SMOOTHING GOCE GRAVITY GRADIENTS BY MEANS OF TOPOGRAPHIC-ISOSTATIC REDUCTIONS

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ABSTRACT

Gravity gradients measured by ESA's satellite mission GOCE contain significant high- and medium-frequency components mainly originating from the attraction of the Earth's topographic and isostatic masses. In order to mitigate the induced instability of downward continuation, the observed gradients should be smoothed by applying topographic-isostatic reductions. This paper provides a detailed description of the suggested smoothing strategy and an overview of the developed parallelized software tool TOISMAT which generates suitable topographic-isostatic reductions for gravity gradient smoothing. Taking a GOCE data set collected in November 2009 as example, the degree of smoothing is assessed by analysing changes in standard deviation and range due to topographic-isostatic reductions.

Key words: GOCE; Topographic-isostatic reductions; Forward gravity modelling; Tesseroid.

1. INTRODUCTION

Analysing the gravity gradients measured by the GOCE satellite, high- and medium-frequency signals can be discriminated which are mainly caused by the attraction of the Earth's topographic and isostatic masses. Therefore, the harmonic downward continuation of these gradients from the satellite altitude to Mean Sea Level (MSL) can be considered as an ill-posed problem in the context of gravity field modelling. Concerning the numerical stability of the downward continuation of gravity gradients it is recommended to smooth them by applying topographic-isostatic reductions using a remove-compute-restore technique [18]. Previous investigations based on simulated GOCE gravity gradients have already shown that significant smoothing effects shall be possible [e.g., 11, 12]. In this paper, for the first time, real GOCE gravity gradient measurements are smoothed by means of topographic-isostatic reductions relying upon a newly developed and advanced concept.

In section 2 the processing strategy is described, including input data preparation. Numerical results obtained by analysing a GOCE data set measured in November 2009 are presented and discussed in section 3. Finally, section 4 provides concluding remarks with an outlook for future research work.

2. PROCESSING STRATEGY

The proposed processing strategy consists of three steps: (1) preparation of GOCE data, (2) evaluation of topographic-isostatic reductions, (3) combination of measurements and reductions. In the following, each processing step is described in detail.

2.1. Preparation of GOCE data

The use of the published GOCE Level 2 Product Data [3] requires the extraction and preparation of three types of information: (1) observed GOCE gravity gradients to be smoothed, (2) the corresponding measurement positions used for computing topographic-isostatic reductions, (3) epoch-wise transformation matrices from the conventional Local North Oriented Frame (LNOF) to the Gradiometer Reference Frame (GRF). All the information mentioned above can be obtained from the daily EGG_NOM_2 and SST_PSO_2 data files.

In the first step, the XML data files have to be converted into ASCII format. For this purpose, the L1b-L2 XML Parser provided by the GOCE High Level Processing Facility [4] is utilized. The converted EGG_NOM_2 files contain time series of GOCE gravity gradients related to GRF, their GPS time tags and Inertial Attitude Quaternions (IAQ) providing additional information on the attitude of the GRF with respect to the Inertial Reference Frame (IRF). The SST_PSO_2 files and their converted subproducts SST_PRD_2 and SST_PRM_2 comprise Cartesian coordinates of a reduced dynamic GOCE orbit and Earth-Fixed Oriented Quaternions (EOQ) describing the orientation of the Earth-Fixed Reference Frame (EFRF) relative to the IRF.

Since the subproducts information is not temporally synchronised with the observed gravity gradients in EGG_NOM_2, they have to be interpolated onto

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the GPS time tags of the gradiometer measurements. Due to a slight shift of the covered time period in comparison to the EGG_NOM_2 data, information from two subsequent SST_PSO_2 files has to be incorporated into the interpolation. For further processing steps, the measurement positions in terms of the interpolated Cartesian coordinates are transformed to spherical geocentric coordinates using conventional formulas [e.g., 8]. The transformation from LNOF to GRF can be realised by a composition of three rotations: (1) LNOF to EFRF using the interpolated spherical coordinates of GOCE, (2) EFRF to IRF based on the interpolated EOQs, (3) IRF to GRF by directly applying the IAQs [cf. 3].

