# Simulations of low-low satellite-to-satellite tracking constellations for the ESA/NASA MAGIC satellite gravity mission

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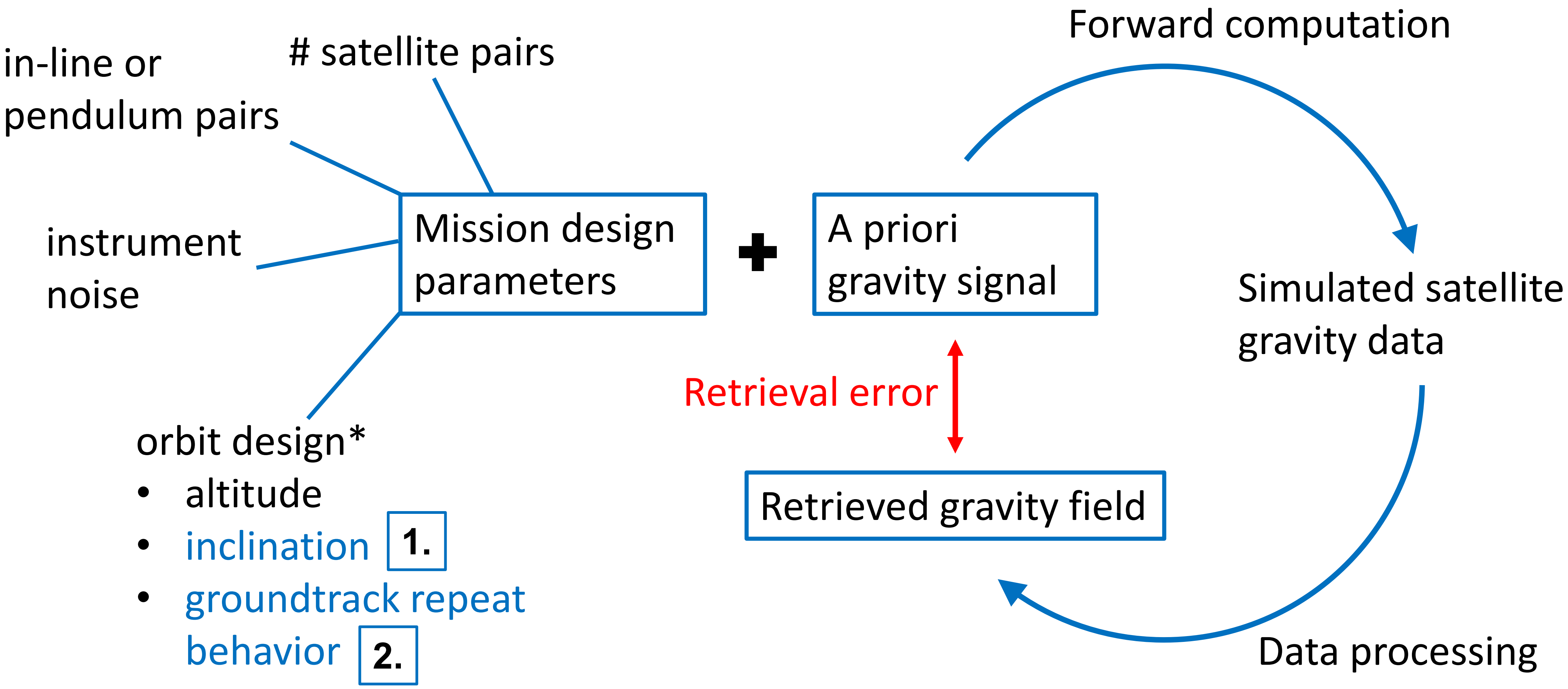
**M**ass-change **A**nd **G**eosciences **I**nternational **C**onstellation (MAGIC)

Aim of the ESA Science Support Study for MAGIC:

- Evaluating the performance of various satellite gravity mission designs

# ESA Science Support Study for MAGIC – Closed-loop simulations




*Massotti et al. (2021)

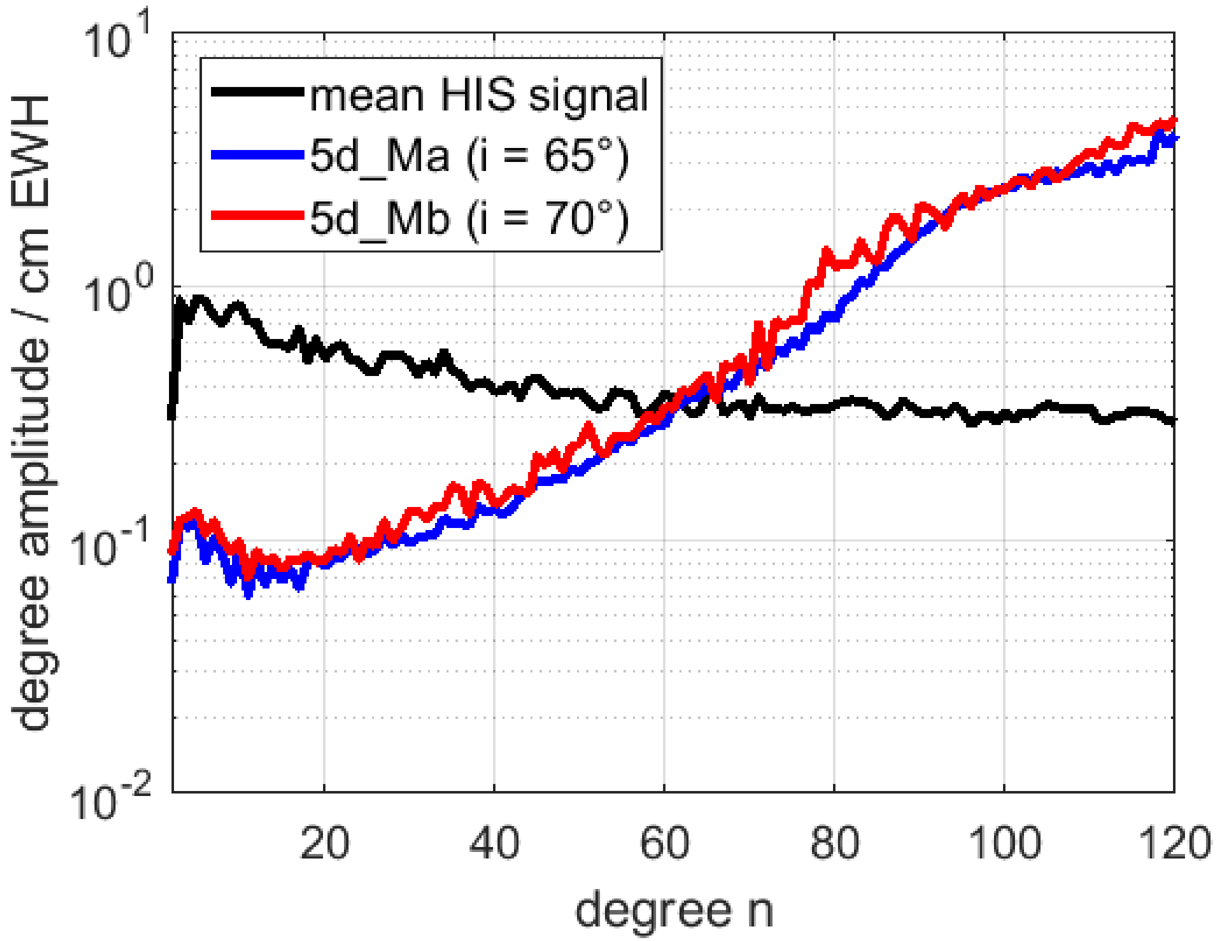
# 1. Impact of the inclination of the second pair

What is the impact of the inclination of the second satellite pair on the retrieval errors of a double-pair mission?

# 1. Impact of the inclination of the second pair – Unfiltered results

Two **double-pair scenarios** with similar altitude
(31-day simulation incl. full signal + noise):

