

On phase unwrapping based on minimum cost flow networks

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ABSTRACT: Phase unwrapping is a key step in the SAR interferometric processing chain as it converts the phase information derived from an interferometric image pair into valuable height information. Many algorithms have been developed to solve the phase unwrapping problem. None of the algorithms implemented so far has met with all the requirements for an optimal solution. Recently, a very promising approach has been introduced by Costantini (1998). The new method formulates the phase unwrapping problem as a global minimisation problem which can be solved by using minimum cost flow (MCF) networks. These MCF networks in general have been well studied and efficient algorithms exist. However, application of the MCF for phase unwrapping is a new approach and requires further research. This paper deals with the investigation of this particular algorithm and focuses on the optimisation of the cost function used to define the MCF network.

Introduction

Three-dimensional movements of the Earth's surface caused by the sea-level rise and land subsidences have serious consequences for countries near sea-level such as the Netherlands. Hence, monitoring and prediction of these motions is of vital importance, e.g., for the coastal management. SAR interferometry serves as an important data source for these kind of studies

since it can provide continuous information about surface changes over large areas.

In the interferometric processing chain, phase unwrapping plays a key role in deriving the valuable height information from SAR interferometric data. Since none of the algorithms implemented so far has met all the requirements for an optimal solution, this complex task is still in the focus of research. This paper deals with the investigation of a recently established approach using minimum cost flow (MCF) networks for solving the phase unwrapping problem and pays special attention to the optimal definition of the cost function used for this approach.

Phase unwrapping using minimum cost flow networks

Over the years, numerous phase unwrapping algorithms have been proposed but none of the implemented methods has been able to provide an optimal solution for the phase ambiguity problem. Recently, a completely different approach from the existing techniques has been proposed by Costantini (1998). Using the fact that phase differences of neighbouring pixels can be estimated with a potential error that is an integer multiple of 2π , he formulated the phase unwrapping problem as a global minimisation problem with integer variables. The underlying global minimisation problem can be efficiently solved by minimum cost flow networks. MCF networks in general have been well

studied and efficient algorithms are available. The RelaxIV algorithm for the minimum cost flow problem was originally developed as a FORTRAN code by Bertsekas and Tseng (1988). The code used for the implementation was entirely rewritten and implemented in C++ by Frangioni and Gentile from the University of Pisa, Italy. For a complete mathematical description of the formulation of phase unwrapping as a global minimisation problem the reader is referred to Costantini (1998).

The design of the minimum cost flow network for phase unwrapping is depicted in Figure 1. The size of the phase image in rows and columns defines the number of nodes which are required in the MCF network. Since the network only works with positive flows two neighbouring nodes are connected via two arcs to enable positive and negative charges. One demand for the optimisation of the MCF network is that the flows within the network are in balance. This requires the definition of a grounding node which is connected to all boundary nodes.