2.2. Modelling topographic-isostatic reductions

Topographic-isostatic reductions can be obtained by forward gravity modelling based on the evaluation of functionals of Newton's integral [14] extending over the domain of the Earth's topographic and isostatic masses. Thereby, global information on the geometry and mass-density distribution of the Earth's crust and upper mantle is required. The proposed reduction concept applies model parameters resulting from the numerical studies presented in [6].

For representation of the topographic masses, the $5' \times 5'$ global digital terrain model DTM2006.0 [16] is utilized which provides information on surface elevation, ocean and lake depth, as well as ice thickness. Moreover, each grid element of the model is classified into one of six terrain types: (1) Dry Land Below MSL, (2) Lake, (3) Oceanic Ice Shelf, (4) Ocean, (5) Grounded Glacier, (6) Dry Land Above MSL. As an advantage over common modelling approaches, in which height information of a digital terrain model is usually used to characterise topographic masses with constant density, the suggested concept makes use of a more sophisticated and self-developed Rock-Water-Ice Model. In this context, the information from DTM2006.0 is used to construct a $5' \times 5'$ three-layer terrain and density model consisting of a rock (R), water (W), and ice (I) component possessing different heights (h_R, h_W, h_I) with respect to MSL and consistent thickness (t_R, t_W, t_I). Furthermore, the specified terrain types in DTM2006.0 have been used to derive layer-specific density values (ρ_R, ρ_W, ρ_I), see Fig. 1.

For the computation of the isostatic reductions, the isostatic compensation masses have to be quantified. Using classical isostatic models, these masses are usually calculated directly from the topographic load by applying the mass equality condition with respect to a particular compensation depth. The Airy-Heiskanen isostatic scheme, which was originally designed to deal with topographic masses of constant density [10, p. 135ff], is adopted to the proposed Rock-Water-Ice concept using layer-specific density values. As it is well-known that isostatic masses also reflect heterogeneities inside the Earth [13], attempts are made to further extend the concept by introducing a depth model for the Mohorovičić discontinuity (Moho).

Modifying the classical Airy-Heiskanen model, the (anti-)root depths are replaced by Moho depths. This modification, however, will no longer fulfill the mass equality condition. As a consequence, [12] found that the resulting topographic-isostatic reductions possess a large magnitude contaminating the aspired smoothing of gravity gradients. In order to nevertheless maintain the mass equality condition, the density contrast between crust and mantle is kept variable (Fig. 2). A similar procedure has been applied to gravity anomalies to determine an improved Austrian geoid [19]. Related to the Rock-Water-Ice approach, the variable density contrast can be calculated by

$$\Delta\rho = \frac{\rho_R \left(h_R^3 - R^3\right) + \rho_W \left(h_W^3 - h_R^3\right) + \rho_I \left(h_I^3 - h_W^3\right)}{\left(R - T\right)^3 - \left(R - h_M\right)^3}$$

h_R : Top of Rock surface
h_W : Top of Water surface
h_I : Top of Ice surface
R : Mean Earth radius
T : Compensation depth
h_M : Moho depth.

Although Fig. 2 illustrates the principle of the modified Airy-Heiskanen model in a planar manner, the derived formulas corresponds to a spherical approximation. Since up to now no global high-resolution Moho depth model is available, a $5' \times 5'$ grid model of smoothed global Moho depths is derived from the $2° \times 2°$ global crust model CRUST 2.0 [2] by means of harmonic analysis and synthesis [20]. The obtained Moho depths are introduced into the modified Airy-Heiskanen model with respect to a normal compensation depth of $T = 28$ km.

