
<tr>
<td>EIGEN-6S</td>
<td>2011</td>
<td>240</td>
<td>S(Goce, Grace, Lageos)</td>
<td>Förste et al, 2011</td>
</tr>
<tr>
<td>GOCC02S</td>
<td>2011</td>
<td>250</td>
<td>S(Goce, Grace, ...)</td>
<td>Goiginger et al, 2011</td>
</tr>
<tr>
<td>AIUB-GRACE03S</td>
<td>2011</td>
<td>160</td>
<td>S(Grace)</td>
<td>Jäggi et al, 2011</td>
</tr>
<tr>
<td>GO_CONS_GCF_2_DIR_R2</td>
<td>2011</td>
<td>240</td>
<td>S(Goce)</td>
<td>Bruinsma et al, 2010</td>
</tr>
<tr>
<td>GO_CONS_GCF_2_TIM_R2</td>
<td>2011</td>
<td>250</td>
<td>S(Goce)</td>
<td>Pail et al, 2011</td>
</tr>
<tr>
<td>GO_CONS_GCF_2_SPW_R2</td>
<td>2011</td>
<td>240</td>
<td>S(Goce)</td>
<td>Migliaccio et al, 2011</td>
</tr>
<tr>
<td>GO_CONS_GCF_2_DIR_R1</td>
<td>2010</td>
<td>240</td>
<td>S(Goce)</td>
<td>Bruinsma et al, 2010</td>
</tr>
<tr>
<td>GO_CONS_GCF_2_TIM_R1</td>
<td>2010</td>
<td>224</td>
<td>S(Goce)</td>
<td>Pail et al, 2010a</td>
</tr>
<tr>
<td>GO_CONS_GCF_2_SPW_R1</td>
<td>2010</td>
<td>210</td>
<td>S(Goce)</td>
<td>Migliaccio et al, 2010</td>
</tr>
<tr>
<td>GOCC01S</td>
<td>2010</td>
<td>224</td>
<td>S(Goce, Grace)</td>
<td>Pail et al, 2010b</td>
</tr>
<tr>
<td>EIGEN-51C</td>
<td>2010</td>
<td>359</td>
<td>S(Grace, Champ), G, A</td>
<td>Bruinsma et al, 2010</td>
</tr>
<tr>
<td>AIUB-CHAMP03S</td>
<td>2010</td>
<td>100</td>
<td>S(Champ)</td>
<td>Prange, L. et al, 2010</td>
</tr>
</tbody>
</table>

- Model coefficients (common format)
- Links to documentation (!)
<table>
<thead>
<tr>
<th>Model</th>
<th>D/O</th>
<th>2M</th>
<th>4M</th>
<th>6M</th>
<th>8M</th>
<th>10M</th>
<th>1Y</th>
<th>2Y</th>
<th>3Y</th>
<th>4Y</th>
<th>5Y</th>
<th>6Y</th>
<th>7Y</th>
<th>8Y</th>
<th>9Y</th>
<th>10Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>GOCE-DIR1</td>
<td>240</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GOCE-DIR2</td>
<td>240</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GOCE-DIR3</td>
<td>240</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GOCE-TIM1</td>
<td>274</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GOCE-TIM2</td>
<td>250</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GOCE-TIM3</td>
<td>250</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GOCE-SPW1</td>
<td>210</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GOCE-SPW2</td>
<td>240</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>EIGEN-6S</td>
<td>240</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>EIGEN-6C</td>
<td>1420</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GOC001S</td>
<td>224</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GOC002S</td>
<td>240</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GOC003S</td>
<td>250</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

GOCE-only and GOCE combination models

- GOCE+GRACE+
- GOCE+GRACE+CHAMP+Terr.
- GOCE+GRACE
- GOCE
- GOCE+GRACE
- GOCE+GRACE+Terr.
- GOCE+GRACE
- GOCE+GRACE+CHAMP

Colors:
- GOCE
- GRACE
- CHAMP
- SLR
- Altimetry/Terrestrial
Methods of gravity field modeling

<table>
<thead>
<tr>
<th>Space-wise method</th>
<th>Time-wise method</th>
<th>Direct method</th>
</tr>
</thead>
<tbody>
<tr>
<td>SPW</td>
<td>TIM</td>
<td>DIR</td>
</tr>
<tr>
<td>• observations distributed in space</td>
<td>• observations distributed along the orbit</td>
<td>• observations distributed along the orbit</td>
</tr>
<tr>
<td>• Large least squares collocation problem</td>
<td>• Large least square adjustment problem</td>
<td>• Large least square adjustment problem</td>
</tr>
<tr>
<td></td>
<td>• pure GOCE models</td>
<td>• combination models</td>
</tr>
<tr>
<td></td>
<td>• full spectral range of gradiometry</td>
<td>• only measurement bandwith</td>
</tr>
</tbody>
</table>

GOCE Gravity Models
Overview & Methods

TUM IAPG eGE
Power spectral density (PSD) of gradiometer error
**GOCE gradients: signal & noise**

**Gravity Signal** (D/O200) and **noise** of gradiometer component $V_{zz}$

**Time domain**

**Frequency domain**
(Power spectral density)

- **Time domain** graph shows the gravity gradients $V_{zz}$ in [Etvöš] over time in hours.
- **Frequency domain** graph displays the gravity gradient PSD in [E/sqrt(Hz)] with measurement bandwidth (MBW) indicated.

