UPDATING EGM08 MEDITERRANEAN GEOID USING LOCAL GOCE DATA FROM THE SPACE-WISE SOLUTION

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ABSTRACT

In the framework of the GOCE-Italy project a solution to improve at local level a global model using the delivered GOCE data has been studied and tested. The combination is based on a collocation procedure considering as observations the spherical harmonic coefficients with their predicted error variances and a local grid of the second radial derivative of the gravitational potential; this grid is estimated from GOCE observations at satellite level by using the space-wise approach. The use of local data, instead of the available global grids, gives the possibility to consider the full error covariance matrix of the gridded data into the collocation combination; this covariance matrix is computed by Montecarlo simulations.

The method has been applied for the update of the Mediterranean geoid computed from the EGM08 model at a resolution of 5’×5’ in the area between 28°N - 46°N and 10°E - 37°W. The geoid corrections have an rms of less than 1 cm, that increases to some cm over specific areas (e.g. Alps, Aegean plate, Cyprus). Regarding the corrections of the global model coefficients, they are concentrated in the degrees from 70 to 220.

These corrections are quite small, probably due to the very good quality of the EGM08 model in Europe and to the fact that only the first two months of GOCE data have been used. A more significant impact is expected when considering a longer data period and when the method is applied in areas where only poor or inadequate ground gravity data are available.

1. INTRODUCTION

The Earth global gravity field model EGM08 [18] is basically a combination of a satellite only GRACE model [7] at low degrees and a 5’×5’ grid of gravity data at ground level. This model has been assessed [1] with independent data showing a comparable accuracy with respect to local geoid models. This implies that the global EGM08 spherical harmonic coefficients (together with a topographic correction) can be directly used to generate a high resolution local geoid (see e.g. [6]).

In this work we investigate whether data from the GOCE mission [4] can contribute to improve a local geoid derived from EGM08.

The most straightforward solution to this problem would probably be to combine the spherical harmonic coefficients of a GOCE-only global gravity field model with those of EGM08, and then use this new model to derive the local geoid of the area under study. However, in order to get an optimal combination of these two sets of coefficients some theoretical and numerical problems have to be faced (see e.g. [20]).

Here we follow a simpler combination strategy, i.e. a subset of GOCE data over the area under study is first selected and then used to update EGM08 coefficients. In this way we aim at improving the accuracy of the functionals of the gravitational potential derived by these updated coefficients (e.g. geoid undulation, gravity anomalies, etc.) only in the considered area.

The main advantages of this approach are to reduce the computational burden and, hopefully, better exploit the local characteristics of the GOCE data that could be "averaged" in a GOCE-only global gravity field model.

Since the gridded data provided by the space-wise approach are used as input for the local update of EGM08, in the next section a brief overview of this approach is presented. In Section 3 the method to combine a set of spherical harmonic coefficients with local data at satellite altitude is described in general terms. Then this method is applied to update the EGM08 Mediterranean geoid and the results are presented in Section 4. On the basis of these results some conclusions are drawn in Section 5.

2. THE SPACE-WISE APPROACH

The space-wise approach [8] is one of the three different strategies that have been implemented in the framework of the High-level Processing Facility (HPF) [21] for the estimation of an Earth global gravity field model from GOCE data. In particular the solution is based both on the satellite-to-satellite tracking (SST) data derived from the on-board GPS receiver and on the satellite gravity gradients (SGG) observed by the on-board electrostatic gradiometer.

The low frequency part of the gravity field is estimated from kinematic orbits [11][25] by means of the energy conservation approach [5][24]. The high frequency part is derived by combining the estimated potential with the

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observed gravity gradients. In particular a multi-step collocation procedure [19] is implemented, basically consisting of a Wiener filter along the orbit [17] to reduce the time correlated noise of the gradiometer, a data gridding by collocation [9] and finally a harmonic analysis by integration [3] to estimate the geo-potential spherical harmonic coefficients. The whole procedure is iterated till convergence, as shown in Figure 1.

