Assimilation of Distributed Targets and PS Information for the Monitoring of Polarimetric SAR Systems

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Outline

- Introduction
  - Traditional polarimetric calibration

- PS-based technique
  - Model
  - Algorithm
  - Integration with external data
    - Calibrated data
    - Distributed targets

- Results on RS2 dataset

- Conclusions
### SAR calibration aims to:

- Remove the radiometric and polarimetric distortion from the target signatures
- SAR instrument health status monitoring
  - T/R Modules, Antenna pattern, Power losses

### System distortion (without Faraday)

\[
\begin{bmatrix}
M_{HH} & M_{VH} \\
M_{HV} & M_{VV}
\end{bmatrix} = A e^{j\phi} \cdot \begin{bmatrix}
1 & \delta_2 \\
\delta_1 & f_1
\end{bmatrix} \cdot \begin{bmatrix}
S_{HH} & S_{VH} \\
S_{HV} & S_{VV}
\end{bmatrix} \cdot \begin{bmatrix}
1 & \delta_3 \\
\delta_4 & f_2
\end{bmatrix}
\]

- **6 COMPLEX PARAMETERS**
  - IMBALANCES: \(f_1, f_2\)
  - CROSS-TALKS (CTs): \(\delta_1, \delta_2, \delta_3, \delta_4\)

**Typical Requirements:**
- **0.2 dB** for \(f_1, f_2\)
- **< -30 dB** for \(\delta_1, \delta_2, \delta_3, \delta_4\)
Introduction
Polarimetric Calibration

Current polarimetric calibration approaches exploit:

A. Network of calibrated active/passive reflectors
   • PARC, corners (3 or more)

B. A Distributed Target (DT)
Introduction
Polarimetric Calibration

ISSUES

- A calibration site is expensive to be deployed and then maintained for the whole mission lifetime. Moreover it demands for dedicated acquisitions that can interfere with the mission operations.
- Approaches based solely on DTs (no point calibrators) can provide only partial information.

$$D_{h} = f_{1}, \frac{\delta_{1}}{f_{1}}, \frac{\delta_{2}}{f_{1}}, \frac{\delta_{3}}{f_{2}}, \frac{\delta_{4}}{f_{2}}$$

- Without a calibrated point target (PT) amplitude and phase imbalances are missing.

$$\langle S_{hh}S_{hv}^{*} \rangle = 0$$
Introduction
Proposed PScal approach

IDEA

- Exploit multiple-image information
- Use the stable targets (Permanent Scatterers) on the scene as if they were calibrated point targets (PT)

E.g.: 1000 PSs with 10 dB SCR are equivalent to 1 PT with 40 dB SCR
PS-Based Technique

**PS model**

\[
H_1(r,x) \quad H_2(r,x) \quad \ldots \quad H_{N_i}(r,x)
\]

Distortion

\[
M_1(n) \quad M_2(n) \quad \ldots \quad M_{N_i}(n)
\]

Stack \( n^{th} \) Imagette

Detected PS

\[
H_i(r,x) \quad H_i^{(n)}
\]

Time (i)
PS-Based Technique

PS model

Distortion

\[ H_1(r,x), H_2(r,x), \ldots, H_{N_i}(r,x) \]

Stack \( n^{th} \) Imagette

\[ M_1^{(n)}, M_2^{(n)}, \ldots, M_{N_i}^{(n)} \]

\[ i^{th} \text{ image} \]

\[ H_i(r,x) \]

\[ H_i^{(n)} \]

\[ p^{th} \text{ target} \]

\[
\begin{bmatrix}
S_{hh}(i,p) \\
S_{hv}(i,p) \\
S_{vv}(i,p)
\end{bmatrix}
\rightarrow
\begin{bmatrix}
S_{hh}(p) \\
S_{hv}(p) \\
S_{vv}(p)
\end{bmatrix}
+ \begin{bmatrix}
w_{hh}(i,p) \\
w_{hv}(i,p) \\
w_{vv}(i,p)
\end{bmatrix}
\]

