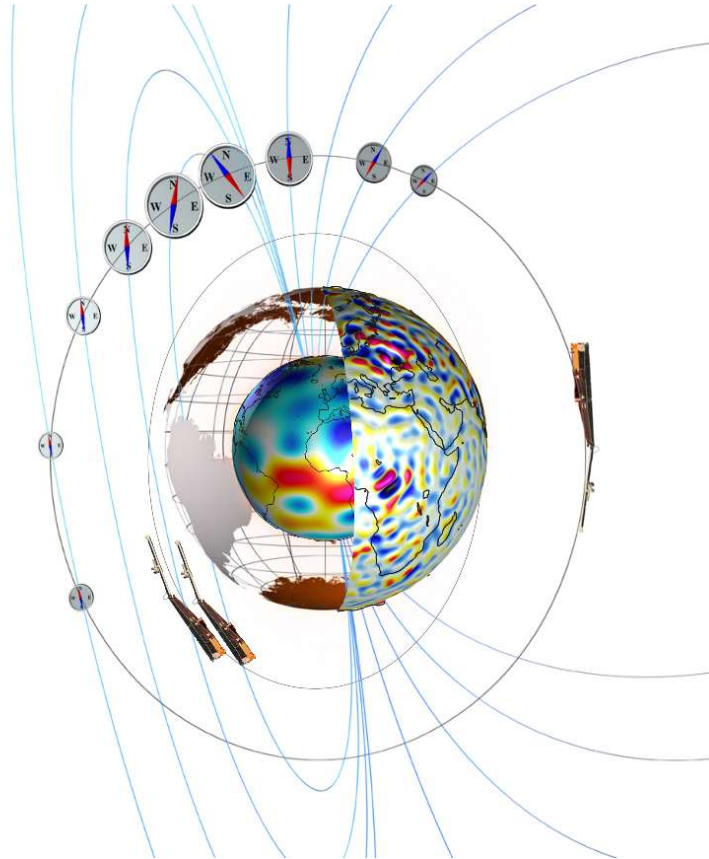


Data, Innovation, and Science Cluster

Swarm FAC_SVD Validation Report



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1 Introduction

1.1 Scope and applicability

This document comprises the summary and main results of the validation activity for the Swarm L2 FAC_SVD product in response to the requirements of [AD-1]

The document and the accompanying Product Definition [AD-2] and Processing Algorithm [AD-3] are available on the SVN at https://smart-svn.spacecenter.dk/svn/smart/SwarmDISC/DISC_Projects/SIFACIT_toolbox/Deliverables

2 Applicable and Reference Documentation

2.1 Applicable Documents

The following documents are applicable to the definitions within this document.

- [AD-1] SW-CO-DTU-GS-032, Rev: 1 Swarm DISC Subcontract with ISS
- [AD-2] SW-TN-ISS-GS-001_FAC_SVD_Product_Definition
- [AD-3] SW-TN-ISS-GS-002_FAC_SVD_Processing_Algorithm

2.2 Reference Documents

The following documents contain supporting and background information to be taken into account during the activities specified within this document.

- [RD-1] Vogt, J., Blagau, A., and Pick, L. (2020). *Robust adaptive spacecraft array derivative analysis*. Earth Space Sci. 7, e00953. doi:10.1029/2019EA000953
- [RD-2] Vogt, J., E. Sorbalo, M. He, A. Blagau, *Gradient estimation using configurations of two or three spacecraft*, Ann. Geophys., 31, 1913-1927, 2013
- [RD-3] Blagau, A., and Vogt, J., SwarmFACE: a Python Package for Field-Aligned Currents Exploration with Swarm, Frontiers in Astronomy and Space Sciences, Volume 9, 2022, <https://doi.org/10.3389/fspas.2022.1077845>
- [RD-4] SW-TN-ISS-GS-002_FAC_LLS_Product_Validation available at https://smart-svn.spacecenter.dk/svn/smart/SwarmDISC/DISC_Projects/SIFACIT_toolbox/Deliverables
- [RD-5] Ritter, P., H. Lühr, J. Rauberg, Determining field-aligned currents with the Swarm constellation mission, *Earth Planets Space*, 65, 1285–1294, 2013.
- [RD-6] SW-TR-GFZ-GS-008, Swarm L2 FAC-dual Product Description
- [RD-7] Toffner-Clausen, L., Lesur, V., Olsen, N., and Finlay, C. C. (2016). In-flight scalar calibration and characterisation of the Swarm magnetometry package, *Earth, Planets, and Space*, 68, 129. <https://doi.org/10.1186/s40623-016-0501-6>

2.3 Abbreviations

A list of acronyms and abbreviations used by Swarm partners can be found [here](#). Any acronyms or abbreviations not found on the online list but used in this document can be found below.