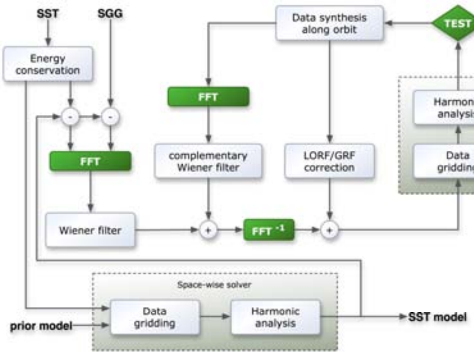

Figure 1. The scheme of the space-wise approach

The complexity of the resulting strategy does not allow for an exact error covariance propagation and therefore the error covariance matrix at each step of the iterative scheme is computed by Montecarlo simulations [10].

Note that one of the intermediate results of the space-wise approach is a spherical grid of the second radial derivative T_{rr} of the anomalous potential at mean satellite altitude (about 250 km). This grid is here used as input for the local update of the EGM08 geoid model. A grid of the anomalous potential T is also available but it is not useful for the purpose of this work. In fact this grid is necessary for the computation of the low degrees of the GOCE-only gravity field model, but cannot improve the accuracy of the corresponding coefficients of EGM08, which are based on GRACE [23].

Note also that the use of gridded data instead of the original observations has the advantage of reducing the noise level and at the same time makes the functional relation between global model coefficients and local GOCE observables much simpler (think e.g. to the fact that the original gradiometric observations are taken in the instrumental reference frame which is in general not oriented in the radial direction).

The first release of the space-wise model [12], based on the first two months of GOCE data, has been delivered during the ESA Living Planet Symposium in Bergen in July 2010, together with the models computed by time-wise approach [15] and the direct approach [2]. See also [16] for details on these delivered models.

From the comparison between the error degree variances of the space-wise solution computed with respect to EGM08 and predicted from the Montecarlo simulations (see Figure 2), it comes out that there are significant differences only from degree 70 to about degree 180, where it is reasonable to expect that GOCE can globally improve the accuracy of EGM08.

On the other hand we hope that locally the information coming from the GOCE data grids could be at higher resolution, e.g. depending on the local data density.

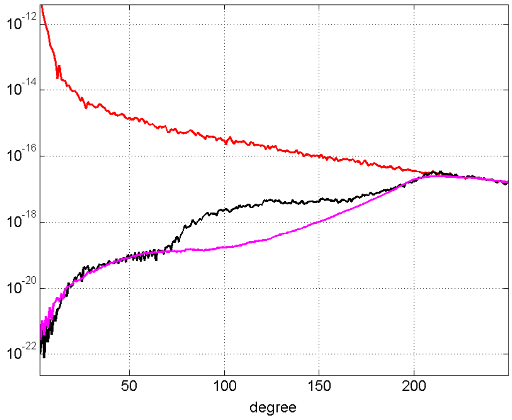

Figure 2. Error degree variances computed with respect to EGM08 (in black) and predicted from Monte Carlo simulations (in magenta); EGM08 reference degree variances in red

The T_{rr} grid at satellite altitude estimated from GOCE data by the space-wise approach and its predicted error standard deviations are displayed in Figures 3 and 4, respectively.

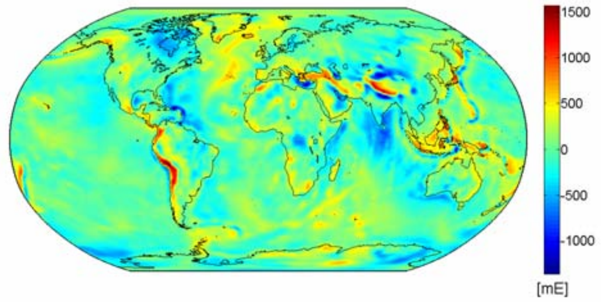

Figure 3. GOCE space-wise T_{rr} grid at satellite altitude

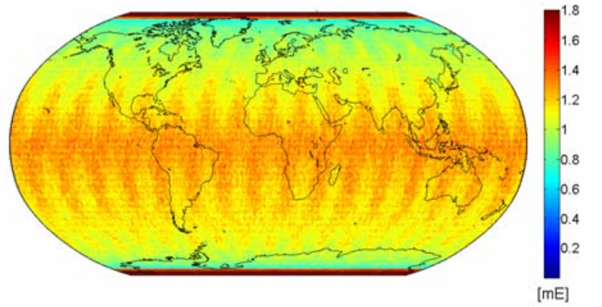

Figure 4. Predicted error standard deviations of the GOCE space-wise T_{rr} grid at satellite altitude