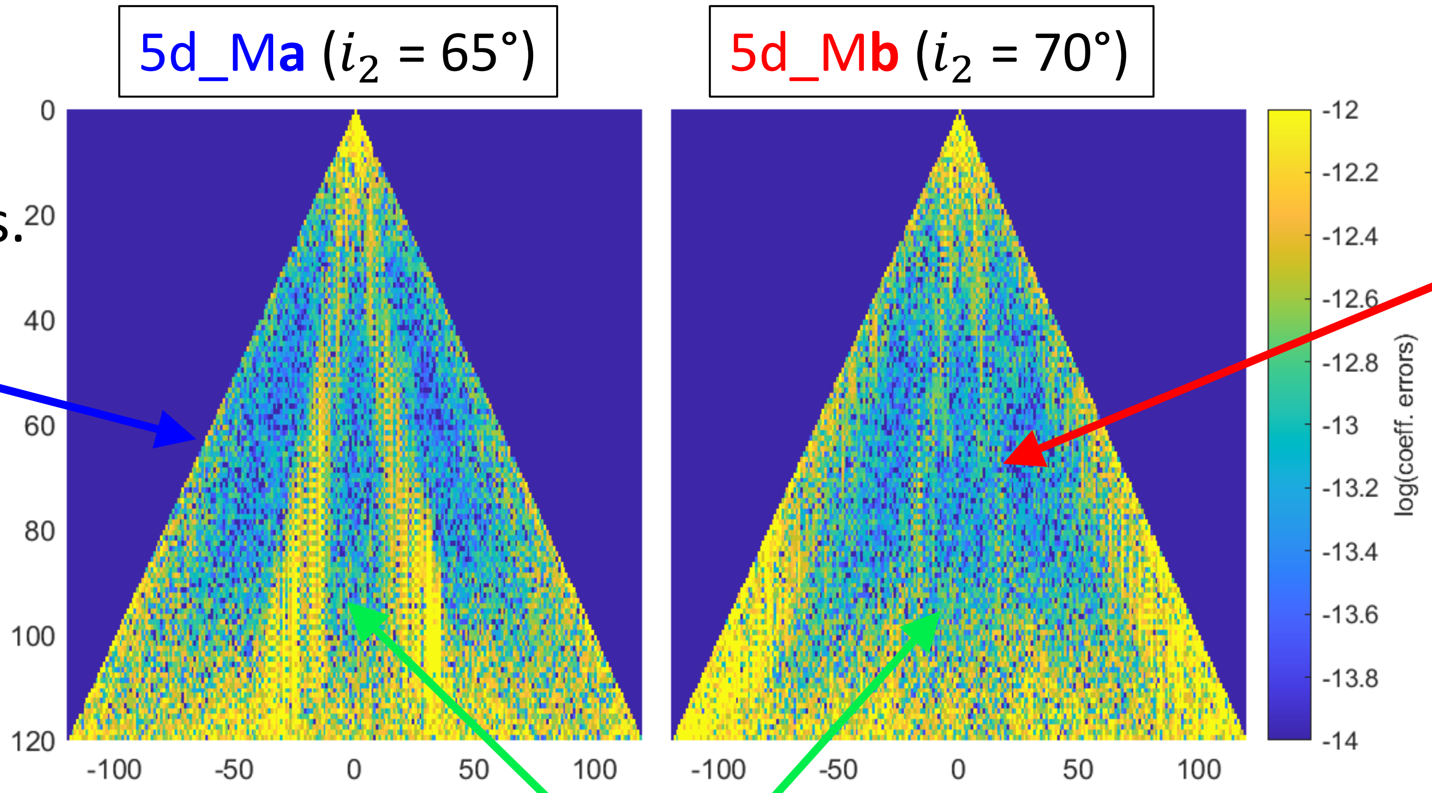
Global performance:



# Impact of the inclination of the second pair – Unfiltered results

Two **double-pair scenarios** with similar altitude
(31-day simulation incl. full signal + noise):

larger east-west component of obs.
⇒ better (near-) sectorials




smaller pol. gap of incl. pair
⇒ better near-zonals

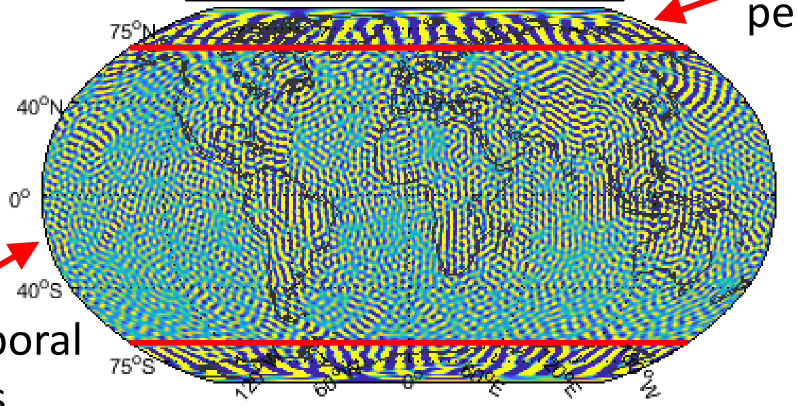
dense grountracks of polar pair over the poles
⇒ zonals similar

# 1. Impact of the inclination of the second pair – Unfiltered results

## EWH grid differences to true HIS signal:

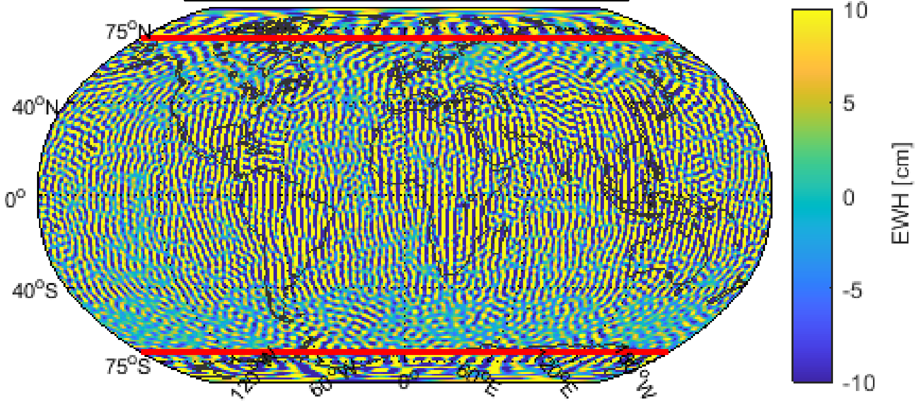
5d_Ma ($i_2$ = 65°)

single-polar pair performance

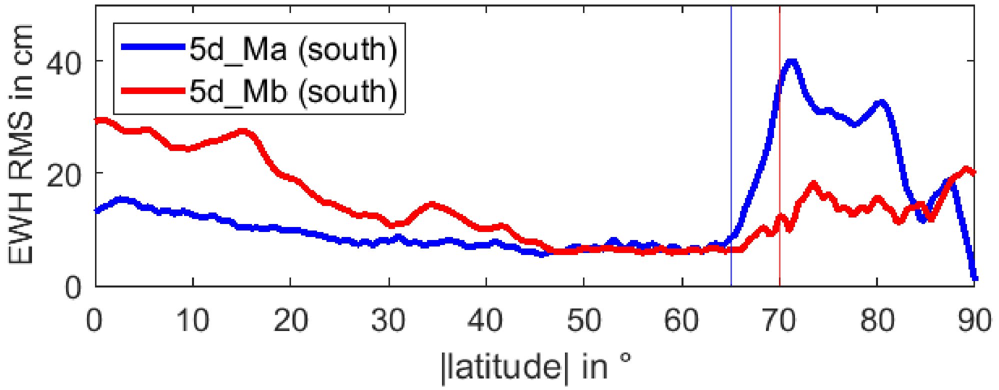



reduced temporal aliasing errors

5d_Mb ($i_2$ = 70°)




## RMS of errors along parallels:



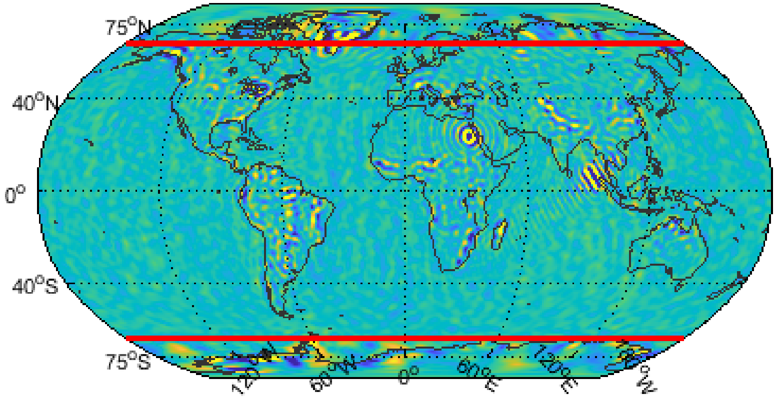

## Area-weighted mean of rms errors in cm:

| | southern hemisphere | | | global |
|---|---|---|---|---|
| | **all $\varphi$** | $\lvert\varphi\rvert < 65°$ | $\lvert\varphi\rvert > 70°$ | **all $\varphi$** |
| 5d_Ma (65°) | 12.56 | 9.76 | 30.94 | 14.81 |
| 5d_Mb (70°) | 17.79 | 18.25 | 14.44 | 18.71 |
| | 65° case 29% better | 65° case 47% better | 70° case 53% better | 65° case 21% better |

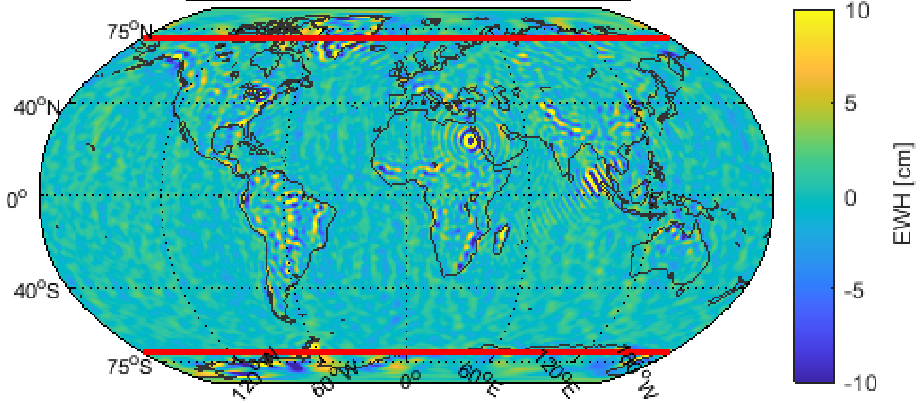
# Impact of the inclination of the second pair – VADER-filtered results