Acronym or abbreviation	Description
dual-s/c	dual spacecraft (method)
EZ	Exclusion Zone
FAC	Field-Aligned Current
FAC_TMS	Swarm L2 FAC-dual product
IRC	Ionospheric Radial Current
L2	Level 2
LLS	Local Least-Squares
MLat	Magnetic Latitude
MLT	Magnetic Local Time
SVD	Singular Value Decomposition
VFM	Vector Field Magnetometer (the Swarm magnetic field instrument)

3 Background

3.1 Context

The algorithm used to generate the FAC_SVD product represents an adaptation of the more general RASADA (from Robust Adaptive Spacecraft Array Derivative Analysis) method and code developed in [RD-1]. RASADA allows to estimate the spatial derivative of physical quantities and their corresponding errors from an array of arbitrary numbers of observation points. Remarkably, the method automatically adapts to possible degenerate geometries of the array configuration, that could be a source of large estimation errors, by identifying and keeping only non-degenerate directions in the analysis.

In the context of dual-s/c methods used to infer the FAC densities from Swarm measurements, such degenerate geometries occur close to the orbital crossing-points (i.e. close to the poles, for the case of nominal orbit configuration), when the satellite's cross-track separation is much smaller than the along-track distance, resulting in a four-point configuration close to a line. Sufficiently far from these regions the quad configuration is considered non-degenerated and the LLS method [RD-2] and SVD provides exactly the same solution. The SVD algorithm automatically adapts close to the poles, when it yields estimates that are close to the average (over the satellites) single satellite estimates. This provides the notable advantage to have dual-s/c based FAC solutions with no data gaps in the region close to the orbital cross-points. The application of dual-s/c SVD method to estimate the FAC densities from Swarm data has been facilitated by the development of the open-source SwarmFACE package [RD-3].

3.2 The dual-s/c FAC_SVD algorithm

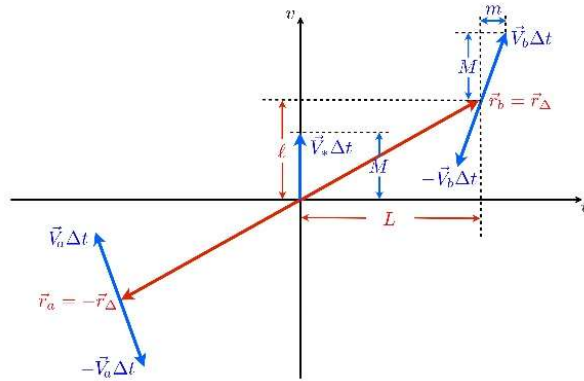


Figure 1. The four point configuration used in the dual-s/c SVD method [RD-2]

Geometry: Figure 1 presents the four-point geometry considered both in the SVD [RD-1] and the LLS [RD-2] [RD-4] methods, i.e. the configuration generated by combining along- and cross-track measurements provided by two Swarm-like satellites a and b. Unlike the method used in the L2 FAC_TMS algorithm [RD-5], restricted to the approximation of rectangles, here a small deviation from parallel satellite orbits as well as skewed configurations are allowed. In the mesocentric frame, spacecraft position vectors are $\mathbf{r}_a = -\mathbf{r}_\Delta$ and, respectively, $\mathbf{r}_b = \mathbf{r}_\Delta$ with $\mathbf{V}_a$ and $\mathbf{V}_b$ the corresponding velocities. The planar four-point configurations is formed by combining points along the orbits separated in time by $2\Delta t$. The coordinate system $(\hat{\mathbf{u}}, \hat{\mathbf{v}})$ has the $\hat{\mathbf{v}}$ axis along the mesocenter velocity $\mathbf{V}_* = (\mathbf{V}_a + \mathbf{V}_b)/2$ and the $\hat{\mathbf{u}}$ axis along the difference vector $\mathbf{V}_\Delta = (\mathbf{V}_a - \mathbf{V}_b)/2$. The quantity $2L = 2\hat{\mathbf{u}} \cdot \mathbf{r}_\Delta$ is the spatial separation between the satellite orbits, $2l = 2\hat{\mathbf{v}} \cdot \mathbf{r}_\Delta$

reflects the satellites' orbital phase separation, $2\mathbf{M} = 2\hat{\mathbf{v}} \cdot \mathbf{V}_* \Delta t$ describes the along track separation and m is defined by $2\mathbf{m} = 2\hat{\mathbf{v}} \cdot \mathbf{V}_\Delta \Delta t$.