Finally the differences between the GOCE space-wise T_{rr} grid and the one computed from EGM08 are shown in Figure 5, clearly indicating the areas where EGM08 could benefit more from GOCE data, i.e. part of South America, Central Africa, Himalayas, etc. The same differences for the Mediterranean area are shown in Figure 6. These differences are partly GOCE error and partly residual signal; the goal of the method described in the next section is to discriminate between these two components so to compute corrections for the EGM08 spherical harmonic coefficients.

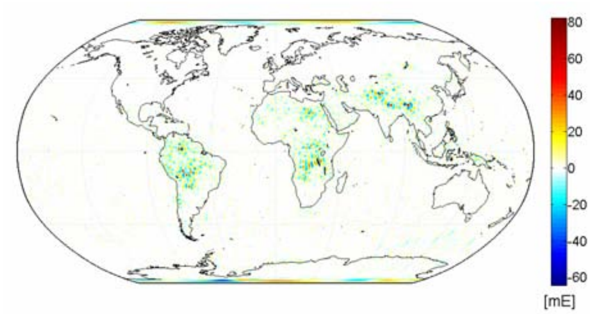

Figure 5. Differences between GOCE space-wise T_{rr} grid and EGM08 T_{rr} grid

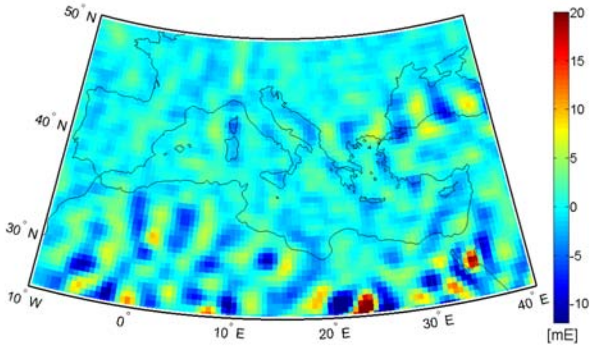

Figure 6. Differences between GOCE space-wise T_{rr} grid and EGM08 T_{rr} grid in the Mediterranean area

3. THE UPDATING METHOD

The method here described faces the general problem of updating a set of spherical harmonic coefficients by using additional local observations. It is based on least-squares collocation [13][22] and it is described in general terms in [14]. Here it is applied to the specific problem of "locally" adapting EGM08 coefficients to GOCE T_{rr} observations.

In particular, defining $\underline{s}$ as the finite vector of spherical harmonic coefficients up to a maximum degree L, the input data to be combined are:

- the EGM08 coefficients $\underline{s}_0$ up to degree $L = 360$:

$$\underline{s}_0 = \underline{s} + \underline{e} \tag{1}$$

where $\underline{e}$ is the coefficient estimation error;

- the GOCE T_{rr} local grid $\underline{\ell}_0$ as computed by the space-wise approach:

$$\underline{\ell}_0 = \underline{\ell} + \underline{\nu} = A\underline{s} + \underline{\nu} \tag{2}$$

where $\underline{\nu}$ is the grid error and A is the linear operator transforming coefficients into second radial derivatives at satellite altitude.

Note that GOCE observations are practically insensitive to coefficients of degree higher than 360; this means that these coefficients cannot be "corrected" by GOCE data and therefore can be reasonably neglected in the problem modeling.

From the stochastic point of view, the signal and error covariance matrices of the coefficients are both diagonal, i.e:

$$C_{ss} = \mathrm{diag}\left(\sigma^2_{s,nm}\right) = \mathrm{diag}\left(\sigma^2_{s,n}\right), \quad C_{ee} = \mathrm{diag}\left(\sigma^2_{e,nm}\right) \tag{3}$$

where n and m are respectively the degree and order of the single coefficient. While modeling the signal in terms of degree variances is standard in geodesy [13], the use of error variances to model coefficient errors is a strong approximation. However this is the only public information on the EGM08 coefficient error.