After VADER filtering, $x_\alpha = (N + \alpha M)^{-1}\, N\, x = W_\alpha\, x$ …

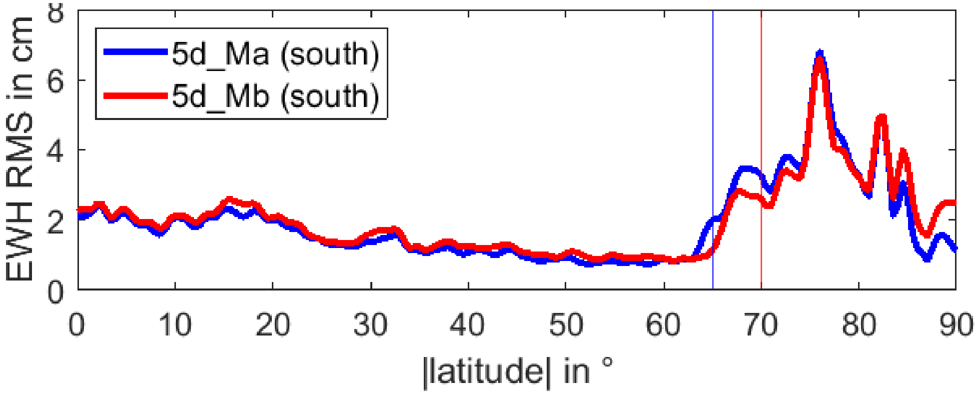
5d_Ma ($i_2$ = 65°)




5d_Mb ($i_2$ = 70°)




RMS of errors along parallels:




Area-weighted mean of rms errors in cm:

| | southern hemisphere | | | global |
|---|---|---|---|---|
| | **all $\varphi$** | $\lvert\varphi\rvert < 65°$ | $\lvert\varphi\rvert > 70°$ | **all $\varphi$** |
| 5d_Ma (65°) | 1.89 | 1.59 | 4.02 | 2.61 |
| 5d_Mb (70°) | 1.94 | 1.72 | 3.88 | 2.64 |
| | 65° case 3% better | 65° case 8% better | 70° case 3% better | 65° case 1% better |

# Impact of the inclination of the second pair – Conclusions

Impact of a smaller inclination $i_2$ of the second pair:

**Unfiltered solutions:**

- larger east-west component of observations

$\Rightarrow$ reduction of sectorial noise for $|\varphi| < i_2$

- larger polar gaps of inclined pair

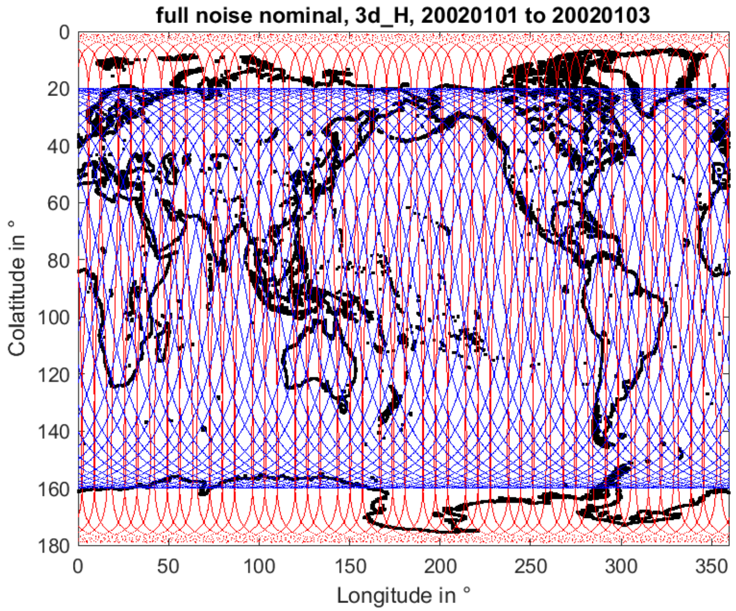
$\Rightarrow$ stronger noise over poles

$\Rightarrow$ larger unobserved region near the poles as stand-alone mission

**VADER-filtered solutions:**

- impact of $i_2$ smaller
- but: need weaker filter if $i_2$ is smaller

# 2. Orbit design aspects for 3-day gravity retrieval




How do the orbits of polar and inclined pair need to be designed for short-term (3-day) gravity retrieval?

# Orbit design aspects for 3-day gravity retrieval

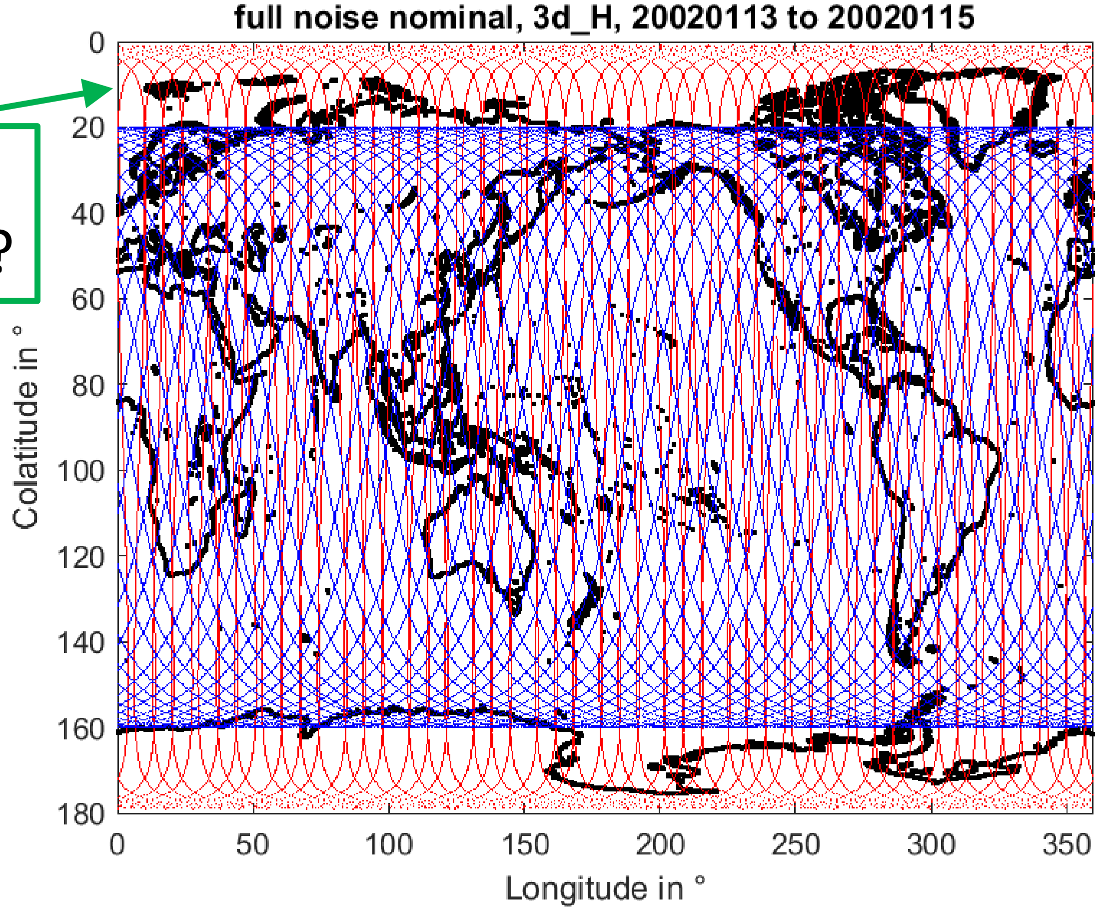
3d_H orbits: both polar and inclined pair have a

- **3-day subcycle**
- with a common longitudinal **drift rate**

-3°/3 days

Aspect 1:

Impact of 3-day subcycle of **polar pair**?



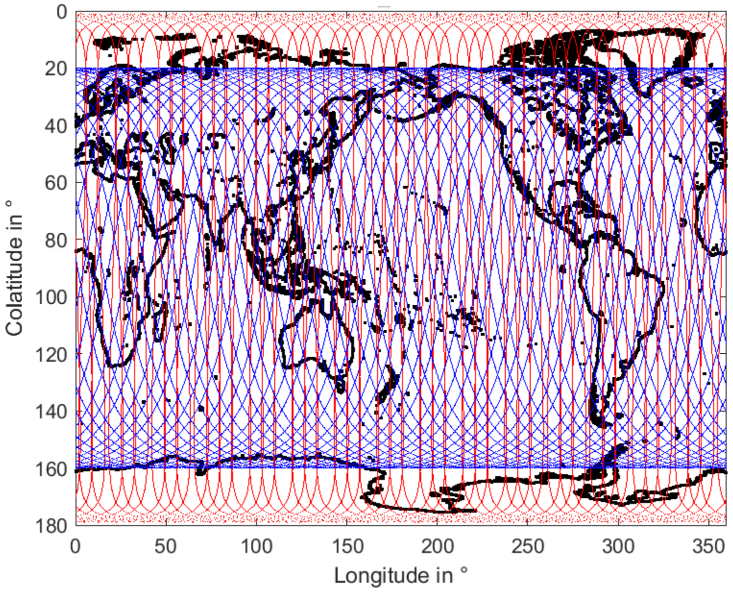

Aspect 2:

Impact of 3-day subcycle of **inclined pair**?