Gradient estimation: The following computation steps are considered in the Swarm FAC_SVD algorithm to compute the radial current and the corresponding estimation error (from [RD-3]; see [RD-1] for more details):

- 1) The quad's position vectors $\mathbf{r}_\sigma$ computed with respect to the quad center (i.e. in the mesocentric frame, where $\sum_{\sigma=1}^4 \mathbf{r}_\sigma = 0$) are stored as rows in the (4×3) position matrix $\mathbf{R}$. The position matrix $\mathbf{R}$ is related to the position tensor $\mathbf{R}_{pos} = \sum_{\sigma=1}^4 \mathbf{r}_\sigma \mathbf{r}_\sigma^t$ by $\mathbf{R}_{pos} = \mathbf{R}^t \mathbf{R}$.
- 2) The SVD of the position matrix is performed, $\mathbf{R} = \mathbf{U} \mathbf{S} \mathbf{V}^t$, where $\mathbf{U}$ is a (4×3) matrix satisfying $\mathbf{U} \mathbf{U}^t = \mathbf{I}$ (with $\mathbf{I}$ the unit matrix) and $\mathbf{V}$ is a (3×3) orthogonal matrix satisfying $\mathbf{V} \mathbf{V}^t = \mathbf{V}^t \mathbf{V} = \mathbf{I}$. The (3×3) matrix $\mathbf{S}$ is diagonal; since in our case the configuration is degenerated to a planar geometry, $\mathbf{S}$ has only two non-zero elements, $\mathbf{S} = \text{diag}(R^{(1)}, R^{(2)}, 0)$. Here $R^{(1)} \geq R^{(2)} \geq 0$.
- 3) Form a stable pseudoinverse of $\mathbf{S}$. For that purpose, compute $\tau = R^{(2)}/R^{(1)}$; if τ is below a set threshold τ_0 , meaning that the quad's configuration is too elongated along the direction associated with $R^{(1)}$ (i.e. close to linear degeneracy), consider only the first eigenvalue during the inversion:

$$\mathbf{S}^- = \begin{cases} \text{diag}(1/R^{(1)}, 1/R^{(2)}, 0) & \text{if } \tau \geq \tau_0 \\ \text{diag}(1/R^{(1)}, 0, 0) & \text{if } \tau < \tau_0 \end{cases}$$

- 4) Compute the (4×3) matrix $\mathbf{Q} = \mathbf{U} \mathbf{S}^- \mathbf{V}^t$. The rows of $\mathbf{Q}$ are the (generalized) reciprocal vectors $\mathbf{q}_\sigma, \sigma = 1, \dots, 4$
- 5) The magnetic field perturbations $\mathbf{b}$ are computed as deviations of VFM (the Swarm magnetic field instrument) magnetic vector data with respect to the magnetic field model. Considering the magnetic field perturbation at the quad's vertices $\mathbf{b}_\sigma, \sigma = 1, \dots, 4$, the $\text{curl } \mathbf{c} = \nabla \times \mathbf{b}$ is estimated through $\mathbf{c} \simeq \sum_{\sigma} \mathbf{q}_\sigma \times \mathbf{b}_\sigma$. The current density along the quad's normal j_n (i.e. the Ionospheric Radial Current) can thus be estimated. The FAC density is subsequently derived from the IRC by considering the local inclination β of the magnetic field with respect to the quad's normal, i.e. $j_{fac} = j_n / \cos \beta$.
- 6) Assuming isotropic and mutually uncorrelated instrumental errors δb for each component of $\mathbf{b}$, the mean square error of the radial current j_n through the spacecraft plane is provided by

$$\langle |\delta j_n|^2 \rangle = \text{trace}(\mathbf{Q}^t \mathbf{Q}) (\delta b)^2 / \mu_0^2$$

Equation 1

Combining solutions: According to third step above, when $\tau \geq \tau_0$ the quad configuration is considered non-degenerated (to a linear geometry) and the LLS and SVD provides exactly the same solution. Near the orbital cross-points, when $\tau < \tau_0$, only the eigenvector associated with the biggest eigenvalue (i.e. the along-track direction) is used in the analysis and the algorithm yields estimates that are close to the average (over the satellites) single-satellite estimates.

Nevertheless, the transitions between different the gradient estimators at the point where the singular ratio threshold τ_0 is crossed leads to discontinuous current density solutions. To avoid that, the transitions may be smoothed by using a pseudoinverse matrix of the form $\mathbf{S}^- = \text{diag}(1/R^{(1)}, f(\tau)/R^{(2)}, 0)$, with the function $f = f(\tau)$ defined in a stepwise manner:

$$f(\tau) = \begin{cases} 1 & \text{if } \tau \geq \tau^* \\ g(\tau) & \text{if } \tau^* > \tau \geq \tau_0 \\ 0 & \text{if } \tau_0 > \tau \end{cases}$$

A second threshold parameter, i.e. $\tau^* \geq \tau_0$, is thus introduced. The function $g = g(\tau)$ allows for continuous connection between the first ($\tau \geq \tau^*$) and third ($\tau < \tau_0$) branches of f . Two options are implemented for g , i.e. the linear interpolation

$$g_{\text{lin}}(\tau) = \frac{\tau - \tau_0}{\tau^* - \tau_0}$$

Equation 2

and cubic (spline) interpolation, which allows for smooth (differentiable) continuation

$$g_{\text{cub}}(\tau) = \left(3 - 2 \frac{\tau - \tau_0}{\tau^* - \tau_0}\right) \left(\frac{\tau - \tau_0}{\tau^* - \tau_0}\right)^2$$

Equation 3

The original behaviour (no interpolation) is recovered with $\tau^* = \tau_0$.