Stable reflectivity

Clutter
PS-Based Technique

**PS Model**

\[
y_{i,p} = \begin{bmatrix} y_{hh}(i,p) \\ y_{hv}(i,p) \\ y_{vh}(i,p) \\ y_{vv}(i,p) \end{bmatrix} \approx e^{j\varphi(i,p)} \cdot H_i \cdot \begin{bmatrix} S_{hh}(p) \\ S_{hv}(p) \\ S_{vh}(p) \\ S_{vv}(p) \end{bmatrix} + \begin{bmatrix} w_{hh}(i,p) \\ w_{hv}(i,p) \\ w_{vh}(i,p) \\ w_{vv}(i,p) \end{bmatrix} + \begin{bmatrix} n_{hh}(i,p) \\ n_{hv}(i,p) \\ n_{vh}(i,p) \\ n_{vv}(i,p) \end{bmatrix}
\]

\(s_p + w_{i,p}\)

**Clutter model**

- Geometrical and temporal decorrelation not accounted
  \[
  E[w_{i_1,p}w_{i_2,p}^*] = 0 \quad \text{with} \quad i_1 \neq i_2
  \]

- ccG behaviour: \(w_{i,p} \sim \text{CN}(0, C_p)\)

**Intrinsic Ambiguity**

\[
H_i \leftrightarrow s_p, C_p
\]

\[
y_{i,p} = e^{j\varphi(i,p)}H_i \cdot (s_p + w_{i,p}) + n_{i,p} = e^{j\varphi(i,p)}H_i \cdot K \cdot K^{-1}(s_p + w_{i,p}) + n_{i,p}
\]

3 by 3 matrix \(K\)
PS-Based Technique

PS Model

\[
y_{i,p} = \begin{bmatrix} y_{hh}(i, p) \\ y_{hv}(i, p) \\ y_{vh}(i, p) \\ y_{vv}(i, p) \end{bmatrix} \approx e^{j\varphi(i, p)} \cdot H_i \cdot \begin{bmatrix} S_{hh}(p) \\ S_{hv}(p) \\ S_{vh}(p) \\ S_{vv}(p) \end{bmatrix} + \begin{bmatrix} w_{hh}(i, p) \\ w_{hv}(i, p) \\ w_{vh}(i, p) \\ w_{vv}(i, p) \end{bmatrix} + \begin{bmatrix} n_{hh}(i, p) \\ n_{hv}(i, p) \\ n_{vh}(i, p) \\ n_{vv}(i, p) \end{bmatrix}
\]

- Clutter model
  - Geometrical and temporal decorrelation not accounted
    \[E[w(i_1, p)w(i_2, p)^*] = 0 \text{ with } i_1 \neq i_2\]
  - ccG behaviour: \(w_{i,p} \sim CN(0, C_p)\) with \(C_p = \begin{bmatrix} v_{hh}(p) & \kappa_{hhv}(p)^* & \kappa_{hvv}(p)^* \\ \kappa_{hhv}(p) & v_{hv}(p) & \kappa_{hvv}(p)^* \\ \kappa_{hvv}(p) & \kappa_{hvv}(p) & v_{vv}(p) \end{bmatrix}\) Covariances dependent on PS and stationary along the stack

- Intrinsic Ambiguity \(H_i \leftrightarrow s_p, C_p\)

\[
y_{i,p} = e^{j\varphi(i, p)}H_i \cdot (s_p + w_{i,p}) + n_{i,p} = e^{j\varphi(i, p)}H_i \cdot K^{-1} \begin{bmatrix} s_p + w_{i,p} \end{bmatrix} + n_{i,p}
\]

3 by 3 matrix \(K\)
PS-Based Technique Scheme

**INITIALIZATION**
- Coarse estimation of the PDMs
- Coarse est. of the target parameters

**PS IDENTIFICATION**
- GLRT-Based detection
  - PS database

**MODEL ESTIMATION**
- SVD-based estim of the PDM
- Estimation of the target parameters
  - Max iterations ?
    - NO
    - YES

