A simultaneous internal tide and mesoscale sea surface height mapping from the global altimetry constellation

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Simultaneous estimation of ocean mesoscale and coherent internal tide sea surface height signatures from the global altimetry record, Ocean Sci., 18, 469–481, https://doi.org/10.5194/os-18-469-2022, 2022.

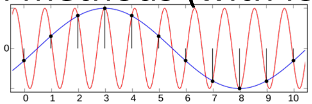
Global Internal tide SSH solution available on Aviso+ : **DOI 10.24400/527896/a01-2022.003**

Context : growing interest for internal tides in Altimetry

- Tackling higher resolution in the Altimetry map reconstructions (motivated by a growing constellation!) confronts us to the coexistence of short mesoscales (**MS**) and the first modes of internal tides (**IT**) at similar spatial scales

- However, **these two signals represent fairly different dynamics** (balanced and unbalanced, non-linear eddies and linear waves)

- The standard Optimal Interpolation methods (with low-pass covariances) for large mesoscale Altimetry leads to severe aliasing of IT in MS



- Need to design mapping techniques to handle more complex covariances, allowing the **separation and the estimation** of **MS** and **IT** Altimetry maps

- The IT estimates can be used as a signal of interest or as a noise correction to study mesoscales…

- **Phase-locked IT in this study**, phase varying IT are also being tackled in the context of SWOT, see F. Le Guillou’s presentation in this session!

The new MIOST tool for Altimetry mapping

(Multi-scale Inversion of Ocean Surface Topography)

- **Why?** The standard Optimal Interpolation mapping method becomes limited and not flexible to invert multiple signals with wide ranges of covariances

$$\mathbf{x_a} = \mathbf{BH^T(HBH^T+R)^{-1}y}$$

Estimate — $\mathbf{x_a}$; (grid,obs) signal cov. — $\mathbf{BH^T}$; (obs,obs) signal cov. — $\mathbf{HBH^T}$; (obs,obs) error cov — $\mathbf{R}$; SLA obs — $\mathbf{y}$

Main issue: prohibitive cost (α n^3) if inversion windows increase or if there are more observations…

- **MIOST : inversion in reduced-basis G** (typically wavelets)

With a variational approach involving the minimization:

$$J = \mathbf{\eta^T Q^{-1} \eta + (y - G\eta) R^{-1} (y - G\eta)}^T$$

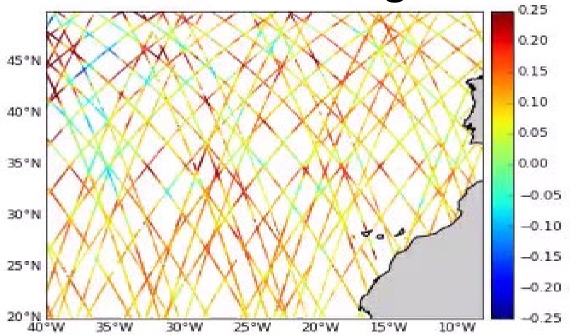
State in param space — $\mathbf{\eta}$; Prescribed variance of the components — $\mathbf{Q}$; Distinct MS and IT basis of components — $\mathbf{G}$

- Also flexible to include surface currents (with derived **G** expression in current)
- Here : well suited to separate and estimate the signatures of IT and MS

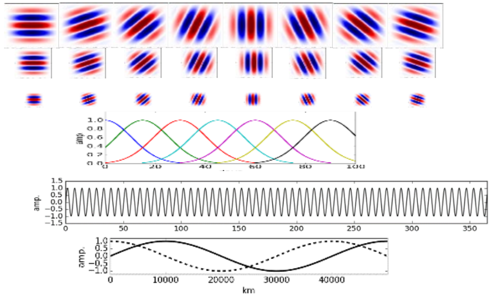
The new MIOST tool for Altimetry mapping

(Multi-scale Inversion of Ocean Surface Topography)

Multi-altimeter along-track SLA



Component basis



Variationnal

0(100) iterations

Minimization

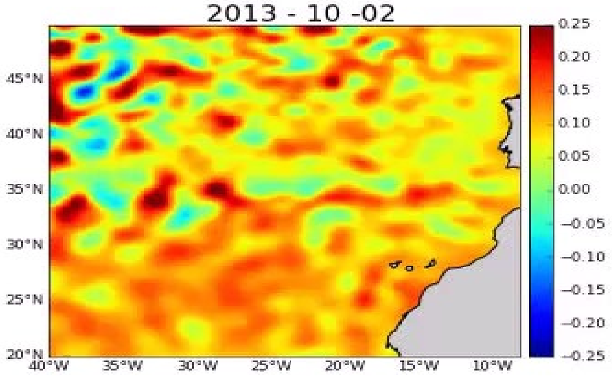
Solution **η**:

5.10^8 adjusted parameters

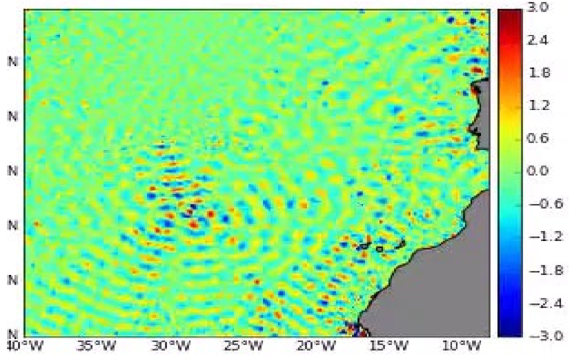
Projection in physical space

30-year worth of :

