



MTF as an indicator of image quality, data efficiency, user friendliness, and product competitiveness

Guoqing (Gary) Lin
NASA

Alana Semple & Bin Tan
Science Systems and Applications, Inc.

VH-RODA 2025 Workshop
Frascati, Italy (virtual)
11/19/2025



Outline

- PSF, LSF (Point/Line Spread Function)
 - FWHM (Full Width at Half Maximum)
 - MTF (modulation transfer function)
- MTF := retaining rate of detail @Nyquist Frequency
- Appearance of small objects
- Effects of over-sampling with a factor > 2 (FWHM_LSF > 2.0 pixels)



A theorem on Nyquist frequency

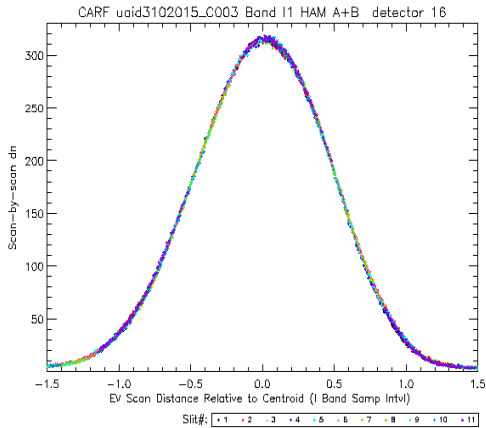
The "Shannon theorem," also referred to as the **Nyquist-Shannon sampling theorem**, states that to accurately reconstruct an analog signal from its digital samples, the sampling rate must be at least twice the **highest frequency present in the signal**, and the "**Nyquist frequency**" is the **maximum frequency** that can be accurately captured based on the sampling rate, that is exactly **half of the sampling rate** itself.

[Nyquist–Shannon sampling theorem - Wikipedia](#)

- Digital sampling is designed to potentially preserve the detail at the Nyquist frequency
- Sensor system PSF/LSF may filter out that potential



LSF \rightarrow FWHM, MTF



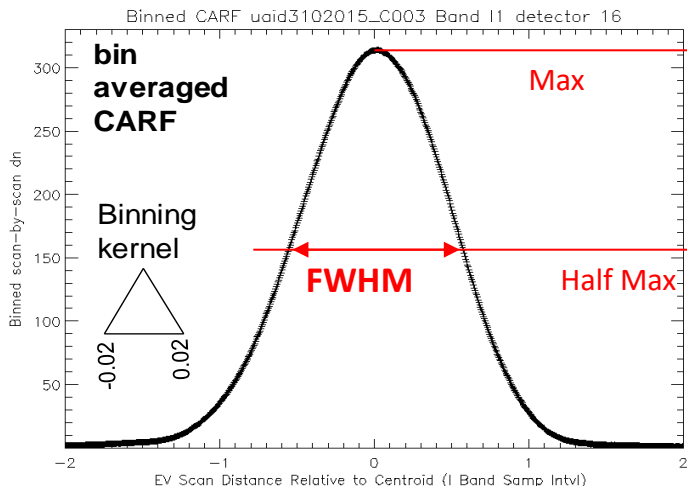
Modulation transfer function (MTF)

$$MTF(\omega) = \frac{|FourierTransform(LSF)|}{|FourierTransform(LSF)|_{\omega=0}}$$

ω is expressed in fraction of Nyquist frequency (NF)

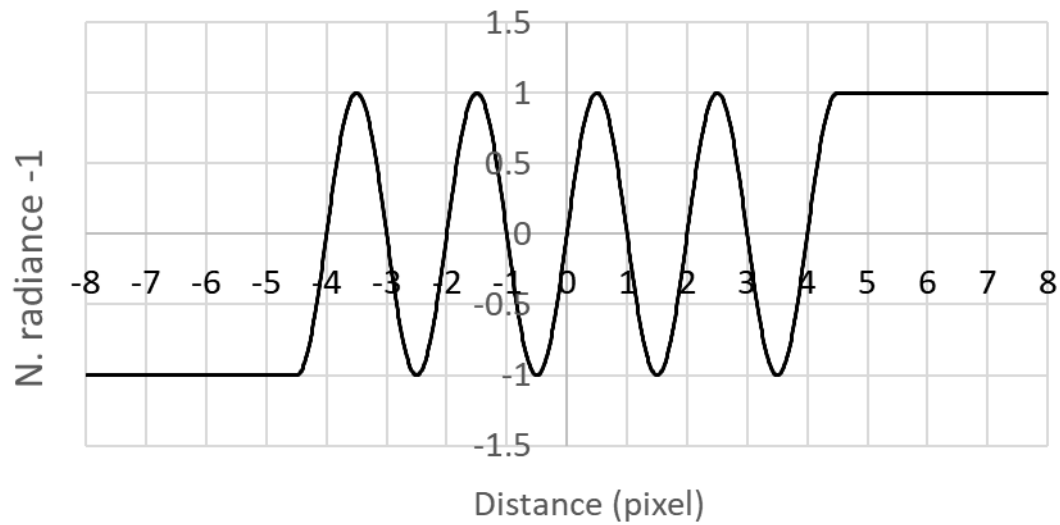
$$NF = \frac{1 \text{ cycle}}{2 \text{ sampling intervals}}$$

FWHM = Full Width at Half Maximum, a metric for an image's true spatial resolution





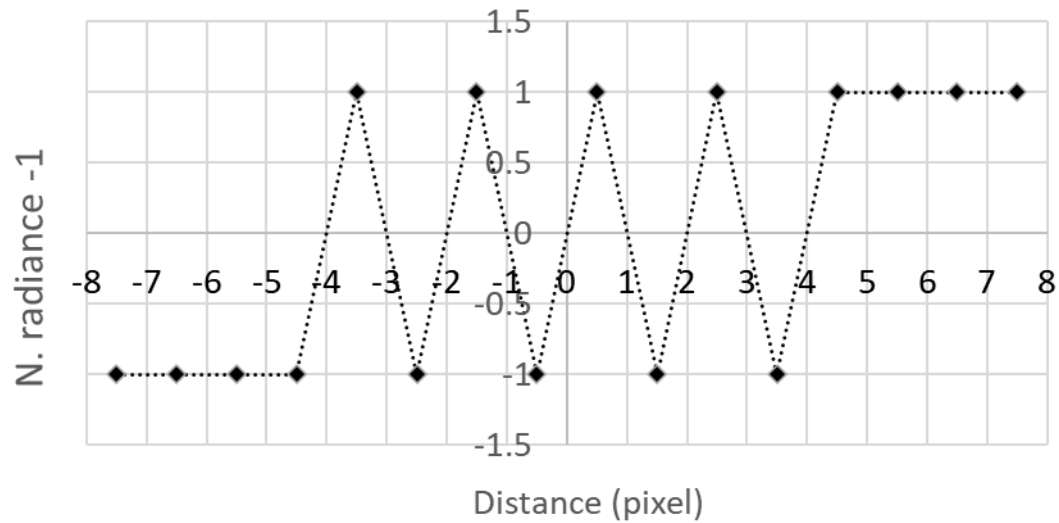
Hypothetical scene with detail at Nyquist frequency