On the other hand the error covariance matrix $C_{\nu\nu}$ of the T_{rr} grid is full and it is computed from the available 400 Montecarlo samples.

The updating procedure is schematized in Figure 7 and it is based on three steps. First of all the original coefficients $\underline{s}_0$ have to be filtered according to their error variances. Defining the Wiener filter as

$$W = C_{ss}\left(C_{ss} + C_{ee}\right)^{-1} = \mathrm{diag}\left(\frac{\sigma^2_{s,nm}}{\sigma^2_{s,nm} + \sigma^2_{e,nm}}\right) , \tag{4}$$

the filtered coefficients result

$$\underline{s}_w = W\underline{s}_0 \ . \tag{5}$$

However, since the EGM08 error variances for the considered degrees are much smaller than the signal variances, practically $W=I$ and no filtering is applied to the original coefficients.

The second step consists in computing the residual signal $\underline{r}$ by removing the contribution of the filtered coefficients from GOCE data:

$$\underline{r} = \underline{\ell}_0 - A\underline{s}_w = \underline{\ell}_0 - A\underline{s}_0 \ . \tag{6}$$

The covariance matrix of $\underline{r}$ and the cross-covariance matrix between $\underline{r}$ and $\underline{s}$ are given by:

$$C_{rr} = AC_{ee}A^T + C_{\nu\nu} \ , \quad C_{rs} = AC_{ee} \ . \tag{7}$$

Once the residual signal is obtained, the last step is to compute by collocation the updated coefficients $\hat{\underline{s}}$:

$$\hat{\underline{s}} = \underline{s}_0 + \delta\underline{s} \ , \quad \delta\hat{\underline{s}} = C_{sr}C_{rr}^{-1}\underline{r} \tag{8}$$

and the covariance matrix of the estimation error $\hat{\underline{\varepsilon}}$:

$$\hat{\underline{\varepsilon}} = \underline{s} - \hat{\underline{s}} \ , \quad C_{\varepsilon\varepsilon} = C_{ee} - C_{sr}C_{rr}^{-1}C_{rs} \ . \tag{9}$$

Some numerical problems could arise when applying the described procedure, so an additional regularization is required. More specifically, two problems have to be overcome. First of all, one has to verify that the number of grid observations is smaller than the number of Montecarlo samples used to compute C_{vv}, otherwise this matrix is not invertible. The second problem is related to the inversion of C_{rr}. The standard approach of adding a diagonal regularization matrix is too simplistic and leads to a result which is too much or too less regularized according to the number of grid observations.

To overcome this problem the following strategy is implemented. A strongly regularized solution based on a diagonal regularization matrix is computed obtaining some corrections $\delta\tilde{s}_{nm}$; the variances of these corrections are simply estimated as $\delta\tilde{s}^2_{nm}$. Then a diagonal covariance matrix

$$C_{\delta s\delta s} = \mathrm{diag}\left(\delta\tilde{s}^2_{nm}\right) , \tag{10}$$

is built and the regularization matrix R to be added to C_{rr} is finally computed as:

$$R = \alpha\, A C_{\delta s\delta s} A^T \tag{11}$$

where α is an empirical parameter which is fixed to 0.1. The chosen value of α is the minimum value that permits an easy inversion of the regularized C_{rr}. By the way, the addition of R changes the diagonal terms of 1.2% in the average.

Note that using this procedure the regularization does not consist in a blind increase of the grid error variances, but it is "calibrated" according to the power of each coefficient correction.

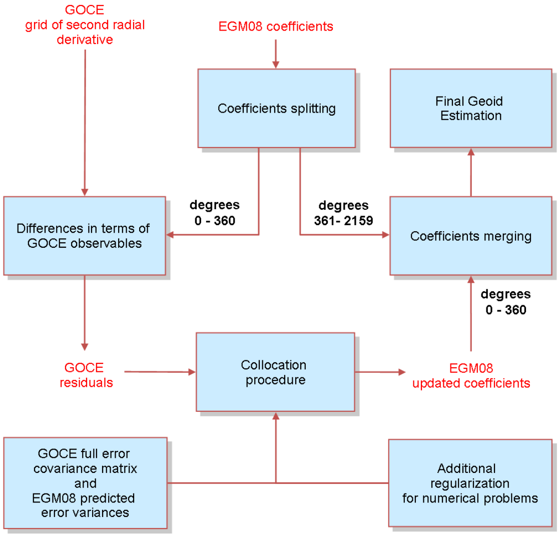

Figure 7. Logical scheme of the updating method

4. RESULTS IN THE MEDITERRANEAN AREA

The method described in the previous section has been here applied for the update of the EGM08 model (up to degree and order 360) in the Mediterranean region.