mesoscale maps

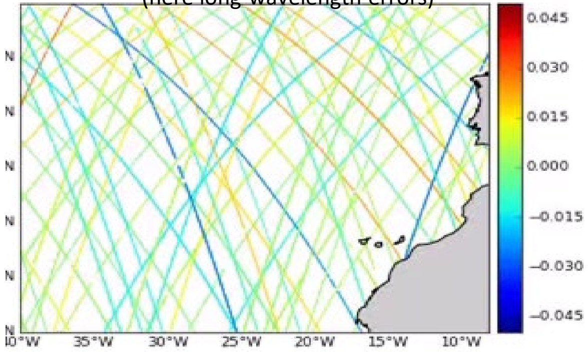


Internal-tides

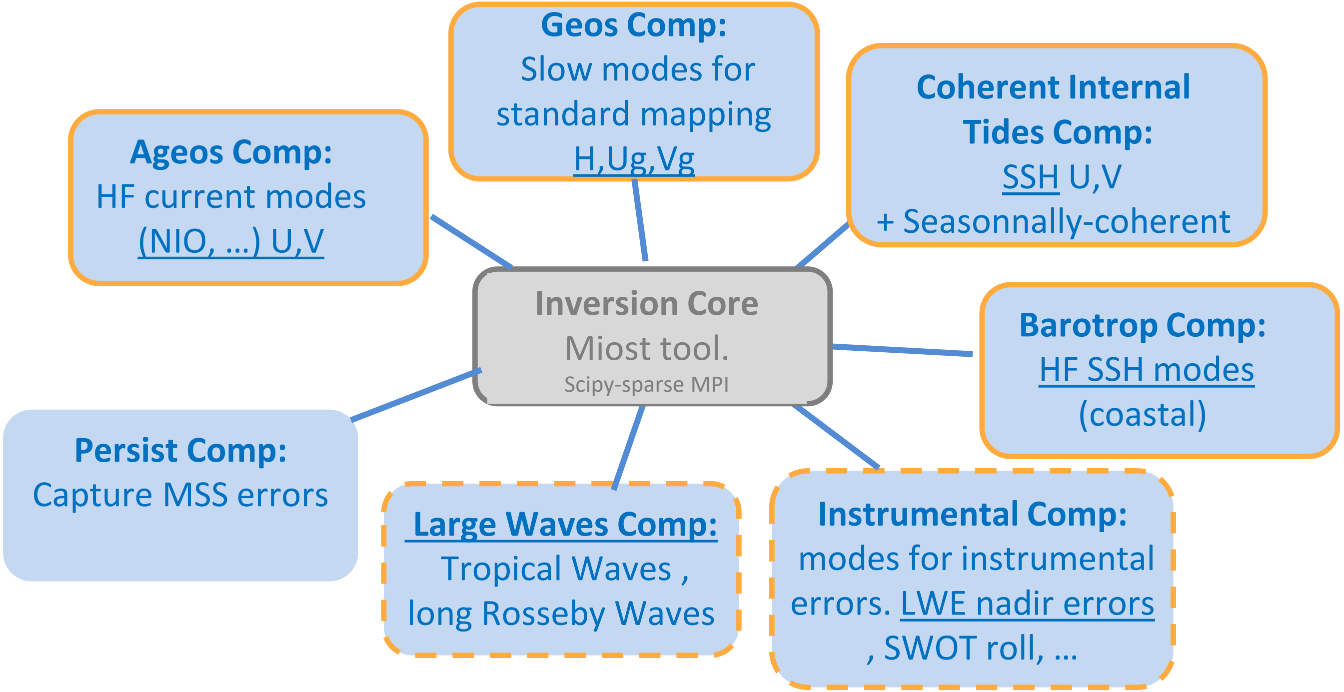


Other modes

(here long-wavelength errors)



The new MIOST tool for Altimetry mapping

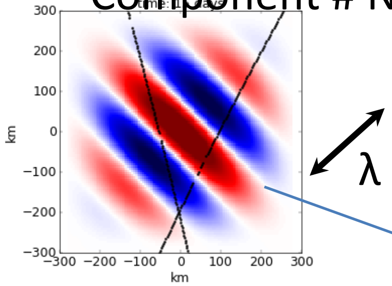


- Release of a 5-year dataset combining Altimetry and Drifters (Aviso+, May 2022, and see M. Ballarotta's poster!)
- **Release of the coherent Internal tide estimation (Aviso+, Sept. 2021, this presentation!)**

Implementation for MS : a 3D wavelet basis

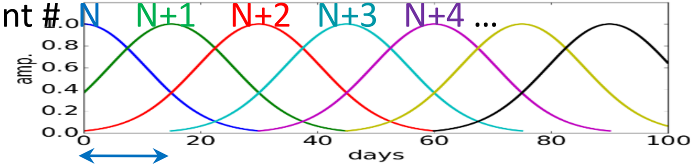
In space:

Component # N



In time:

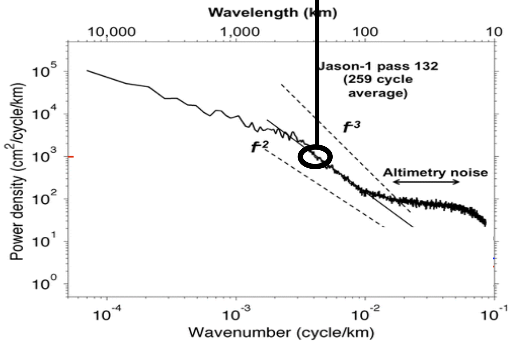
Component # N N+1 N+2 N+3 N+4 …



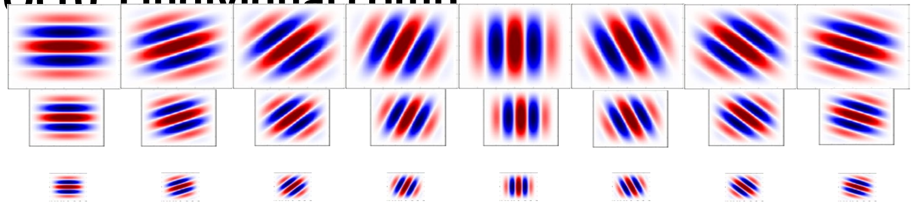
Time dec. set to expected MS dec. at λ

A column of **G** matrix is the value of the component at obs point.

Amplitude-match with the observed mesoscale altimetry spectrum specified in **Q**



- A full decomposition over a 30°x30° domain , 20 years, for λ between 1,000km and 80km gives **O(10⁸) individual comp.**



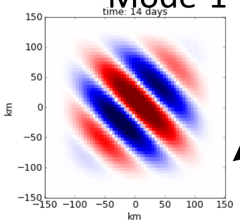
- We don't want to write **G** !! (nobs x ncomp) but only its non-zero segments sequentially for each column to get the product $\mathbf{G}\mathbf{R}^{-1}(\mathbf{y}-\mathbf{G}\boldsymbol{\eta})$

Implementation for IT : plane-wave basis

A local plane-wave basis, following Zhao et al., 2016, is considered for mode 1 and mode 2

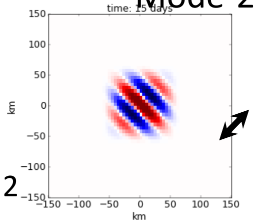
In space:

Mode-1



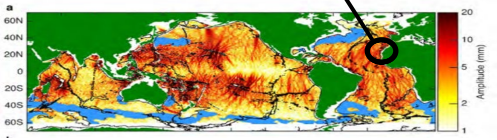
$\lambda = c_1 / f_{M2}$

Mode-2



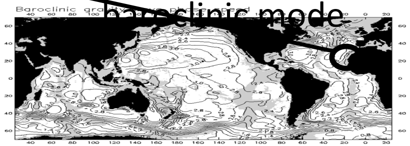
$\lambda = c_2 / f_{M2}$

Expected IT variances specified in **Q**



Zhao et al., 2016

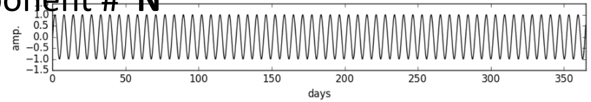
Phase-speed of 1&2 baroclinic mode



Chelton et al., 1998

In time:

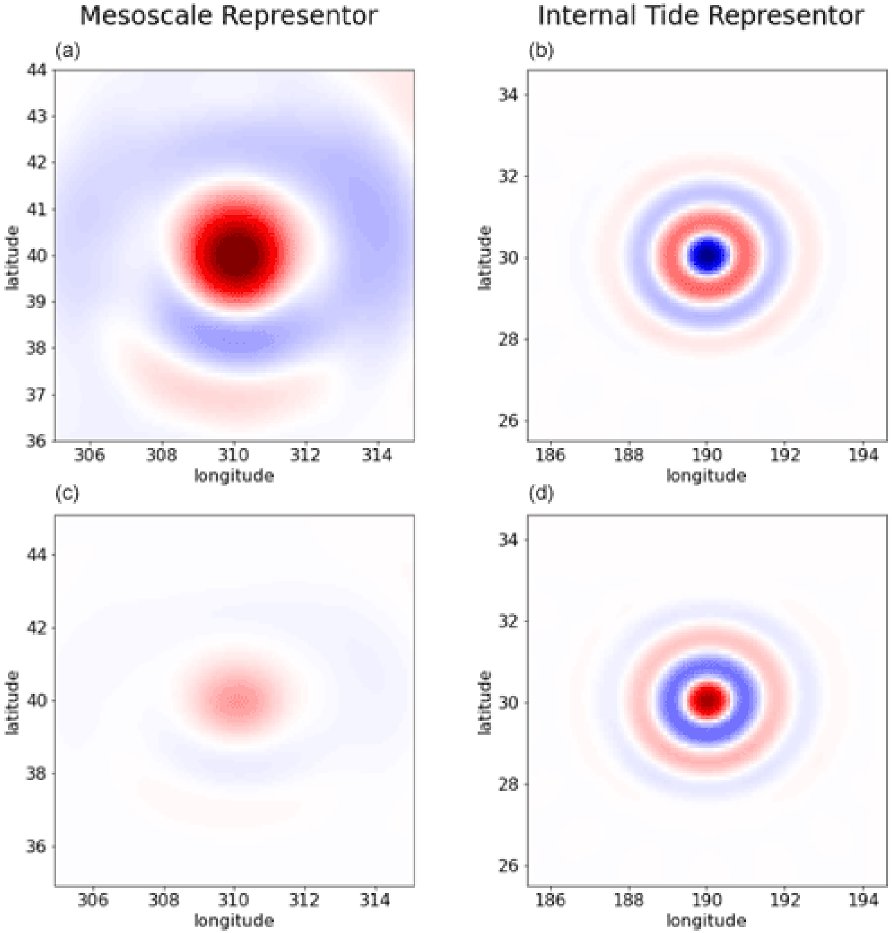
Component # **N**



- A decomposition over a 30°x30° domain, for mode-1 and mode-2, gives **O(10^5) individual comp.**

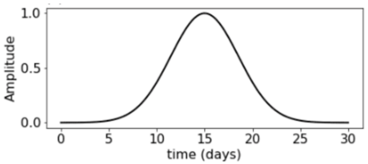
- The non-zero segments of the **G** columns are much longer (pluri-annual comp.) but the number of modes is low: we can still easily compute sequentially $\mathbf{G}\mathbf{R}^{-1}(\mathbf{y} - \mathbf{G}\boldsymbol{\eta})$

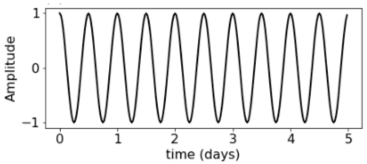
The equivalent covariance models for MS and IT



0 days
(time lag)

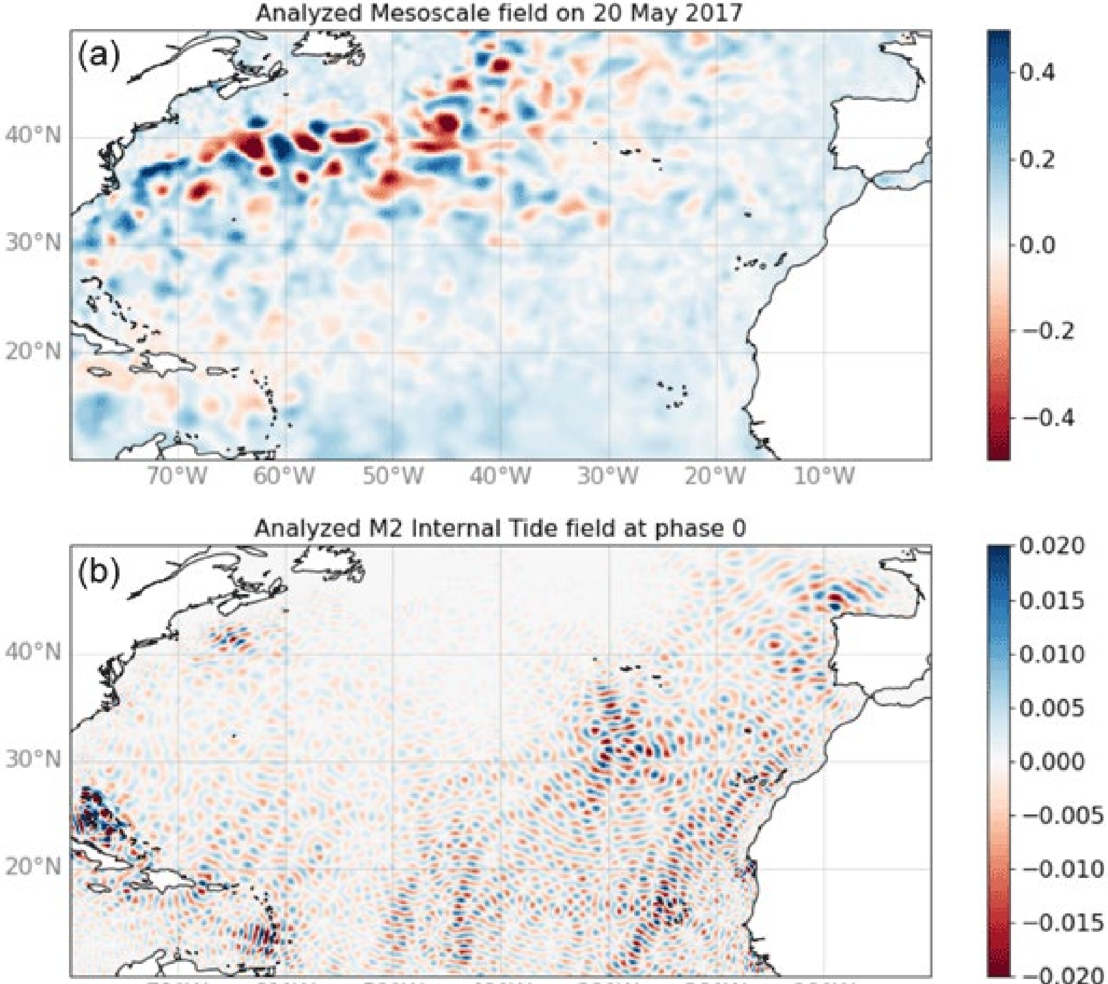
8 days
(time lag)