**MBW ... Measurement bandwidth**
Gradient observations are highly correlated along the orbit track.
Differences: TIM vs. DIR

**TIM**

- The **full spectral range** of the gravity gradients is used.
- Stochastic observation model is introduced for a proper weighting in the adjustment procedure.

\[
\hat{x} = \left( A^T \sum^{-1}_\ell A \right)^{-1} A^T \sum^{-1}_\ell \ell
\]

**DIR**

- Only gravity gradient signal content **within MBW** is used.
Effect of disregarding signal outside MBW

This information must come from a different source !!!
(prior information, GRACE, ...)
Effect of disregarding signal outside MBW

Gravity anomaly signal

Errors (limitation to MBW)

mGal

-60 -40 -20 0 20 40 60
Further consequences

There is no 1-to-1 correspondence of frequency at SH domain!

- One should not cut/add different gravity models!
- One should not simply replace a certain frequency range by complementary gravity information!
Performance evaluation: internal and external validation
GOCE-only and GOCE combination models

<table>
<thead>
<tr>
<th>Model</th>
<th>D/O</th>
<th>2M</th>
<th>4M</th>
<th>6M</th>
<th>8M</th>
<th>10M</th>
<th>1Y</th>
<th>2Y</th>
<th>3Y</th>
<th>4Y</th>
<th>5Y</th>
<th>6Y</th>
<th>7Y</th>
<th>8Y</th>
<th>9Y</th>
<th>10Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>GOCE-DIR1</td>
<td>240</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GOCE-DIR2</td>
<td>240</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GOCE-DIR3</td>
<td>240</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GOCE-TIM1</td>
<td>274</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GOCE-TIM2</td>
<td>250</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GOCE-TIM3</td>
<td>250</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GOCE-SPW1</td>
<td>210</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GOCE-SPW2</td>
<td>240</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>EIGEN-6S</td>
<td>240</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>EIGEN-6C</td>
<td>1420</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GOCO01S</td>
<td>224</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GOCO02S</td>
<td>240</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GOCO03S</td>
<td>250</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

GOCE-only and GOCE combination models:

- GOCE+GRACE+
- GOCE+GRACE+
- GOCE+
- GOCE+GRACE
- GOCE
- GOCE+GRACE
- GOCE+GRACE+
- GOCE+GRACE+
- GOCE+GRACE+
- GOCE+GRACE+
- GOCE+GRACE+
- GOCE+GRACE+

Legend:
- GOCE
- GRACE
- CHAMP
- SLR
- Altimetry/Terrestrial
Estimated coefficient errors
Estimated coefficient errors

<table>
<thead>
<tr>
<th>Model</th>
<th>Deg.</th>
<th>Data period</th>
<th>effective</th>
</tr>
</thead>
<tbody>
<tr>
<td>TIM_R1</td>
<td>224</td>
<td>01/11/2009 – 11/01/2010</td>
<td>2 months</td>
</tr>
<tr>
<td>TIM_R2</td>
<td>250</td>
<td>01/11/2009 – 05/07/2010</td>
<td>6 months</td>
</tr>
<tr>
<td>TIM_R3</td>
<td>250</td>
<td>01/11/2009 – 17/04/2011</td>
<td>12 months</td>
</tr>
</tbody>
</table>
Estimated geoid height errors

- Release 1 (D/O 200)
- Release 2 (D/O 200)
- Release 3 (D/O 200)

Full covariance propagation (D/O 200):
- Geoid: 4.6 cm
- Grav. Anom. 1.3 mGal
Comparison with EGM2008: Gravity anomaly differences at degree/order 200 (100 km spatial wavelength)

→ no systematic effects; correct stochastic modelling
TIM models: external validation

<table>
<thead>
<tr>
<th></th>
<th>South America</th>
<th>North America</th>
</tr>
</thead>
<tbody>
<tr>
<td>[mGal]</td>
<td>TIM</td>
<td>GOCO</td>
</tr>
<tr>
<td><strong>Signal</strong></td>
<td>41.90</td>
<td></td>
</tr>
<tr>
<td><strong>Diff. R1</strong></td>
<td>8.21</td>
<td>8.21</td>
</tr>
<tr>
<td><strong>Diff. R2</strong></td>
<td>7.67</td>
<td>7.62</td>
</tr>
<tr>
<td><strong>Diff. R3</strong></td>
<td>7.59</td>
<td>7.54</td>
</tr>
</tbody>
</table>

