Towards a better understanding of the Earth’s interior and geophysical exploration research

AAS, AUT, DIAS, GIS, TUD & UWB Consortium

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GOCE Solid Earth

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Preliminary and final GGs

spaceborne GRACE/GOCE GGs for geophysics:

- preliminary combined GOCE/GRACE GGs in the initial stage
- final combined GGs from GOCE+ GeoExplore (after 8 mths)

final GGs – validation strategy:

- according to the conceptual line of action of DGFI
- no software synchronization
- GOCO03S GGs used as a benchmark in validation
- validation through independent ground information?
Differences of combined GGs in LNOF

- PSD of combined GG – GOCO03S GG differences:
Validation of gravitational gradients by ground data

- Green’s integral formula for gradient evaluation

\[
\partial_i \partial_j T(x) = \frac{1}{|S|} \int \int_S \delta g(y) \partial_i \partial_j K(x,y) \, dS(y)
\]

- Spectral form of the integral kernel for \(|x| > |y| = R\)

\[
\partial_i \partial_j K(x,y) = \sum \frac{2n + 1}{n + 1} \partial_i \partial_j \left[ \left( \frac{|y|}{|x|} \right)^{n+1} P_n \left( \frac{x \cdot y}{|x||y|} \right) \right]
\]

- Similar form for gravity anomalies (but the eigenvalue differs)

- Radial and polar distance derivatives of the kernel required

- Global integration (no modification, no truncation errors)
Test 1 – gradients from noiseless ground gravity

- Input gravity and diagonal components \( V_{xx} - V_{yy} - V_{zz} \)
comparison of gradients at the orbital elevation of 250 km:

<table>
<thead>
<tr>
<th>component</th>
<th>min</th>
<th>max</th>
<th>mean</th>
<th>std</th>
<th>EGM08</th>
</tr>
</thead>
<tbody>
<tr>
<td>xx</td>
<td>-0.291</td>
<td>0.272</td>
<td>0.001</td>
<td>0.051</td>
<td>0.013</td>
</tr>
<tr>
<td>xy</td>
<td>-0.121</td>
<td>0.125</td>
<td>0.000</td>
<td>0.030</td>
<td>0.004</td>
</tr>
<tr>
<td>xz</td>
<td>-0.329</td>
<td>0.363</td>
<td>0.001</td>
<td>0.063</td>
<td>0.022</td>
</tr>
<tr>
<td>yy</td>
<td>-0.398</td>
<td>0.271</td>
<td>0.000</td>
<td>0.061</td>
<td>0.011</td>
</tr>
<tr>
<td>yz</td>
<td>-0.310</td>
<td>0.357</td>
<td>0.000</td>
<td>0.069</td>
<td>0.006</td>
</tr>
<tr>
<td>zz</td>
<td>-0.540</td>
<td>0.708</td>
<td>-0.001</td>
<td>0.094</td>
<td>0.020</td>
</tr>
</tbody>
</table>

global integration over 15 arc-min ground/EGM gravity
Step 1: From orbit to a mean sphere

- simplification of further GGs processing and applications
- GOCE orbits $\vec{r}$ is nearly circular ($|\Delta \vec{r}| \leq 16$ km)
- this step preserves GGs as a time series
- gradient approach (by a 9 point stencil):

![Diagram showing real orbit, mean orbit, and geocentric radius](image)
Gradient method – height vs. accuracy

- estimated from synthetic GGs derived from a GOCE-only model
- estimated RMS of the differences – $\sim 10^{-6}$ E

$\Rightarrow$ this continuation preserves accuracy (e.g., the Laplacian)
Step 2: Continuation from mean sphere to ground

- for the time being – gridded data needed (time series is lost)
- regular grid is more suitable for surface quadratures
- two strategies adopted (spherical approximation):
  1. iterative approach (Xu 2007) based on the Poisson kernel
  2. direct integration with a reciprocal Poisson kernel (Novák 2002)

- **iterative approach** based on Xu et al. (2007)
- upward continuation applies iteratively
- this method can also be applied locally
Iterative approach & noise-free example

Tzz: signal at GOCE altitude, (1)

Tzz: model signal, EGM2008(180), (2)

Tzz: continued signal for h= 250 km, (3)

Convergence of the iteration, ps=1.8, (4)

dTzz=(3)−(2), RMS=0.002 E

dTzz: EGM2008–TImr1, RMS=0.605 E
Iterative approach & noiseless GGs – Laplacian

Laplacian from EGM2008, [E]

Laplacian from the DC data (EGM2008), [E]
Real data example – no filtering

Txx: signal at GOCE altitude, (1)
Txx: model signal, EGM2008(180), (2)
Txx: continued signal for h= 250 km, (3)

Convergence of the iteration, ps=2.9, (4)
dTxx=(3)–(2), RMS=0.976 E
dTxx: EGM2008–TIMr1, RMS=0.282 E
Real data example – anisotropic filtering

Txx: signal at GOCE altitude, (1)

Txx: model signal, EGM2008(180), (2)

Txx: continued signal for h= 250 km, (3)