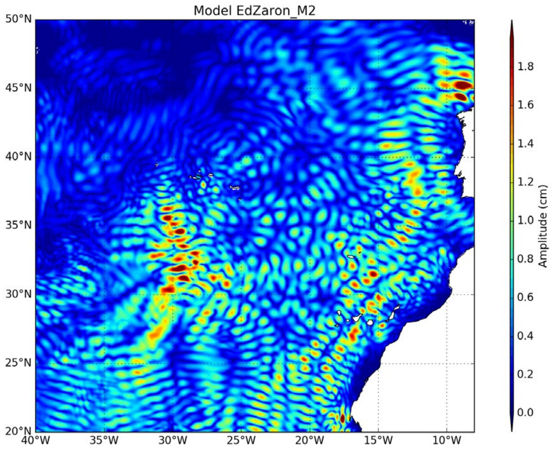
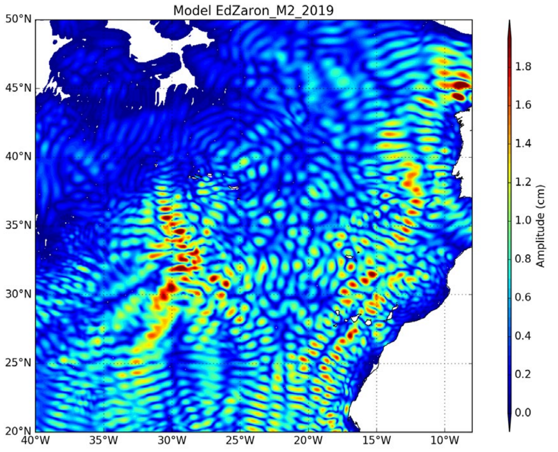
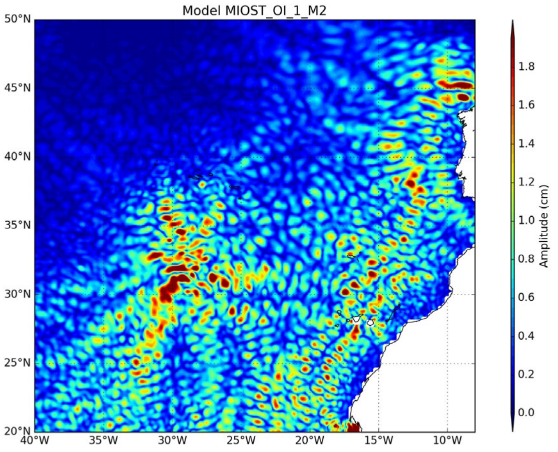
The appropriate wavelet and plane-wave bases allows to represent a covariance model well suited for each dynamics, in particular with persistant time-oscilating patterns for internal tides

Results of the 30-year processing : illustration



Wave-pattern vizualization

Comparison with HRET (Zaron) solutions

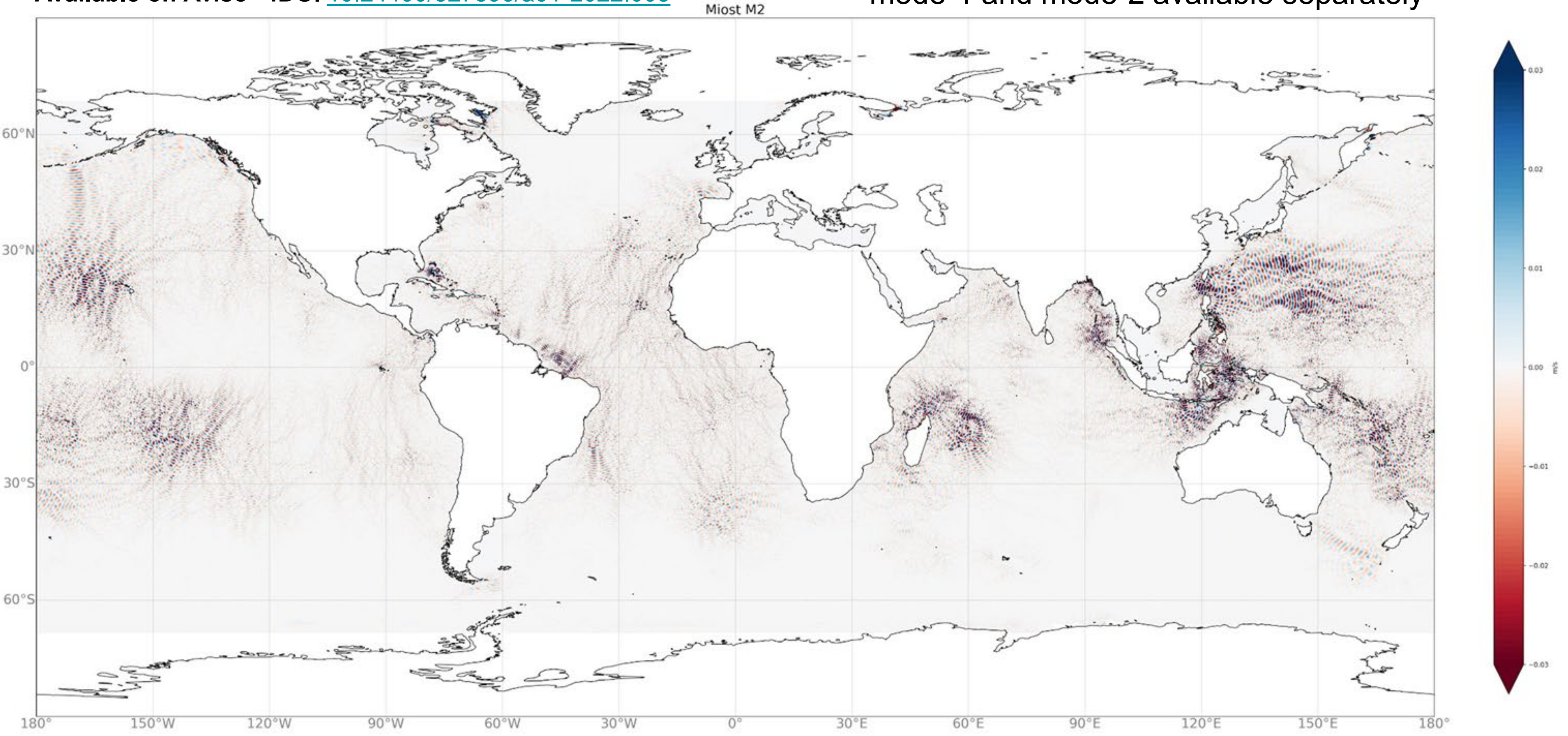
M2		
Zaron 2017	Zaron 2019	MIOST
		

Lateral extension of wave trains is more limited in Miost

Global solution for M2:

Available on Aviso+ :DOI 10.24400/527896/a01-2022.003

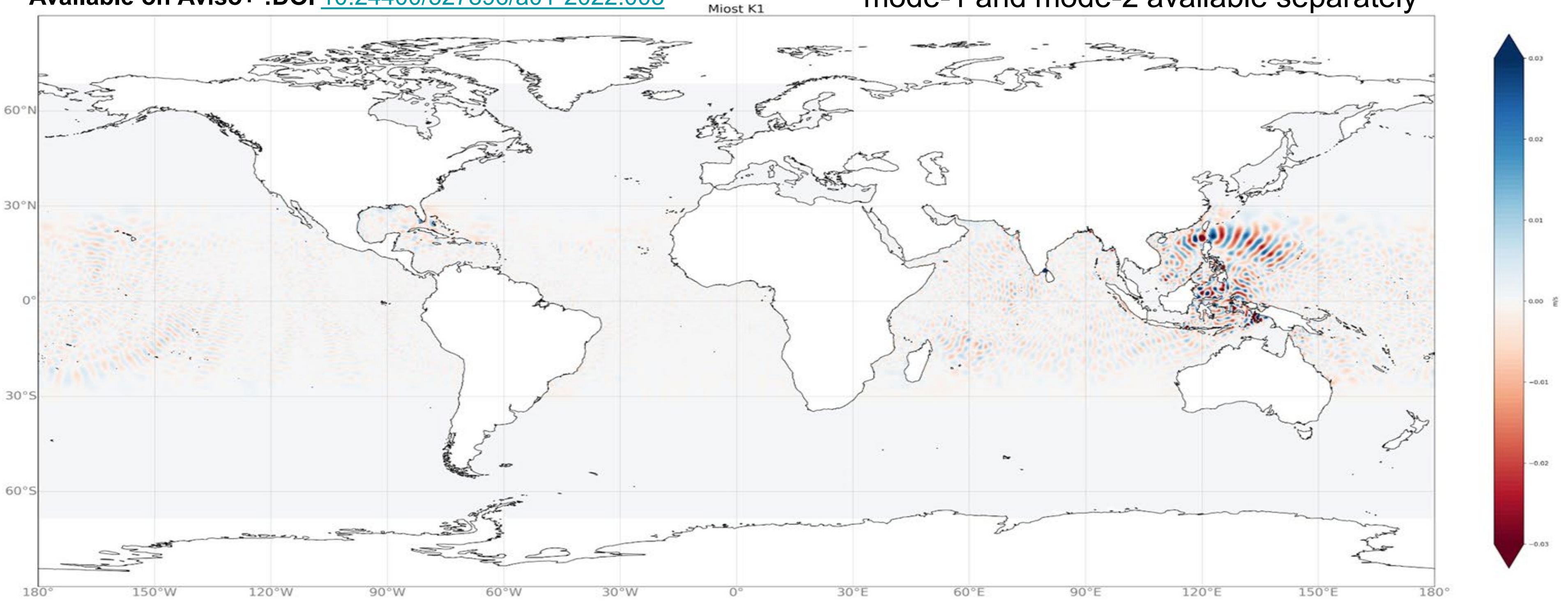
mode-1 and mode-2 available separately



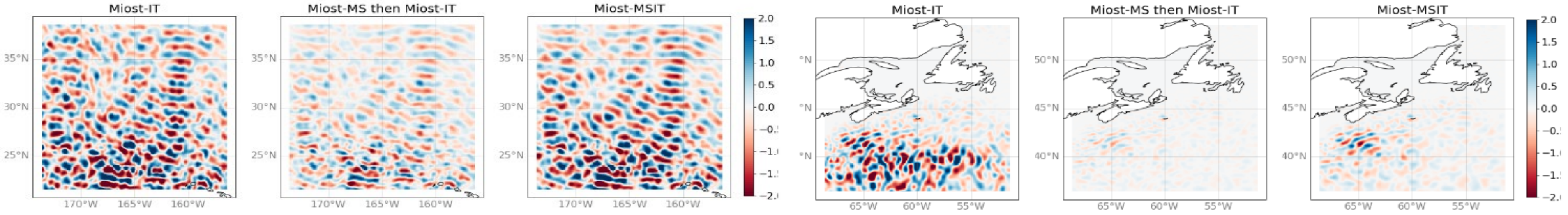
Global solution for K1:

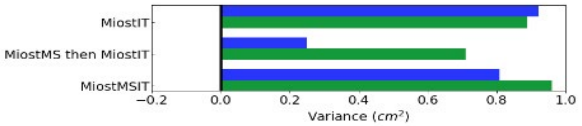
Available on Aviso+ :DOI 10.24400/527896/a01-2022.003

mode-1 and mode-2 available separately

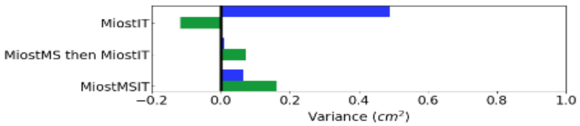


On the benefit of the simultaneous inversion : mitigation of aliasing effects



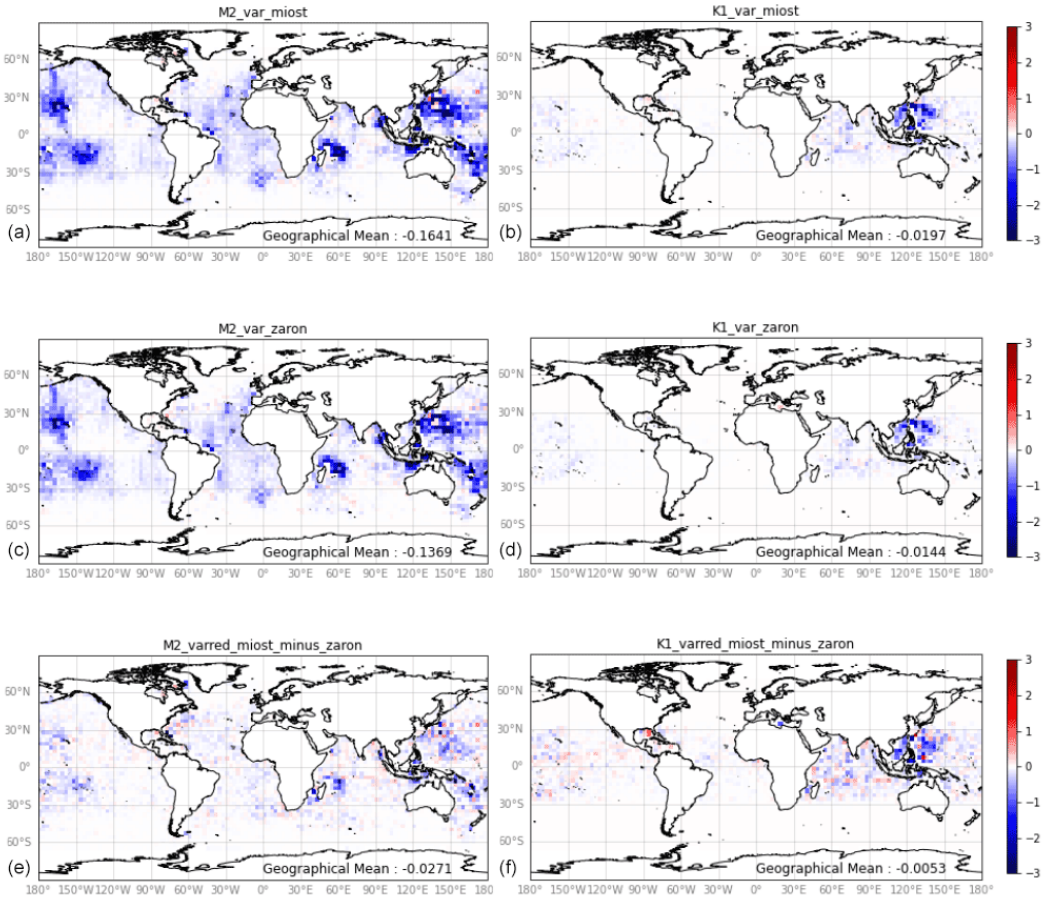


Signal variance
Explained variance reduction on independant data



- IT estimation without mesoscale : overestimation, mesoscale leak (very strong in Gulf Stream)
- MS estimation removed before IT estimation: underestimation because of IT leak in MS first estimation
- IT-MS simultaneous estimation: leakages are minimized

Quantitative validation



- We compute the variance of independent Altimetry nadir tracks before and after applying the IT solution
- Blue indicates a variance reduction : the IT signal is validated from the independent data
- On global average, MIOST-IT reduces slighly more variance (and contains more energy) than other IT estimations.