In particular the considered area covered by GOCE data extents from 28°N to 46°N and from 10°E to 37°W. The grid resolution of the T_{rr} GOCE data is 0.5°×0.5°, corresponding to a total amount of about 2000 grid points.

We recall that all the computations presented in this paper are based on the first two months of GOCE data. An improvement is obviously expected when a longer data period will be analysed.

The results of the updating procedure can be seen at two different levels. One is the global level, i.e. the impact of the local GOCE data on the set of spherical harmonic coefficients; the other is the local level, i.e. the impact in terms of the geoid corrections in the Mediterranean region.

The corrections to the EGM08 coefficients are shown in Figure 8 and, for the sake of readability, are added degree by degree over all orders in Figure 9. It can be seen that these corrections are concentrated in the degrees from 70 to 220. In particular, low-degree and low-order coefficients are very slightly updated, because for these coefficients GOCE is known to be weaker than GRACE. The same holds for the highest degree coefficients that cannot be corrected by GOCE because of the signal damping with the satellite altitude. Note also that sectorial and almost-sectorial coefficients are practically unchanged above degree 150.

Since the global model is combined with a local dataset, the corrections in terms of geoid are concentrated over the considered area (see Figure 10).

The geoid differences over the Mediterranean region (see Figure 11) have a standard deviation of 0.7 cm with a maximum value of 27 cm over Cyprus. Some statistics of these corrections are reported in Table 1.

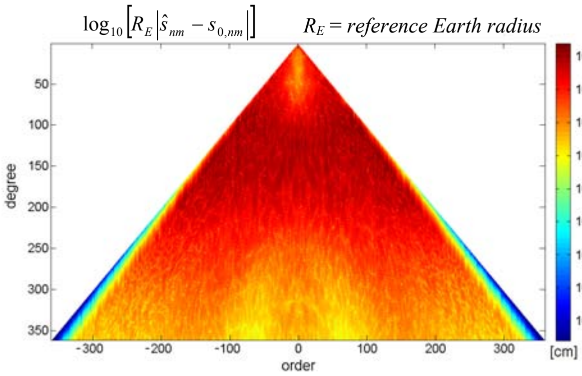

Figure 8: Corrections to each EGM08 coefficient using GOCE data over the Mediterranean region

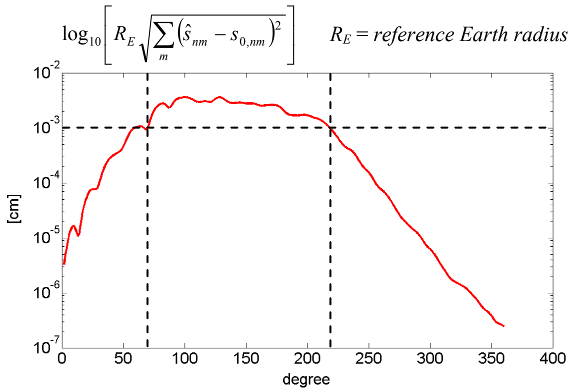

Figure 9: Degree corrections to EGM08 coefficients using GOCE data over the Mediterranean region