The computation of topographic-isostatic reductions is performed by the self-developed C++ program TOISMAT (TOpographic-ISostatic MAss reductions using Tesseroids) applying forward gravity modelling in space domain. Topographic-isostatic reductions are calculated along the real orbit of the GOCE satellite using the measurement positions as computation points. The derived Rock-Water-Ice Model and Isostatic Model serve as input data for the description of the topographic and isostatic masses. For the numerical evaluation of functionals of Newton's integral, a mass discretisation of the integration domain according to the gridded input models is necessary. The decomposition is carried out using tesseroid bodies which are naturally created by the grid lines of geographical coordinates [1]. In view of precision and computation time, previous investigations have verified the high numerical efficiency when utilizing tesseroids instead of conventional rectangular prisms [9, 5]. The algorithm implemented in TOISMAT for

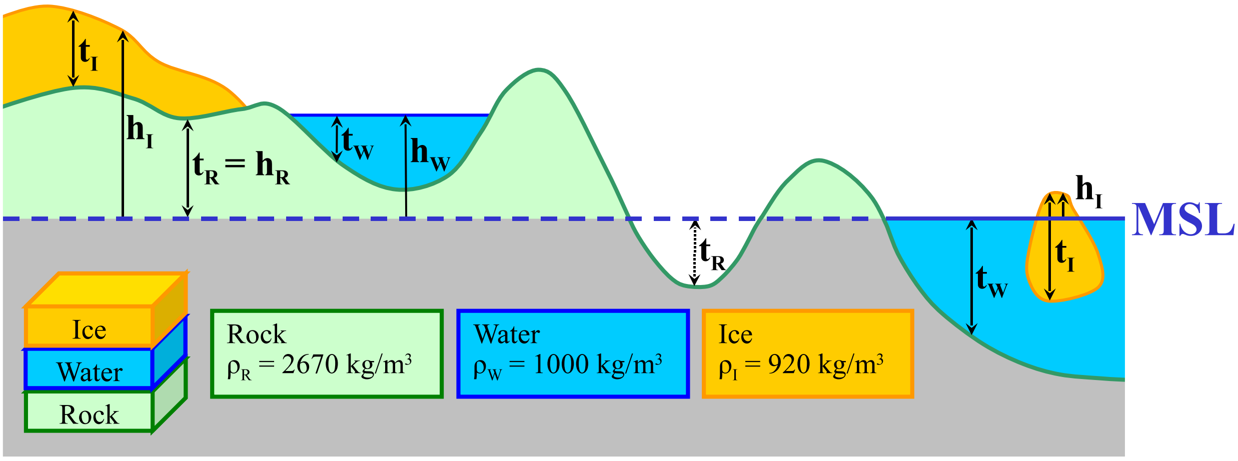

Figure 1. Schematic presentation of the three-layer Rock-Water-Ice Model

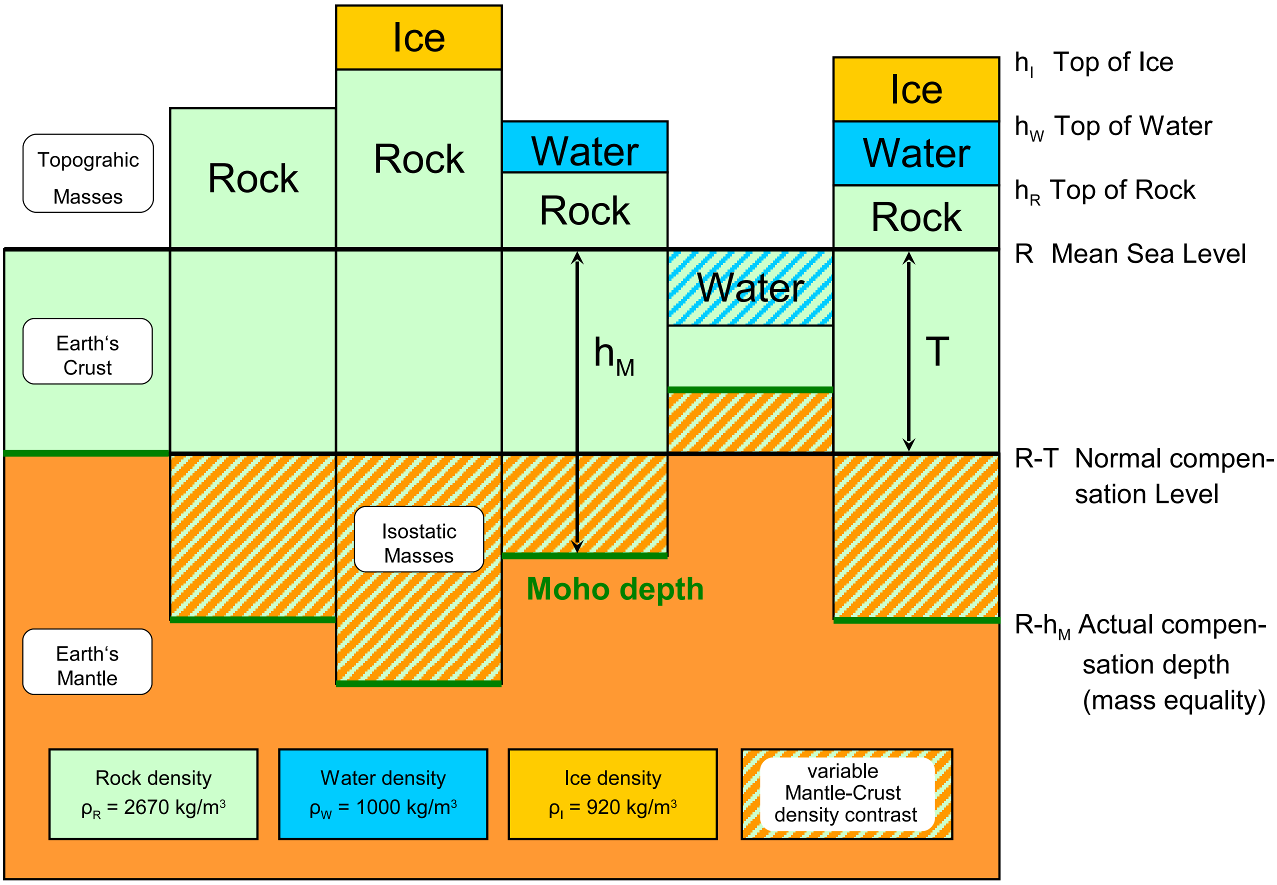

Figure 2. Schematic presentation of the modified Airy-Heiskanen model introducing Moho depths

determining the impact of tesseroids on gravity gradients is based on optimised, singularity-free tesseroid formulas in LNOF [cf. 5]:

$$\frac{\partial^2 V(x,y,z)}{\partial i\,\partial j} = G\rho \iiint\limits_{\Omega} \left(\frac{3\Delta x_i \Delta x_j}{\ell^5} - \frac{\delta_{ij}}{\ell^3} \right) \mathrm{d}\Omega$$

$$\begin{aligned} \Delta x_1 &= r' \cos\varphi \ \sin\varphi' - \sin\varphi \ \cos\varphi' \ \cos(\lambda' - \lambda) \\ \Delta x_2 &= -r' \cos\varphi' \sin(\lambda' - \lambda) \\ \Delta x_3 &= r' \cos\psi - r. \end{aligned}$$

In these formulas G denotes Newton's gravitational constant and ρ is the constant density value within the integration domain Ω extending over the dimensions of the tesseroid. The Euclidean distance between the computation point $P(r, \varphi, \lambda)$ and the running integration point $Q(r', \varphi', \lambda')$ is denoted by ℓ. As the volume integrals appearing in these formulas cannot be solved in an analytical way, a second-order approximation of the integral kernel based on a Taylor series expansion is applied in combination with a subsequent term-wise integration. Numerical evaluation rules related to the optimised tesseroid formulas are explicitly presented on pages 25ff in [5]. Although the tesseroid formulas are given in a spherical approximation, the particular tesseroid bodies are arranged on the surface of a GRS80 ellipsoid of revolution [15] approximating MSL. Applying the super-positioning principle, the total topographic-isostatic reduction at each computation point is finally represented by summation over the impact of all tesseroids.

