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## Introduction

GOCE is expected to provide homogenous, highquality information on the global gravity field with a spatial resolution of about 100 km . External validation of the satellite derived gravity field is possible in densely and accurately surveyed areas. In this study we try to evaluate, if the terrestrial data available in Norway is suitable for validation of GOCE satellite data. In addition we give some indications of possible systematic distortions in the terrestrial gravity field which might be detected with the help of GOCE data products.

## Methodology

In order to validate the final GOCE gravity field, it is necessary to provide a terrestrial data set with corresponding spatial and spectral resolution. Therefore we estimate the accuracy of block mean values computed from terrestrial gravity data in Norway.
There are three error sources for the final blockmean values:

- Measurement error of the individual data points.
- Computation error. In order to avoid aliasing, one usually carries out high pass filtering of the data in a remove-restore procedure. Errors, e.g in computation of topographic effects map into the final block mean values.
- Representation error. This error occurs when block mean values are computed from data points, which are not representative with respect to signal variations inside the block. This is the largest error contribution.
We use least-squares collocation for the error assessment, because it takes the representation error into account (in form of signal-covariance matrices $C$ ), when giving an estimate $\bar{\sigma}$ for the accuracy of a block mean value. The corresponding equation reads:

$$
\bar{\sigma}_{P}=\overline{\bar{C}}_{0}-\bar{C}_{P i} C_{i j}^{-1} \bar{C}_{P i}^{T}
$$

The covariance matrices are constructed from a degree variance model, blended from the model of Tscherning and Rapp (for longer wavelength) and the model by Flury (2006) which describes the short wavelength of the terrain reduced gravity field. Figure 1 shows a comparison of different degree variance models.


Figure 1: Degree variance models

## Accuracy of the mean terrestrial field

The accuracy of gravity block mean values is derived from least-squares collocation where the isotropic Pellinen coefficients (Pellinen, 1966) are used to describe the averaging process. Averaging is carried out in a spherical cap of radius $\psi_{0}$. The relation between $\psi_{0}$ and maximum spherical harmonic degree $L_{\max }$ is given in Table 1.
Accuracy estimates of the mean terrestrial field are shown in Figure 2. Computation was carried out on about 350 homogenously distributed points followed by areal interpolation to create the continuous illustration shown in Figure 2. About 150.000 data points (including marine data) were used. The figure shows the accuracy of the smoothed field, which can be expected from the given data distribution and accuracy of data points, assuming no systematic distortions of the data. No gravity values have been used for this error assessment.

Table 1: Relation between sh-degree and cap radius of a spherical cap of (a) constant area and (d) constant geographic degree spacing

| $L_{\max }$ | 50 | 100 | 150 | 200 | 250 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\left.\psi_{0, \mathrm{a}}{ }^{[ }\right]$ | 2.2 | 1.1 | 0.8 | 0.6 | 0.5 |
| $\left.\psi_{0, \mathrm{~d}}{ }^{[ }\right]$ | 2.0 | 1.0 | 0.7 | 0.5 | 0.4 |



Figure 2: Estimated accuracy of the mean terrestrial gravity field in Norway (including ocean areas). The small white dots indicate locations of gravimetric data points. A smoothing cap of $\psi_{0}=0.5^{\circ}$ was used. The figure is generated by areal interpolation between about 350 homogenuously distributed computation points.


## Systematic errors in terrestrial data sets

Figure 3 shows standard deviations of gravity values as given in the available data sets. Obviously a large amount of values does not reflect the actual accuracy of gravimetric measurements but are rather assigned to the data points as a measure of systematic distortions in various data sets of different origin. The situation is very similar for marine data, but with larger values (mostly ranging between 2 and 10 mGal ).


Figure 3: Standard deviations assigned to land gravity data in the available data sets.

## Conclusions

GOCE is expected to provide gravity anomalies with an accuracy of some few mGal at $L_{\text {max }}$ around 200-250. Comparison with terrestrial block mean values of corresponding spatial resolution shows, that these can (in absence of systematic errors) be determined with almost one order of magnitude higher accuracy. Therefore the terrestrial field in Norway is theoretically well suited for GOCE validation. In real application however one must consider systematic distortions, which might add up to several mGal in Norway. Further investigations need to be carried out to extract data of high quality for GOCE validation and to correct the remaining systematic errors with GOCE products.

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