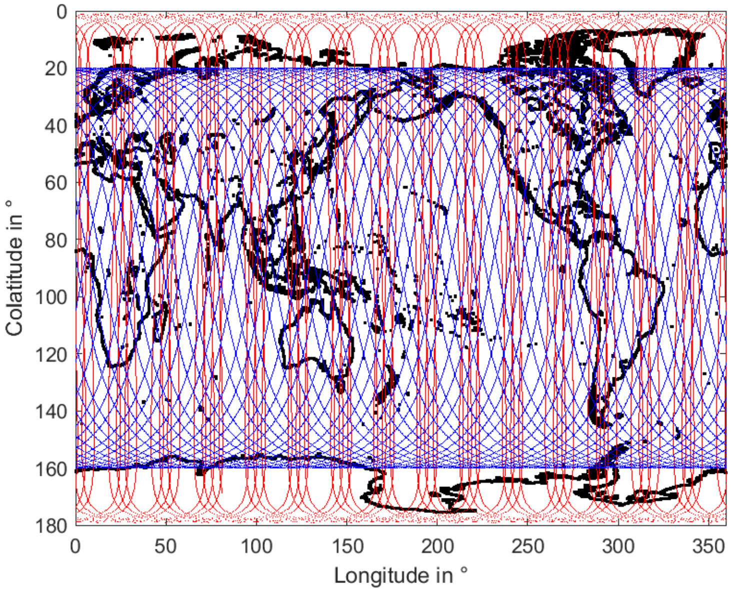
# 2. Orbit design aspects for 3-day gravity retrieval

## Subcycles of satellite orbits - test scenarios

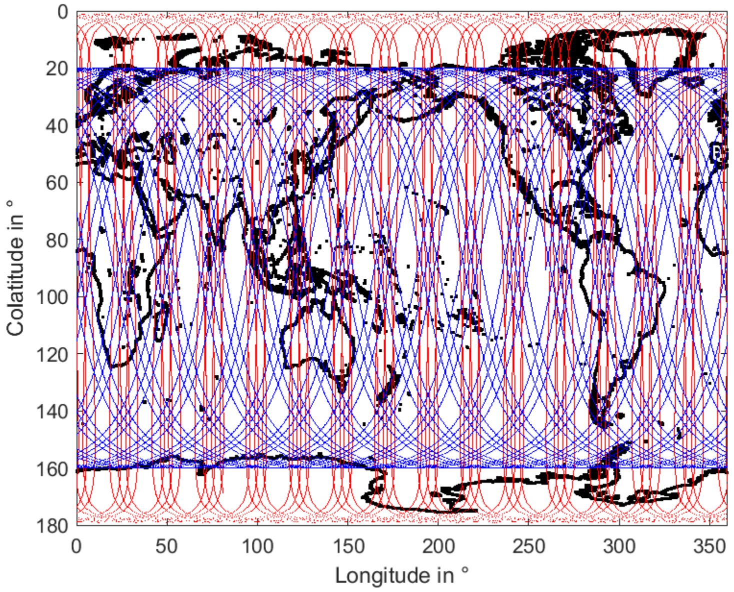
3d_H




U3d5d_H




U5d_H




**PP: 3-day** subcycle → **PP: 5-day** subcycle | PP: 5-day subcycle

IP: 3-day subcycle | **IP: 3-day** subcycle → **IP: 5-day** subcycle

Aspect 1:

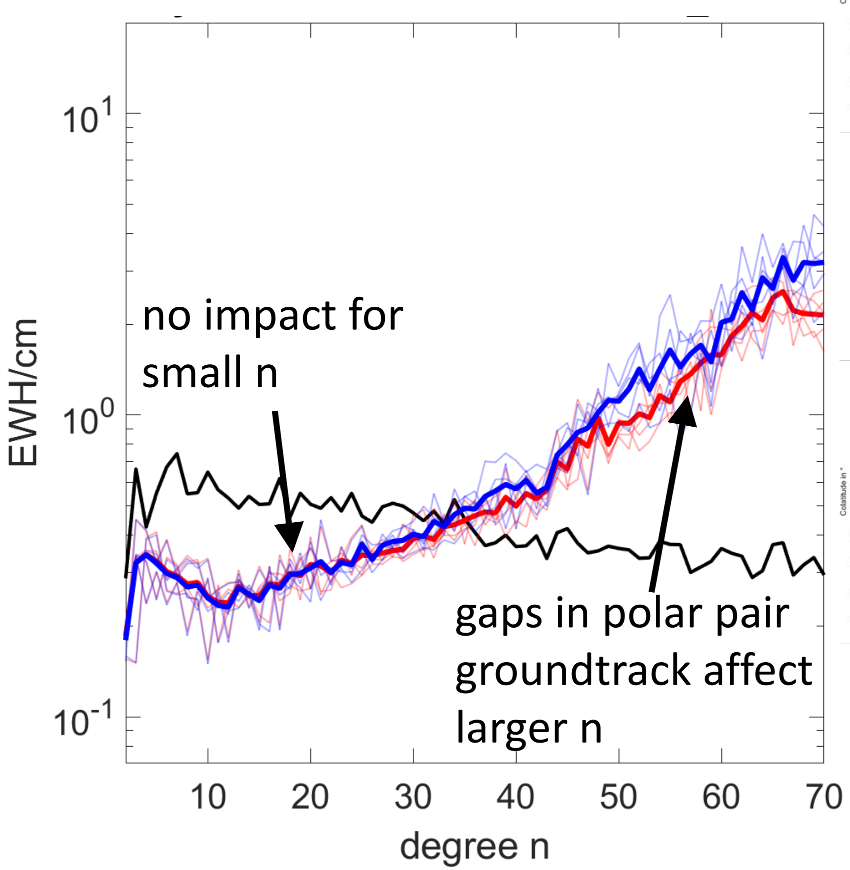
Impact of 3-day subcycle of **polar pair**?

Aspect 2:

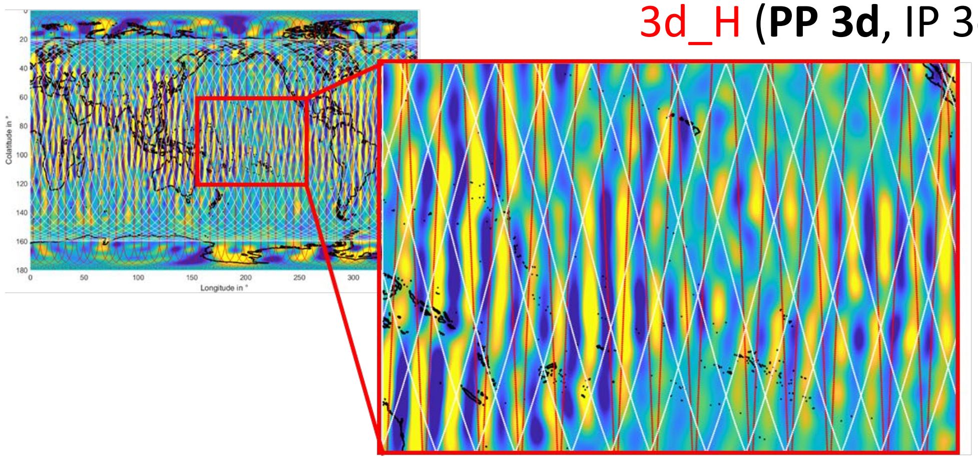
Impact of 3-day subcycle of **inclined pair**?

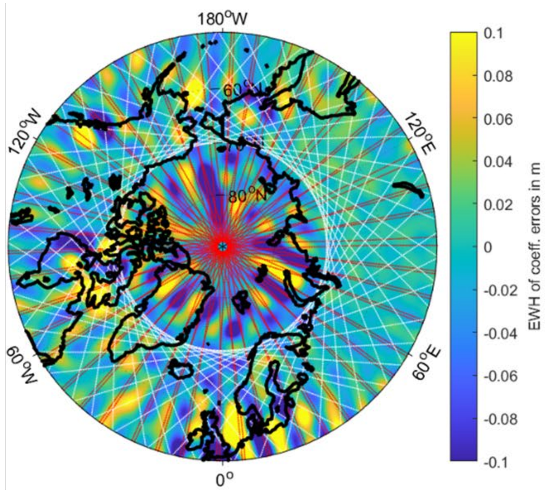
Aspect 1: Impact of 3-day subcycle of **polar pair**

3d_H (**PP 3d**, IP 3d)
U3d5d_H (**PP 5d**, IP 3d)

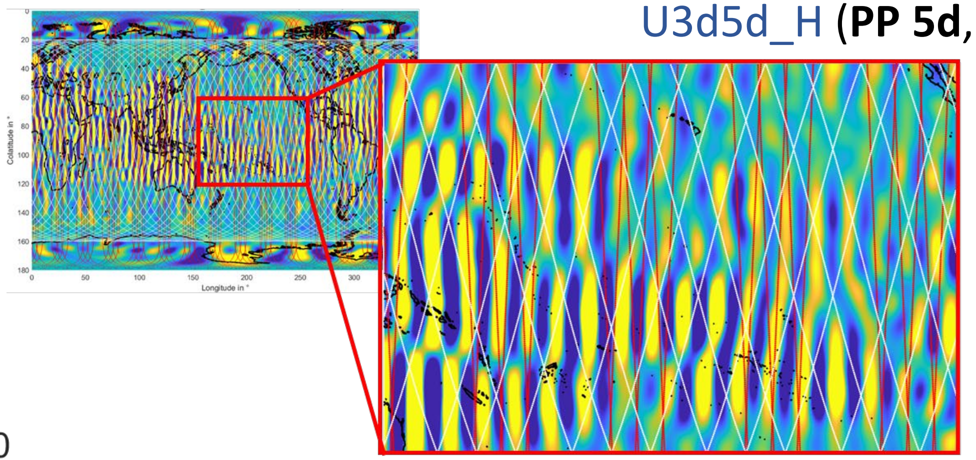


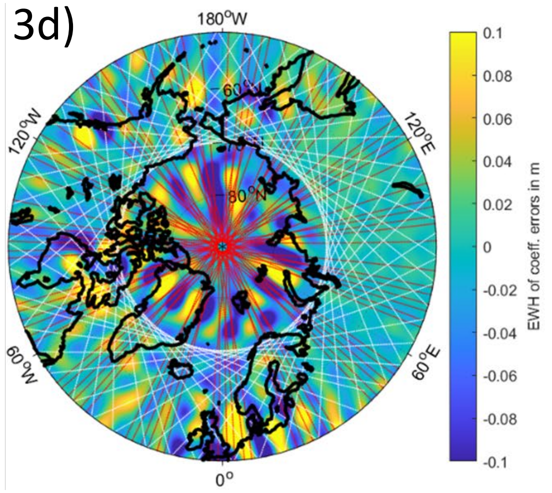

3d_H (**PP 3d**, IP 3d)