Conclusions and perspectives

The new MIOST tool, now implemented in the DUACS system, allows flexibility to tackle a wide range of Altimetry signals.

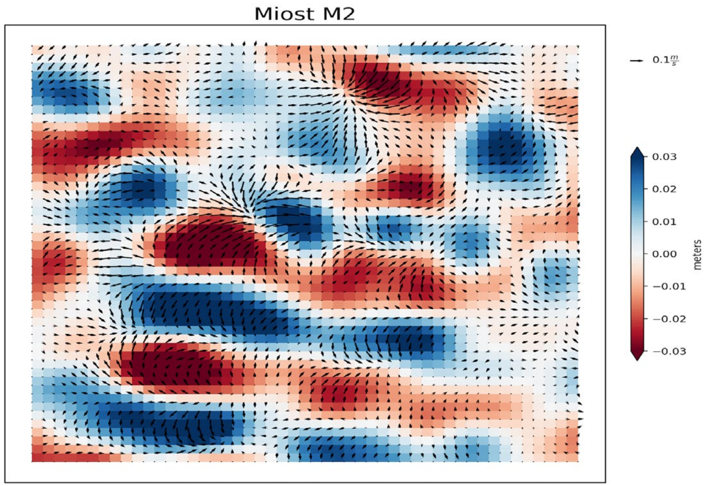
Here in particular, coherent internal-tides have been globally mapped in an optimal way minimizing mesoscale aliasing

Solution available in Aviso+, new versions will be released in the coming years with (and for) SWOT

In the context of improving surface current estimates (e.g. WOC project, future Doppler missions, …) the SSH IT signatures can be converted in current (validations soon)

$$u = \frac{-i\omega_\tau g\partial_x\eta + fg\partial_y\eta}{\omega_\tau^2 - f^2},$$

$$v = \frac{-fg\partial_x\eta - i\omega_\tau g\partial_y\eta}{\omega_\tau^2 - f^2},$$



IT surface current around Hawaiian ridge

Backup

Implementation

- Standard OI formula to map altimetry : $\mathbf{x_a} = \mathbf{BH}^T(\mathbf{HBH}^T + \mathbf{R})^{-1}\mathbf{y}$

Estimate | (grid,obs) signal cov. | (obs,obs) signal cov. | (obs,obs) error cov | SLA obs

Main issue: prohibitive cost (α n^3) if we extend time window to include a wide range of signals. Limited to typically 1,000km, 30 days in DUACS

- We propose a variational approach involving the minimization:

$$J = \boldsymbol{\eta}^T \mathbf{Q}^{-1} \boldsymbol{\eta} + (\mathbf{y} - \mathbf{G}\boldsymbol{\eta})\mathbf{R}^{-1}(\mathbf{y} - \mathbf{G}\boldsymbol{\eta})^T$$

State in param space

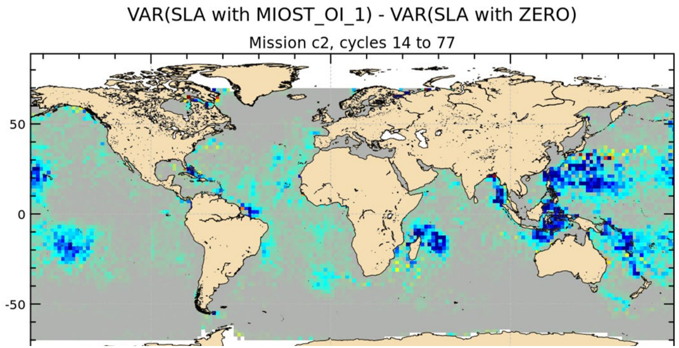
Distinct MS and IT basis of components

Prescribed variance of the components

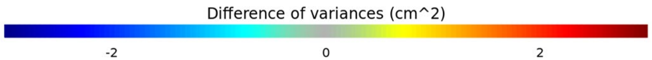
Benefits: We can extend the inversion window (cost α n) up to decades and introduce IT coherent modes

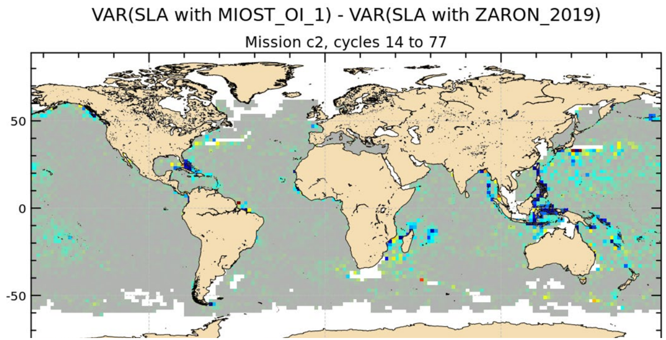
- ✓ Equivalence with OI (provided GQG^T matches the same covariance model)
- ✓ With this setup, MS and IT can be optimally estimated with respect to their prescribed covariances
- ✓ The solution strongly relies on a suited basis of components for MS and IT (see next

Diagnostics de validation pour l'onde M2 : Along track SLA for C2

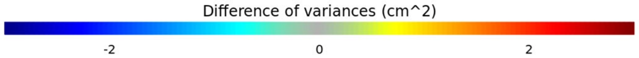


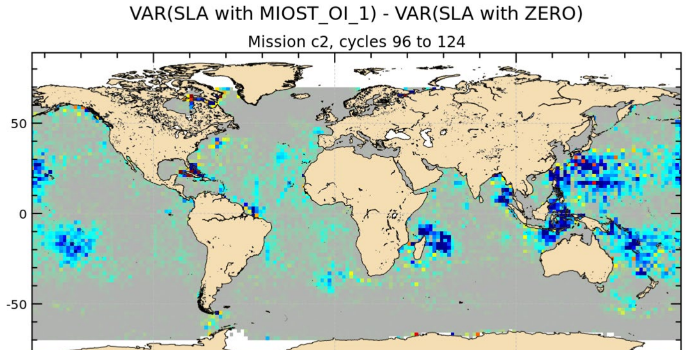
Min = - 79.4966 ; Max = 23.6215 ; NbObs = 9560



