

GOCE GRAVITY GRADIENTS: A NEW SATELLITE OBSERVABLE

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ABSTRACT

The Gravity field and steady-state Ocean Circulation Explorer (GOCE) is one of the flagships in ESA's Living Planet Programme. With the help of the on-board, very precise gravitational gradiometer the Earth's gravity field is to be determined with unprecedented accuracy. The gradiometer measures gravity field differences that are contained in six gravity gradient (GG) tensor components. Because of the instrument characteristics, four out of six tensor elements are very accurate, whereas the other two are less accurate.

We will concentrate on the noise characteristics of the accurate GGs by a spectral analysis of the residuals of a semi-analytical gravity field solution for the first measurement phase (November - December 2009). Formal errors of spherical harmonic coefficients are estimated in a semi-analytical way by means of a GOCE combined gravity field solution. In addition they are then compared with formal errors of recent GOCE-only models.

As in the measured GG tensor there are two less accurate components the rotation of the GG tensor from the instrument frame to a local Earth related frame is not straightforward. A possible solution is to replace the less accurate components with model gradients. We will discuss the trade-off between measured and model GGs and assess the information content of the rotated gradients in different local frames, specifically the local north-oriented frame and the local orbital frame.

Furthermore we will present a quality assessment of the GOCE GGs over time. We will show that the errors of V_{xx} and V_{zz} are constant over time, but that the errors in V_{yy} increase after each calibration event, which is due to a drift in the differential scale factor associated with V_{yy} . Finally, the V_{zz} error level seems to have decreased by 25% after the switch from CPU A to B in February 2010.

Key words: Spectral gravity gradient characteristics; Formal GOCE-only error models; Tensor rotation; Quality assessment.

1. INTRODUCTION

One of the first steps in GOCE data analysis should be to look into the characteristics of the new data type of GGs. Our goal therefore is to obtain a comprehensive understanding of signal and error characteristics of the individual measured GGs. This should be the basis of their use in science and application. The GOCE GGs are given in the instrument frame. The analysis of the GGs may require rotation of the GOCE GGs to other local reference frames. As observations now are available in a time series of a few months it is also important to analyze the GGs quality evolution over time.

2. SPECTRAL CHARACTERISTICS OF THE DIAGONAL GRADIOMETER COMPONENTS

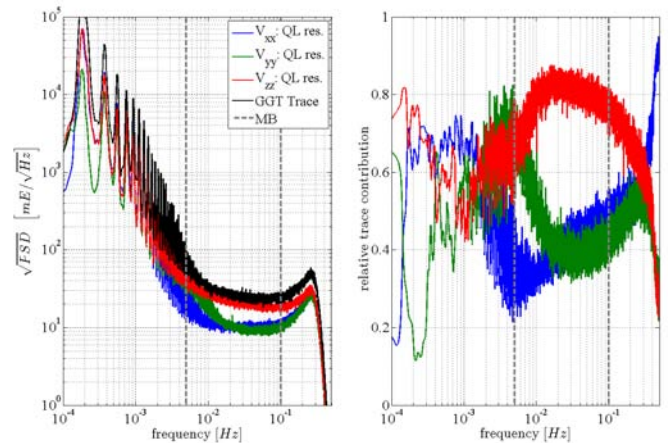


Figure 1. Left: PSDs of the GOCE Quick-look residuals (three diagonal components of the GG tensor) together with the PSD of the observed trace (black). Right: relative contribution of the three diagonal components to the trace, i.e. $V_{ii}/\sqrt{V_{xx}^2 + V_{yy}^2 + V_{zz}^2}$, $i \in \{x, y, z\}$

The spectral characteristics of the GOCE gravity gradients (GG) were analyzed in the frequency domain. A first idea of the noise in the GGs is given by the trace

of the measured GG tensor and its power spectral density (PSD). This trace PSD is shown in Figure 1 (left, black) and should in principle carry all the noise of the three diagonal components. A realistic estimation of the noise in these diagonal GG can be achieved with a least squares adjustment. A Quick-look solution [6] of MOP1 provides residuals (adjusted – measured observations) of the three diagonal components as well. Their PSDs are also shown in Figure 1 on the left. A closer look at the individual contributions of these components to the total noise of the trace can be seen on the right. The noise level in the upper measurement bandwidth (MB) of V_{zz} is nearly twice the levels of V_{xx} and V_{yy} . Therefore it contributes most to the trace in this frequency range (around 80%) whereas V_{yy} contributes least. Towards the lower frequencies the noise in V_{yy} increases relatively to the trace and at the lower end of the MB it gets even larger than the noise in V_{zz} . This can also be seen on the left in Figure 1 where the flat nearly white parts of the noise PSDs of V_{zz} and V_{xx} are larger than for V_{yy} .