Data filtering: In order to reduce the influence of local small-scale fluctuations and to match the scale of the magnetic perturbations with the configuration scale, the magnetic field perturbations recorded by the two satellites are pre-processed by applying a low-pass filter. To generate the L2 FAC_TMS product, a 2-pole low-pass filter with -3dB cutoff period of 20 s is used ([RD-5], [RD-6]), which concentrates the analysis on the largescale FACs (scale length larger than ~ 150 km). Similarly, in the case of the FAC_SVD product a 20-s low-pass Butterworth filter has been used, which provides the flattest frequency response within the pass band. This choice also keeps compatibility with the data filtering used for the FAC_LLS product ([RD-4]).

4 Product validation

4.1 Validation using synthetic data

The more general RASADA method and code have been extensively tested on synthetic data, see for that [RD-1] and its supplementary material. In the context of Swarm, the dual-sat FAC_SVD algorithm has undergone the same type of tests described in [RD-4], to acquire information on its performance on various current distributions.

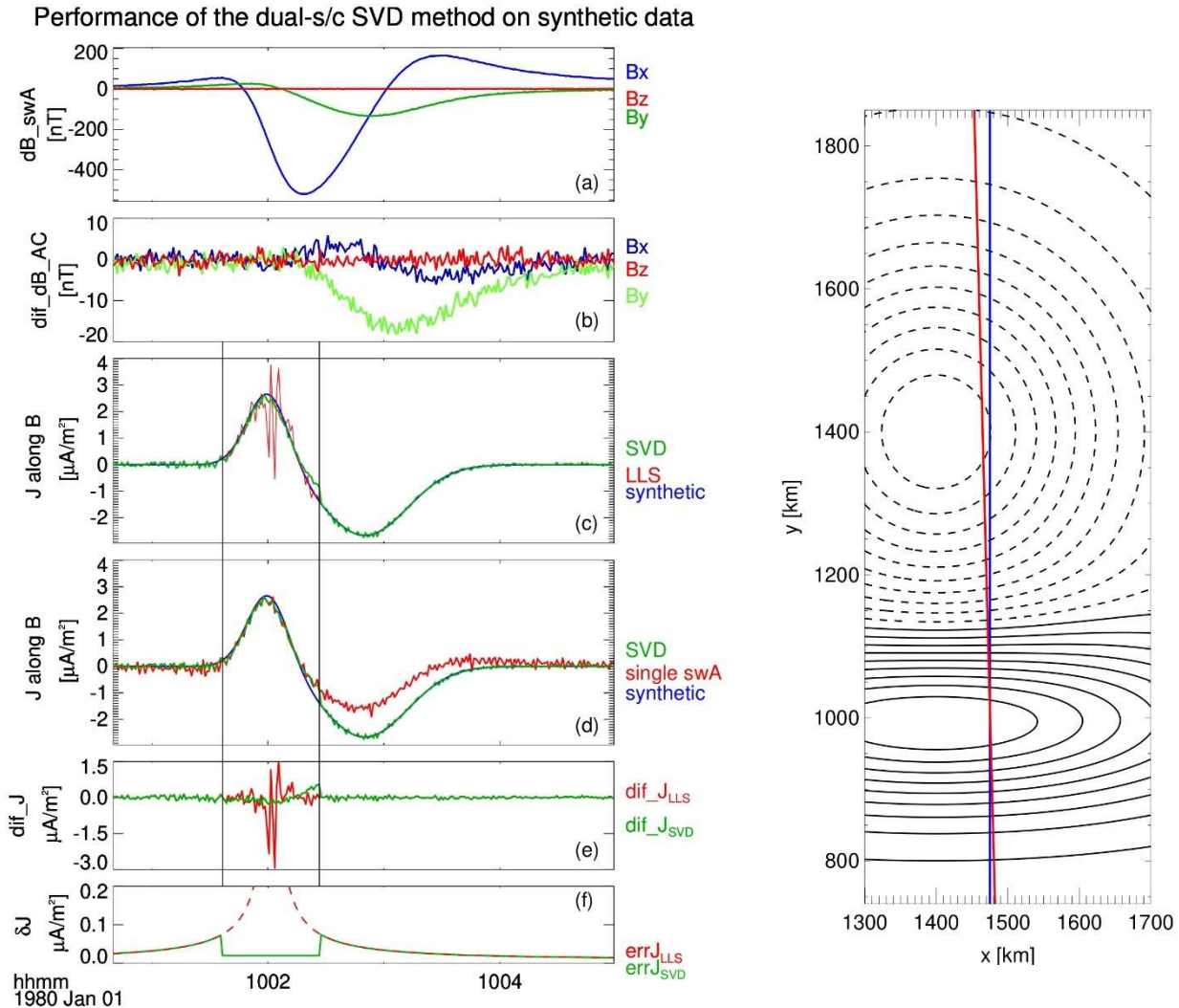


Figure 2. Dual-s/c SVD FAC algorithm performance on synthetic data

Figure 2 presents the results of one such test, when two Swarm-like sensors (A and B) are sampling a sequence of upward and downward cylindrical current structures of different scale lengths. The satellite orbits are shown on the right as solid lines that crossed each other, overlaid on the current density contour plot. All the parameters of the simulation (spatial scales, orbits, current intensity, etc.) were chosen to be close to the Swarm context. An artificial noise δb (randomly distributed values of 0.5 nT standard deviation, in accordance with Swarm VFM instrumental noise, see [RD-7]) has been added to the data to simulate the measurement errors. No data filtering has been used in the simulation.

Panel (a) presents the vector magnetic field perturbation recorded by sensor A, with x , y , and z components corresponding to the horizontal, vertical, and out of the paper directions for the orbit plot on the right. Panel (b) shows the difference in magnetic field perturbation measured by the two sensors. In panel (c), the dual-s/c FAC estimations based on SVD (green) and LLS (red) methods are compared with the simulated current density at the quad's mesocenter (blue). Similarly, panel (d), compares the dual-s/c FAC SVD estimation (green) and the FAC density provided by the single-s/c method (red) with the simulated current density (blue). The two branches of the SVD solution are clearly visible: within the interval between the vertical lines (adjustable by setting different values for the parameter τ), the quad configuration is considered too elongated and the SVD solution is computed using only the along track gradients of the magnetic field perturbation. There, the SVD method clearly provide more stable results than the LLS method (see panel (c)) but in the same time, the SVD estimation becomes less accurate when the cross-track gradient in the magnetic field (more evident in panel (b), the green trace) starts to increase, as can be seen close to the second vertical line. It is therefore beneficial to use the full (planar) magnetic field gradient as much as possible along the orbit. Note the discontinuity in FAC density, more visible at the second vertical line; this can be avoided by implementing a transition zone between the different SVD gradient estimators (see Equation 2 and Equation 3 above).

