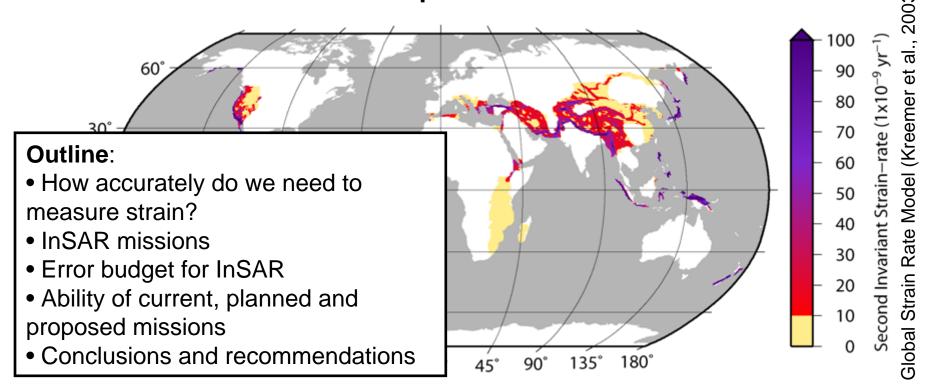
How accurately can current and futureInSAR missions map tectonic strain?

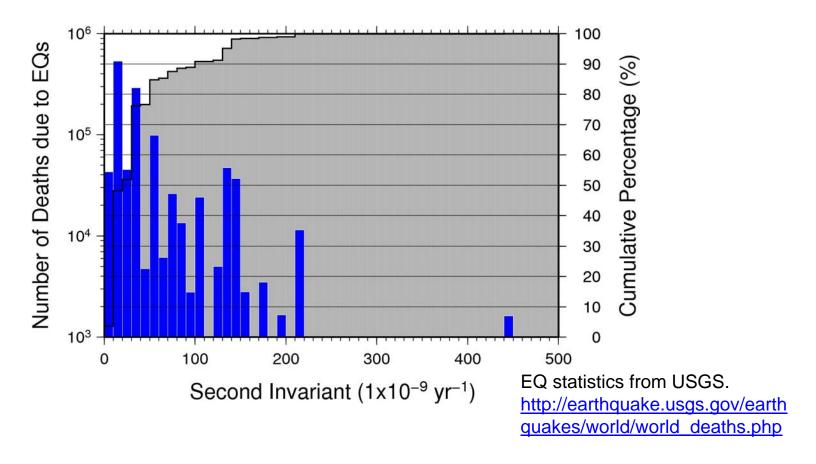


Tim Wright¹, Matthew Garthwaite¹, Hyung-Sup Jung², Andrew Shepherd¹ (1) University of Leeds, UK; (2) University of Seoul, South Korea



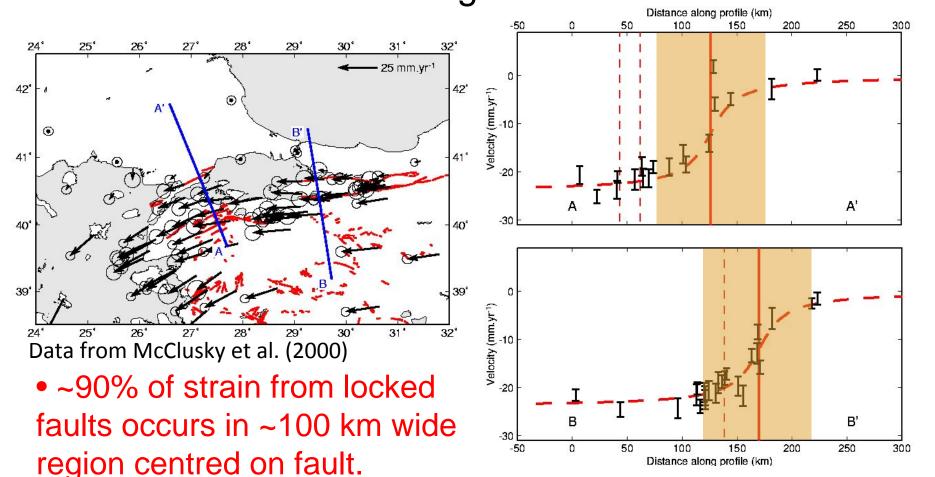


Required Accuracy for Tectonic Deformation: 1. Strain and EQ deaths



• 90% of all earthquake-related deaths occur in regions which are straining at rates above 1.2 x 10⁻⁸ yr⁻¹

Required Accuracy for Tectonic Deformation: 2. Length scale



Target threshold for measuring tectonic strain: velocity gradients of 1.2 mm/yr over 100 km

Distance along profile (km)

Current/Planned/Proposed InSAR Missions

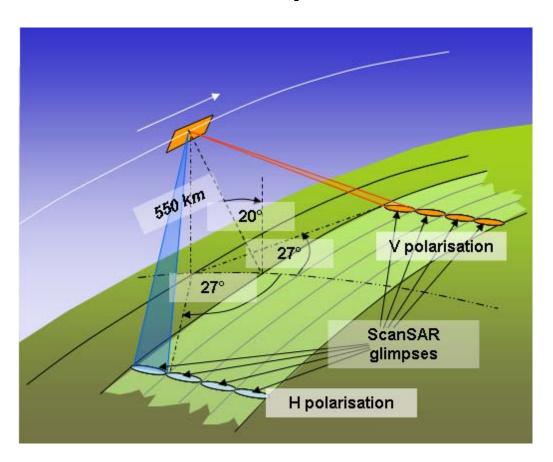
Mission	Λ	Revisit Time	% Aquis.	Geometry
Envisat 2003-2010	C (~6 cm)	35 days	~50%	R-looking, usu. 23° inc, mostly descending
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None of the other current/planned missions have global acquisition strategies or data policies that could allow them to be useful for global strain mapping.