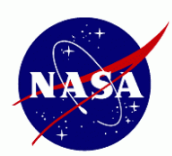




Hypothetical scene with detail at Nyquist frequency

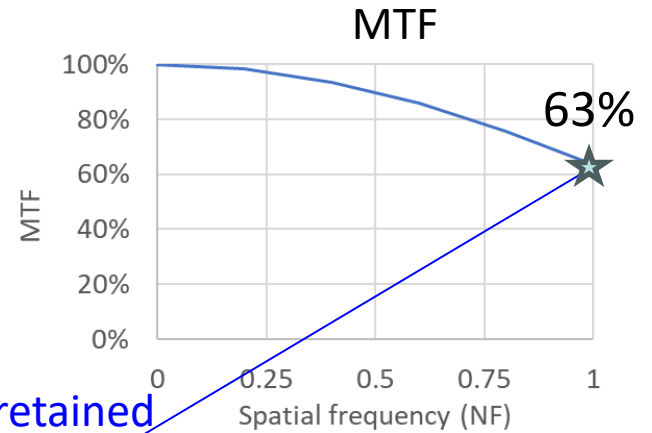
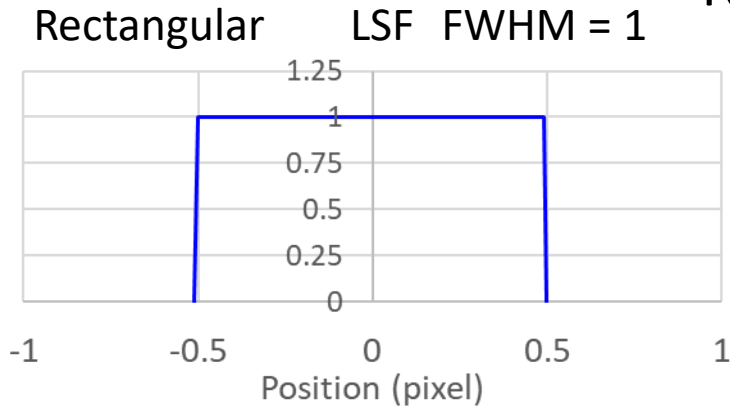
Point sampling