Panel (e) presents the deviation of FAC SVD (green) and FAC LLS (red) estimations from the actual, i.e. simulated, current density. The last panel presents the corresponding FAC estimation errors based on Equation 1 (SVD, in green) and similar the equation from [RD-4] (LLS, in red). It is clear from this panel that the errors in current density are evaluated by taking into account only the instrumental noise, although the possible presence of magnetic cross-track gradients close to the cross-orbital point could have a significant impact.

4.2 Choosing the appropriate processing parameters

As mentioned in Section 3.2, the FAC_SVD algorithm requires some input parameter(s) to combine the solution based on the full (planar) gradient of the magnetic field perturbation and the solution based solely on the along-track gradient. Instead of using just one parameter τ that switches between the two solutions when the quad configuration becomes too elongated, leading thus to discontinuous current density estimations, we decided to use a combination of two parameters, i.e. τ_0 and τ^* that establishes a transition region where the two solutions are combined to ensure a smooth evolution.

In order to decide the optimum values of the parameters and, at the same time, to validate the FAC_SVD product we compared the dependence with latitude of the average FAC_LLS and single-sat FAC densities with that of average FAC_SVD density obtained for different combinations of τ_0 and τ^* . For the same reasons explained in [RD-4] (i.e. cross-orbital points located mainly inside the polar cap, with no significant currents expected at the highest geographic latitudes apart from those caused by the presence of local small-scale amplitude waves), only the region close to the North pole have been analyzed.

Figure 3, based on magnetic field data provided by Swarm lower satellites between 17 April and 31 December 2014, presents four such comparisons. In each panel, the orange trace shows the evolution of average FAC_LLS density, with roughly constant values up to $\sim 86.6^\circ$ geographic latitude (i.e. the validity range of the product, indicated by the vertical dashed line in each panel, see [RD-4]), followed by a rapid increase due to the instability of this solution close to the orbital crossing point. The green trace presents the average single-sat FAC density based on the magnetic field observations provided by Swarm A. We generated these estimates with an in-house algorithm that applies the same data filter (i.e. Butherworth order 5, 20 s, low-pass filter) used in the pre-processing stage for the FAC_SVD and FAC-LLS products and computes the magnetic gradient using an along-track distance of 5 seconds (i.e. not 1 sec., as in the case of the official FAC L2

product, see [RD-5]). The average single-sat FAC estimation does not take into account the magnetic field gradients in the cross-track direction and slightly decreases with increasing latitude. (The similar quantity based on Swarm C data assumes essentially the same values and therefore an average over these two data sets – as in the case of SVD algorithm at the highest latitudes - is very well represented by just Swarm A FAC density).