Since the whole calculation process is very time-consuming, especially when using such high-resolution input data, TOISMAT is designed for parallel computing on high-performance computer systems, such as the HP XC3000 (HC3) operated at the Karlsruhe Institute of Technology. According to the specification of EGG_NOM_2 gravity gradients, the output of TOISMAT are LNOF-related topographic-isostatic reductions in form of time series.

2.3. Combination of measurements and reductions

Keeping the main purpose of smoothing GOCE gravity gradients in mind, the topographic-isostatic reductions obtained from TOISMAT have to be applied to the measured gravity gradient data. Being different from the observations which are related to the GRF, the reductions are computed in the conventional LNOF. As the combination of both data sets can only be accomplished in the same reference frame, the gravity gradient tensor of the reduction values has to be transformed from LNOF to GRF. The so-called Key transformation can be performed by a direct point-wise rotation using the transformation matrices derived in the data preparation step. Afterwards, by subtracting the transformed reductions from the measurements, the topographic-isostatically reduced GOCE gravity gradients are obtained.

3. FIRST RESULTS

To get an impression of the achievable degree of smoothing, the presented processing strategy has been applied to a real GOCE data set measured on November 2, 2009. For the computation of topographic-isostatic reductions by means of the TOISMAT program on a daily basis, a run-time of about four hours is required using parallel computing on 80 processors on HC3 (Intel Xeon E5540, 2.53 GHz).

It should be noted that the proposed processing strategy based on topographic-isostatic reductions only smooths the high- and medium-frequency signal components of the GOCE observations, since these signals are especially responsible for the instability of the harmonic downward continuation process. The GOCE gradiometer is particularly sensitive within a frequency range of 5 to 100 mHz, the so-called measurement bandwidth (MBW). However, gravity gradients in EGG_NOM_2 products contain all measured spectral information, also including long-wave signals below the MBW which implicate maximum amplitudes of 2500 E (Eötvös unit, $1\text{E} = 10^{-9}\ \text{s}^{-2}$) in the V_{zz} component, for instance. The derived high- and medium-frequency topographic-isostatic reductions vary within about ±1 E and are therefore in a totally different scale.

In order to investigate the smoothing effects, the signal content of the GOCE gravity gradients before and after reduction should be compared only within a limited frequency bandwidth, for example in the MBW. To extract the signal content within the bandwidth of interest, a symmetric non-recursive bandpass filter according to [7] has been applied to the original GOCE time series as well as to the topographic-isostatically reduced GOCE time series. In doing so, the bandpass-filtered measurements are widely de-trended and possess the same order of magnitude as the topographic-isostatic reductions.

For a representative graphical comparison of the filtered time series, one particular 90-minute orbit cycle starting at GPS time tag 941157414.3945 is selected, since in this time period a wide range of different topographic surface types have been passed by GOCE. In Fig. 3 the corresponding ground track of the GOCE satellite is visualised. Comparing the original signal with the reduced one, significant smoothing effects can be detected for the main diagonal tensor elements V_{xx}, V_{yy}, V_{zz}, as well as for the off-diagonal element V_{xz} (see Fig. 4). This is particularly obvious when regions with highly variable topography are crossed, for example the Rocky Mountains (time: 10-15 min) and the Himalayas (time: 40-50 min). However, for areas over oceans and especially over the ice masses

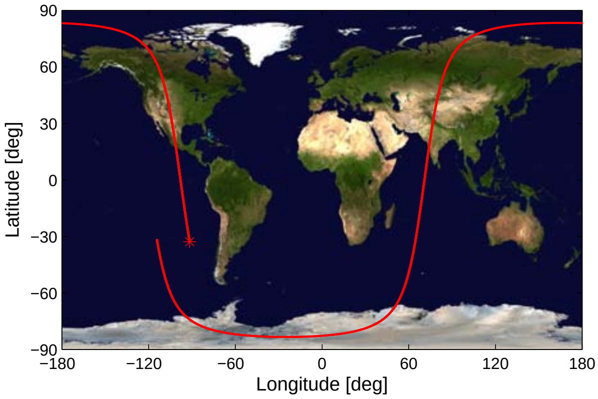

Figure 3. GOCE ground track of the selected orbit cycle (start position indicated by a asterix)