3. ANALYSIS OF RECENT GOCE-ONLY FORMAL ERROR MODELS

In the next step these GG noise information have been propagated to the SH spectrum with a semi-analytical processing [7]. Furthermore normal equations for GOCE GPS observations are included as well. Their stochastic behavior is assumed to be white in terms of gravity potential on a level of $1m^2/s^2/\sqrt{Hz}$, but the focus is on the GG performance. With this setting one gets a combined GOCE formal error estimation based on the noise PSDs from above. These formal errors have been compared with recent GOCE-only models (See table 1). Figure 2 shows these comparisons in terms of the matching of formal errors (left) and in terms of SH degree medians (right). The relative formal error equality is computed from the log10 of the differences of the simulated formal errors and the one given from the model divided by the model formal errors. In other words it tells up to which decimal the two error sets match. As the GG noise PSDs are derived from the Quick-look model its formal errors fit the simulated ones best. Several aspects can be seen from the other models. As no of these three remaining error models (TIM, SPW and DIR) contains the peaks in the PSDs of all GG components at multiples of the orbit frequency f_0 ($1, f_0 \approx 1.86 \cdot 10^{-4} Hz$) they underestimate the sensitivity of the GGs for the lower SH degrees

| Model | Period [days] | L_{max} | |
|-------|------------------|-----------|-----|
| QSG | 61 | 200 | [3] |
| TIM | 71 | 224 | [5] |
| SPW | 73.3 | 210 | [4] |
| DIR | 72 | 240 | [1] |

Table 1. Parameters of GOCE-only models for MOP1. The mean values for the altitude of 259.5km and the inclination of 96.65° hold for all four models.

at multiples of the order 16. Mainly at higher SH degrees it can be seen in red where regularization was applied (SPW only over the poles, TIM Kaula regularization for high SH degrees). In general it can be said that the white and light blue areas mark the SH domain where the formal errors describe the real GOCE noise behavior best. In the red SH areas other information than pure GOCE was used to derive the formal error sets (e.g. regularization, a priori model).

This becomes even clearer if one looks at the matching of the error sets and the coefficient differences in terms of SH degree median (Figure 2, right). In all four cases one sees the SH degrees where GOCE can improve recent combined models as the here used EIGEN-51C. These are the SH degrees between 110 and 170 where all the differences (red curves) become larger than the GOCE errors. The DIR model uses prior information in terms of EIGEN-5C. Therefore the differences for higher degrees to EIGEN-51C get far lower as the formal error estimates.