Average FAC_SVD densities near the N pole for different τ parameters

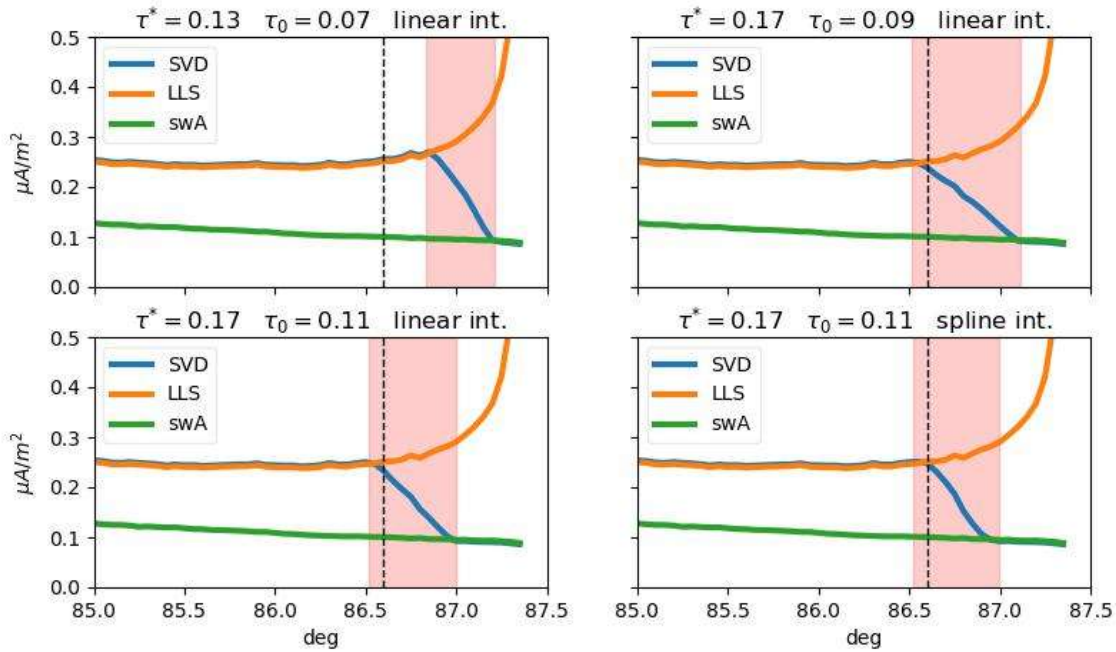


Figure 3. Latitude dependence of averaged FAC_SVD for different processing parameters

The average FAC_SVD estimations for different parameters τ_0 and τ^* , and ways of combining the two branches of the SVD solution ('Linear' and 'Cubic' interpolation) are presented with blue traces in different panels of Figure 3. Outside the transition region, shown by the pink vertical band, the FAC_SVD solutions provide the same current density estimations as either the FAC_LLS product (at the lower latitude) or the single-sat FAC estimations (at higher latitude). This aspect provides a strong validation for the FAC_SVD product.

From the first panel (upper left in Figure 3), a value of 0.13 for τ^* (parameter associated with the left margin of the transition zone) has been considered too small due to the slight increase above $\sim 86.6^\circ$ geographic latitude. Similarly to the case of FAC_LLS product (that predict the same values), one can regard the increase as artificially caused by the satellite close separation in the cross track direction (see [RD-4]). Similarly, from the second panel (upper right in Figure 3), a value of 0.09 for τ_0 (parameter associated with the right margin of the transition zone) has been considered too small since the artificial high values of the FAC_LLS solutions have still a big influence at the end of the transition zone.

A pair of values that seems to balance different aspects (e.g. use as much as possible the full planar magnetic field gradient, but avoid significant contribution in the FAC_SVD product from artificial effects) has been achieved for $\tau_0 = 0.11$ and $\tau^* = 0.17$, see the last two panels (at the bottom of Figure 3). The final choice was obtained by imposing (within the transition zone) a mixture of the two SVD branches based on cubic spline interpolation (see Section 3.2), i.e. the panel at the bottom right in Figure 3. This ensures also a smoother individual FAC_SVD solutions, as can be seen in the two events shown in Figure 4.