SuperSAR Concept



- L-band, ScanSAR
- Forward and Rear beams
- Achieved through phased array antenna
- Optimised for mapping tectonic strain
- Proposed to ESA's EE8 call in 2010

$$\sigma_{def}^2 = \sigma_{gm}^2 + \sigma_{topo}^2 + \sigma_{atm}^2 + \sigma_{coh}^2 + \sigma_{sys}^2 + \sigma_{unw}^2$$

- Orbital errors ⇒ long-wavelength ramps.
- Envisat: ~0.3 mm/km (across-track) and 0.1 mm/km (along-track) [Wang, Wright and Biggs, GRL 2009].
- Can correct by processing long strips and tying to GPS (see. Fringe presentations by Wang, Pagli and Hamlyn)
- Should be negligible for future missions with onboard GPS receivers.

$$\sigma_{def}^2 = \sigma_{gm}^2 + \sigma_{topo}^2 + \sigma_{atm}^2 + \sigma_{coh}^2 + \sigma_{sys}^2 + \sigma_{unw}^2$$

$$\sigma_{topo} = \frac{\overline{r}_{slant}B_{\perp}}{\sin\theta_{inc}}\sigma_{DEM}$$

 SRTM error ~ 4 m absolute, of which 2.5 m is not spatially correlated [Rodriguez et al., PERS 2006]

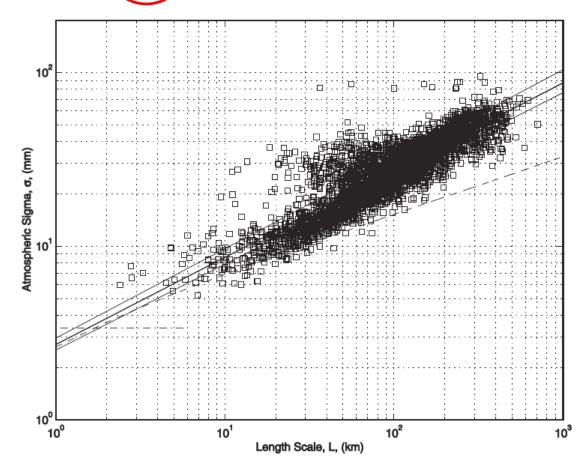
B _{perp}	σ _{topo} (40° incidence)		
150 m	1.1 mm		
300 m	2.3 mm		
1000 m	7.8 mm		

$$\sigma_{def}^2 = \sigma_{gm}^2 + \sigma_{topo}^2 + \sigma_{atm}^2 + \sigma_{coh}^2 + \sigma_{sys}^2 + \sigma_{unw}^2$$

Troposhere

Emardson et al., 2003: $\sigma = cL^{\alpha}$ [c~2.5, α ~0.5] $\sigma = 25$ mm at 100 km

(assume no corrections)



$$\sigma_{def}^2 = \sigma_{gm}^2 + \sigma_{topo}^2 + \sigma_{atm}^2 + \sigma_{coh}^2 + \sigma_{sys}^2 + \sigma_{unw}^2$$

- Ionosphere (1/f² dependence). Important at L-band, but not at C-band.
- Can correct with split band processing (e.g. 1200 and 1260 MHz) in future missions
- Ionospheric error on 100 km wavelength ~
 1mm after spatial averaging

$$\sigma_{def}^2 = \sigma_{gm}^2 + \sigma_{topo}^2 + \sigma_{atm}^2 + \sigma_{coh}^2 + \sigma_{sys}^2 + \sigma_{unw}^2$$

Coherence, γ

important at short wavelengths, but can be averaged through multilooking to < 1 mm for most ground cover types

$$\sigma_{def}^2 = \sigma_{gm}^2 + \sigma_{topo}^2 + \sigma_{atm}^2 + \sigma_{coh}^2 + \sigma_{sys}^2 + \sigma_{unw}^2$$

- Coherence, γ
 - important at short wavelengths, but can be averaged through multilooking to < 1 mm for most ground cover types
- System (thermal) modifies coherence
 - reduces effective coherence, but still insignificant after spatial averaging.

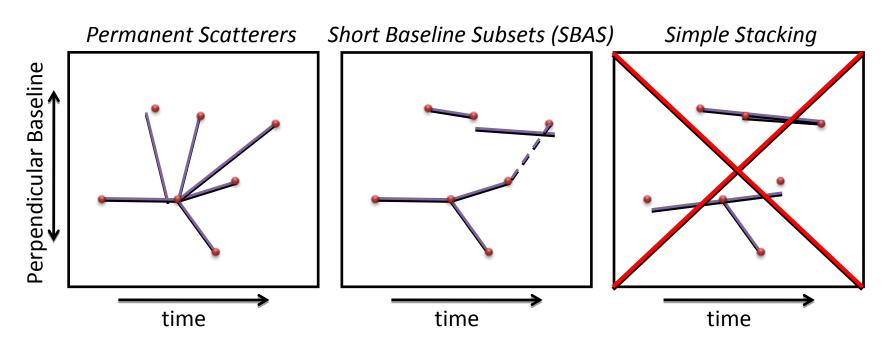
$$\sigma_{def}^2 = \sigma_{gm}^2 + \sigma_{topo}^2 + \sigma_{atm}^2 + \sigma_{coh}^2 + \sigma_{sys}^2 + \sigma_{unw}^2$$