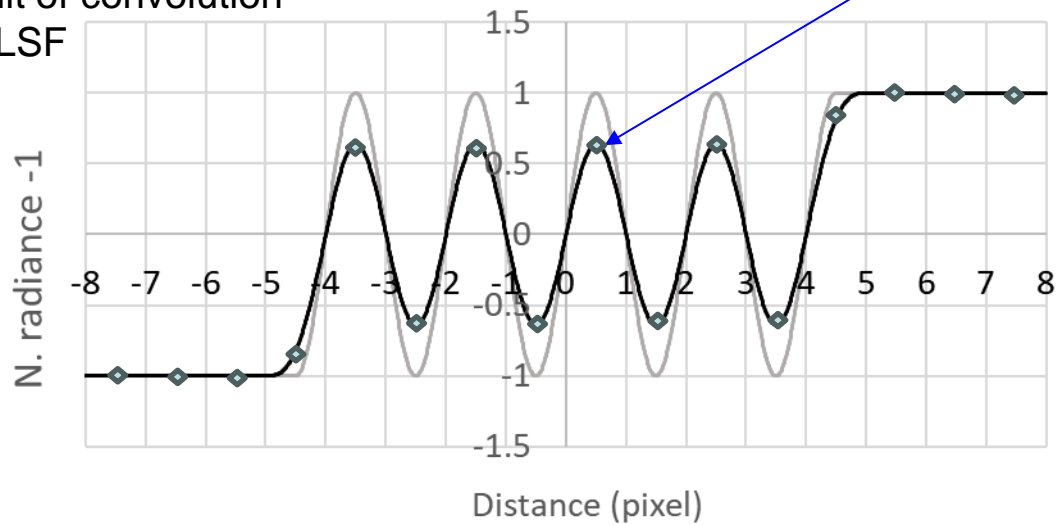


Hypothetical scene with detail at Nyquist frequency

Nyquist sampling



Black curve is the result of convolution of gray curve with the LSF



63% amplitude retained

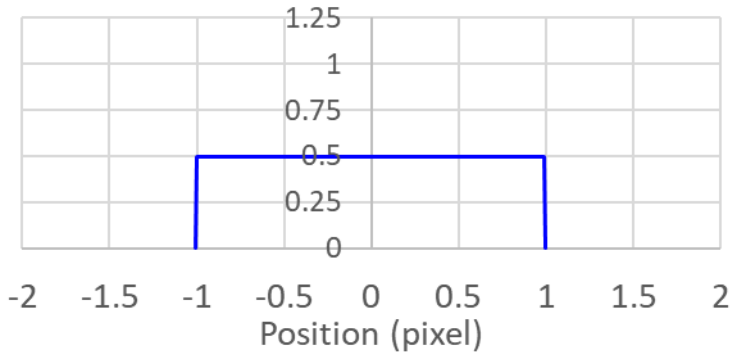


Hypothetical scene with detail at Nyquist frequency

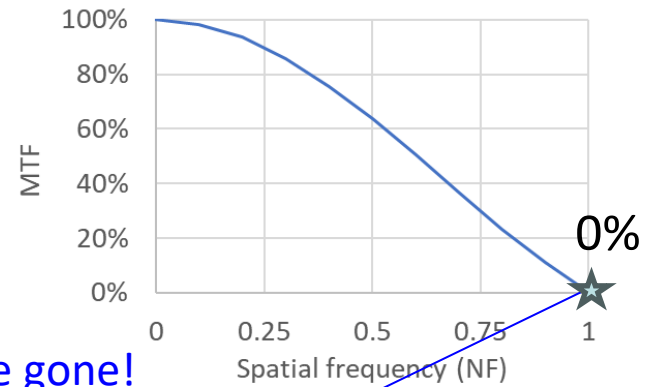
Over-sampling, factor = 2

Rectangular

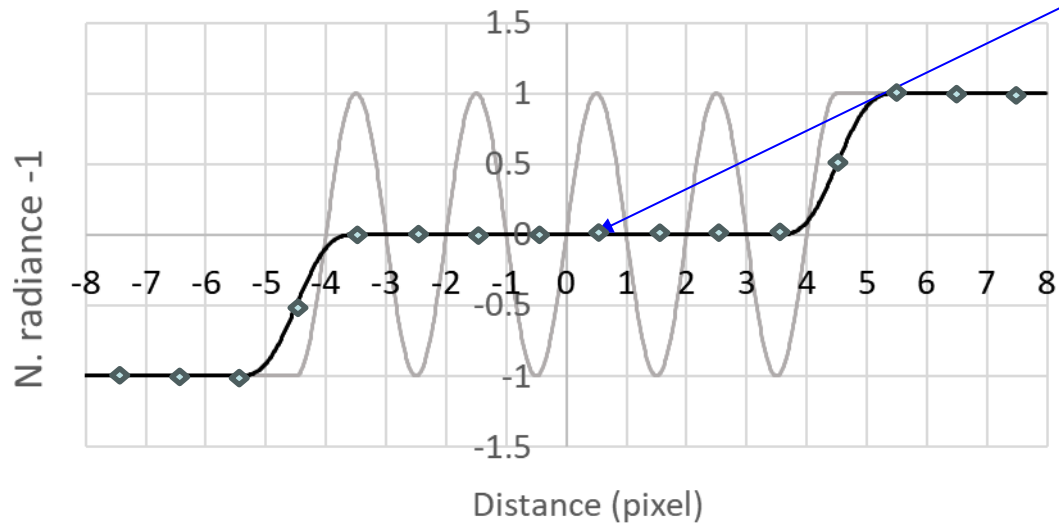
LSF FWHM = 2



MTF



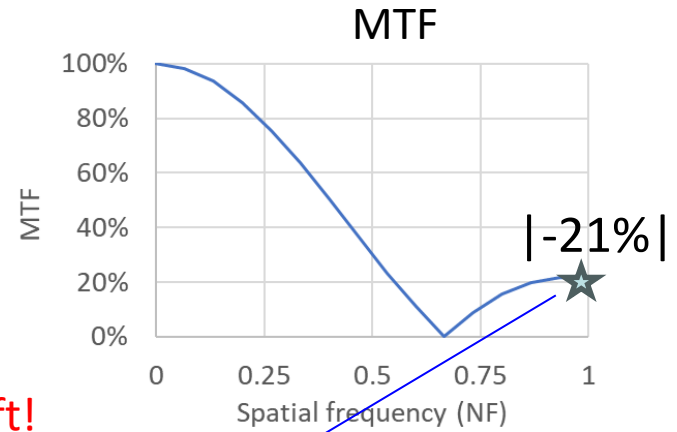
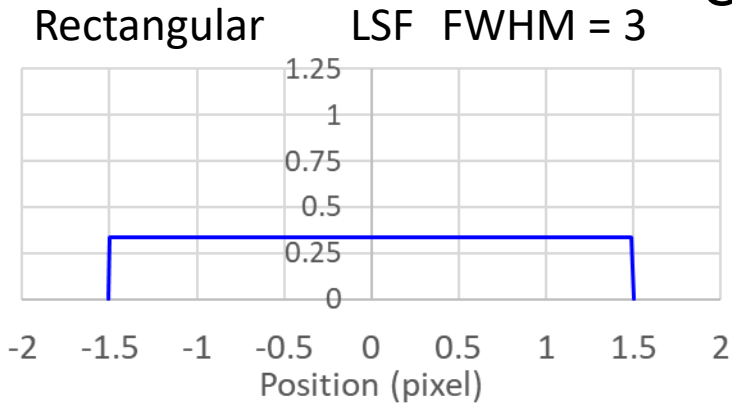
Amplitude gone!



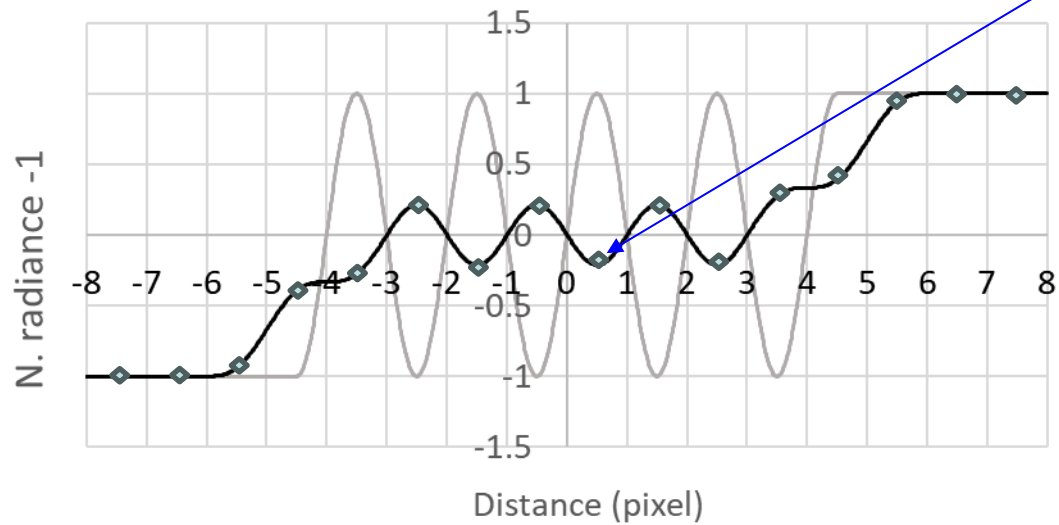


Hypothetical scene with detail at Nyquist frequency

Over-sampling, factor = 3



Phase shift!

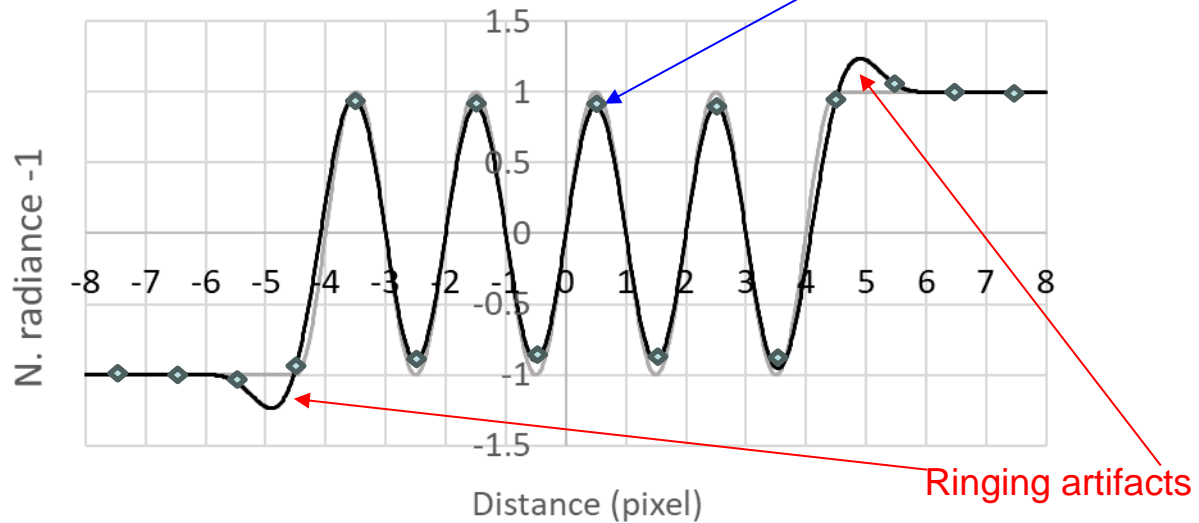
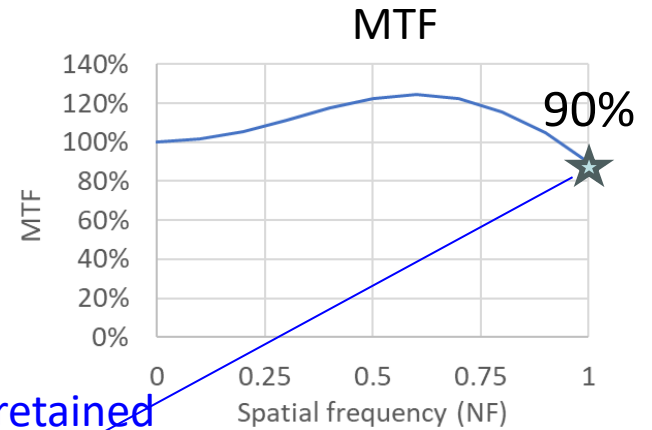
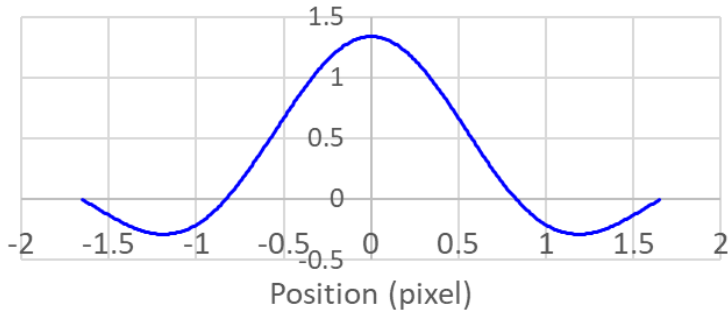