U3d5d_H (**PP 5d**, IP 3d)



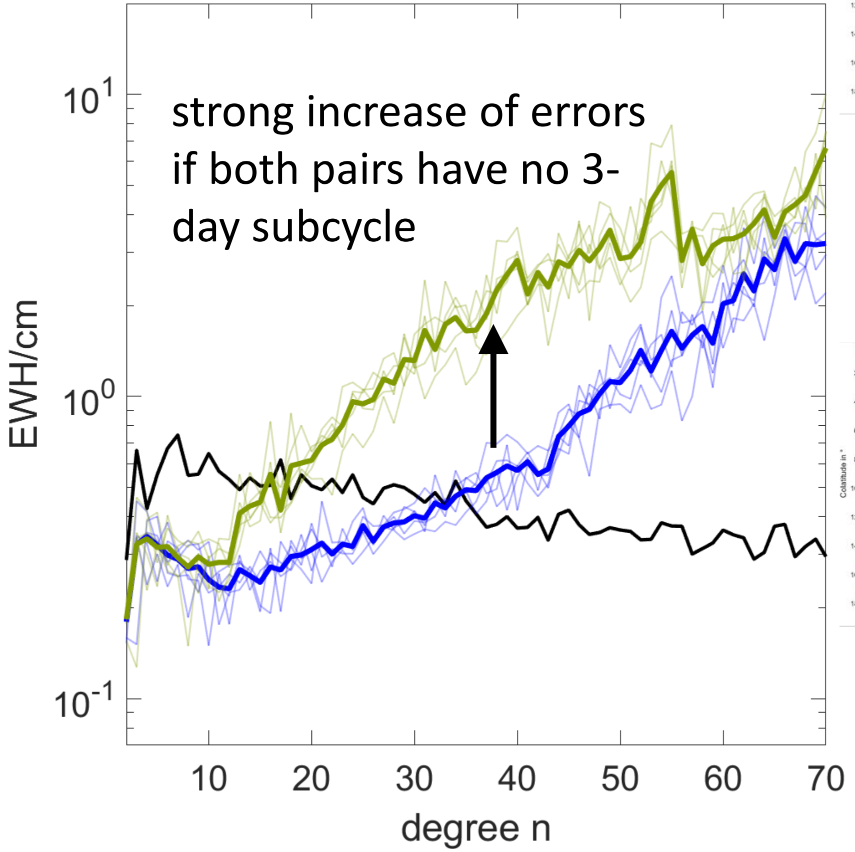


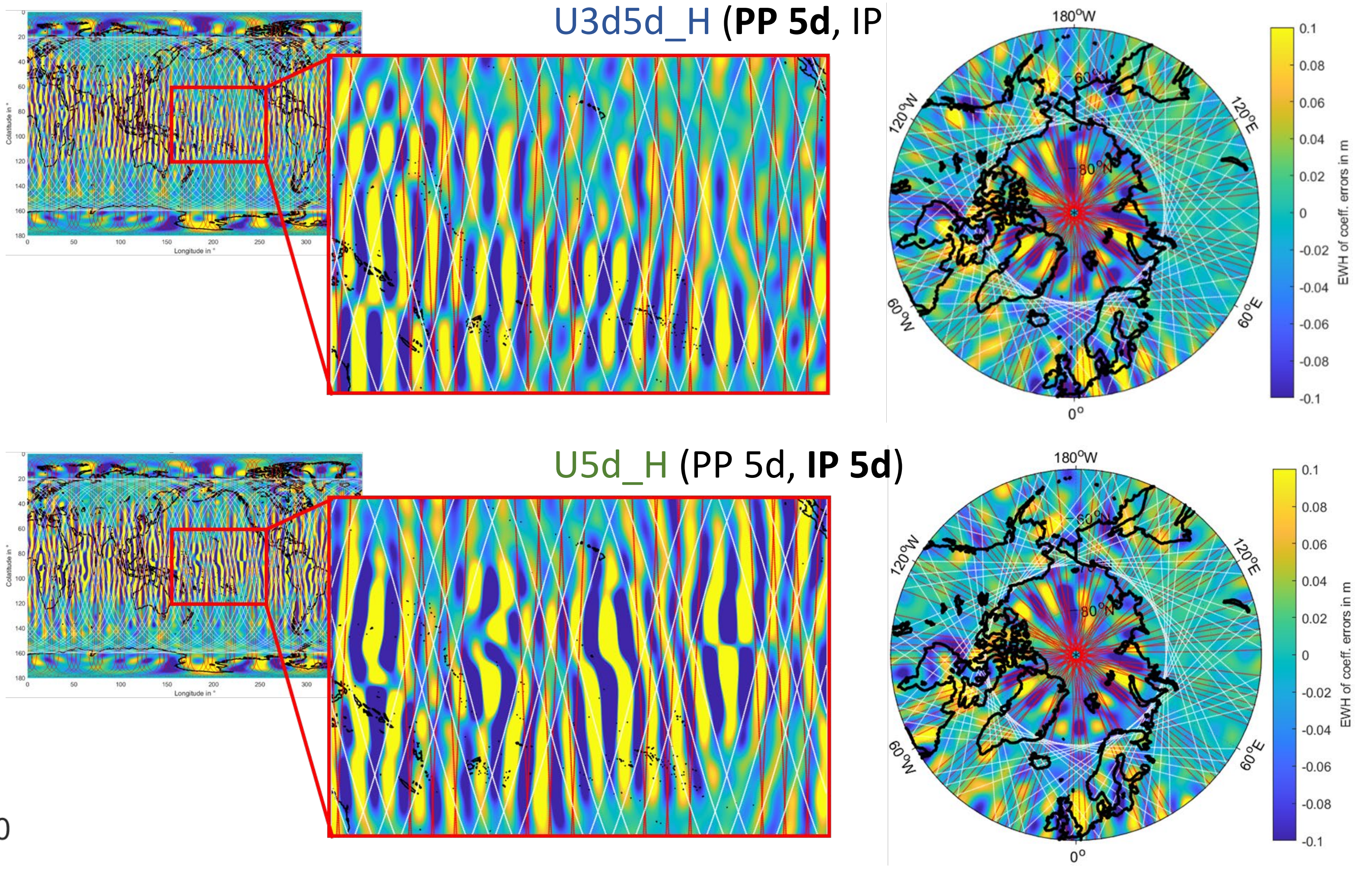

low lat. => more stripes in groundtrack gaps

poles: smaller impact

## Aspect 2: Impact of 3-day subcycle of **inclined pair**

U3d5d_H (PP 5d, **IP 3d**)
U5d_H (PP 5d, **IP 5d**)




strong noise correlated with groundtrack gaps

poles: no impact

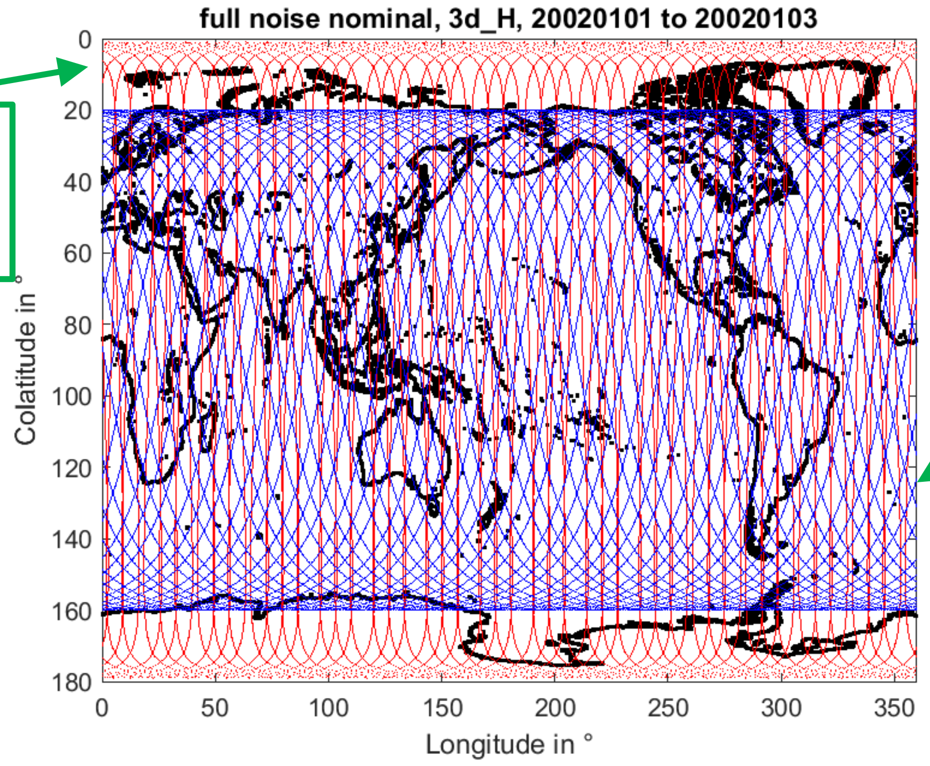
# 2. Orbit design aspects for 3-day gravity retrieval – Conclusions

Impact of orbit subcycles on short-term gravity retrieval:

Aspect 1:

Impact of 3-day subcycle of **polar pair**?

- missing 3-day subcycle of polar pair acceptable, but resolution of small-scale features at low latitudes is affected




Aspect 2:

Impact of 3-day subcycle of **inclined pair**?

- if polar pair orbit is uncontrolled, orbit control for the inclined pair to maintain the groundtrack subcycles is crucial for the homogeneous quality of short-term solutions

$\Rightarrow$ Match between subcycle length and retrieval period affects short-term solutions!