- Unwrapping errors difficult to quantify.
- Assume = 0 in this analysis (probably OK for L-band missions with short revisits).

$$\sigma_{def}^2 = \sigma_{gm}^2 + \sigma_{topo}^2 + (\sigma_{atm}^2) + \sigma_{coh}^2 + \sigma_{sys}^2 + \sigma_{unw}^2$$

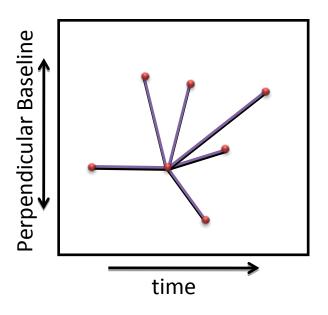
Atmospheric (tropospheric) error dominates at 100 km length scales, at which single interferograms have error of ~25 mm.

Optimum determination of Linear Deformation Rates



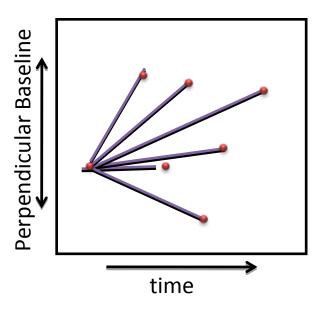
For the determination of linear deformation rates, optimum errors are determined through a connected network, since noise terms are associated with individual acquisitions not interferograms.

Optimum determination of Linear Deformation Rates



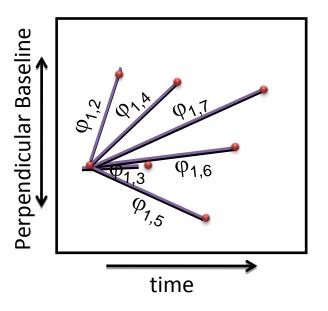
• Error on linear rate is independent of how network is connected (but of course short-baseline, short-time interferograms are best).

Optimum determination of Linear Deformation Rates



- Error on linear rate is independent of how network is connected (but of course short-baseline, short-time interferograms are best).
- To simplify mathematics, assume all connections to date d1...

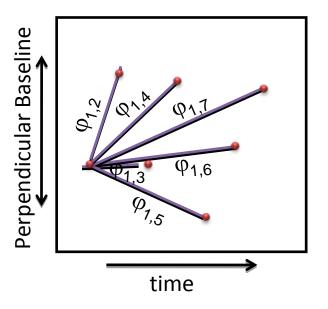
Optimum determination of Linear Deformation Rates



- Error on linear rate is independent of how network is connected (but of course short-baseline, short-time interferograms are best).
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...and regular acquisition spacing, t_m

Optimum determination of Linear Deformation Rates

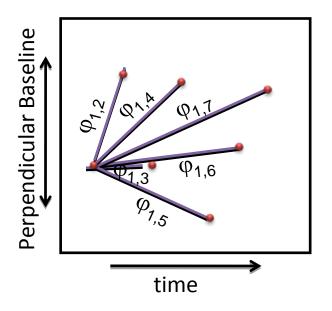


- Error on linear rate is independent of how network is connected (but of course short-baseline, short-time interferograms are best).
- To simplify mathematics, assume all connections to date d1...
- ...and regular acquisition spacing, t_r
- We can determine the best-fit linear rate of phase change due to deformation, $\frac{d\varphi}{dt}$, using weighted least squares:

$$\mathbf{\Sigma}_{\mathbf{P}}^{-1}\mathbf{T}\frac{d\boldsymbol{\varphi}}{dt} = \mathbf{\Sigma}_{\mathbf{P}}^{-1}\mathbf{P}$$

where $\mathbf{T} = [t_r, 2t_r, ... Nt_r]^{\mathsf{T}}$, $\mathbf{P} = [\phi_{1,2}, \phi_{1,3}, ... \phi_{1,N}]^{\mathsf{T}}$, and $\Sigma_{\mathbf{P}}^{-1}$ is the inverse of the variance-covariance matrix for the range change observations, \mathbf{P} .

Optimum determination of Linear Deformation Rates



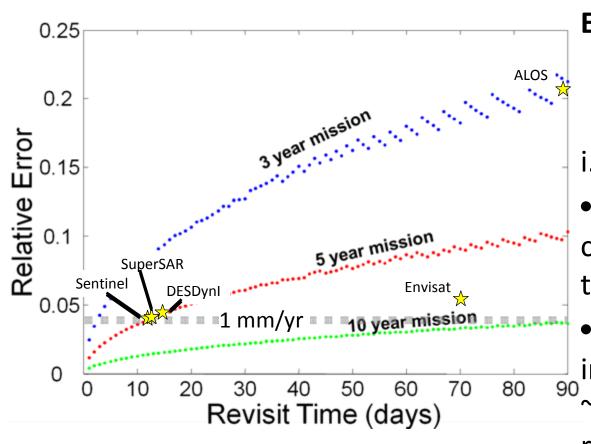
- ullet Using the correct VCM, $\Sigma_{
 m P}$, is essential.
- In this particular network, all interferograms share a common acquisition (epoch 1).