Convergence of the iteration, ps=2.9, (4)

dTxx=(3)–(2), RMS=0.846 E

dTxx: EGM2008–TIMr1, RMS=0.282 E
Conclusions on iterative downward continuation

Step 1: From real orbit to mean sphere
- GOCE orbit: $\delta r \in \langle -16, 16 \rangle$ km
- gradient method can be applied in any frame (GRF, LNOF)
- gradients can be computed from existing models
- accuracy of continued gradients does not deteriorate

Step 2: From mean sphere to ground level
+ DC of the GGs for $h = 250$ km is possible:
  - basic features recovered
  - for $N = 180$ an agreement at 0.5 – 1 E level
+ algorithm performs at mE level for noise-free data
- gridding and LNOF (time series lost) required
Downward continuation by inverse Poisson kernel

- downward continuation as forward integration

\[
\partial_i \partial_j V(y) = \frac{1}{|S|} \int \int_S \partial_i \partial_j V(x) \mathcal{J}(x, y) \, dS(x)
\]

- spectral form of the integral kernel for \(|x| > |y| = R\)

\[
\mathcal{J}(x, y) = \sum (2n + 1) \left( \frac{|x|}{|y|} \right)^{n+3} P_n \left( \frac{x \cdot y}{|x| |y|} \right)
\]

- only for band-limited data (kernel has no analytical form)

- kernel is diverging but no problems for specs of spaceborne data

- degree-dependent exponential amplification vs. integral smoothing
Test 1 – continuation of noiseless gradients

- degree 50-250 components $V_{zz}$ / 15 arc-min / 250 km
Downward continuation "per partes"

- Downward continuation (mE) over increasing height (km):

<table>
<thead>
<tr>
<th>height</th>
<th>min</th>
<th>max</th>
<th>mean</th>
<th>std</th>
</tr>
</thead>
<tbody>
<tr>
<td>250 km</td>
<td>-5.053</td>
<td>4.642</td>
<td>0.001</td>
<td>1.000</td>
</tr>
<tr>
<td>200 km</td>
<td>-6.044</td>
<td>5.828</td>
<td>0.000</td>
<td>1.051</td>
</tr>
<tr>
<td>150 km</td>
<td>-32.750</td>
<td>30.335</td>
<td>0.001</td>
<td>5.555</td>
</tr>
<tr>
<td>100 km</td>
<td>-186.339</td>
<td>176.249</td>
<td>0.006</td>
<td>32.610</td>
</tr>
<tr>
<td>050 km</td>
<td>-1129.775</td>
<td>1055.271</td>
<td>0.052</td>
<td>203.490</td>
</tr>
<tr>
<td>000 km</td>
<td>-7225.613</td>
<td>6756.083</td>
<td>0.388</td>
<td>1324.988</td>
</tr>
</tbody>
</table>
Forward modelling – volumetric mass layers

- atmosphere
- ice
- topography
- sediments
- water
- upper crust
- lower crust
- Moho
- mantle
Gravitational potential by harmonic series expansion

- spherical harmonic synthesis

\[ V(r, \Omega) = \frac{GM}{R} \sum_{n,m} \left( \frac{R}{r} \right)^{n+1} V_{nm} Y_{nm}(\Omega) \]

- required harmonic coefficients

\[ V_{nm} = \frac{3}{2n+1} \frac{1}{\rho^e} \sum_i \left[ F_{i,nm}^{(i)} - F_{e,nm}^{(i)} \right] \]

- coefficients of a volumetric mass layer

\[ F_{n,m}^{(i)} = \sum_k \binom{n+2}{k} \frac{(-1)^k}{k+i+1} \frac{f_{nm}^{(k+i+1)}}{R^{k+1}} \]

- coefficients through spherical harmonic analysis

\[ f_{nm}^{(k)} = \int \int_\Theta \beta(\Omega) \alpha_i(\Omega) H^k(\Omega) Y_{nm}(\Omega) \, d\Omega \]
Topographic potential (5 arc-min / 9,335,520 nodes)
Sea water potential (5 arc-min / 9,335,520 nodes)
CRUST2.0 thickness (2 arc-deg / 16,380 nodes)
Ice sheets gradients (2 arc-deg / 16,380 nodes)
Soft sediment gradients (2 arc-deg / 16,380 nodes)
Hard sediment gradients (2 arc-deg / 16,380 nodes)
Upper crust gradients (2 arc-deg / 16,380 nodes)
Middle crust gradients (2 arc-deg / 16,380 nodes)
Lower crust gradients (2 arc-deg / 16,380 nodes)
PN combined gradients (1 arc-deg / 64,800 nodes)
KIT vs. PN combined gradients

- **KIT**: three layers evaluated separately through NI (tesseroids)
  - three effects combined at the regular geographic grid
  - combined effect processed by spherical harmonic analysis

- **PN**: three layers evaluated separately by the spectral approach
  - three effects combined at the regular geographic grid

- Result: values seem to match very well (but ice sheets)

- Ice sheet effect still under investigation (but not critical)
Conclusions

- project in the middle of its duration (20 months)
- re-processed GGs will be applied next
- geodesy – several data products (global grids)
- geophysics – improved modelling over two areas:
  - Zdeněk Martinec – Africa (later today)
  - Wouter van der Wal - Reykjanes Ridge (later today)