Hypothetical scene with detail at Nyquist frequency

Nyquist sampling

Sinc function LSF FWHM = 1

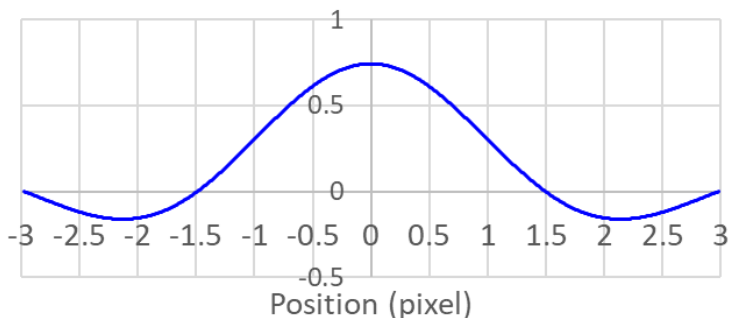




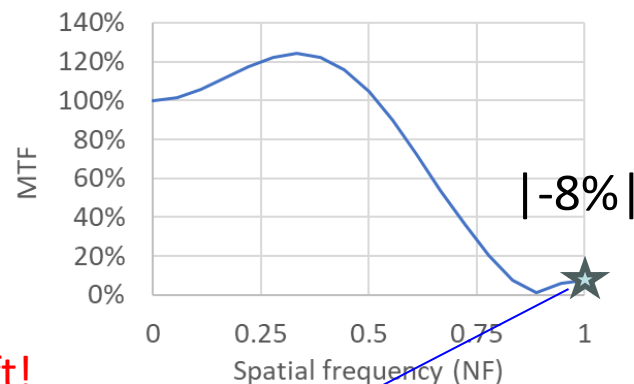
Hypothetical scene with detail at Nyquist frequency

Over-sampling, factor = 1.8

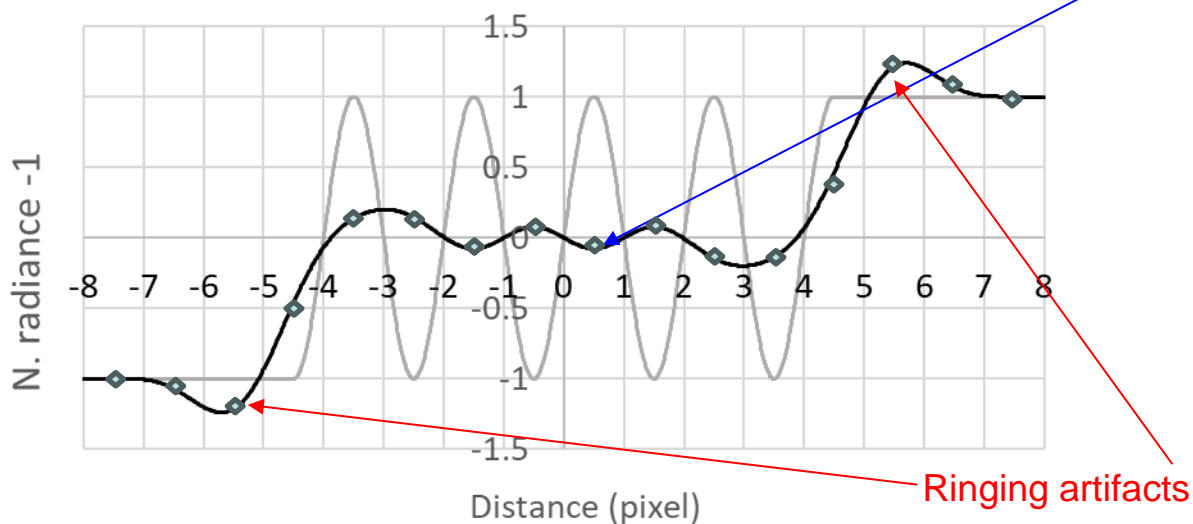
Sinc function LSF FWHM = 1.8



MTF



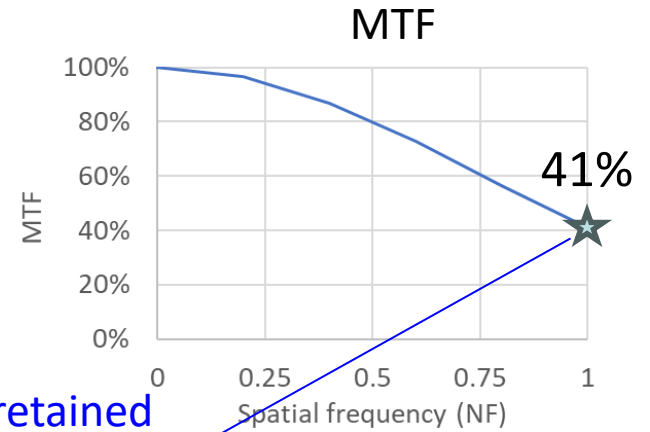
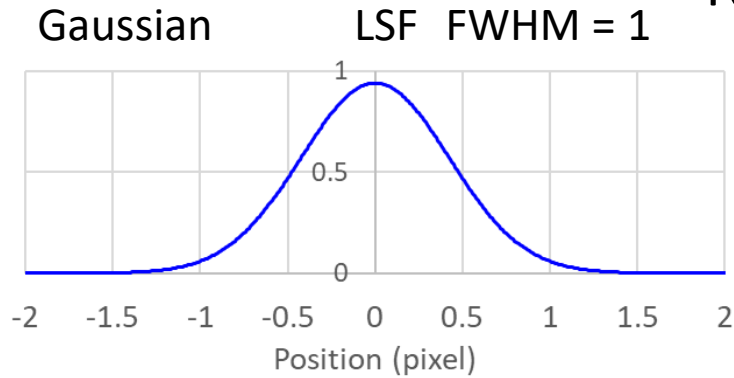
Phase shift!



