Deriving High-Resolution Non-Linear Deformation Time Series from TerraSAR-X Interferograms with the Method of Least Squares

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Outline

- Adjustment calculation in the conventional Small Baseline (SB) method
- Motivation
- An SB method for phase observations with different accuracy
- Data processing
- Conclusions and future perspectives
Adjustment calculation in the conventional SB method

- Functional model \[ Ax = l + v \]

- Stochastic model \[ \sum_{ll} = \sigma_0^2 Q_{ll} = E \]

- Optimization criterion \[ v^T P v \Rightarrow \min \]

- Solution \[ \hat{x} = (A^T P A)^{-1} A^T P l \]

- Error of estimation \[ \hat{\sigma}_0^2 = \frac{v^T P v}{n - u} \]

Motivation

TerraSAR-X data is suitable for the conventional SB method

- Large critical deformation gradient
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- High spatial resolution
- Short revisit time
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Motivation

**TerraSAR-X data is suitable for the conventional SB method**

- Large critical deformation gradient
- High spatial resolution
- Short revisit time
- Limited spatial baselines

**But why not in every case?**

- Temporal decorrelation => difficult to retrieve unambiguous phase
Motivation


(c)
An SB method for phase observations with different accuracy

- Functional model
  \[ Ax = l + v \]

- Stochastic model
  \[ \sum_{ll} = \sigma_0^2 Q_{ll} \neq E \]

- Optimization criterion
  \[ v^T P v \Rightarrow \min \]

- Solution
  \[ \hat{x} = (A^T PA)^{-1} A^T P l \]

- Error of estimation
  \[ \hat{\sigma}_0^2 = \frac{v^T P v}{n - u} \]
An SB method for phase observations with different accuracy

Phase variance

- Variance of interferometric phase

\[
\text{var}(\phi) = E\left((\phi - E(\phi))^2\right)
\]
An SB method for phase observations with different accuracy

Phase variance

- Variance of interferometric phase
  \[ \text{var}(\phi) = E\left((\phi - E(\phi))^2\right) \]

- Error propagation in a 2D path-following phase-unwrapping network
  \[ \Delta\phi_i = W^{-1}(\phi_i) - W^{-1}(\phi_{i-1}) = \phi_i - \phi_{i-1} + 2\pi k_i \]

  \[ \text{var}(\Delta\phi_i) = \text{var}(\phi_i) + \text{var}(\phi_{i-1}) - \text{cov}(\phi_i, \phi_{i-1}) \]
An SB method for phase observations with different accuracy

Phase variance

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  \]

  \[
  W^{-1}(\phi_i) = \sum_{j=1}^{M-1} \Delta \phi_j
  \]

  \[
  \text{var}(W^{-1}(\phi_i)) = \sum_{j=1}^{M-1} \text{var}(\Delta \phi_j)
  \]
An SB method for phase observations with different accuracy

Phase variance

- Variance of interferometric phase
  \[
  \text{var}(\phi) = E( (\phi - E(\phi))^2 )
  \]

- Error propagation in a 2D path-following phase-unwrapping network

This could be simplified as

\[
W^{-1}(\phi_i) = \sum_{j=1}^{M-1} \Delta \phi_j
\]
\[
W^{-1}(\phi_i)' = W^{-1}(\phi_i) - W^{-1}(\phi_0)
\]
\[
\text{var}(W^{-1}(\phi_i)) = \sum_{j=1}^{M-1} \text{var}(\Delta \phi_j)
\]
\[
\text{var}(W^{-1}(\phi_i)) = \text{var}(\phi_i) + \text{var}(\phi_0) - \text{cov}(\phi_i, \phi_0)
\]
An SB method for phase observations with different accuracy

Outlier test

- What could an outlier in this case be?
An SB method for phase observations with different accuracy

Outlier test

- What could an outlier in this case be?

- Detection of possible outliers
  - The normalized refinements
    \[ w_i = \frac{v_i}{\sigma_i} \]
An SB method for phase observations with different accuracy

Outlier test

- What could an outlier in this case be?

- Detection of possible outliers

  - The normalized refinements

    \[ w_i = \frac{v_i}{\sigma_v} \]

  - The assumptions

    \[ l \sim N(Ax, \sigma^2_0 Q_{ll}) \]

    \[ H_0 : E(l) = Ax \]
An SB method for phase observations with different accuracy

Outlier test

- What could an outlier in this case be?

- Detection of possible outliers
  - The normalized refinements
    \[ w_i = \frac{v_i}{\sigma_{v_i}} \]
  - The assumptions
    \[ l \sim N(Ax, \sigma_0^2 Q_{ll}) \]
    \[ H_0 : E(l) = Ax \]
  - Which lead to
    \[ H_0 : w_i \sim N(0,1) \]
Simulated data processing

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Simulated data processing with outlier test

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Identification of coherent pixels

- Coherence magnitude model

\[ |\gamma| = \left( |\gamma_{\text{geom}}| \cdot |\gamma_{\text{DC}}| \cdot |\gamma_{\text{thermal}}| \right) \cdot \left( |\gamma_{\text{temporal}}| \cdot |\gamma_{\text{vol}}| \right) \cdot |\gamma_{\text{process}}| \]

acquisition-related components \quad object-related components


Identification of coherent pixels

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acquisition-related components \quad object-related components

- Weighted mean of decomposed object-related coherence magnitude

\[ |\hat{\gamma}_{\text{object}}| = \frac{\sum_{i=0}^{n-1} w_i \left( |\hat{\gamma}_{\text{temporal},i}| \cdot |\hat{\gamma}_{\text{vol},i}| \right)}{\sum_{i=0}^{n-1} w_i} \]


Real data processing

- Phase decomposition result of a soccer stadium and its surrounding vs. Google Earth™
And their deformation time series
And their deformation time series
And their deformation time series
Another deformation time series of a subsidence trough
Another deformation time series of a subsidence trough
Another deformation time series of a subsidence trough
Conclusions

- Phase accuracy to be taken into account as weight in adjustment calculation

- Only object-related coherence components to identify coherent pixels
Conclusions

- Phase accuracy to be taken into account as weight in adjustment calculation
- Only object-related coherence components to identify coherent pixels

And future perspectives

- Developing error propagation models for phase-unwrapping algorithms
- Improving phase quality by unwrapping phase in (pseudo-)3Ds
- Validating with reference data
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