(TIM models: external validation Performance)
External validation with GPS/levelling data

GPS-Levelling Data as a Tool to Validate Global Gravity Field Models

EGM2008 Data Coverage & Coverage of GPS/Levelling Data

External validation with GPS/levelling data

RMS of Geoid Differences at GPS-Levelling Points
(Omission Error estimated from EGM2008)

Germany

Acknowledgement: GPS levelling data for have been provided for validation purposes by BKG, Frankfurt/Main
RMS of Geoid Differences at GPS-Levelling Points
(Omission Error estimated from EGM2008)

Japan

[Graph showing RMS differences for different models]

Acknowledgement: GPS levelling data for have been provided for validation purposes by the Japanese Geographical Survey Institute.
RMS of Geoid Differences at GPS-Levelling Points
(Omission Error estimated from EGM2008)

Saudi-Arabia

Acknowledgement: GPS levelling data for have been provided for validation purposes by King Abdulaziz City for Science and Technology KACS.
RMS of Geoid Differences at GPS-Levelling Points
(Omission Error estimated from EGM2008)

Brazil

Acknowlegdment: GPS levelling data for have been provided for validation purposes by Brazilian Institute of Geography and Statistics - IBGE, Directorate of Geosciences - DGC, Coordination of Geodesy - CGED
GOCE – achievable accuracies

Geoid height accuracy [cm]

- GOCE-TIM1: 2 months
- GOCE-TIM2: 6 months
- GOCE-TIM3: 12 months
- Extended mission *)

*) Dec. 2012
GOCE – achievable accuracies

Gravity anomaly error [mGal]

Cum. grav. anom. error

degree n

- GOCE-TIM1: 2 months
- GOCE-TIM2: 6 months
- GOCE-TIM3: 12 months
- Extended Mission

*) Dec. 2012
GOCE Status

There is more to come ...

• Level 1b data reprocessing
• GOCE orbit lowering
Level 1b data reprocessing

PSD of gravity gradient trace

- A: current L1b processing
- B: plus upgraded ARR
- C: plus STR combination
- D: plus ICM interpolation

Frequency [Hz]

PSD GGT trace [mE/Hz^{1/2}]
Level 1b data reprocessing

SGG-only: gravity anom. diff. to GOCO03S (d/o 180)

<table>
<thead>
<tr>
<th>$\sigma_{\Delta g}$ [mGal]</th>
<th>Original L1b</th>
<th>Reproc. L1b</th>
<th>Gain%</th>
</tr>
</thead>
<tbody>
<tr>
<td>SGG-only</td>
<td>2.12</td>
<td>1.45</td>
<td>46</td>
</tr>
<tr>
<td>GOCE-only</td>
<td>1.51</td>
<td>1.32</td>
<td>15</td>
</tr>
<tr>
<td>GOCE+GRACE</td>
<td>1.25</td>
<td>1.10</td>
<td>14</td>
</tr>
</tbody>
</table>

Pail et al. (2012), Stud. geophys. geod.
GOCE Gravity Models

Signal (D/O 200)

Gradiometer noise

Signal + noise

REPROCESSED GRADIENTS

[Eötvos Units]
Lowering of the satellite improves the estimates for the high degrees, and thus **increases spatial resolution**.

- **255 km**
  - **-8.6 km**: August 2012 → 1 cycle (62 days)
  - **-15 km**: November 2012 → 1 cycle (67 days)
  - **-20 km**: February 2013 → at least 1 cycle (TBD days)
  - April 2013 ???
Some facts:

• Global gravity models are a globally optimum solution (= optimum separation of signal and noise).

• We do our best to squeeze out as much information as possible from the gradients to generate global models.

• With regional models maybe 5-10% added value (???) but: signal-to-noise separation imposes an x00 % problem!
Gravity models vs. measured gradients

Some numbers: accuracy of $V_{zz}$ gradients:

Pay attention: wir error measures such as $10 \text{ mE}/\sqrt{\text{Hz}}$ you cannot directly work with (do not disregard the $\sqrt{\text{Hz}}$ !!!)

$$\sigma^2 = \int_{f_1}^{f_2} S(f) \, df$$

- full spectrum: $\sim 700 \text{ mE}$ (original)
  $\sim 100 \text{ mE}$ (reprocessed)
- MBW: $\sim 3-5 \text{ mE}$ → but: not full signal!
- synthetized from a global GOCE (+GRACE) model (Release 3)
  (full spectrum): $\sim 0.4 \text{ mE}$
Currently, we have reached with global GOCE models the mission goal of 1 mGal gravity anomaly error at 100 km wavelength (proven by external validation).

Gravity gradients at orbit altitude synthesized from R3 model have an accuracy of 0.4 mE (d/o 200).

Further improvements are expected due to data reprocessing.

Due to lowering the satellite altitude, the spatial resolution can be further increased.