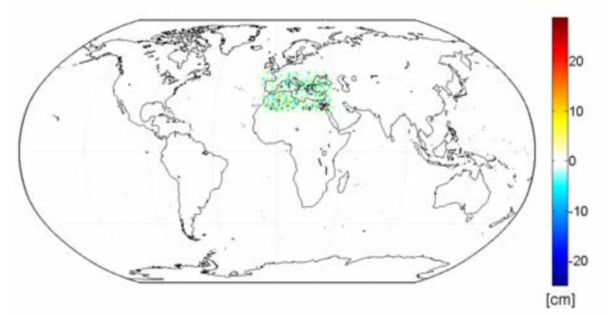

Figure 10: Geoid corrections on a global scale

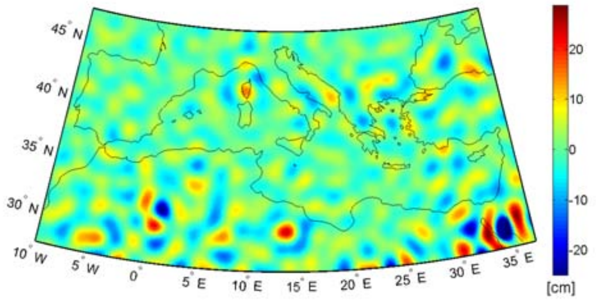

Figure 11: Geoid corrections in the Mediterranean area

Table 1: Statistics on the geoid corrections in the Mediterranean area

mean	std	min	max
$8.5\cdot10^{-6}$ cm	0.7 cm	-23.5 cm	27.5 cm

Moreover it can be seen that the zones where the geoid corrections are higher approximately correspond to the zones where the predicted error of EGM08 is higher too (see Figure 12). This geoid predicted error is available on the EGM08 website [18], however it is not consistent with the error variances of the spherical harmonic coefficients, that basically give rise to geoid error variances depending on the latitude only.

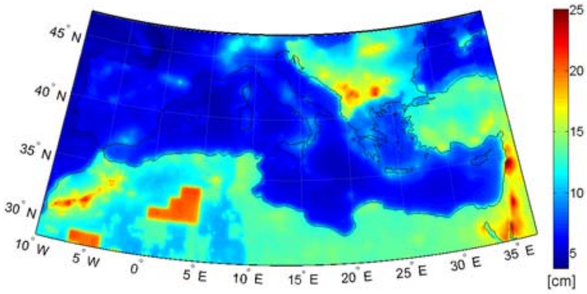

Figure 12: Predicted error standard deviations of EGM08 geoid in the Mediterranean area

5. DISCUSSION AND CONCLUSIONS

In this work GOCE data in the form of grids at satellite altitude are used to locally update EGM08 geoid. These grids are computed as a by-product of the space-wise approach and they are easier to handle than the original track-wise data. Local patches of these global grids can be used as "observations" together with their full error covariance matrix which is much smaller than the non-localized error covariance matrix of the GOCE spherical harmonic coefficients.

A GOCE-EGM08 combined model, adapted to the Mediterranean area, has been computed. Altogether the correction of EGM08 is not very significant (standard deviation of 0.7 cm), probably due to the very good quality of the model in Europe and to the short time span of GOCE data used in this study (2 months). However the corrections seem to be somehow correlated with the EGM08 predicted geoid errors and they are more significant over delimited area like the Alps, the Aegean plate and Cyprus.

Spectrally speaking the correction is concentrated in the degree interval 70–220, so at a higher resolution than the one expected from the global error degree variances. This justifies the choice of using local GOCE data instead of a global GOCE gravity field model.

To conclude it has to be underline that the method is quite general and could be useful especially where only poor or inadequate ground gravity data are available. Furthermore it could benefit from the knowledge of the block diagonal error covariance matrix of the EGM08 coefficients, since a modeling based on error variances only is quite approximated.