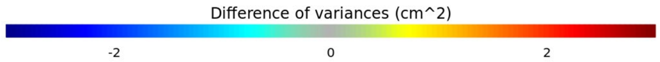


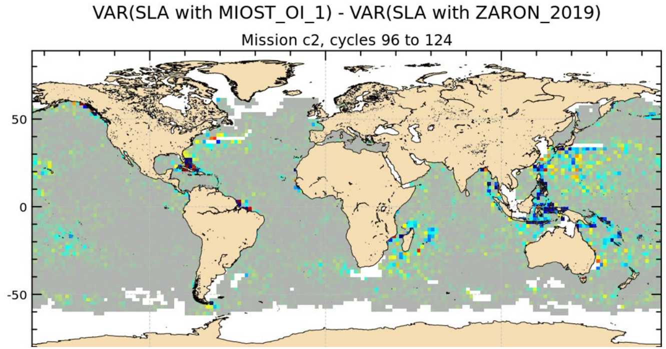
Min = - 110.8839 ; Max = 9.8898 ; NbObs = 7793



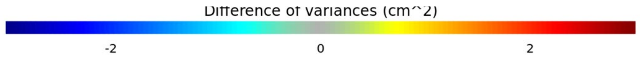


Min = - 75.5514 ; Max = 18.6341 ; NbObs = 9569





Min = - 123.6731 ; Max = 19.8016 ; NbObs = 7787



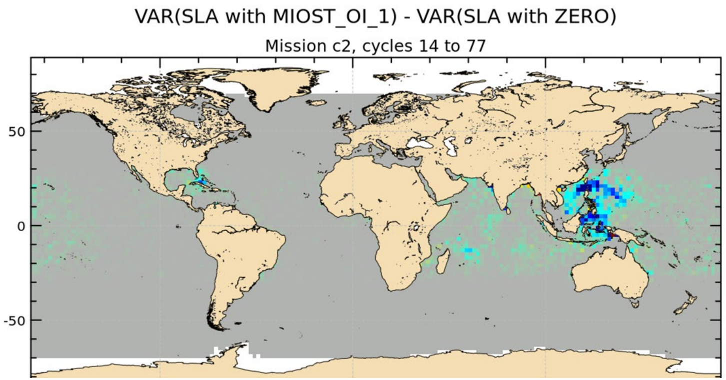
Conclusions

- The simultaneous inversion of mesoscales and internal tides was successful to provide an estimation of coherent internal tides for the major components (M2, K1, S2,)
- The solution compares well with Zaron 2019:
  → Slighly different wave structures with shorter spatial variations
  → Explains slighly more variance reduction with independant data on global average, although in some regions it would perform not as well

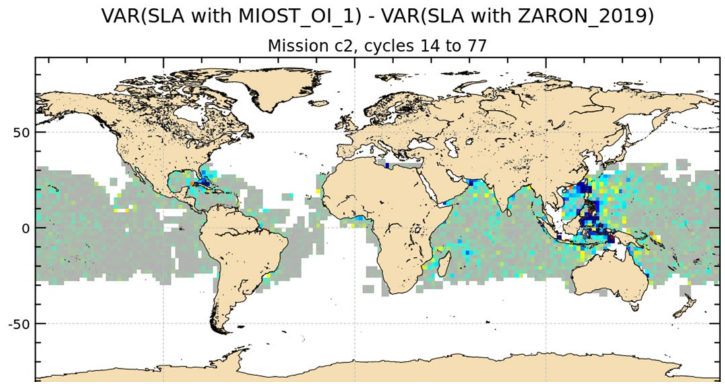
- May constitute an additional model to be used for SWOT

- The inversion algorithm can be easily improved to handle some non-stationarity, without (seasonality?) or with future SWOT data (eddy-scale instationarity?)

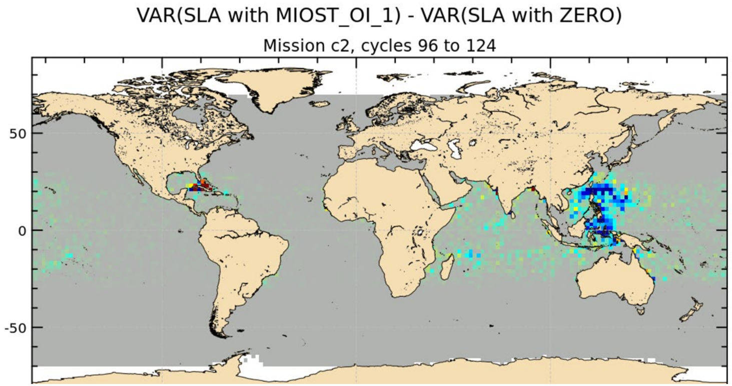
Diagnostics de validation pour l’onde K1 : Along track SLA for C2



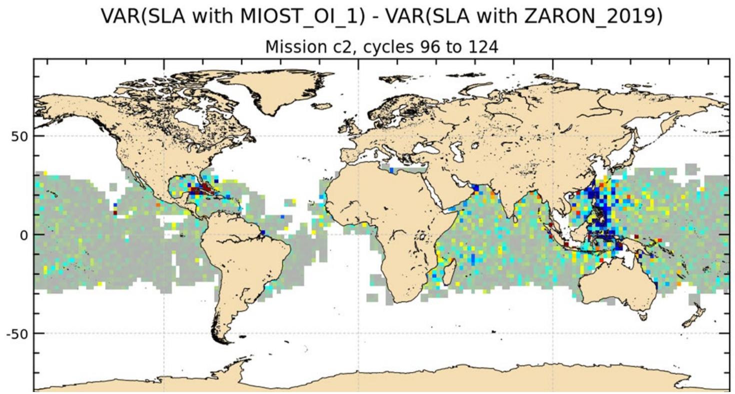
Min = - 17.7548 ; Max = 16.9863 ; NbObs = 9560



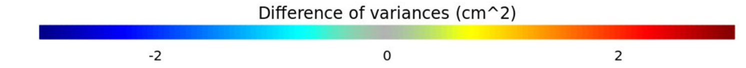
Min = - 17.2023 ; Max = 16.4664 ; NbObs = 3569

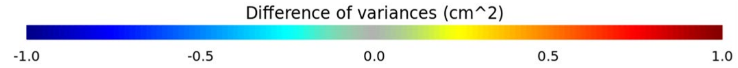


Min = - 39.7620 ; Max = 10.8737 ; NbObs = 9569



Min = - 24.7727 ; Max = 10.0753 ; NbObs = 3565





Diagnostics de validation : statistiques moyennes sur SSH aux points de croisement C2

Cycles 96-124

M2

SSH Crossovers

Modèles comparés	Global	Tahiti	Hawaii	Madagascar	Gulf of Guinea	Luzon	NATL	NPAC
MIOST – ZERO	- 1.26cm²	- 2.19cm²	- 1.93cm²	- 2.20cm²	-	- 4.26cm²	- 0.24cm²	- 0.60cm²
%	- 2.40%	- 9.07%	- 7.13%	- 3.86%	-	- 5.00%	- 0.91%	- 2.74%
MIOST – ZARON_2019	- 0.16cm²	- 0.43cm²	- 0.16cm²	0.04cm²	-	- 1.12cm²	- 0.10cm²	- 0.02cm²
%	- 0.39%	- 1.92%	- 0.63%	0.07%	-	- 1.42%	- 0.38%	- 0.09%