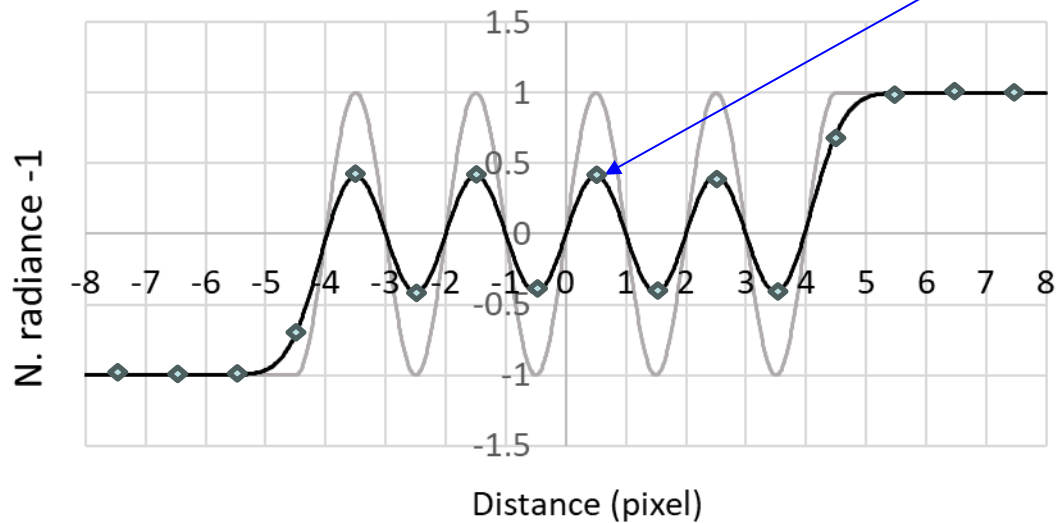


Hypothetical scene with detail at Nyquist frequency

Nyquist sampling



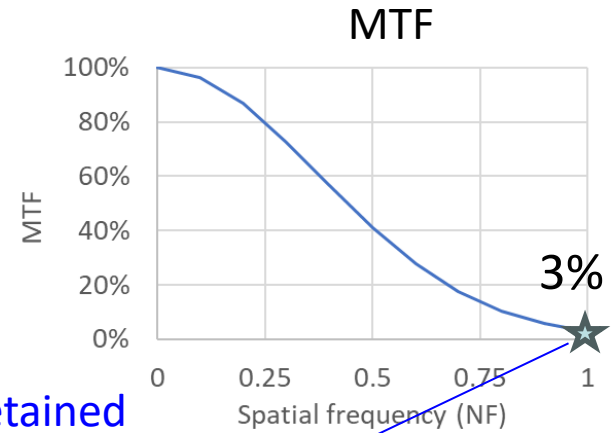
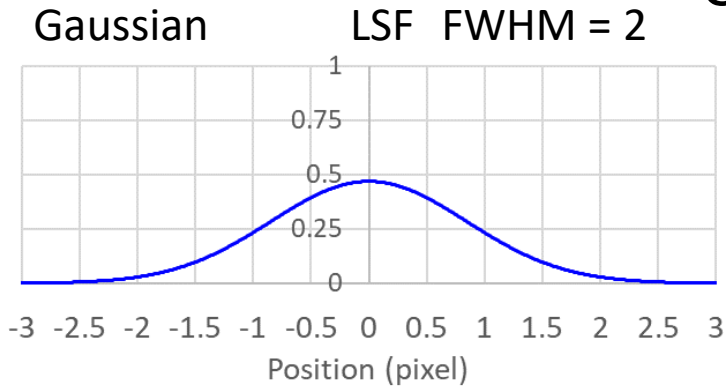
41% amplitude retained



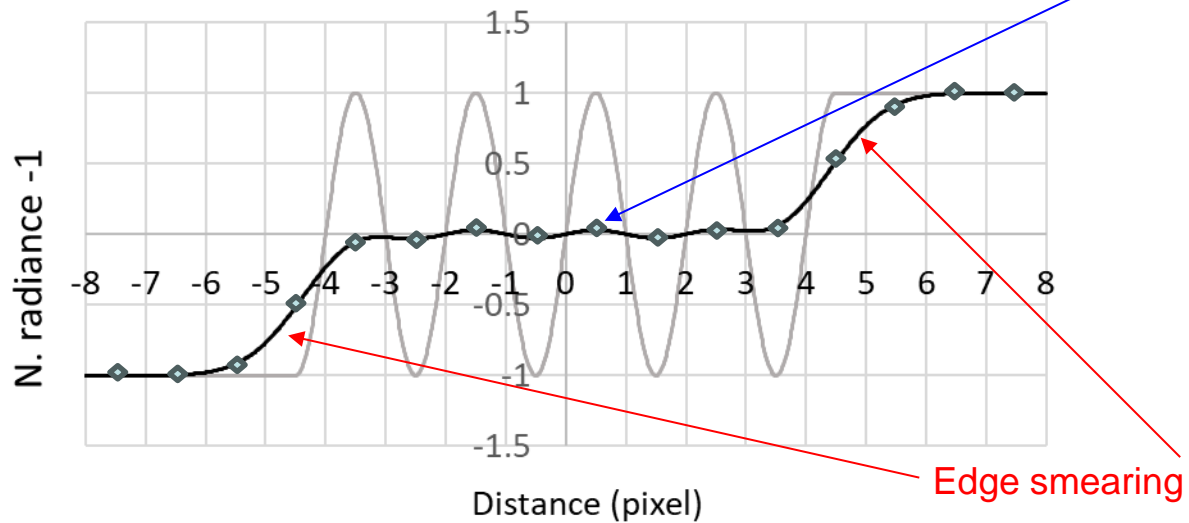


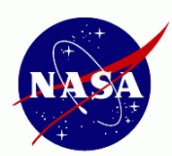
Hypothetical scene with detail at Nyquist frequency

Over-sampling, factor = 2



3% amplitude retained

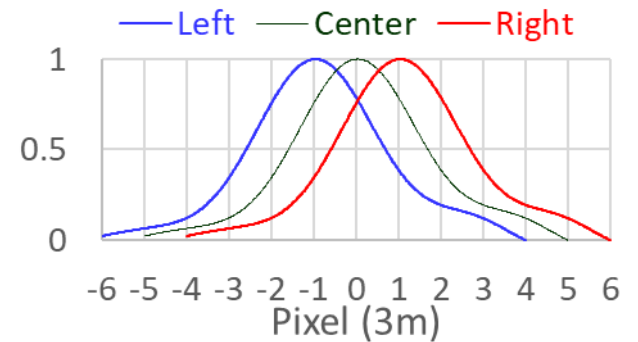
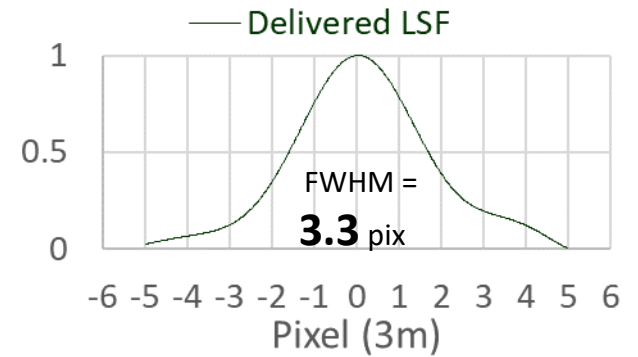
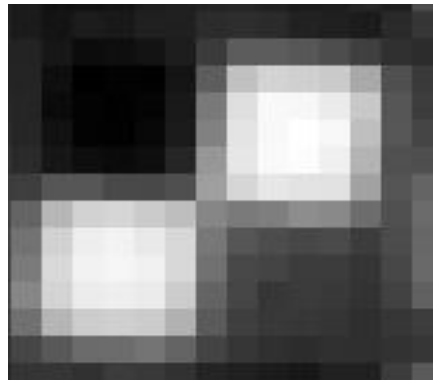
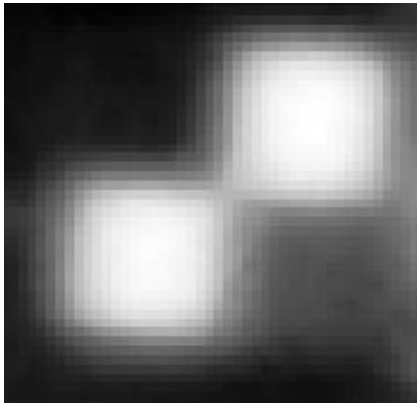