# Summary

## 1. Impact of the inclination of the second pair

Smaller inclination $i_2$ of the second pair
$\Rightarrow$ reduction of sectorial noise for $|\varphi| < i_2$
$\Rightarrow$ increase of noise over poles

} Trade-off between improvement at low latitudes and increasing polar gap

## 2. Orbit design aspects for 3-day gravity retrieval

Non-homogeneous ground track pattern…
… of the polar pair
$\Rightarrow$ increased striping for low latitudes

… of the inclined pair
$\Rightarrow$ severe degradation over whole spectral range

} For short-term solutions, match between orbit subcycles and retreival period should be considered

- Orbit specifications:

Massotti, L.; Siemes, C.; March, G.; Haagmans, R.; Silvestrin, P. Next Generation Gravity Mission Elements of the Mass Change and Geoscience International Constellation: From Orbit Selection to Instrument and Mission Design. Remote Sens. 2021, 13, 3935. https://doi.org/10.3390/rs13193935

- MAGIC Mission Requirements Document:

Next Generation Gravity Mission as a Mass change And Geosciences International Constellation (MAGIC). A joint ESA/NASA double-pair mission based on NASA's MCDO and ESA's NGGM studies. Mission Requirements Document. ESA, Earth and Mission Science Division.

# Appendix

# 2. Impact of the inclination of the second pair

**Table 3.** Candidate orbit sets for inclined and polar pairs recommended for further investigation. The ID shows the number of subcycle days for which the set is optimized as a first step and additional information about the altitudes: mid (M) and high (H).

| ID | Sats 1 (IP) | | Sats 2 (PP) | | $h_{l,1}$ (-) | $h_{l,2}$ (-) | $\Delta\lambda_{shift,1}$ (deg) | $\Delta\lambda_{shift,2}$ (deg) | Subcycle (days) |
|---|---|---|---|---|---|---|---|---|---|
| | Alt. (km) | Incl. (deg) | Alt. (km) | Incl. (deg) | | | | | |
| 3d_M | 409 | 70 | 440 | 89 | 1.368 | 1.383 | 2.308 | 2.384 | 2, 3, 8, 11, 30 |
| 3d_H | 432 | 70 | 463 | 89 | 1.451 | 1.449 | −3.076 | −3.067 | 3, 7, 31 |
| 5d_Ma | 396 | 65 | 434 | 89 | 1.397 | 1.383 | −1.499 | −1.458 | 2, 3, 5, 13, 18, 31 |
| 5d_Mb | 397 | 70 | 425 | 87 | 1.168 | 1.167 | 0.736 | 0.733 | 2, 5, 27, 32 |
| 5d_H | 465 | 75 | 488 | 89 | 1.185 | 1.190 | 0.762 | 0.781 | 4, 5, 29 |
| 7d_M | 389 | 70 | 417 | 87 | 1.238 | 1.253 | 0.743 | 0.786 | 2, 7, 30 |
| 7d_H | 432 | 70 | 463 | 89 | 1.218 | 1.226 | 0.672 | 0.692 | 3, 7, 31 |

Massotti et al. (2021)

# Impact of the inclination of the second pair – VADER-filtered results

Post-processing of the retrieved fields

- VADER filtering (time-**va**riable **de**co**r**relation, Horvath et al (2018)):

$$x_\alpha = (N + \alpha M)^{-1} N x = W_\alpha x$$

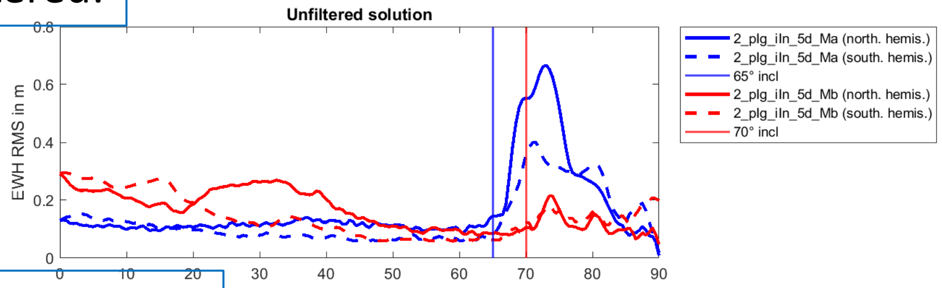
- $x_\alpha$: vector of filtered SH coefficients
- $x$: vector of unfiltered SH coefficients
- $N$: NEQ-Matrix $(A^T PA)$ (= inverse of error VCV)
- $M$: inverse of signal variance matrix (computed from monthly HIS signal)
- $\alpha$: scaling factor (determined such that RMS of global EWH errors is minimized)
- $W_\alpha$: filter matrix

# Impact of the inclination of the second pair – VADER-filtered results

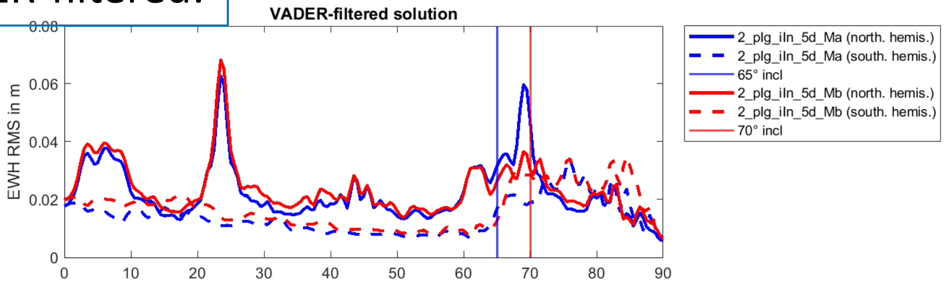
## Impact of VADER filtering:

### EWH RMS values along parallels:

unfiltered:




VADER-filtered:

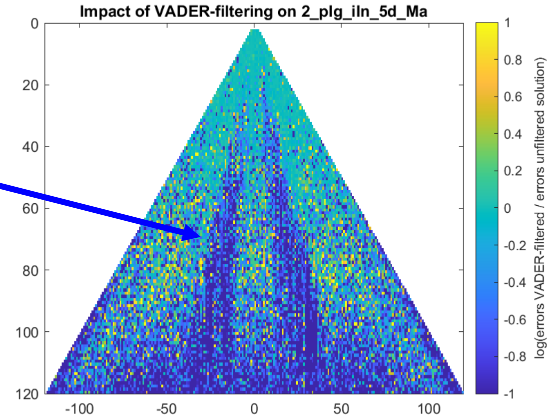



- errors reduced and **less latitude-dependent**
- difference 5d_Ma vs. 5d_Mb reduced
- impact of filtering larger in **areas of larger errors**

### Errors(filtered sol.) / errors(unfiltered sol.):

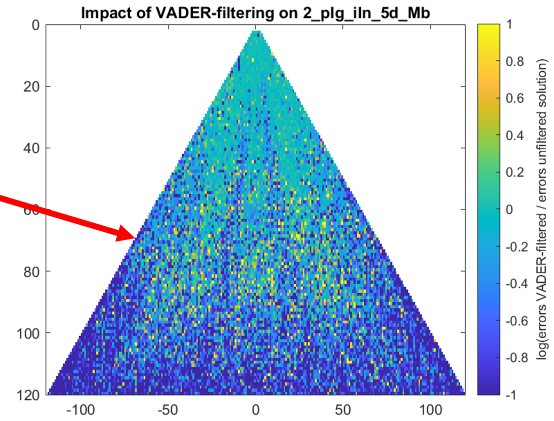
5d_M**a** ($i_2$ = 65°)

- improvement of near-zonal polar gap coefficients




5d_M**b** ($i_2$ = 70°)