$$\Rightarrow$$
 Cov $(\phi_{1,i}, \phi_{1,j}) = \sigma_1^2$ (the variance on epoch 1)

and Var
$$(\phi_{1,i}) = \sigma_1^2 + \sigma_i^2$$

= $2\sigma^2$ (assuming noise is identical on all epochs)

Optimum determination of Linear Deformation Rates

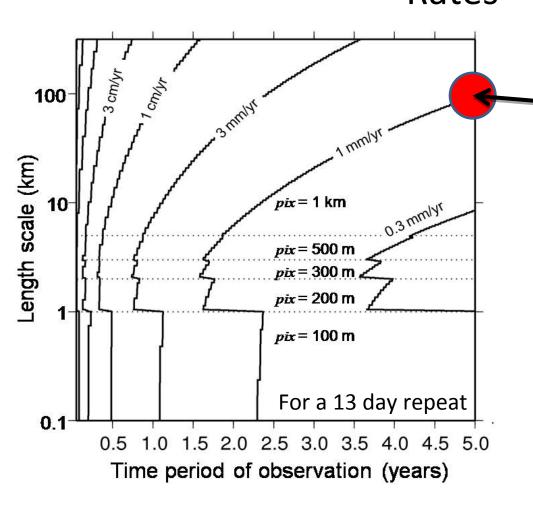


Error ∞ (revisit time)^{0.5} ∞ (mission length)^{-1.5}

i.e.

- For a **fixed length mission**, cut revisit time by 4 to halve the linear rate error.
- For a fixed revisit time,
 increase mission length by
 60% to halve the linear
 rate error.

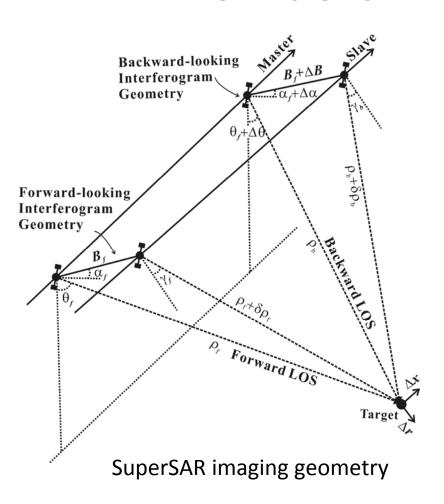
Optimum determination of Linear Deformation Rates



Reaching the target precision is tough!

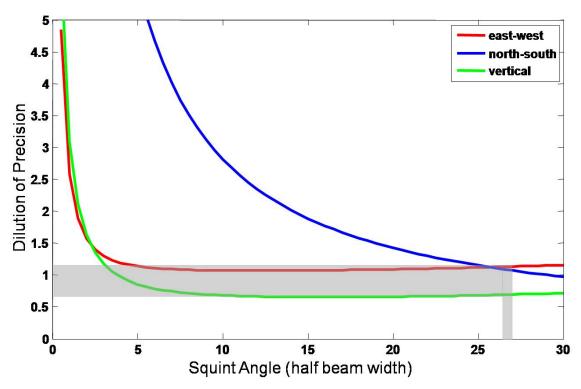
Everything so far has been for Line-of-sight deformation

Error Budget (3) 3D deformation retrieval



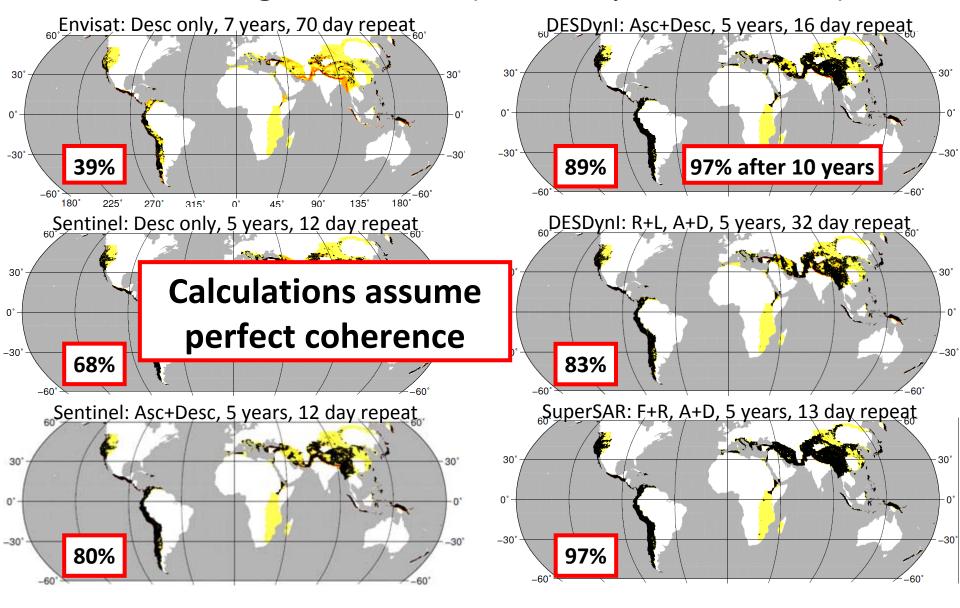
- SuperSAR and DESDynI were designed to retrieve 3D deformation.
- SuperSAR forward and backward looking beams. 3D from 1 Asc + 1 Desc pass.
- DESDynI L & R looking capability (although routine acquisitions were not planned). 3D from e.g 1 Asc + 2 Desc passes.