Room for improvement

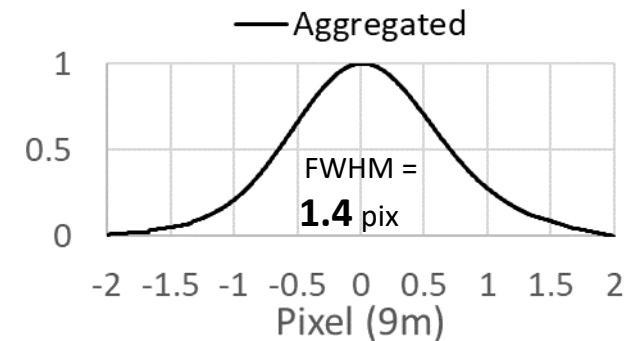
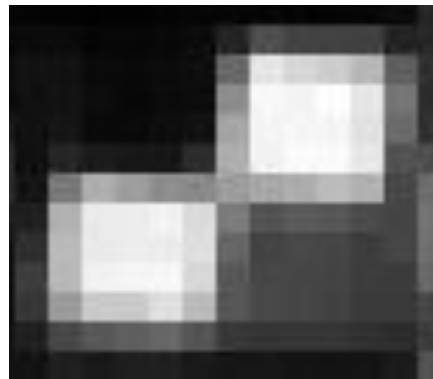
A Commercial image