- improvement of sectorials




- improvement largest for coefficients of larger errors

## Subcycles of satellite orbits - test scenarios

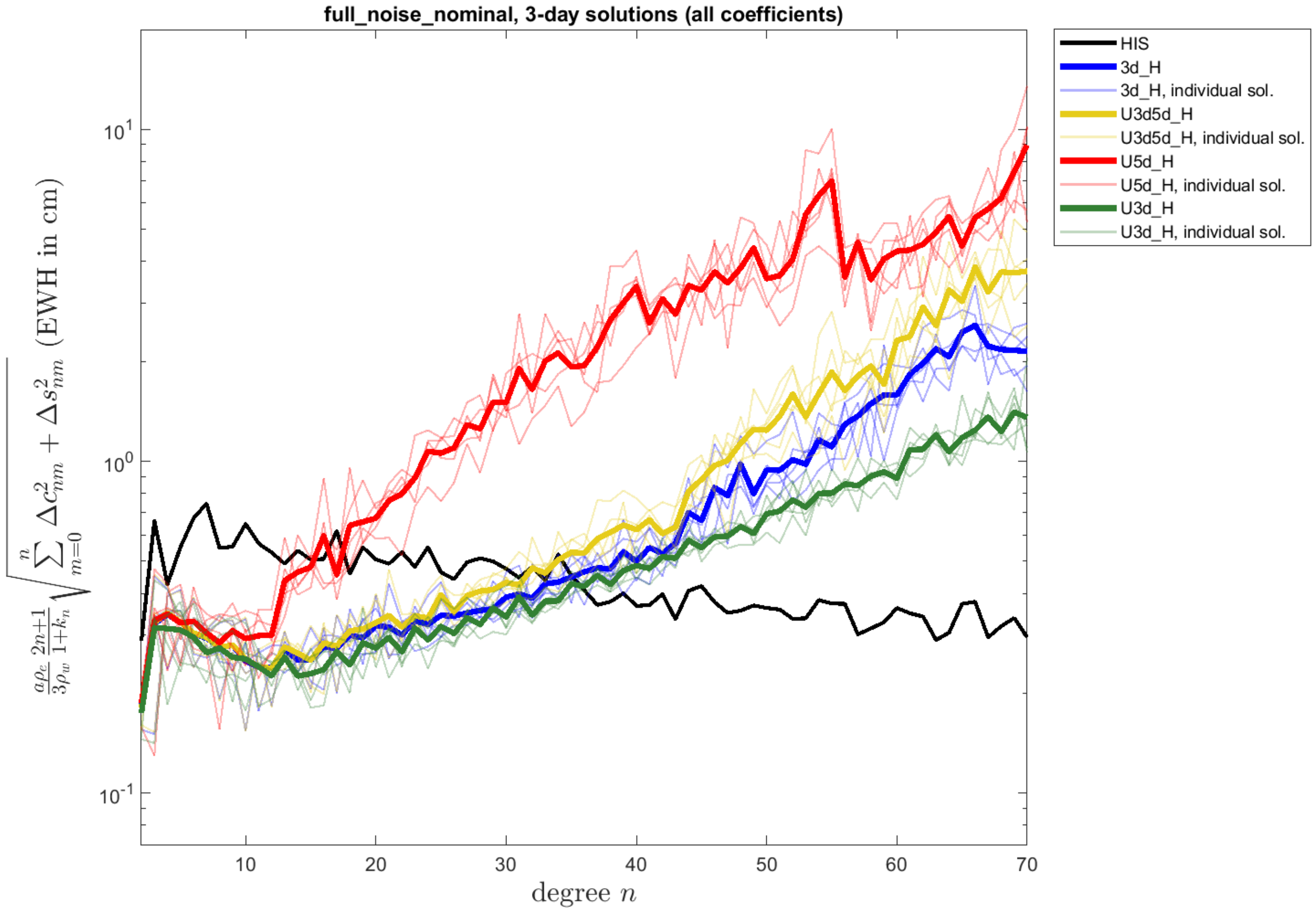
- **double-pair low-low satellite-to-satellite constellations**
  - polar pair: 89° inclination
  - inclined pair: 70° inclination
- **numerical closed-loop simulations** including
  - AO model errors
  - ocean tide model errors
  - HIS signal
  - orbit, ACC, LRI sensor noise
- **ACC noise** assumption:
  - polar pair: GRACE-type noise (~ $10^{-10}$ $m/s^2/\sqrt{Hz}$)
  - inclined pair: NGGM-type noise (~ $10^{-11}$ $m/s^2/\sqrt{Hz}$)
- 5 subsequent d/o 120 **3-day solutions**

# 2. Orbit design aspects for 3-day gravity retrieval

| ID | Sats 1 (IP) | | Sats 2 (PP) | | $h_{l_1}$ [-] | $h_{l_2}$ [-] | Δ(Lon)$_1$ [deg] | Δ(Lon)$_2$ [deg] | Sub-cycles [days] | Retrieval period [days] | MRD |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Alt. [km] | Incl. [deg] | Alt. [km] | Incl. [deg] | | | | | | | Requirements |
| **Coordinated** | | | | | | | | | | | |
| 3d_H | 432 | 70 | 463 | 89 | 1.451 | 1.449 | -3.076 | -3.067 | 3, 7, 31 | 3 | MRD-040 – **Fulfilled**<br>MRD-050 – **Fulfilled**<br>MRD-060 – **Fulfilled** |
| **Uncoordinated** | | | | | | | | | | | |
| U3d5d_H | 432 | 70 | 492 | 89 | 1.451 (3d) | 1.172 (5d) | -3.076 (3d) | -0.790 (5d) | IP: 3, 31<br>PP: 5, 31 | 3 | MRD-040 – **Violated**<br>MRD-050 – **Violated**<br>MRD-060 – **Violated** |
| U5d_H | 460 | 70 | 492 | 89 | 1.061 (5d) | 1.172 (5d) | -0.284 (5d) | -0.790 (5d) | IP: 5<br>PP: 5, 31 | 3 | MRD-040 – **Violated**<br>MRD-050 – **Violated**<br>MRD-060 – **Violated** |
| U3d_H | 402 | 65 | 463 | 89 | 1.382 | 1.449 | 2.380 | -3.067 | IP: 3, 29-30<br>PP: 3, 7, 31 | 3 | MRD-040 – **Fulfilled**<br>MRD-050 – **Violated**<br>MRD-060 – **Violated** |

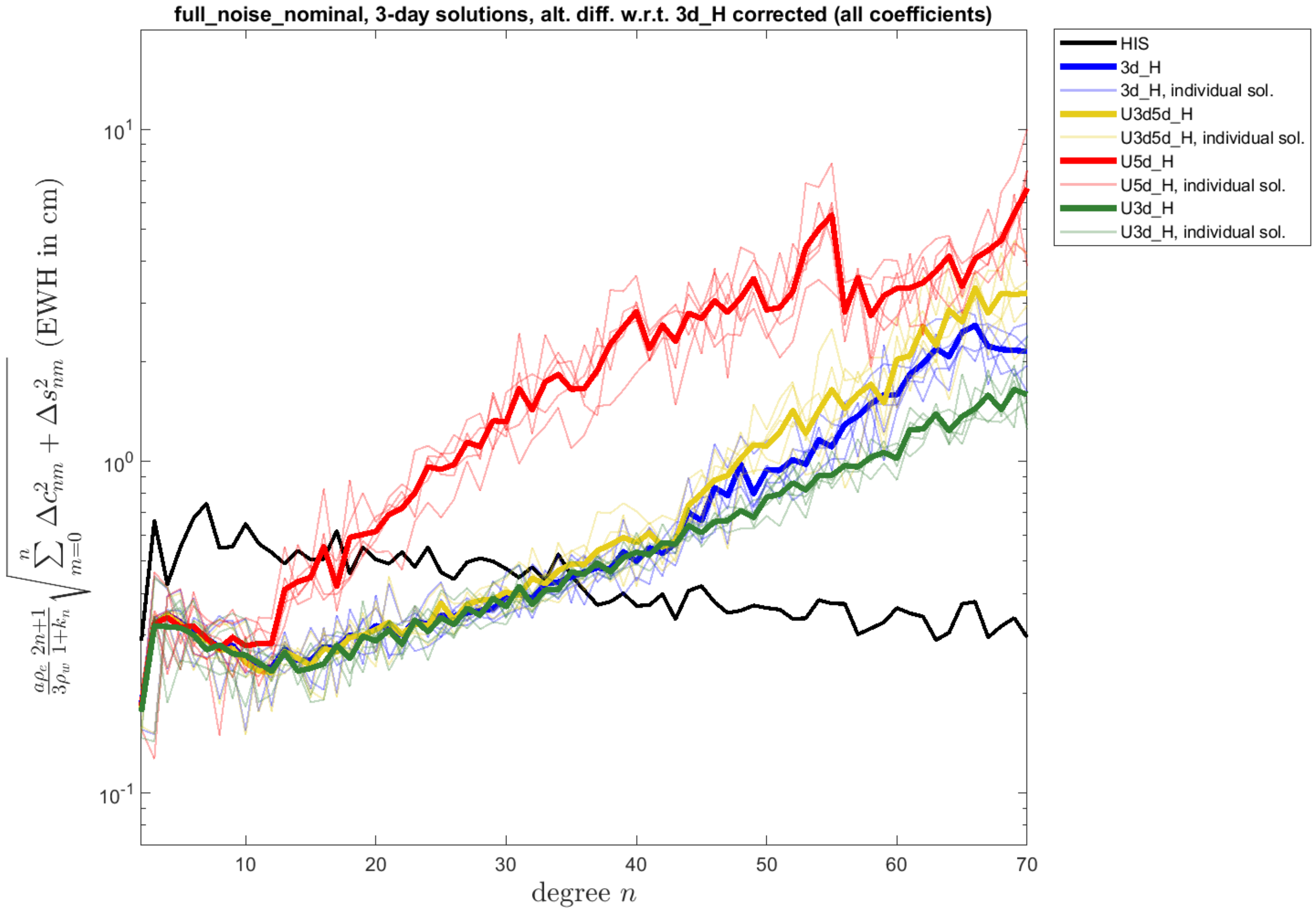
# Orbit design aspects for 3-day gravity retrieval – Correction for altitude effect

## Degree amplitudes (not corrected for altitude):



# Orbit design aspects for 3-day gravity retrieval

## Degree amplitudes (corrected for altitude):



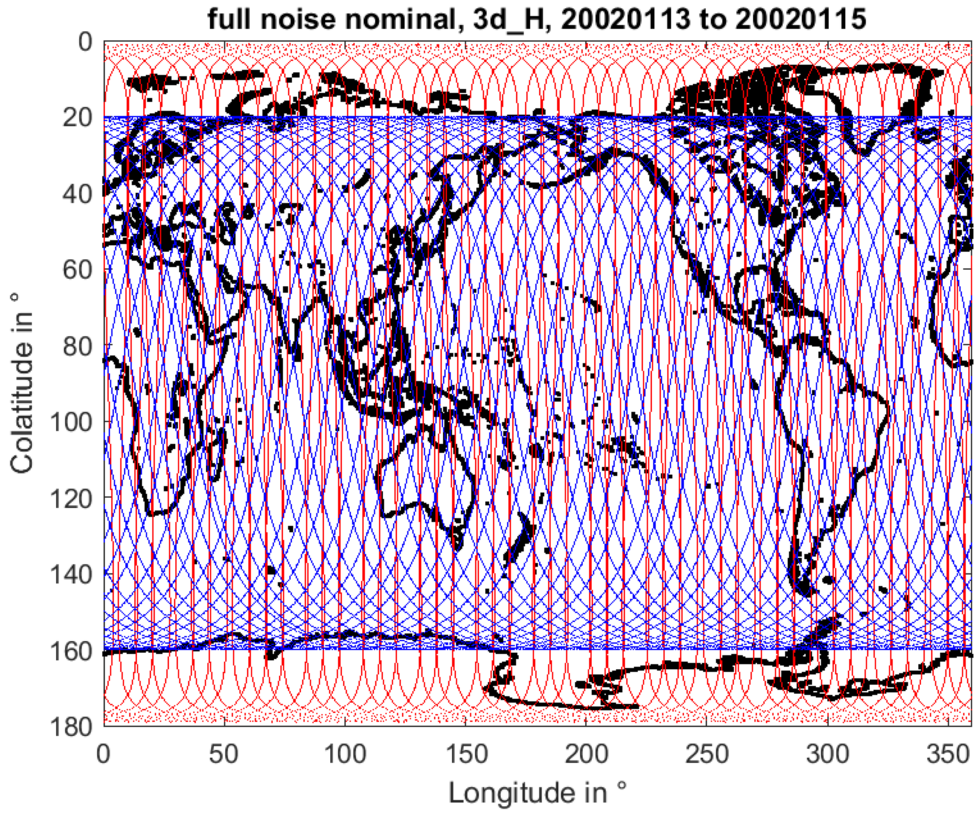

Altitude correction relative to 3d_H (here: in blue)