of Antarctica (time: 70-80 min), only very small smoothing effects and irregularly occurring phase displacements can be observed. On the one hand, this may indicate uncompensated topographic masses that are not captured by the applied isostatic model as well as insufficient modelling of the density distribution in the ice masses. On the other hand, it should also be noted that the errors in the input data are considerably larger in regions like Antarctica. For all of these four gravity gradient components, a highly regional character concerning the smoothing behaviour can be recognized. Regarding the remaining elements V_{xy} and V_{yz} only insignificant impact of smoothing is visible (see Fig. 4). This is caused by the fact that these two off-diagonal tensor elements are observed less accurately by the GOCE gradiometer. The signal content in the MBW is superposed by high-frequency noise which makes effective smoothing by means of topographic-isostatic reductions inapplicable.

Table 1. Percentage changes in standard deviation and range before and after reduction

[%]	V_{xx}	V_{yy}	V_{zz}	V_{xy}	V_{xz}	V_{yz}
P_{std}	29.0	29.6	29.8	1.0	29.3	0.9
P_{range}	43.0	22.7	47.5	11.7	50.5	1.8

To quantify the degree of smoothing, the percentage changes in standard deviation and range before and after reduction can be calculated. In Tab. 1 numerical values are given for each gravity gradient component, derived using the bandpass-filtered time series of the whole day. Considering the standard deviations, V_{xx}, V_{yy}, V_{zz}, and V_{xz} can be reduced by nearly 30%. Large percentage changes in range are found in the V_{zz} and V_{xz} components (about 50%) as well as in the V_{xx} component (about 40%). For V_{yy}, a value of only about 20% is dedected which may be explained by a smaller signal amplitude in comparison to the other gradient components. Taking a look at the off-diagonal components V_{xy} and V_{yz}, no significant improvements can be seen. Summarising, the statistical analysis confirms the previous findings gained by visual inspection.

4. CONCLUSIONS AND OUTLOOK

To mitigate the numerical instability of the harmonic downward continuation of observed GOCE gravity gradients, a smoothing approach based on topographic-isostatic reductions obtained form forward gravity modelling has been presented in this paper.

Being advantageous over previous approaches a new concept for reduction has been suggested which is based on a three-layer Rock-Water-Ice decomposition of the topography with variable density values. Geometry and density information is derived from the global topographic data base DTM2006.0. Additionally, a modified Airy-Heiskanen model has been applied, which is improved by introducing a Moho depth model obtained from the global crust model CRUST 2.0. The computation of topographic-isostatic reductions is performed by the self-developed C++ program TOISMAT which is designed and optimised for parallel computing on high-performance computer systems.

Preliminary results from processing a real GOCE daily data set collected in November 2009 show a significant smoothing potential for the V_{xx}, V_{yy}, V_{zz}, and V_{xz} gradient components within the measurement bandwidth of the gradiometer. The degree of smoothing quantified by assessing changes in standard deviations amounts to about 30%, while the range can be reduced by 20-50%. Generally, the smoothing effect is strongly dependent on the actual topographic surface that is crossed by the satellite. This dependence makes the proposed procedure particularly suitable for regional applications. However, the suggested processing strategy seems to be inapplicable to the two less accurately observed components V_{xy} and V_{yz} due to high-frequency noise in the original gradient measurements.

Further research work will concentrate on the enhancements in the reduction concept by comparing the influence of different isostatic models as well as on improvements by means of a more detailed mass-density distribution model. Apart from the suggested smoothing strategy for GOCE gravity gradients by applying topographic-isostatic reductions in the space domain, alternative calculations in the frequency domain as well as other smoothing approaches, for example based on Residual Terrain Modelling or EGM08 information [17], will be analysed, compared, and probably combined. Moreover, investigations on harmonic downward continuation of GOCE gravity gradients with and without smoothing will be performed to verify the assumed improved stability.