Delivered pixel size, 3 m \rightarrow Aggregated, 9 m



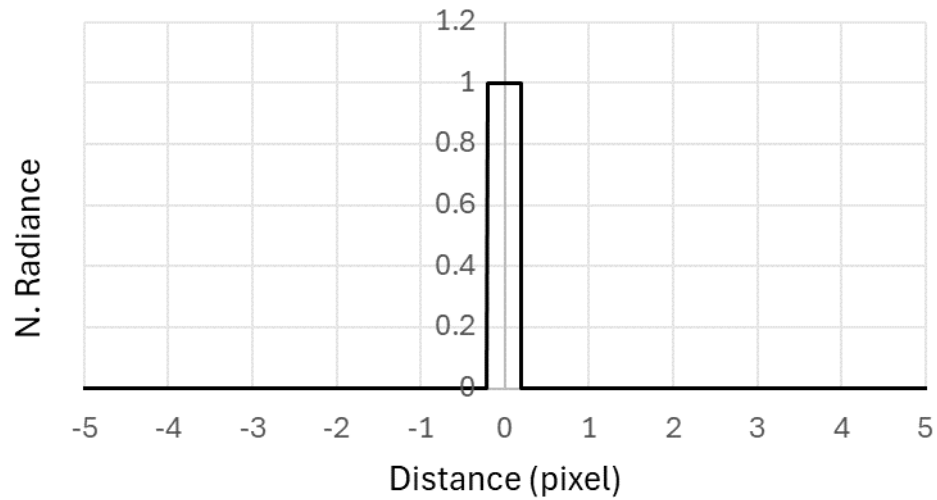
Sentinel-2

Resampled, 3 m \leftarrow Delivered pixel size, 10 m





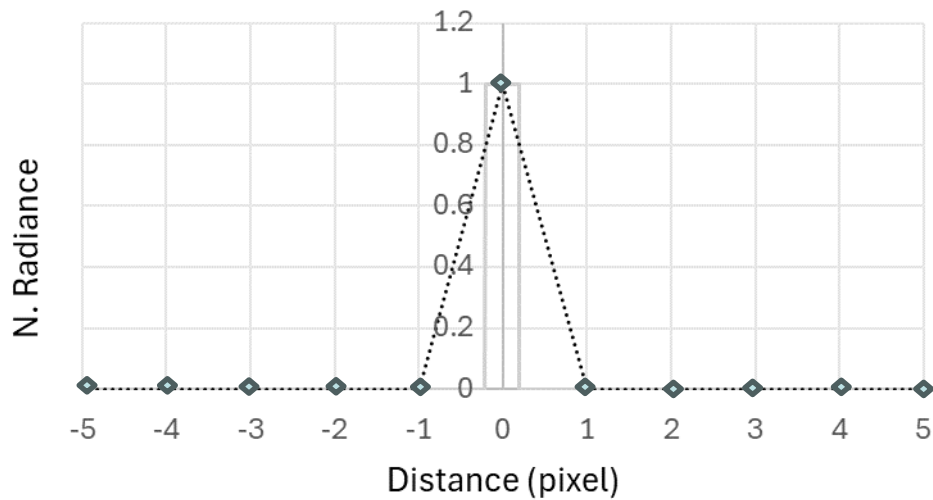
An isolated small object





An isolated small object

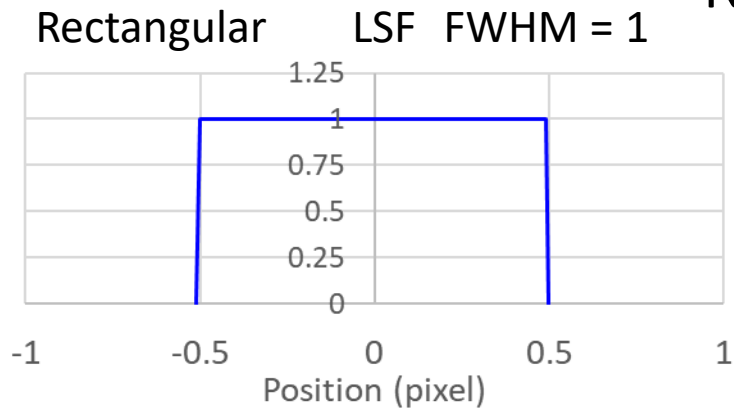
Point sampling



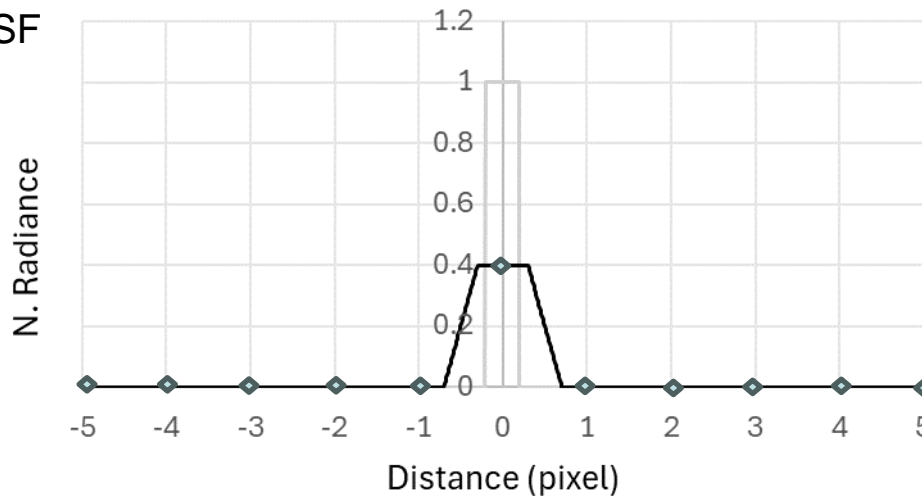


An isolated small object

Nyquist sampling



Black curve is the result of convolution of gray curve with the LSF

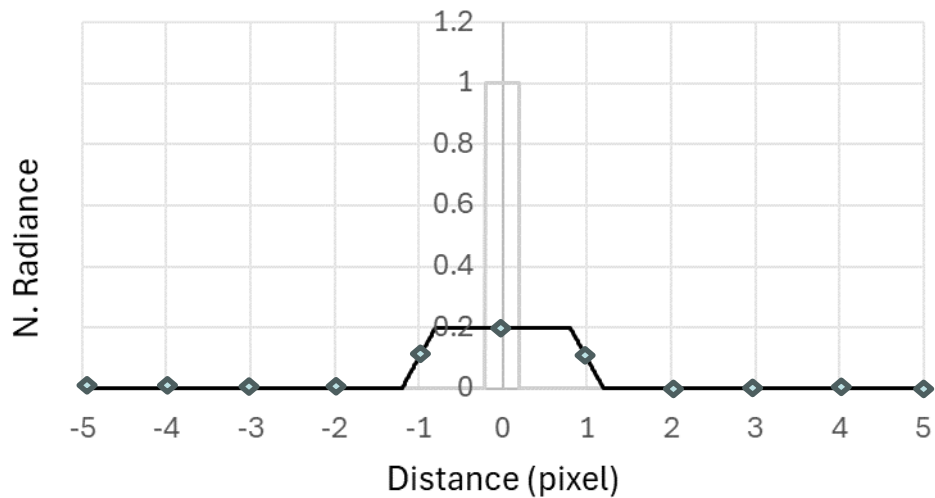
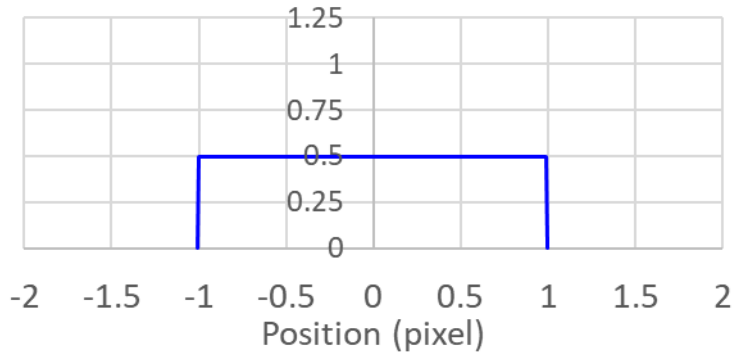




An isolated small object

Over-sampling, factor = 2

Rectangular LSF FWHM = 2

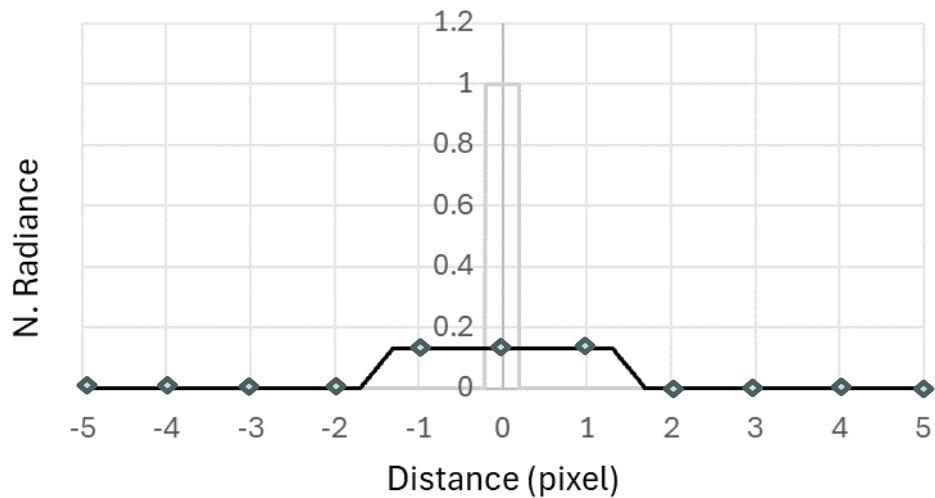
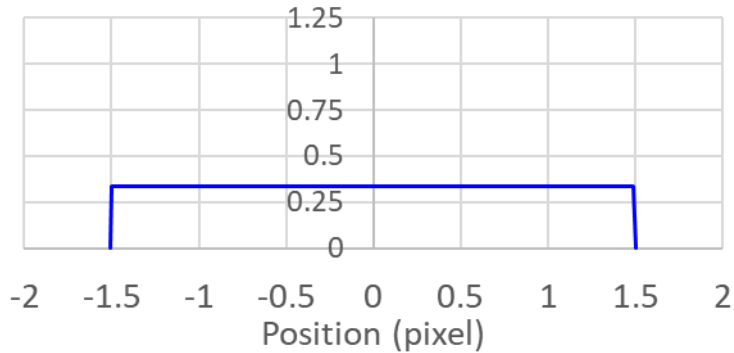




An isolated small object

Over-sampling, factor = 3

Rectangular LSF FWHM = 3



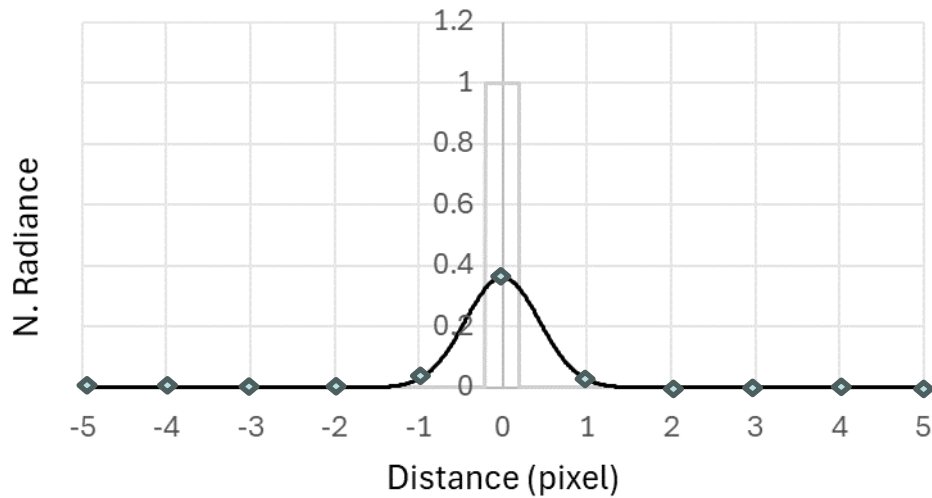
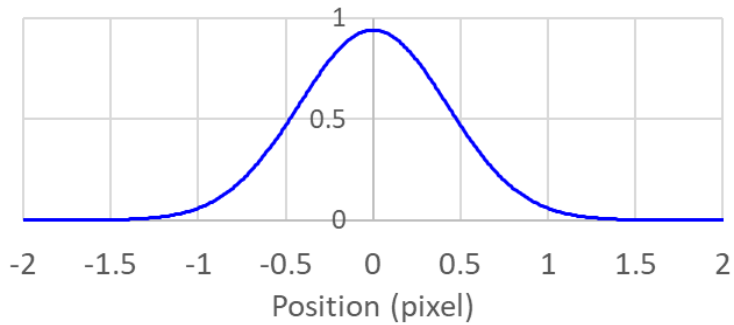