Error Budget (3) 3D deformation retrieval

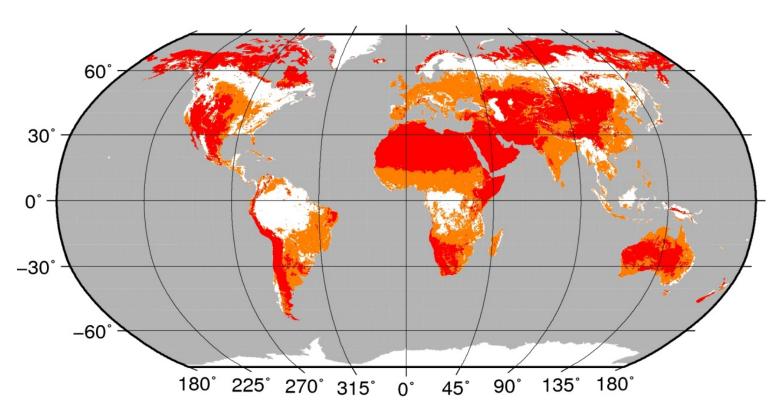


- Dilution of precision for SuperSAR ~1 for all 3 components if angle between beams > ~50 degrees
- Dilution of precision for DESDynI is ~1.1/5.1/0.9 in East/North/Up using 3 acquisitions (~0.8/3.6/0.7 using 4)

Abilities of missions to map tectonic strain above target threshold (1.2 mm/yr over 100 km)

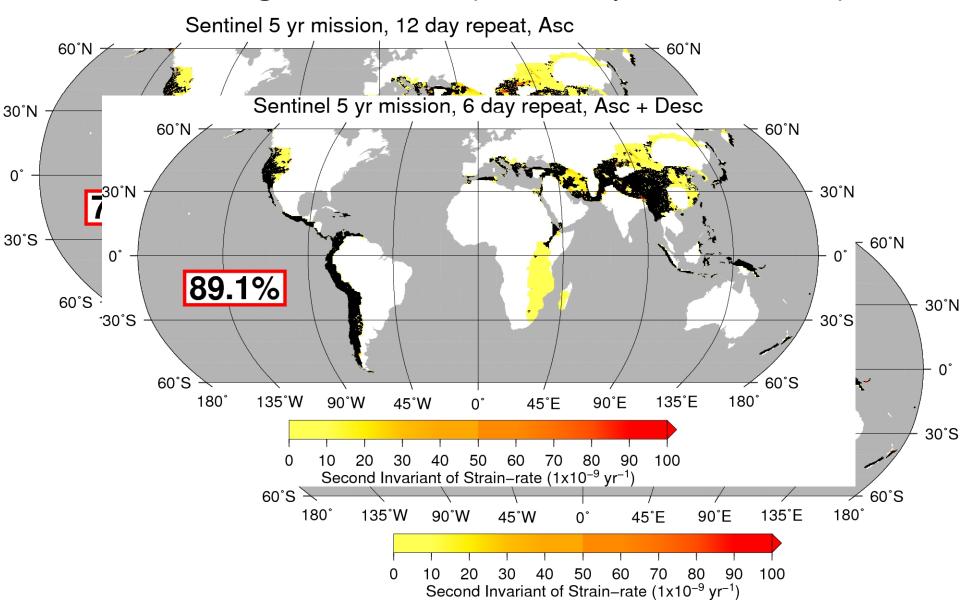


Abilities of missions to map tectonic strain: Coherence at C-band

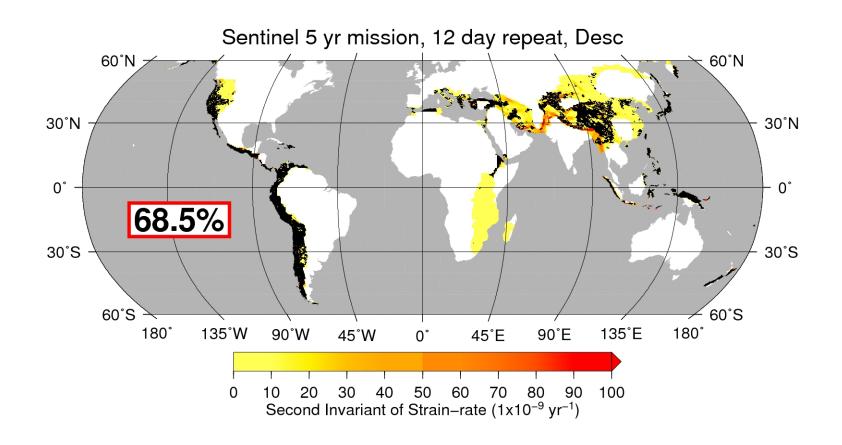


C-band coherence (1 year = red; 1 cycle = red+orange)
L-band should be coherence in **most** places over 13 days

Abilities of missions to map tectonic strain above target threshold (1.2 mm/yr over 100 km)