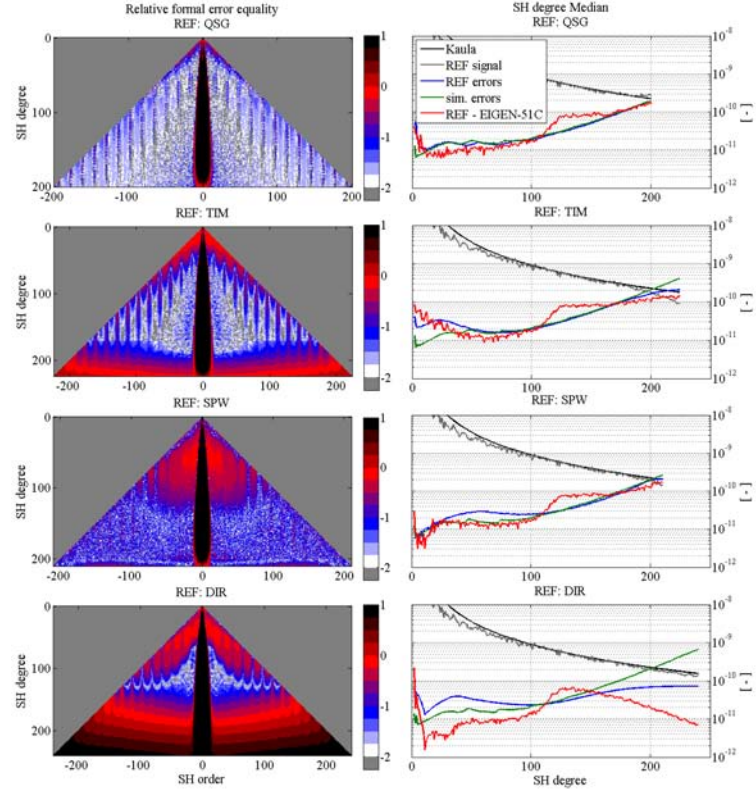


Figure 2. Comparison of simulated formal errors with the four models in table 1. Left: $\log_{10}(|\Delta\sigma|/\sigma_{REF})$, decimal up to which the two error sets match. Right: SH degree medians of the two error sets compared with signals and coefficient differences w.r.t. EIGEN-51C.

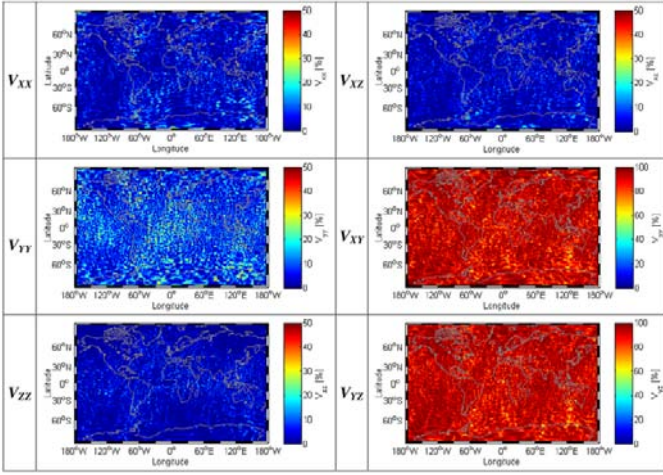


Figure 3. Geographical plot of the relative model content in the rotated GGs for different components in the LORF-v [2].

4. ROTATION OF THE GRAVITATIONAL TENSOR

GOCE measurements are used in many geophysical applications in local frames such as e.g. the local orbital reference frame (LORF) or the local north oriented reference frame (LNOF). Since the GOCE only measurements include also inaccurate measured GGs the tensor rotation must be enhanced using prior information. An alternative method is to substitute the inaccurate measured GGs in the measurement bandwidth (MB) with model information. To avoid leakage inside the MB, model gradients are computed also for the lower frequencies. Therefore two gravity tensor matrices, one for the model information and one for the GOCE GGs can be set up and rotated combined to any reference frame chosen. One important question concerning the rotated gradients is the amount of model information left in the rotated gradient tensor. The relative model content has been determined integrating the signal energy from the lower MB limit to the limit where the gradient signal equals the signal to noise ratio of one (Noise and signal energy are equal). Both energies, one for the model and one for the measured tensor elements, can then finally be used to compute a relative model content for each gradient component respectively. It is seen from Figure 3 and 4 that the model content in the rotated gradients differ significantly w.r.t. the frame chosen. Table 2 gives an overview of the global average values for each rotated frame.

5. QUALITY ASSESSMENT OF THE GOCE GRAVITY GRADIENTS OVER TIME

The GOCE gradient quality changes over time where the gradient components are affected differently. The V_{xx} and V_{xz} gradient components have an almost constant quality level where in contrast V_{yy} and V_{zz} show tem-

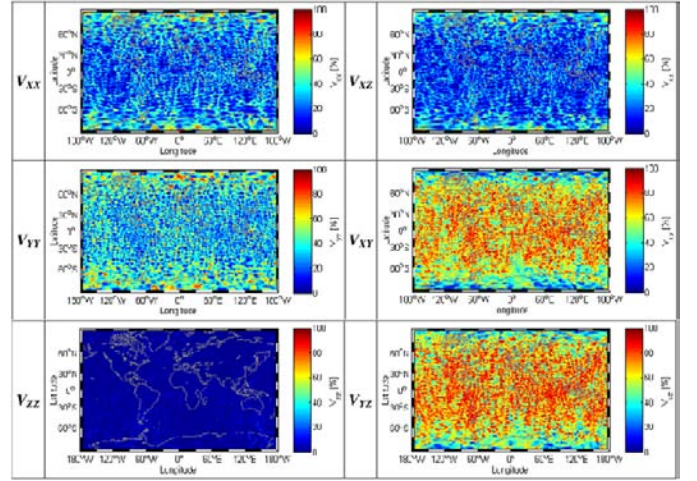


Figure 4. Geographical plot of the relative model content in the rotated GGs for different components in the LNOF [2].