**CALIBRATION**
- Estimation of the ambiguity matrix K
- Stack Calibration

**LEGEND**
- ... Processing of all the targets
- ... Processing on selected PSs

**Outputs**
- Calibrated SLC stack
- PS and model parameters
- DT information
- Calibrated image(s)
PS-Based Technique
Resolving the ambiguity

- The PDM estimates before calibration are affected by the $K$ ambiguity

Polarimetric Distortion Matrix

$$\hat{H}_i = H_i \cdot K$$

Backscatter vector

$$\hat{s}_p = K^{-1} \cdot s_p$$

- $K$ can be estimated:

**Completely** by $\geq 1$ calibrated image of the stack

$$H_{i_0} = H_{\text{ref}}$$

**Partially** by integration with the DT information

$K$ ambiguity is same for all the stack
PS-Based Technique

Estimation of ambiguity by means of DT

- K can be resolved up to a gain $A_0$ and an imbalance complex factor $f_0$ common to all the stack estimates.

Data calibration is still unfeasible
- Channels are decoupled but still hampered by imbalance

$$
S_{HH}^{cal} \approx A_0 \cdot S_{HH}
$$

$$
S_{HV}^{cal} \approx A_0 f_0 \cdot S_{HV}
$$

$$
S_{VH}^{cal} \approx A_0 f_0 \cdot S_{VH}
$$

$$
S_{VV}^{cal} \approx A_0 f_0^2 \cdot S_{VV}
$$

A complete temporal monitoring of the distortion is achievable
- PS stability is able to interlink phase and amplitudes of the distortion alongside the stack

$$
\tilde{H} \xrightarrow{\text{A}} \frac{A}{A_0}, \frac{f_1}{f_0}, \frac{f_2}{f_0}, \delta_1, \frac{\delta_2}{f_0}, \delta_3, \frac{\delta_4}{f_0}
$$
Application to RS2
Case study

RADARSAT-2 Dataset

<table>
<thead>
<tr>
<th>Mode</th>
<th>Fine Quad-Pol</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam</td>
<td>FQ9</td>
</tr>
<tr>
<td>Angle</td>
<td>28°-29.8°</td>
</tr>
<tr>
<td>Gr. Res.</td>
<td>10.5 – 11.1 m</td>
</tr>
<tr>
<td>Width</td>
<td>25 x 25 km</td>
</tr>
</tbody>
</table>

10 images | 2008-Apr-12 to 2008-Dec-08
16 images | 2010-Feb-13 to 2011-Mar-28
Application to RS2
DT estimates

- Constrained to areas which feature orientation symmetry.
- Supervised selection of the most fitted areas

Quegan technique is adopted

Average values throughout the dataset

<table>
<thead>
<tr>
<th></th>
<th>$\delta_1$ [dB]</th>
<th>$\delta_2/f_1$ [dB]</th>
<th>$\delta_3$ [dB]</th>
<th>$\delta_4/f_2$ [dB]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mount 1</td>
<td>-46.8</td>
<td>-49.3</td>
<td>-51.5</td>
<td>-52.1</td>
</tr>
<tr>
<td>Mount 2</td>
<td>-43.9</td>
<td>-43.2</td>
<td>-47.7</td>
<td>-46</td>
</tr>
<tr>
<td>Fields</td>
<td>-44.6</td>
<td>-45.4</td>
<td>-43</td>
<td>-42.9</td>
</tr>
</tbody>
</table>
Application to RS2
PolIPS detected

100000 PolIPSs
Good quality

Details
PS-Based Technique
Estimation of ambiguity by means of DT

- Example of the calibration effects (on a single imagette)

\[
H = \begin{bmatrix}
H_{11} & H_{12} & H_{13} \\
H_{21} & H_{22} & H_{23} \\
H_{31} & H_{32} & H_{33} \\
H_{41} & H_{42} & H_{43}
\end{bmatrix}
\]