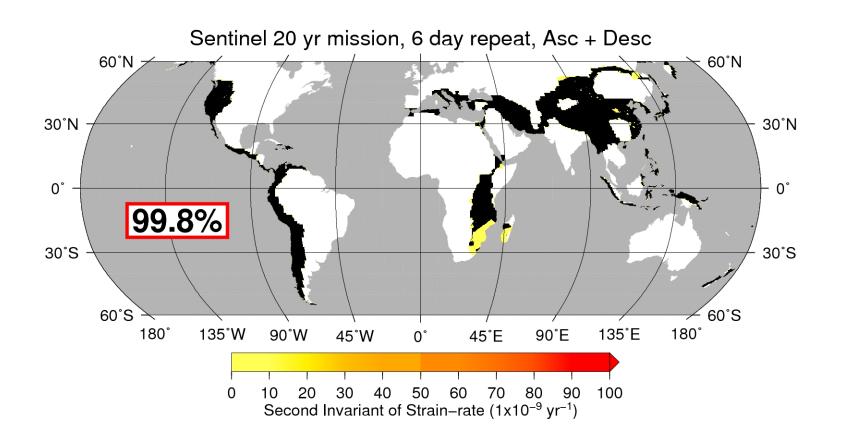
Abilities of missions to map tectonic strain above target threshold (1.2 mm/yr over 100 km)



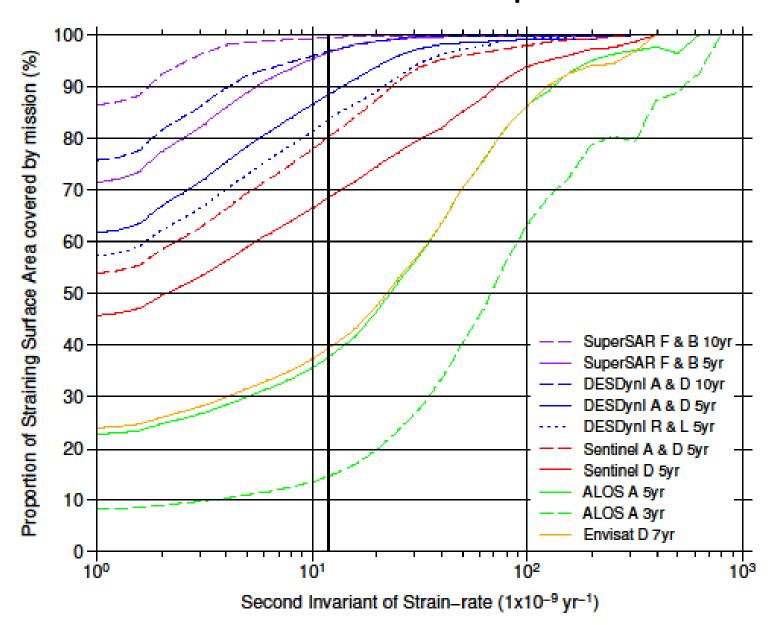
Conclusions and Recommendations

- Atmospheric errors are limiting factor for using InSAR to map strain accumulation
 - Further research on routine adoption of weather models required
- Sentinel-1 will greatly improve capability
 - It should acquire ascending + descending data
- DESDynI mission would have further improvements
 - But there is no great benefit (for tectonic strain) in having left- and right- looking capability
 - Maximising the mission length is vital
- SuperSAR's forward and rear squinted beams would enable 3D deformation to be retrieved with comparable accuracies in all three dimensions
 - Future missions should consider adopting this concept

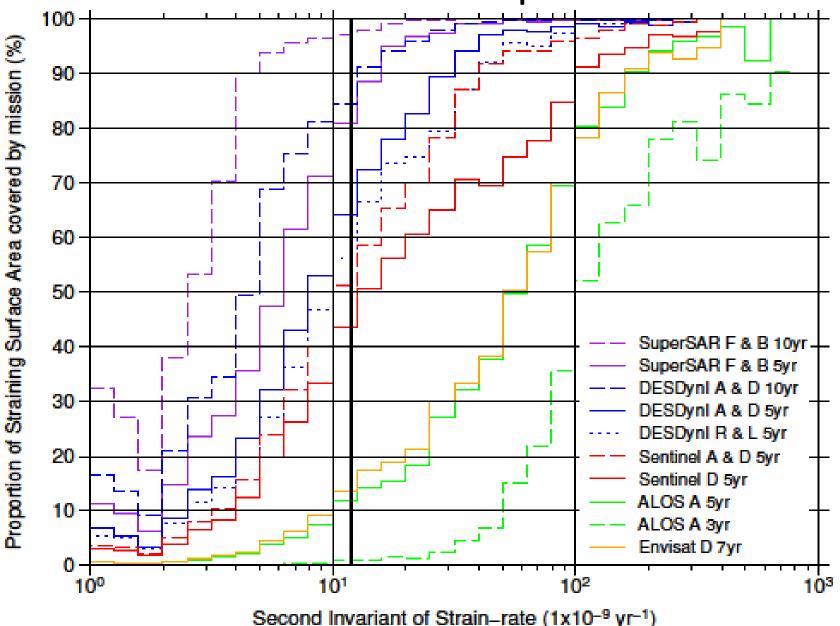
Abilities of missions to map tectonic strain above target threshold (1.2 mm/yr over 100 km)