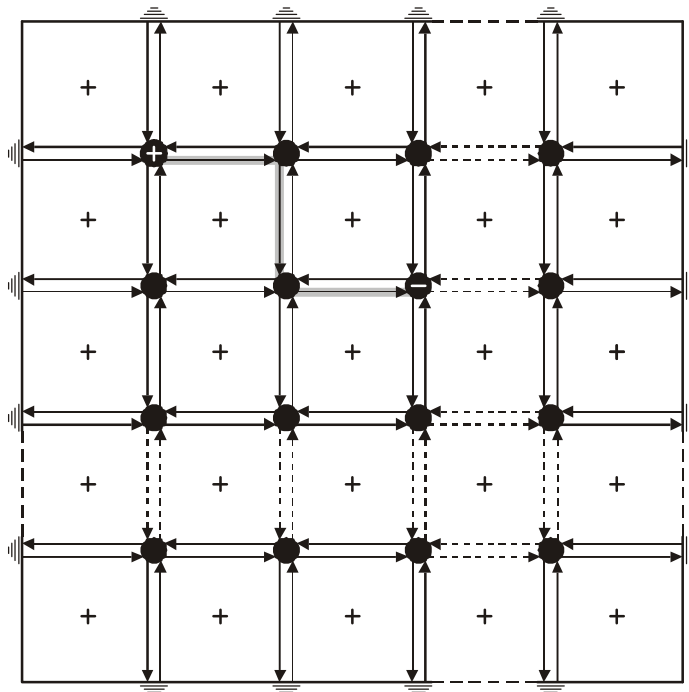


Figure 1. Design of the minimum cost flow network for phase unwrapping. In the centre of four phase values (indicated with crosses) a node (represented by a circle) is defined. Each node is connected its neighbouring node by two arcs. The boundary arcs are connected to the grounding node. The energy is flowing from the positive residue (plus node) to the negative residue (minus node).

For the input of the MCF network the number of nodes and arcs are needed. Furthermore, a capacity and a cost is assigned to each arc, defined by its starting and ending node. The capacity on an arc is the maximum flow that is allowed. A value of 20 is larger than any likely flow and allows to connect several residue pairs along a common path. Reducing the capacity would prevent more than two cuts from lying on top of each other and increase the probability to lie side by side on parallel arcs (Wilkinson, 1999). The

cost serves as a weighting factor for the optimisation of the flows and can be derived from various sources. This is described in the next chapter in further detail.

As output the MCF network determines an optimal objective function value, optimal flows and optimal reduced costs. For the phase unwrapping problem only the optimal flows are relevant as they define the position of the branch cuts which are used during the actual phase unwrapping.

Cost function in the minimum cost flow network

The processing of the phase unwrapping approach is shown in figure 2. In a first step, the residues are located in the interferogram. The costs for each flow can be derived from various sources such as the interferometric coherence, the image amplitudes, the phase gradients and the residue density.

The optimisation using the MCF network provides the positions of the branch cuts. For the actual phase unwrapping a flood-fill algorithm is used, unwrapping the interferogram pixel by pixel circumnavigating all defined branch cuts.

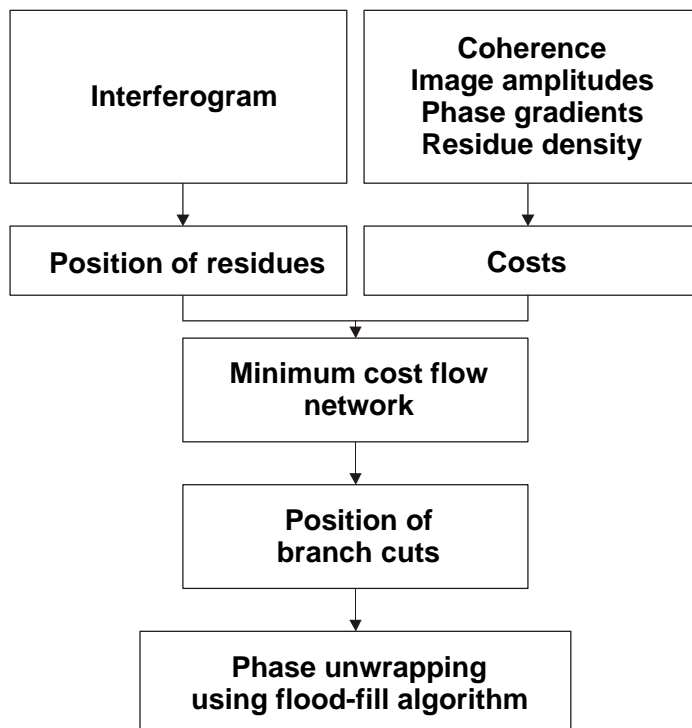


Figure 2. Schematic processing chain for phase unwrapping using a minimum cost flow network. The input for the MCF network is defined by the position of the residues found in the interferogram and the costs derived from various sources. The optimisation of the MCF network determines the position of the branch cuts used during the phase unwrapping.

For the definition of the costs several methods have been proposed in the literature.