IDEAL PDM

\[
H = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

~ From -22 dB to -30/-40 dB
Application to RS2
Amplitude stability on imagettes

- 500 PS – Low Quality (~ SCR > 8 dB)
- 5000 PS – High Quality (~ SCR > 12 dB)
- Quegan technique

**Image gain A**

**Imbalance f₁**

**Imbalance ratio f₁/f₂**
Application to RS2
Amplitude stability on imagettes

Amplitude estimates

Number of PSs

Stability ($3\sigma$) [dB]

- A - Low quality
- A - High quality
- f - Low quality
- f - High quality
- $f_1/f_2$ - Low quality
- $f_1/f_2$ - High quality

Application to RS2
Amplitude stability on imagettes
Application to RS2
Phase stability on imagettes

Imbalance Ratio

Imbalance f₁

Imbalance f₂

500 PS – Low Quality (~ SCR > 8 dB)
5000 PS – High Quality (~ SCR > 12 dB)
Quegan technique
Application to RS2 CT results

- CROSS-TALKS
  - Hard test for PolPSCal
  - The number and quality of PSs must provide higher stability than the RS2 CT level (< -40 dB) to be effective

![Graph showing cross-talk δ₁ and δ₂ with data points for 2008 and 2010-2011, comparing 500 PS - Low Quality (~ SCR > 8 dB) and 5000 PS - High Quality (~ SCR > 12 dB) with Quegan technique.](image-url)
Conclusions

- An algorithm for a joint radiometric and polarimetric calibration based on the natural stable scatterers in the scene has been proposed.

- Full calibration requires a single calibrated image out of the whole stack.

- System distortion fluctuations (temporal monitoring) can be assessed without a calibrated target just through assimilation with DT
  - Technique stability was tested on a 26-images RS2 dataset registering values of stability below 0.3 and 0.2 dB (3σ) for radiometric gain and imbalances.
  - Cross-talk estimation also managed to provide good results. The - 40 dB level was indeed effectively achieved by high quality imagettes.

- Extension to other systems is indeed feasible. The potential for carrying out Faraday compensation seems for instance one the most promising directions for further research.
Thank you
Summary

1 Image

Partial monitoring of the distortion

1 Image

Full monitoring and calibration

Stack

1 Image

Full monitoring and calibration

Stack

1 Image

Full temporal monitoring of the distortion (only partial data calibration)
PS-Based Technique
PS detection

- Best 10000 PS are compared

- Common PS: 52%

Result
- Best quality is reached with the GLRT selection method
**DT information**

\[
\alpha = \frac{r_{22}}{r_{11}} t_{11}, \quad u = \frac{r_{21}}{r_{11}} t_{21}, \quad v = \frac{r_{12}}{r_{22}} t_{22}, \quad w = \frac{z_{12}}{r_{11}} t_{11}
\]

**PS information**

\[
\left( T_{Tot}^T \otimes R_{Tot}^T \right) \cdot P \cdot B \quad \text{with} \quad P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{B = ambig. matrix}
\]

---

**Scenarios**

<table>
<thead>
<tr>
<th>Null FRA</th>
<th>DT Monitoring</th>
<th>Issues</th>
<th>DT+PS monitoring</th>
<th>Issues</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Full model:</strong> (A, f_1, f_2, \delta_1, \delta_2, \delta_3, \delta_4)</td>
<td>The parameter set (f_1/f_2, \delta_1, \delta_2/\delta_4) is unambiguously retrieved.</td>
<td>An unknown radiometric and phase factor between between the different polarizations must be accounted. It is related to (f_{1,2}), which can be non-stationary along time.</td>
<td>The (f_{1,2}) uncertainty is resolved up to a complex factor common for the whole stack. Full temporal system monitoring and Intra-stack calibration are now possible.</td>
<td>Inter-stack data processing still requires the common scale factor information that can only be provided by external calibrators.</td>
</tr>
</tbody>
</table>