Abilities of missions to map tectonic strain



Abilities of missions to map tectonic strain



Error Budget 4. Unobserved uncertainties b. Other

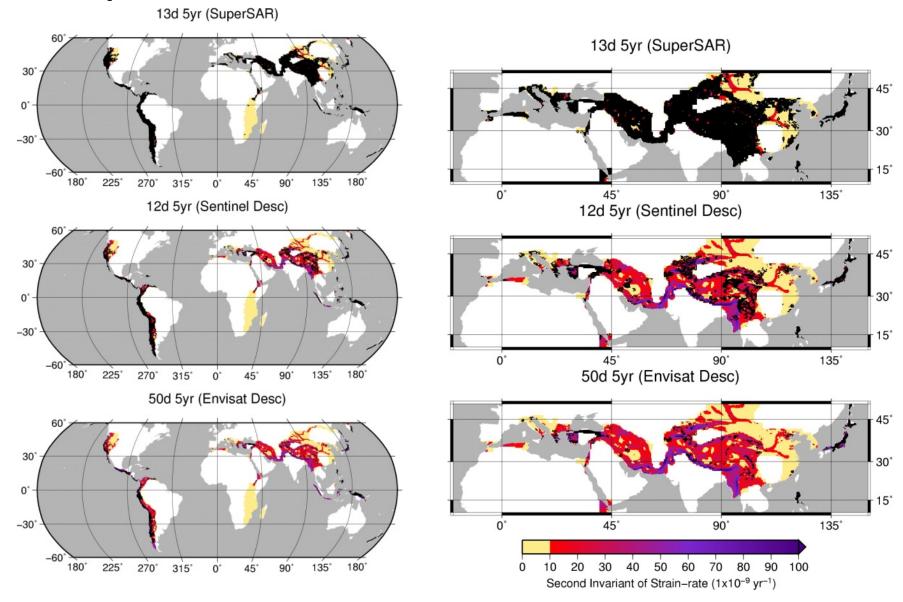
- Snow cover reduces accuracy
- Water no strain in oceanic plates can be observed
- Orbit no observations north of 81.5°
- Pixel size limits max gradient to 60 cm per kilometre (17 m per year).
- Viewing geometry (layover/shadow), impacts on < 1% of straining zones.

Duty cycle

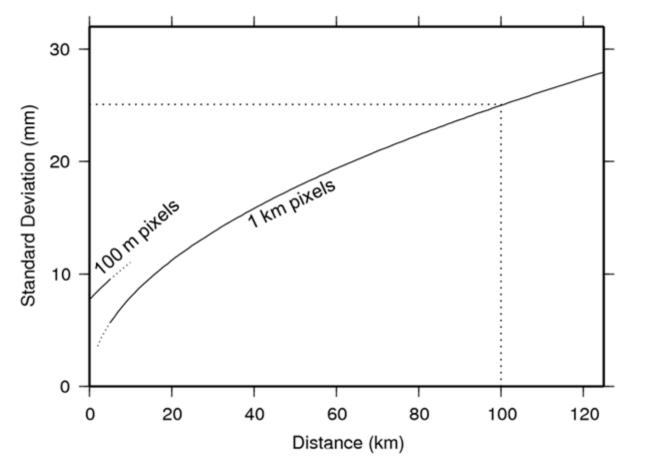
Target	Frequency of Observation	Duty Cycle	Notes
Tectonic Strain	Every pass, Asc + Desc	7.1%	All areas straining above 10 ⁻⁸ / year
Volcanoes	Every pass, Asc + Desc	0.14%	~300 volcanoes outside tectonic strain zones
Ice	Two passes from four, Asc + Desc	0.6%	Complete spatial coverage
Background Archive	One image per year, Asc + Desc	0.9%	All remaining areas
Economic/Other	450 targets, every pass, Asc + Desc	1.25%	Each target covers an area 100 x 310 km. # targets could be increased by decreasing the revisit time
Total		10%	An increase or decrease in this value would directly impact on the number of economic/other targets that could be imaged.

Table D4.3.1: Estimates for the total duty cycle load for each of our scientific targets

SuperSAR vs Envisat and Sentinel-1



$$\sigma_{def}^2 = \sigma_{gm}^2 + \sigma_{topo}^2 + \sigma_{trop}^2 + \sigma_{ion}^2 + \sigma_{coh+sys}^2 + \sigma_{unw}^2$$



Error Budget

1. Single interferogram

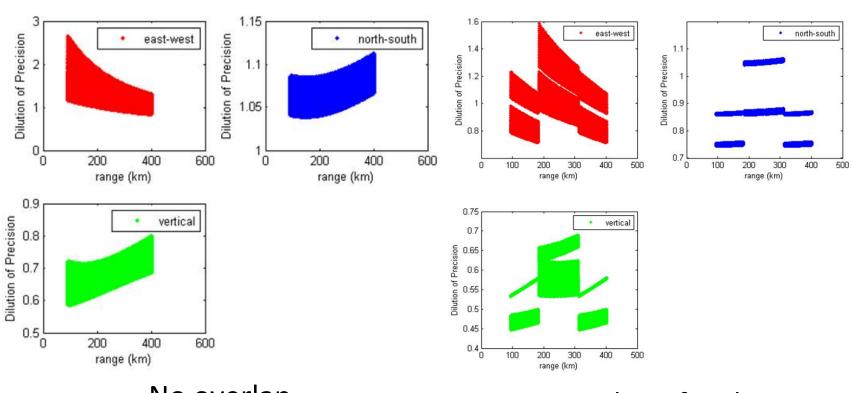
$$\sigma_{def}^2 = \sigma_{gm}^2 + \sigma_{topo}^2 + \sigma_{trop}^2 + \sigma_{ion}^2 + \sigma_{coh+sys}^2 + \sigma_{unw}^2$$