K1

SSH Crossovers

Modèles comparés	Global	Tahiti	Hawaii	Madagascar	Gulf of Guinea	Luzon	NATL	NPAC
MIOST – ZERO	- 0.46cm²	- 0.29cm²	- 0.08cm²	- 0.16cm²	-	- 1.04cm²	0.00cm²	- 0.03cm²
%	- 0.88%	- 1.20%	- 0.30%	- 0.28%	-	- 1.22%	0.02%	- 0.14%
MIOST – ZARON_2019	- 0.02cm²	0.01cm²	- 0.04cm²	- 0.13cm²	-	0.52cm²	0.15cm²	- 0.07cm²
%	- 0.05%	0.04%	- 0.13%	- 0.23%	-	0.67%	0.93%	- 0.35%

M2 surface current near Hawaii: up to 0.20m/s

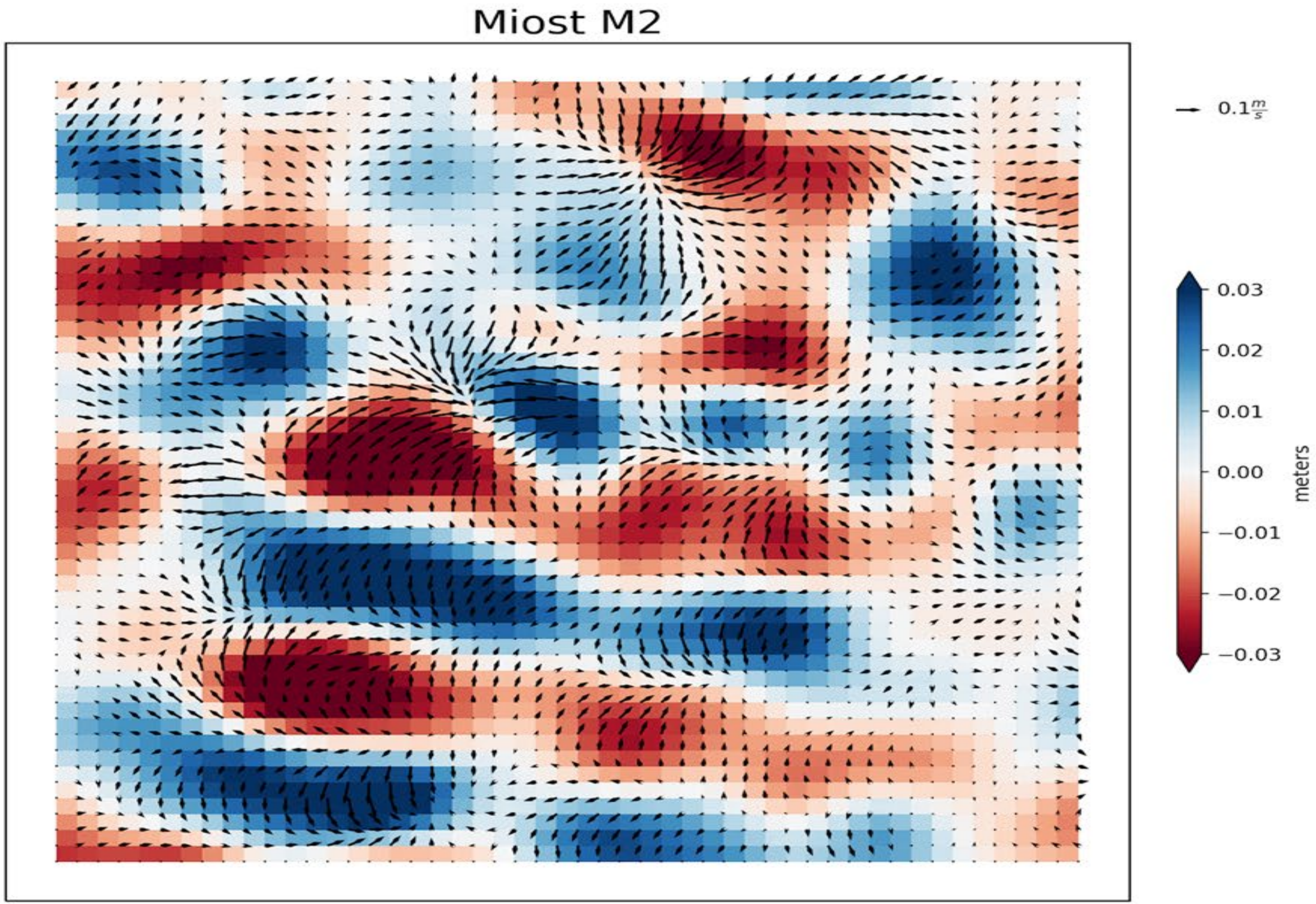
Momentum Equations:

$$-i\omega u - fv = -g\partial_x \eta - u/\tau,$$

$$-i\omega v + fu = -g\partial_y \eta - v/\tau,$$

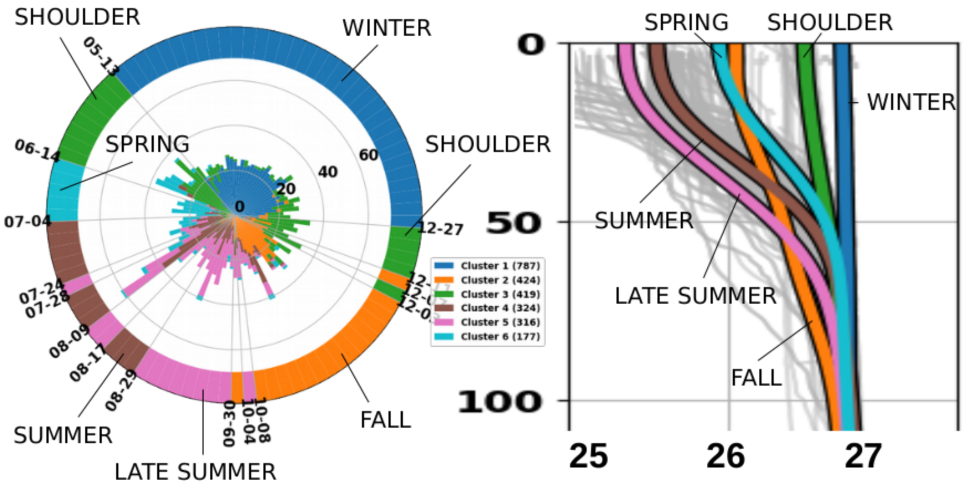
$$u = \frac{-i\omega_\tau g\partial_x \eta + fg\partial_y \eta}{\omega_\tau^2 - f^2},$$

$$v = \frac{-fg\partial_x \eta - i\omega_\tau g\partial_y \eta}{\omega_\tau^2 - f^2},$$



A next step: resolve Internal Tides according to vertical profile type

Work from Simon Barbot (LEGOS)



Mean cumulative histogram over the all period (1980-2015) with a time step of roughly 4 days
→ Mean time distribution of the cluster along a year

→ We can solve the MIOST inversion on each different clusters