Eineder *et al.* (1998) used two different approaches. First, they defined the costs from amplitude, residue density and flatness. High costs were assigned to low amplitude values (to force the branch cut definition in foreshortening and layover areas) and to low residue density (to prevent branch cuts in flat terrain areas). Both values were combined by a maximum operation to obtain the final costs. The thresholds

used for this approach were calculated by minimising the absolute value of the sum of absolute distances of the unwrapped phase. Their second approach tried to maximise the smoothness of the unwrapped phase that is defined as the sum of the absolute values of the phase gradients. The absolute difference of the values of the phase estimator is directly used as the cost function.

Carballo and Fieguth (1999) formulated phase unwrapping as a Maximum Likelihood estimation problem. This

approach is based on the knowledge of the probability of phase discontinuities that can be derived as a function of coherence and topographic slope from known statistical properties of the SAR phase.

Refice *et al.* (1999) proposed an automatic inference methodology considering coherence, image intensities, residue density, phase gradients and an indicative image based on mean-field annealing and relying on some a priori assumptions about the regularity of the absolute phase function.

Wilkinson (1997) achieved satisfactory results by defining the cost by enhanced

coherence values, rescaling the coherence value to the power of four to the appropriate value range.

Proposed method

The approaches found in the literature are only partly convincing. The approach of Eineder *et al.* (1999) is based on the empirical estimation of some threshold values. Carballo and Fieguth (1999) as well as Refice *et al.* (1999) have used methods based on mathematical approaches without proving that the theory behind these approaches is applicable to the phase unwrapping problem. Wilkinson's method (1997) already leads to promising results but makes only use of the coherence values.



Figure 3. Branch cut images from different phase unwrapping algorithms. Left: The classical Goldstein algorithm shows one part completely isolated by branch cuts which leads to an incorrect unwrapping result. Centre: Branch cuts from MCF network with constant costs also show some erroneous connections. Right: Branch cuts from MCF network with costs derived from enhanced coherence values pass the visual inspection.

Figure 3 shows branch cut images using different algorithms. The images are calculated from an interferogram that has been simulated from a digital elevation model of the mountainous terrain around Long's Peak, Colorado, provided by Ghiglia and Pritt (1998) via the Wiley ftp server. The left image shows the result of the classical Goldstein algorithm. It clearly shows problems in layover regions where even one part is completely isolated by branch cuts. The centre image shows the MCF solution using constant costs. It clearly indicates that minimising the total geometric length of the branch cuts does not always lead to plausible results. The MCF solution with costs derived from coherence image by rescaling the fourth power of the coherence values offers visually better results.

The last result needs to be further enhanced. The major problem areas for any phase unwrapping algorithm are in the foreshortening and layover areas. Based on the local phase gradient a smooth terrain surface needs to be reconstructed. This simulated terrain surface can serve as the basis for defining the weighting of the costs.

Conclusions

The implementation using the RelaxIV algorithm for the minimum cost flow network is computationally not practical for larger data sets since it requires huge amount of memory. Part of the current research is focusing on designing MCF networks more adapted to the specific constraints of the phase unwrapping problem instead of being operational for general purpose. It can be anticipated that MCF network more suitable for the specific task of phase unwrapping are available soon but this does not affect the search for an optimal definition of the cost function used for this approach.

The first results look very promising. Nevertheless, it needs a lot more processing of a variety of scenes to gain the experience required for finding a better definition of the cost function. It appears quite logical that the result is supposed to become better the more relevant information derived from various sources is considered for the definition of the cost function. Since the phase unwrapping problem is an ambiguity problem there is generally no true solution to it. The way to find an optimal approximation to it leads to the use of interferograms simulated from real elevation models since they offer a reference for comparison.

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For the calculations of the MCF network the C⁺⁺ implementation of the RelaxIV algorithm provided by Antonio Frangioni and Claudio Gentile (University of Pisa, Italy) have been used. For the implementation of the phase unwrapping algorithm partly C codes provided by Dennis Ghiglia and Mark Pritt (1998) via the Wiley ftp server have been adapted.

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