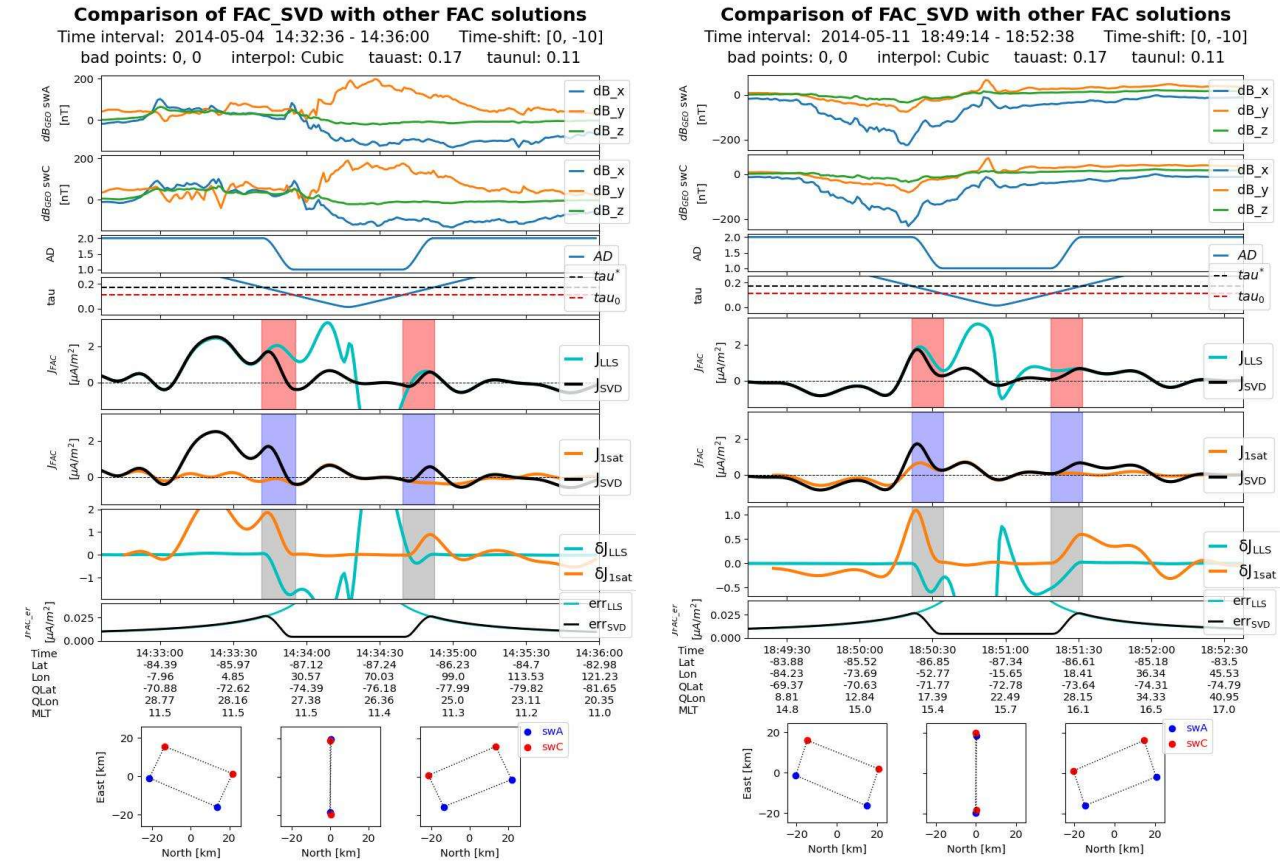


Figure 4. Comparison between SVD, LLS, and single-s/c FAC densities on high latitude events

Both events illustrated in Figure 4 refer to the relevant highest latitudinal portion of the orbit. The first two panels show the magnetic field perturbation observed by Swarm A and Swarm C respectively. The third and fourth panels present the evolution of AD and τ parameters. A comparison between the FAC_SVD (black) and FAC_LLS (cyan) current densities is provided in the fifth panel (note that the FAC_LLS density for the entire interval, i.e. also within the so-called exclusion zone, is shown). As can be observed, the transition between the two solutions is very smooth at the low-latitude edge of the (poleward and equatorward) transition zones (vertical pink bands). The next panel compares the FAC_SVD (black) and single-sat FAC (orange) current densities; the smooth transition between the two solutions at the higher-latitude edge of the transition zones (here shown by vertical gray band) is evident. The difference between the FAC_SVD and FAC_LLS solution (cyan) and between the FAC_SVD and single-sat FAC solutions (orange) is presented in the following panel while the last panel shows the mean square error in FAC_SVD (black) and FAC_LLS (cyan) current estimations assuming isotropic and mutually uncorrelated instrumental errors of 0.5 nT for each component of the magnetic field perturbation (see Sections 3.2 and [RD-4]).