| Non-null FRA | All the parameters can be retrieved with accuracy depending on the FRA and on the imbalance ratio phase (see section ref:sec). | When Faraday is null or close to 0 or the imbalance ratio phase is \(\approx \pm \pi/2\) the quality of the estimates is poor. | All the stack distortion can be retrieved when at least one DT estimate has good quality. | When good quality DT estimates cannot be found the user should refer to the previous scenario. |
| CTs reciprocal: \(A, f_1, f_2, \delta_1 = \delta_3, \delta_2 = \delta_4, \Omega\) | | | |

| Non-null FRA | The distortion model is intrinsically ambiguous. The DT-based parameter set \(\{u, v, w, z\}\) can be nonetheless computed. The parameters become functions of \(\Omega\) as well. | Nor system monitoring nor accurate Faraday estimation is possible. | Improvements are eventually possible only if additional model constraints are accounted, such as slow distortion fluctuations along the time-series (hint for future research). | To be investigated. |
| Full model: \(A, f_1, f_2, \delta_1, \delta_2, \delta_3, \delta_4, \Omega\) | | | | |
PS-Based Technique
Estimation of ambiguity by means of DT

- $K$ can be resolved up to an imbalance factor

$$K_{res} = A_0 \begin{bmatrix} 1 \\ f_0 \\ f_0^2 \end{bmatrix}$$

$$K' = K \cdot K_{res}$$

is also a valid solution for $K$

- So that the estimated PDMs are:

$$\tilde{H}_i = \tilde{H}_i \cdot K^{-1} = \frac{A}{A_0} \cdot \begin{bmatrix} 1 & \delta_2 + \delta_4 & \delta_2\delta_4 \\ \delta_1 & f_1 + \delta_1\delta_4 & \delta_4f_1 \\ \delta_3 & f_2 + \delta_2\delta_3 & \delta_2f_2 \\ \delta_1\delta_3 & (\delta_1 + \delta_3)f_2 & f_1f_2 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ f_0^{-1} \\ f_0^{-2} \end{bmatrix}$$

A complete temporal monitoring of the distortion is achievable

$$\tilde{H} = \frac{A}{A_0}, \frac{f_1}{f_0}, \frac{f_2}{f_0}, \delta_1, \frac{\delta_2}{f_0}, \delta_3, \frac{\delta_4}{f_0}$$

- PS stability is able to interlink phase and amplitudes of the distortion alongside the stack

Data calibration is still unfeasible

- Channels are decoupled but still hampered by imbalance

$$M_{HH}^{cal} \cong A_0 \cdot M_{HH}$$

$$M_{HV}^{cal} \cong A_0f_0 \cdot M_{HV}$$

$$M_{VH}^{cal} \cong A_0f_0 \cdot M_{VH}$$

$$M_{VV}^{cal} \cong A_0f_0^2 \cdot M_{VV}$$
PolPSCal Technique
PS detection

- Theoretical performance
  - Best 10000 PS are compared
  - 51% PS common
  - Qualitative comparison shows better performance for GRLT (few PSs are found in vegetated area)

- Application to real dataset

\[ N_I = 26 \]

- Probabilities of false alarm (Pfa) and missing detection (Pmd)
  - Based on Monte Carlo simulations:
    - 1-Pol Dispersion: \( \mu / \sigma \)
    - SPAN Dispersion: \( \mu_{\sqrt{SPAN}} / \sigma_{\sqrt{SPAN}} \)
    - Likelihood Ratio: \( \log(1 + s^{\mu \sigma^{-1}s}) \)
PS-Based Technique
Calibration by means of a known PDM

- Simulated performance using an image calibrated without error
- Measured through Maximum Normalized Error (Wang-Ainsworth-Lee\(^*\))
  - Distortion experienced by the most unfavourable pol

\[ MNE = \max_x \left| \frac{H^{cal}}{x} \right| \]

\[ N_t = 10 \text{ images} \]

\[ \text{Calibration Quality} \]