An isolated small object

Nyquist sampling

Gaussian

LSF FWHM = 1



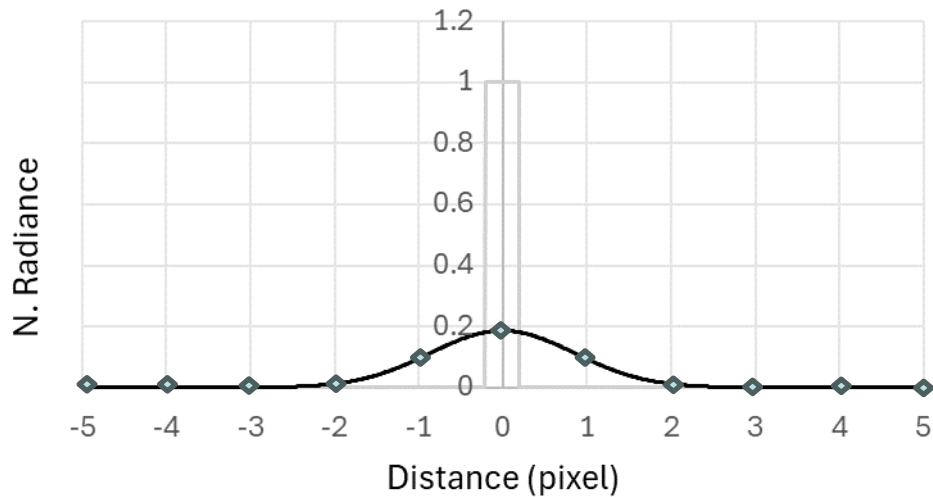
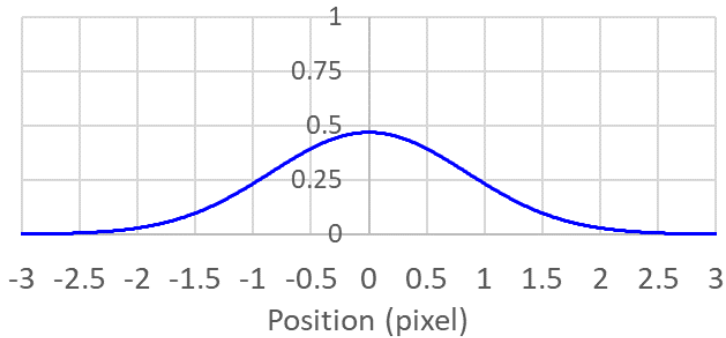


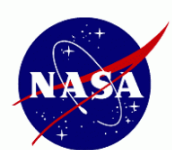
An isolated small object

Over-sampling, factor = 2

Gaussian

LSF FWHM = 2



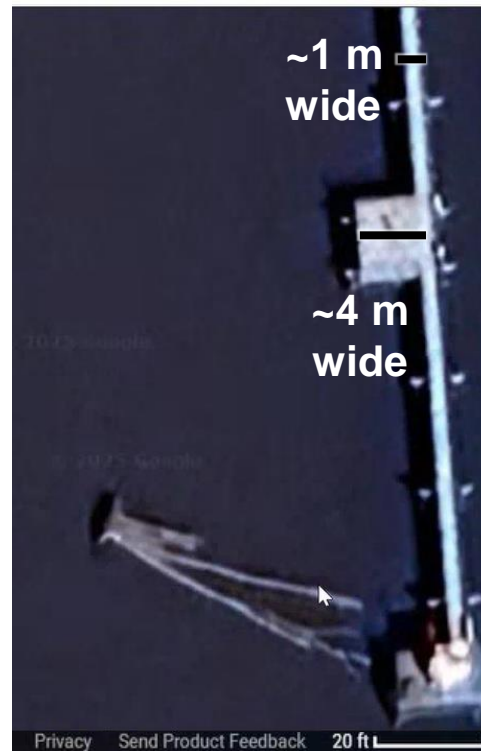


Google Earth images

Ronald Reagan Airport (Washington, DC)

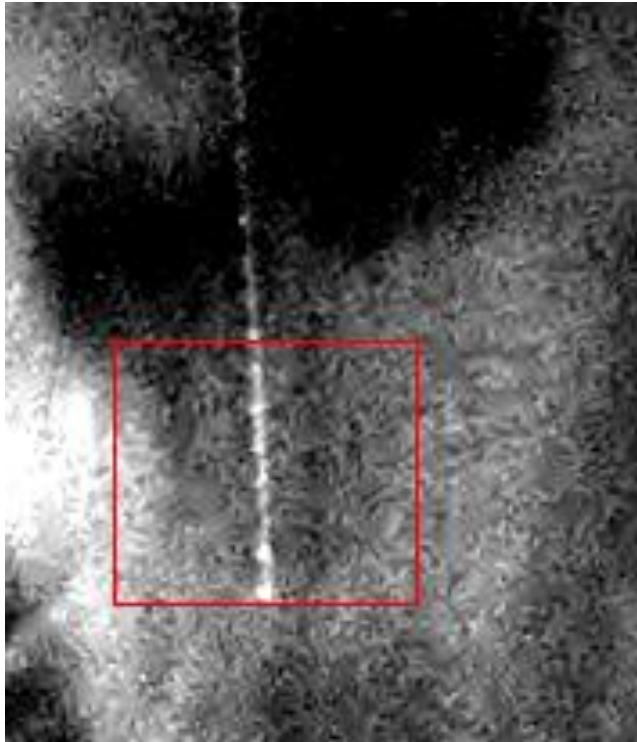


Zoomed in on the guide





A commercial image of Reagan Airport Guide



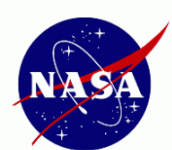
*Color scaled to bring out the guide

Zoomed in on guide (3m pixels)



The 1.5 m guide appears 3-4 pixels wide, 10 m in the image. That results in a Sensor spatial resolution 3.3 pixels wide, 10 m

Brightness change is where the guide is wider, but the underlying width is the same. The wider guide in the image is due to much larger sensor spatial resolution than the pixel size.



Another set of Commercial satellite images



Generation-1



1m pixels, FWHM ~ 1.4 pixels = 1.4m



Generation-2



0.7m pixels, FWHM ~ 2.8 pix = 2.0m
(vehicle width ~ 3.5 m?)



WorldView-2 image

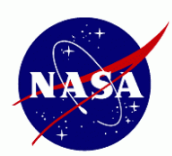
~1.8 m pixels

FWHM = 1.5 pixels = 2.7 m.

Slanted edges show up in saw tooth shapes

→ excellent sharp image!





Updated grading on FWHM & MTF

- FWHM to pixel-size ratio

	Ideal	Excellent	Good	Basic	Poor
FWHM/pixel ratio	(0.75, 1.25]	(1.25, 1.5]	(1.5, 1.75]	(1.75, 2.0]	> 2.0

- MTF based on Gaussian LSF

	Ideal	Excellent	Good	Basic	Poor
MTF@NF	[0.25, 0.6)	[0.13, 0.25)	[0.07, 0.13)	[0.03, 0.07)	< 0.03

Acronyms:

FWHM: full-width at half-maximum of an LSF

LSF: sensor system line spread function

MTF: modulation transfer function

NF: Nyquist frequency



Concluding remarks

Effects of over-sampling with a factor >2 ($\text{FWHM_LSF} > 2.0$ pixels)

1. Filters out MTF@Nyquist Frequency
2. Lowers data efficiency, $< 25\%$
3. Poses unfriendliness to users
 - 1) Wastes data storage, $> 75\%$
 - 2) Increases data transfer time and data processing time
 - 3) Mis-identifies small objects
4. Loses product competitiveness