4.3 Product dependence of the magnetic field model

The influence of the magnetic field model used on FAC_SVD estimation has been tested, with the results presented in Figure 5. The format of the plots is similar to the one used in Figure 4, with the only change in the last panel, that now presents the difference between two FAC SVD estimations, i.e. one that uses only the CHAOS-Core magnetic field model and the other that uses the full CHAOS model (includes contributions from the core, lithosphere, and magnetosphere). Note that the values on the y axes are measured in nA/m^2 ,

i.e. three orders of magnitude smaller than the typical values for FAC density within the auroral oval. For the orbit section that estimates the current density based on the full (planar) magnetic field gradient, the differences are vanishingly small. Here the application of the Ampère's law implies the use of a close contour and, since the magnetic field models are potential fields, one indeed expects basically the same current estimates regardless of the particular magnetic field model being used. For the orbit section that estimates the current density using only the along-track magnetic gradients (assuming no variation in the cross-track direction) no close contour is actually involved and the discrepancies are relatively larger. However, by looking at many events, one concludes that these differences are still smaller than $\sim 10 \text{ nA/m}^2$ and therefore the FAC_SVD estimations are basically independent on the magnetic field model used in the algorithm.

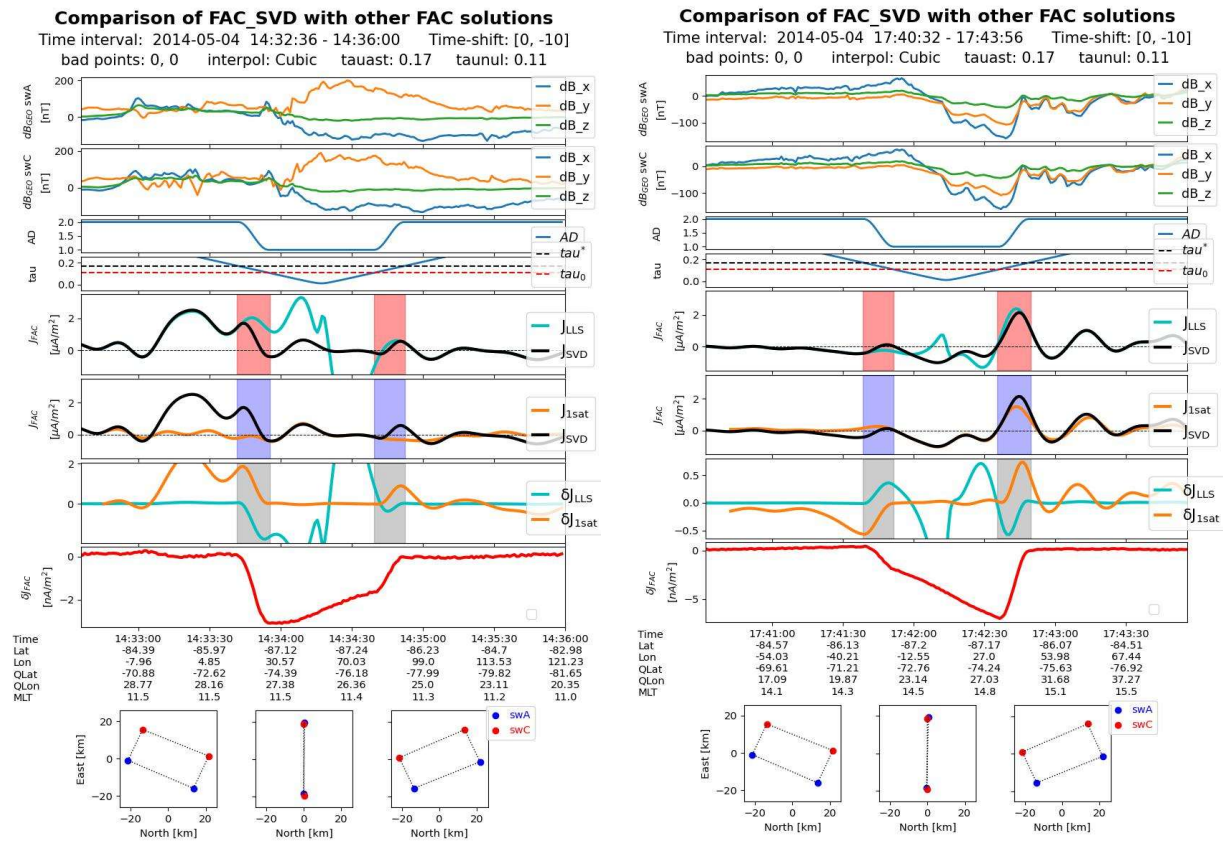


Figure 5. Dependence of FAC_SVD product on the magnetic field model