- \( N_p \) vs. Mean Squared MNE [dB]
  - SCR = 5 dB
  - SCR = 10 dB
  - SCR = 15 dB
  - SCR = 20 dB

\[^{*}\] Yanting Wang; Ainsworth, T.L.; Jong-Sen Lee; , "Assessment of System Polarization Quality for Polarimetric SAR Imagery and Target Decomposition," TGRS, May 2011
Results
PolPSCal

- **CROSS-TALKS**
  - Hardest test for PolPSCal -> The number and quality of PSs must provide higher precision than the RS2 CT level (< -40 dB) to be effective
  - Furthermore, the normalization step must prove to be effective enough in the removal of the ambiguity that impacts notably on the out-of-diagonal PDM elements.
Without calibrator

- Exploit information provided by traditional techniques

\[ H_{i_0} = A_{i_0} \cdot H_{i_0}^{DT} = H_{i_0}^{PS} \cdot K \]

- We then arbitrarily set:
  \[ \hat{A}(i_0) = 1 \]
  \[ \hat{f}_2(i_0) = 1 \]

\[ \hat{H}_i = \frac{A}{A_0} \cdot \begin{bmatrix} 1 & \delta_2 + \delta_4 & \delta_2 \delta_4 \\ \delta_1 & f_1 + \delta_1 \delta_4 & \delta_4 f_1 \\ \delta_3 & f_2 + \delta_2 \delta_3 & \delta_2 f_2 \\ \delta_1 \delta_3 & (\delta_1 + \delta_3) f_2 & f_1 f_2 \end{bmatrix} \begin{bmatrix} 1 \\ f_0^{-1} \\ f_0^{-2} \end{bmatrix} \]

- We can retrieve from each PDM the distortion parameters:

Problem is underdetermined. A residual ambiguity \( K_{res} \) cannot be resolved

\[ K_{res} = A_{res} \begin{bmatrix} 1 \\ f_{res} \\ f_{res}^2 \end{bmatrix} \]

It is readily verified that

\[ F_{i_0} = F_{i_0} \cdot K_{res} \]
\[ K' = K \cdot K_{res} \]

are valid solutions of the problem

Apart from a constant scale factor, we are now able to monitor the amplitude and phase trends of the imbalances.
PolPSCal Technique
Theoretical Performance

- SVD provides LS optimization for $H$ and $S$
  \[ z_{i,p} = e^{-j\phi(i,p)}y_{i,p} \]
  \[ \hat{s}_p, \hat{H}_i = \arg\min_{s_p, H_i} \left( \sum_i \sum_p \left\| z_{i,p} - H_i s_p \right\| \right) \]

- Approximate theoretical performance can be derived
  - Unbiased for $SCR \to \infty$
  - Covariance:
    \[
    C_H(n) = E \left[ \text{vec}\left( \hat{H}_n - H_n \right) \text{vec}\left( \hat{H}_n - H_n \right)^H \right] \\
    = \kappa \left[ \sum_p \sum_i \left( s_p^* \otimes H_i^H \right) \Gamma_{i,p} \left( s_p^* \otimes H_i^H \right)^H \right] + \\
    \sum_p \left( S_{\text{inv}}^T s_p^* \otimes I_{[4\times4]} \right) \Gamma_{n,p} \left( S_{\text{inv}}^T s_p^* \otimes I_{[4\times4]} \right)^H + \\
    2 \text{Re} \left\{ \kappa \sum_p \left( s_p^* \otimes H_n^H \right) \Gamma_{n,p} \left( S_{\text{inv}}^T s_p^* \otimes I_{[4\times4]} \right)^H \right\}
    \]

The same that would be recursively attained by the estimators

\[
\hat{H}_i = \sum_p z_{i,p} \hat{s}_p \left( \sum_p \hat{s}_p \hat{s}_p^H \right)^{-1} \\
\hat{s}_p = \left( \sum_i \hat{H}_i^H \hat{H}_i \right)^{-1} \sum_i \hat{H}_i^H z_{i,p}
\]