- Apply degree-dependent factor $\left(\frac{r_{ref}}{r}\right)^{n+2}$ to the SH coefficients
- r: mean radius of polar and inclined pair of the considered scenario
- $r_{ref}$: mean radius of polar and inclined pair of scenario 3d_H
- Background: measurement quantity is range rate, related to gravity accelerations, proportional to $\left(\frac{1}{r}\right)^{n+2}$
- Effect: scenarios U3d_H, 3d_H, U3d5d_H move together for n<40

Aspect 3: Impact of **common drift rate** of polar and inclined pair

3d_H (PP 3d, IP 3d)

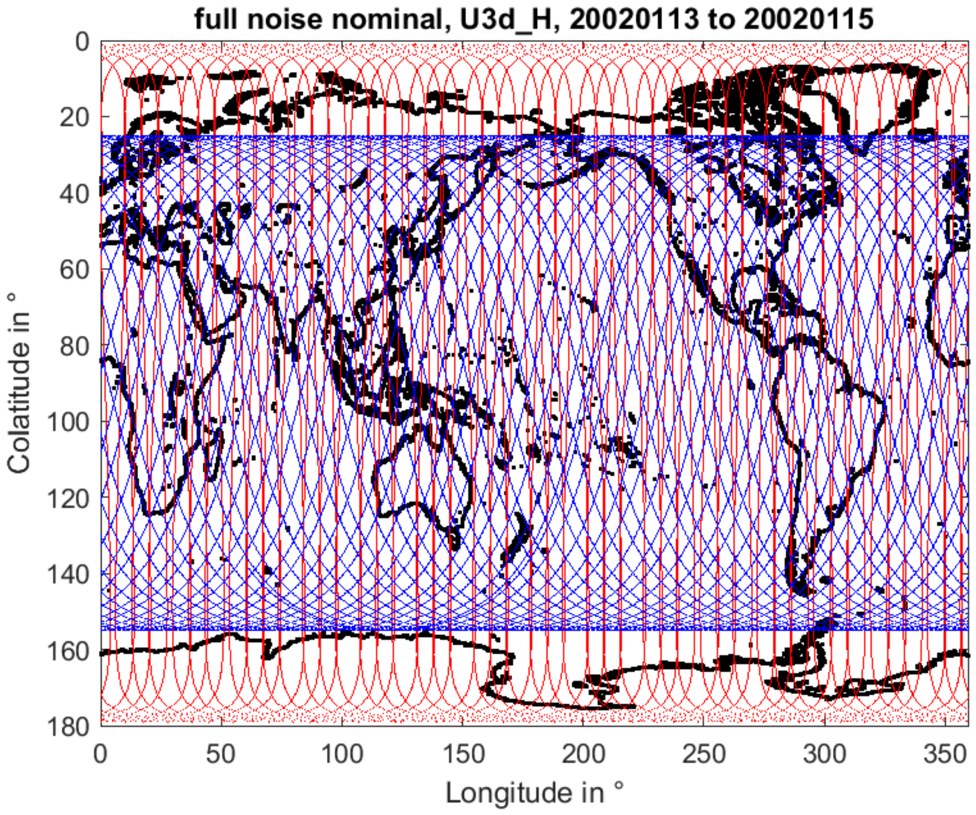
-3°/3 days




U3d_H (PP 3d, IP 3d)

-3°/3 days

2°/3 days

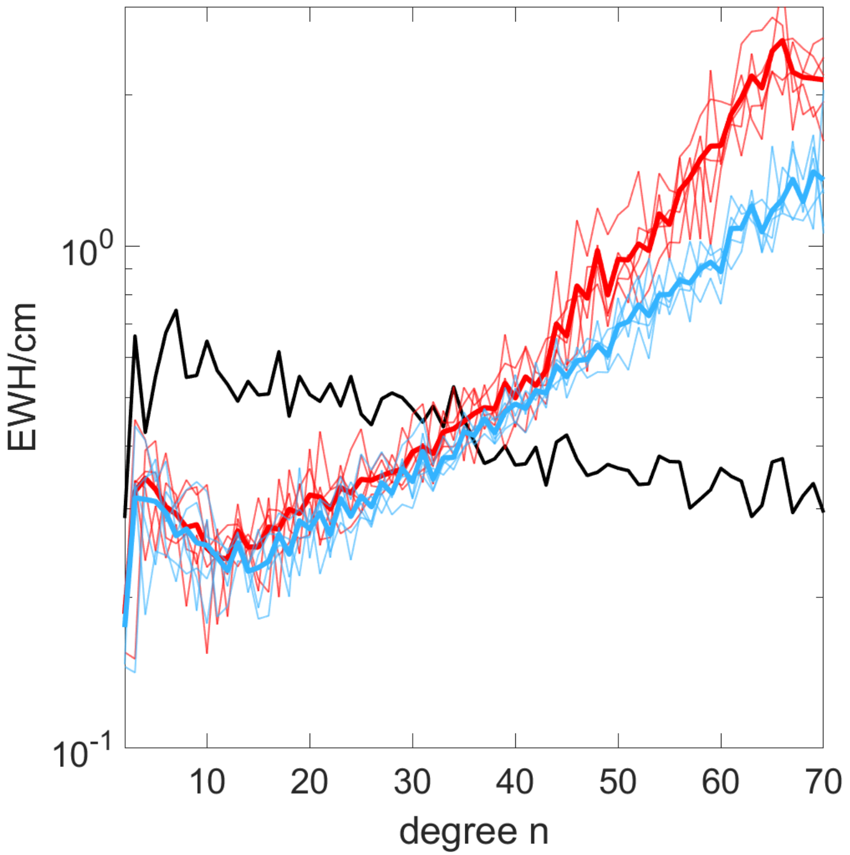


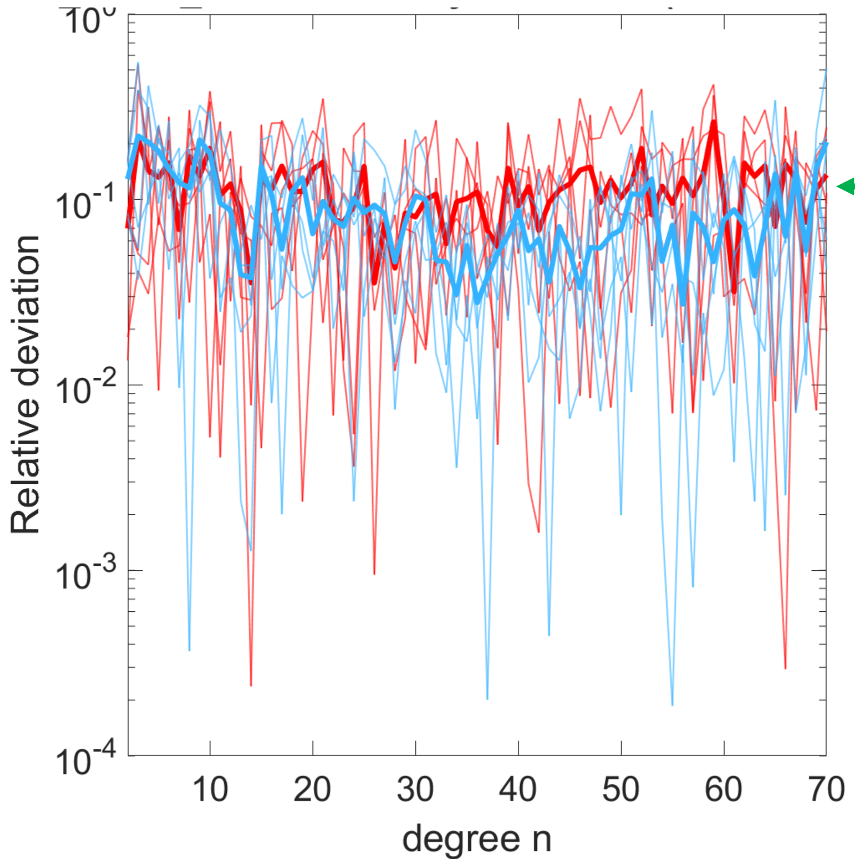
# 2. Orbit design aspects for 3-day gravity retrieval

**Aspect 3: Impact of common drift rate of polar and inclined pair**

3d_H (common drift rate)

U3d_H (uncommon drift rate)




no impact of (un)common drift on error spread of subsequent 3-day solutions visible

- thin: error of individual 3-day solutions
- thick: mean of thin curves

- thin: relative deviation of ind. curves from their mean
- thick: mean of thin curves