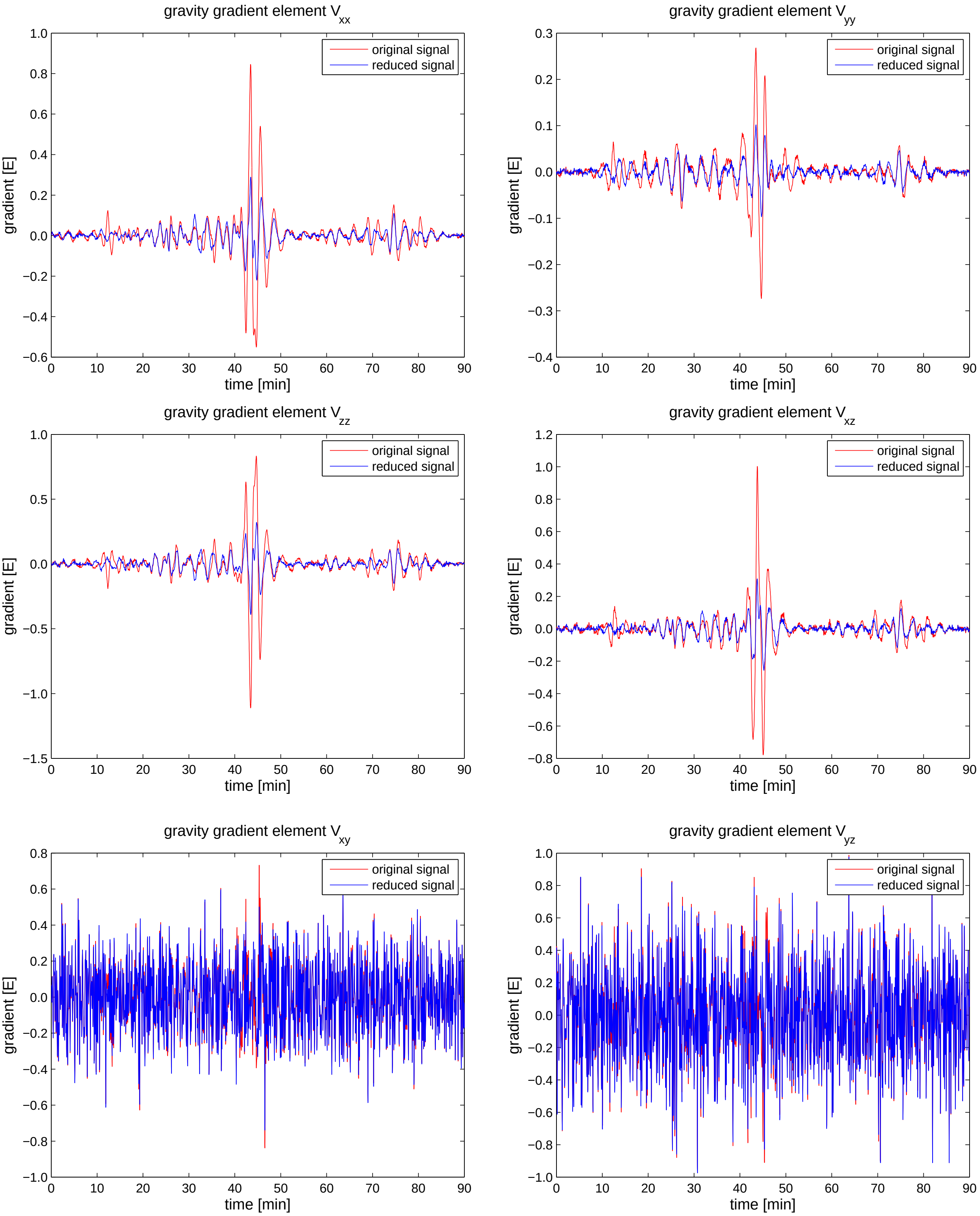

Figure 4. Comparison of original and topographic-isostatically reduced GOCE gradient signals within the MBW

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