**LEGEND**

\[ \kappa = S_{\text{inv}}^T \otimes H_n H_{\text{inv}} \]
\[ S_{\text{inv}} = \left( \sum_p s_p s_p^H \right)^{-1} \]
\[ H_{\text{inv}} = \left( \sum_i H_i^H H_i \right)^{-1} \]
PolPSCal Technique
Theoretical Performance

- SVD provides LS optimization for $H$ and $S$

$$z_{i,p} = e^{-j\phi(i,p)}y_{i,p}$$

$$\hat{s}_p, \hat{H}_i = \text{argmin}_{s_p,H_i} \left( \sum_i \sum_p \| z_{i,p} - H_i s_p \| \right)$$

- Approximate theoretical performance can be derived
  
  - Unbiased for SCR $\rightarrow \infty$
  
  - Covariance: $C_{..}(n) = E\left[\text{vec}(\hat{H}_- - H_+), \text{vec}(\hat{H}_- - H_+)^H\right]$  

- $N_I$ is not influential for estimation (it is only important for detection)

- PS density and quality determine:

  $$\Delta_{[\text{dB}]}MSE \approx \Delta_{[\text{dB}]}\text{SCR}$$
  $$\Delta_{[\text{dB}]}MSE \approx -\Delta_{[\text{dB}]}N_p$$

$$\kappa = S_{\text{inv}}^T \otimes H_n H_{\text{inv}}$$

$$S_{\text{inv}} = \left( \sum_p \hat{s}_p \hat{s}_p^H \right)^{-1}$$

$$H_{\text{inv}} = \left( \sum_i \hat{H}_i^H \hat{H}_i \right)^{-1}$$

Not straightforward

$$\text{Tr}\{.\} \approx \text{Total MSE}$$

Simplification:

$$H = I, \quad \Gamma_{i,p} = \Gamma_W$$

$$\text{Tr}\{C_H\} = \left( \frac{1}{N_I} + 2 \right) \cdot \text{Tr}\{S_{\text{inv}}\}(\text{Tr}\{\Gamma_W\} - \sigma_n) + \text{Tr}\{S_{\text{inv}}\}\text{Tr}\{\Gamma_W\}$$

Impact of the number of images $N_I$ and of the clutter to thermal noise ratio

Dependence on the number of targets $N_p$ and on the SCR
Estimation Step

Theoric performance

<table>
<thead>
<tr>
<th>PS</th>
<th>Clutter (temporally uncorrelated)</th>
<th>Thermal et al.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{hh}$ [dB]</td>
<td>$P_{hv}$ [dB]</td>
<td>$P_{vv}$ [dB]</td>
</tr>
<tr>
<td>0</td>
<td>-7</td>
<td>1.5</td>
</tr>
<tr>
<td>coher. $c_{HH,VV}$</td>
<td>$c_{HV,HH} = c_{HV,VV}$</td>
<td>NESZ [dB]</td>
</tr>
<tr>
<td>0.2·exp(j·20°)</td>
<td>0</td>
<td>-15</td>
</tr>
</tbody>
</table>

Faraday Rotation $\Omega$

Image Gain $A$

Imbalances Amplitude $|f|$

Imbalances Phase $\angle f$
Estimation Step
Theoric performance – C-band (no Faraday)

<table>
<thead>
<tr>
<th>Images</th>
<th>$P_{hh}$ [dB]</th>
<th>$P_{hv}$ [dB]</th>
<th>$P_{vh}$ [dB]</th>
<th>SCR [dB]</th>
<th>NESZ [dB]</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>0</td>
<td>-12</td>
<td>0</td>
<td>from 5 to 20</td>
<td>-25</td>
</tr>
</tbody>
</table>

Image Gain $A$

Imbalances Amplitude $|f|$

Imbalances Phase $\angle f$

Imbalances Ratio Amplitude $|f_1/f_2|$

Imbalances Ratio Phase $\angle f_1/f_2$