| GG | LORF-v | LORF-r | LNOF |
|----------|--------|--------|------|
| V_{xx} | 3.8 | 3.8 | 25.8 |
| V_{yy} | 9.2 | 7.9 | 35.4 |
| V_{zz} | 2.6 | 1.8 | 1.8 |
| V_{xy} | 91.7 | 92.6 | 61.2 |
| V_{xz} | 3.1 | 2.9 | 21.7 |
| V_{yz} | 92.2 | 93.1 | 64.4 |

Table 2. Average model content due to mixing of the tensor elements [%] [2].

poral changes. In Figure 5 the standard deviation w.r.t. a reference gravity field model [5] is shown over time. For V_{yy} a relation to calibration events is visible. The change in gradient quality is probably related to a small drift of differential scale factors. The noise increases after a calibration event. The V_{zz} residuals decrease significantly after the change of the CPU-A to the CPU-B side. In the frequency space it can be seen that the gradient quality over time also changes. Inside the MB the gradient quality is good while below the MB the gradient quality degrades and periodic residuals, which are multiple of the orbital period, dominate. In addition higher residuals which are correlated with data periods that are processed with the second star tracker STR2 onboard of GOCE are visible (See Figure 6).

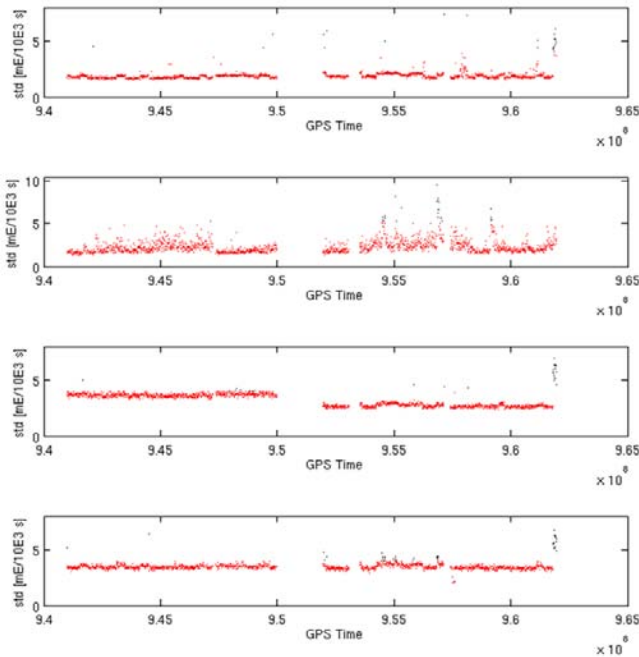


Figure 5. GG standard deviation over time (From top to bottom: V_{xx} , V_{yy} , V_{zz} , V_{xz}).

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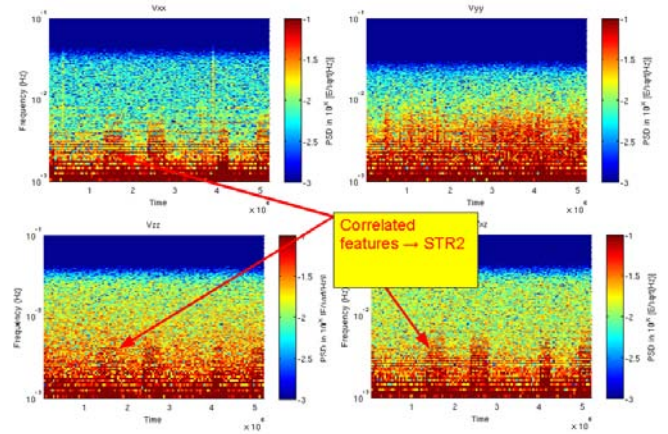


Figure 6. Windowed FFT-Analysis over time for the differences to the time-wise gravity model [5].

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