Component	Error (1 sigma	a)		Comments
	L = 100 m	L = 1 km	L = 100 km	
$\sigma_{ m gm}$	Negligible	Negligible	≤ 1.6 mm (future missions)	Short wavelength orbital error is negligible;
σ_{topo}	1.1 mm	1.1 mm	0.9 mm	Assuming SRTM elevation model and 1 km pixels for 100 km error.
σ_{trop}	0.8 mm	2.5 mm	25 mm	
$\sigma_{\rm ion}$	Negligible	Negligible	0.9 mm	After correction using dual frequencies, and filtering over 10 km length scale.
σ _{coh+sys}	7.6 mm	7.6 mm	0.76 mm	Assuming 100 m pixels for $L \le 1$ km; 1 km pixels for $L = 100$ km, coherence of 0.9, and system of 6.9dB.
$\sigma_{ m unw}$	Negligible	Negligible	Negligible	High coherence, long wavelength, and short repeat times should minimise unwrapping errors.
$\sigma_{ m def}$	7.7 mm	8.1 mm	25.1 mm	The phase noise is dominated by $\sigma_{coh+sys}$ over short distances and σ_{trop} at long lengthscales
Table D3.1.1: Error budget for SuperSAR at different lengthscales				

Error Budget

3. Accuracy of 3D retrieval

DOP for all positions within swath (27 degree half-squint)

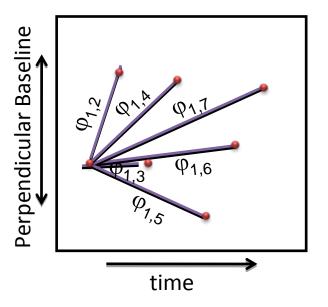


No overlap

Overlap of 90 km

Error Budget

2. Optimum determination of Linear Deformation Rates



- Error on linear rate is independent of how network is connected (but of course short-baseline, short-time interferograms are best).
- To simplify mathematics, assume all connections to date d1...
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Therefore:
$$\frac{d\varphi}{dt} = (\mathbf{T}^{\mathrm{T}}\boldsymbol{\Sigma}_{\mathrm{P}}^{-1}\mathbf{T})^{-1}\boldsymbol{\Sigma}_{\mathrm{P}}^{-1}\mathbf{P}$$
, and $\boldsymbol{\sigma}_r = \sqrt{(\mathbf{T}^{\mathrm{T}}\boldsymbol{\Sigma}_{\mathrm{P}}^{-1}\mathbf{T})^{-1}}$

Current/Planned/Proposed InSAR Missions

Mission	λ	Revisit Time	% Aquis.	Geometry
ERS-1/2 1991-2000	C (~6 cm)	35 days	Variable, usu. low	R-looking, ~23° inc, mostly descending
Envisat 2003-2010	L (~20 cm)	35 days	~50%	R-looking, usu. 23° inc, mostly descending
Radarsat-1/2 1995-	С	24 days	Low (usually)	R-looking, usu. 23° inc, mostly descending
ALOS 2006-	L	46 days	40-60%	R-looking, usu. 34° inc, mostly ascending
Terrasar-X 2008-	X (~2 cm)	12 days	Very Low	R-looking, Variable acquisition modes.
Sentinel-1A 2012/13-	С	12 days	100%	R-looking, 25-45° inc, Mostly descending??
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$$\sigma_{def}^2 = \sigma_{gm}^2 + \sigma_{topo}^2 + \sigma_{atm}^2 + \sigma_{coh}^2 + \sigma_{sys}^2 + \sigma_{unw}^2$$

Coherence, γ

C-band (λ =60 mm)

$$\gamma = 0.7$$

-> ~5 mm for ~100 m pixels

-> 0.5 mm for 1 km pixels

$$\gamma = 0.9$$

 \rightarrow ~4 mm for ~100 m pixels

-> 0.4 mm for 1 km pixels

L-band (
$$\lambda$$
=240 mm) $\sigma_{coh} = \left(\frac{\lambda}{4\pi}\right) \frac{1}{\sqrt{N_L}} \frac{\sqrt{1-\gamma^2}}{\gamma}$

$$\sigma_{def}^2 = \sigma_{gm}^2 + \sigma_{topo}^2 + \sigma_{atm}^2 + \sigma_{coh}^2 + \sigma_{sys}^2 + \sigma_{unw}^2$$

Coherence, γ

C-band (
$$\lambda$$
=60 mm)

$$\gamma = 0.7$$

-> ~5 mm for ~100 m pixels

-> 0.5 mm for 1 km pixels -> 0.4 mm for 1 km pixels

L-band (λ =240 mm) $\sigma_{coh} = \left(\frac{\lambda}{4\pi}\right) \frac{1}{\sqrt{N_{\star}}} \frac{\sqrt{1-\gamma^2}}{\gamma}$

$$\gamma = 0.9$$

-> ~4 mm for ~100 m pixels

• System (thermal) - modifies coherence

$$\gamma_c = \frac{\gamma}{1 + SNR^{-1}}$$

- e.g. Noise of 6.9dB (L-band SuperSAR)
 - -> 7.6 mm for 100 m pixels
 - -> 0.76 mm for 1 km pixels (coh + sys)