Statistical test for a change detector based on the optimized power ratio

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Motivation

- An algorithm was developed for:
  - Testing the ESM hypothesis
  - Change detection

- We want to find a statistical test to set the threshold of the change detector
Change Detector
Pol-InSAR coherence

6x6 PolInSAR Coherency matrix

\[
\begin{bmatrix}
T_{11} & \Omega_{12} \\
\Omega_{12}^T & T_{22}
\end{bmatrix}
\]

Equi-Scattering Mechanisms Hypothesis

\[
[T_{11}] \approx [T_{22}] \quad [T] = \frac{[T_{11}] + [T_{22}]}{2}
\]

\[
\gamma_{ESM} = \frac{\omega^* [\Omega_{12}] \omega}{\omega^* [T] \omega}
\]
Please note, the mathematical expression of the power optimisation here presented, was already derived by Novak. It is refereed as Polarimetric Match Filter (PMF).

The PMF is used to find the scattering mechanisms which has the best contrast with the background clutter (in the same polarimetric acquisition).

Same solutions of ESM test

Proof that the eigenvectors are the same

Power ratio: \[ A' = \begin{bmatrix} T_{22} \end{bmatrix}^{-1} T_{11} \]

Error factor: \[ A = \frac{1}{4} \left( \begin{bmatrix} T_{22} \end{bmatrix}^{-1} T_{11} + 2[I] + \begin{bmatrix} T_{11} \end{bmatrix}^{-1} T_{22} \right) = \frac{1}{4} \left( A' + 2[I] + \begin{bmatrix} A' \end{bmatrix}^{-1} \right) \]

\[ \Sigma = \frac{1}{4} [U] \left( A' + 2[I] + \begin{bmatrix} A' \end{bmatrix}^{-1} \right) [U]^T \]

\[ \Sigma' = [O] A' [O]^T \]

\[ \Sigma = \frac{1}{4} \left( [U] A' [U]^T + 2[U] [I] [U]^T + [U] \begin{bmatrix} A' \end{bmatrix}^{-1} [U]^T \right) = \frac{1}{4} \left( [U] A' [U]^T + 2[I] + [U] \begin{bmatrix} A' \end{bmatrix}^{-1} [U]^T \right) \]

If \([U]\) diagonalise \([A']\) then it will diagonalise \([A]\) as well. But because the eigenvector basis is unique, we must have \([O]=[U]\).

i.e. they have the same eigenvectors.
Statistical Test
pdf: Intensity Ratio

The pdf of the **Power/Intensity Ratio** was already derived by Lee et al. It assumes complex Gaussian distributions for the SLC pixel (i.e. the texture is not taken into account).

\[
 f^{(n)}_R (r) = \frac{\tau^n \Gamma(2n) \left(1 - |\rho_c|^2\right)^n (\tau + r) r^{n-1}}{\Gamma(n) \Gamma(n) \left[(\tau + r)^2 - 4\tau |\rho_c|^2 r^2\right]^{(2n+1)/2}} \]

\[
 = \frac{1}{r \beta(n,n)} \left[\frac{\tau \left(1 - |\rho_c|^2\right) r}{\left((\tau + r)^2 - 4\tau |\rho_c|^2 r^2\right)^{1/2}}\right]^n \left[(\tau + r)^2 - 4\tau |\rho_c|^2 r^2\right]^{-1/2}
\]

**Formulation used to implement in Matlab**

pdf: Intensity Ratio

- Rho = 0
  - N = 1
  - N = 21
  - N = 41
  - N = 61
  - N = 81

- Rho = 0.5
  - N = 1
  - N = 21
  - N = 41
  - N = 61
  - N = 81

- Rho = 0.25
  - N = 1
  - N = 21
  - N = 41
  - N = 61
  - N = 81

- Rho = 0.75
  - N = 1
  - N = 21
  - N = 41
  - N = 61
  - N = 81

- Rho = 0.999
  - N = 1
  - N = 21
  - N = 41
  - N = 61
  - N = 81
Setting the threshold

A Constant False Alarm Rate (CFAR) and Constant Probability of Detection are applied to set the threshold.

\[
Opt_{\omega \in \mathbb{C}^3} \begin{bmatrix} \omega^* T_{11} \omega \\ \omega^* T_{22} \omega \end{bmatrix} \Rightarrow \Lambda = \max [\lambda_{\text{max}}, 1/\lambda_{\text{min}}] \quad \begin{cases} H_0 : \Lambda < T \\ H_1 : \Lambda \geq T \end{cases}
\]

Two possible strategies can be used:

Anomaly detector:
The Probability of False Alarm is fixed

\[
P_f = 1 - \int_0^T f_R^{(n)}(r) dr = 10^{-6}
\]

“Above the threshold”:
The Probability of Missing Detection is fixed

\[
P_m = \int_0^T f_R^{(n)}(r) dr = 10^{-6}
\]
Anomaly detector vs Above the Threshold

Target

Guard

Clutter:

Clutter

pdf Intensity Ratio

$P_f$

$P_m$
SARTOM 2006

DLR: E-SAR

Quad-pol
L-band
RGB Pauli images

Master

Slave: 4 days after
Plotting the pdf

Ground 1 | Ground 2
---|---
Forest 3 | Forest 4

pdf Intensity Ratio

80x80 pixels

Ratio

Data
Plotting the CDF

Ground 1

Ground 2

Forest 3

Forest 4
Kolmogorov-Smirnov test

They all passed the test, with a defined number of samples

Considering tails (as in the previous pictures)

<table>
<thead>
<tr>
<th></th>
<th>Ground1</th>
<th>Ground2</th>
<th>Forest3</th>
<th>Forest4</th>
</tr>
</thead>
<tbody>
<tr>
<td>n</td>
<td>50</td>
<td>15</td>
<td>5</td>
<td>18</td>
</tr>
</tbody>
</table>

Removing the tails of the distribution

<table>
<thead>
<tr>
<th></th>
<th>Ground1</th>
<th>Ground2</th>
<th>Forest3</th>
<th>Forest4</th>
</tr>
</thead>
<tbody>
<tr>
<td>n</td>
<td>70</td>
<td>43</td>
<td>25</td>
<td>48</td>
</tr>
</tbody>
</table>

\[ n = \frac{N_1 N_2}{N_1 + N_2} \quad N_1 = N_2 \]

\( N_1 \) : number of samples for theoretical distribution

\( N_2 \) : number of samples for real (data) distribution
Anomaly detector
RGB Pauli images: SARTOM2006
Results of optimisation

Here, we concentrate on this.

Maximum Ratio
Changes from 1 -> 2

Minimum Ratio
Changes from 2 -> 1
Anomalies detector

\[ P_f = 10^{-6} \]
Anomalies detector

\[ P_f = 10^{-8} \]
“Above the threshold” detector
“Above the threshold” detector:

**Above 5**

Detection Mask

\[ P_m = 10^{-6} \]

**Above 3**

Detection Mask

\[ P_m = 10^{-6} \]
“Above the threshold” detector:

Above 2

Detection Mask

\[ P_m = 10^{-6} \]
Conclusions and Future Work
Conclusions and Acknowledgments

- Two statistical tests for the power change detector were devised:
  - “Anomalies” detection
  - “Above the threshold” detection:

- Future work:
  - To make some simulation to understand if the eigenvectors can be used as well.

- Special thank to all the DLR’s SARTOM2006 team: e.g. Ralf Horn, Matteo Nannini and Rolf Scheiber.
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