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REFERENCES

1. BGI & IGeS (2010). External Quality Evaluation Reports of EGM08. *Newton's Bulletin*, Issue n.4, April 2009.
2. Bruinsma S.L., Marty J.C., Balmino G., Biancale R., Förste C., Abrikosov O. & Neumayer H. (2010). GOCE gravity field recovery by means of the direct numerical method. In: *Proc. of the ESA Living Planet Symposium*, Bergen (Norway), ESA SP-686, ISBN 978-92-9221-250-6, ISSN 1609-042X.
3. Colombo O.L. (1981). Numerical methods for harmonic analysis on the sphere, *Report 310*, Department of Geodetic Science and Surveying, The Ohio State University, Columbus, Ohio.
4. Drinkwater M.R., Floberghagen R., Haagmans R., Muzi D. & Popescu A. (2003) GOCE: ESA's first Earth Explorer Core mission. In: *Earth Gravity Field from Space - from Sensors to Earth Science*, Space Sciences Series of ISSI, Beutler G.B., Drinkwater M.R., Rummel R. & von Steiger R. (eds), Vol. 18, Kluwer Academic Publishers, Dordrecht, Netherlands, pp. 419-432.
5. Jekeli C. (1999). The determination of gravitational potential differences from satellite-to-satellite tracking. *Celestial Mechanics and Dynamical Astronomy*, vol. 75, pp. 85-101.
6. Martin A., Anquela A.B., Padín J. & Berné J.L. (2010). Ability of the EGM2008 high degree geopotential model to calculate a local geoid model in Valencia, Eastern Spain. *Studia Geophysica et Geodaetica*, Vol. 54, N. 3, pp. 347-366.
7. Mayer-Guerr T. (2006). *Gravitationsfeldbestimmung aus der Analyse kurzer Bahnbögen am Beispiel der Satellitenmissionen CHAMP und GRACE*, Dissertation, University of Bonn.
8. Migliaccio F., Reguzzoni M. & Sansò F. (2004). Space-wise approach to satellite gravity field determination in the presence of coloured noise. *Journal of Geodesy*, Vol. 78, N. 4-5, pp. 304-313.
9. Migliaccio F., Reguzzoni M., Sansò F. & Tselfes N. (2007). On the use of gridded data to estimate potential coefficients. In: *Proc. of the 3rd International GOCE User Workshop*, ESA SP-627, pp. 311-318.
10. Migliaccio F., Reguzzoni M., Sansò F. & Tselfes N. (2009). An error model for the GOCE space-wise solution by Monte Carlo methods. In: *Observing our Changing Earth*, IAG Symposia, Sideris M.G. (ed), Vol. 133, Springer-Verlag, Berlin, pp. 337-344.
11. Migliaccio F., Reguzzoni M. & Tselfes N. (2010). A simulated space-wise solution using GOCE kinematic orbits. *Bullettin of Geodesy and Geomatics*, N. 01/2010, pp. 55-68.
12. Migliaccio F., Reguzzoni M., Sansò F., Tscherning C.C. & Veicherts M. (2010). GOCE data analysis: the space-wise approach and the first space-wise gravity field model. In: *Proc. of the ESA Living Planet Symposium*, Bergen (Norway), ESA SP-686, ISBN 978-92-9221-250-6, ISSN 1609-042X.
13. Moritz H. (1989). *Advanced Physical Geodesy*. 2nd Edition. Wichmann Verlag. Karlsruhe.
14. Pail R., Reguzzoni M., Sansò F., Kühtreiber N. (2010). On the combination of global and local data in collocation theory. *Studia Geophysica et Geodaetica*, Vol. 54, N. 2, pp. 195-218.
15. Pail R., Goiginger H., Mayrhofer R., Schuh W.D., Brockmann J.M., Krasbutter I., Höck E., Fecher T. (2010). Global gravity field model derived from orbit and gradiometry data applying the time-wise method. In: *Proc. of the ESA Living Planet Symposium*, Bergen (Norway), ESA SP-686, ISBN 978-92-9221-250-6, ISSN 1609-042X.
16. Pail R., Bruinsma S., Migliaccio F., Förste C., Goiginger H., Schuh W.D., Höck E., Reguzzoni M., Brockmann J.M., Abrikosov O., Veicherts M., Fecher T., Mayrhofer R., Krasbutter I., Sansò F. & Tscherning C.C. (2011). First GOCE gravity field models derived by three different approaches. *Journal of Geodesy*, in print.
17. Papoulis A. (1984). *Signal analysis*. McGraw Hill, New York.
18. Pavlis N.K., Holmes S.A., Kenyon S.C. & Factor J.K. (2008). An Earth Gravitational Model to Degree 2160: EGM2008. *Presented at the 2008 General Assembly of the EGU*, Vienna, April 13-18, 2008. EGM08 website: http://earth-info.nga.mil/GandG/wgs84/gravitymod/egm2008/
19. Reguzzoni M. & Tselfes N. (2009). Optimal multi-step collocation: application to the space-wise approach for GOCE data analysis. *Journal of Geodesy*, Vol. 83, N. 1, pp. 13-29.
20. Reguzzoni M. & Sansò F. (2011). On the combination of high-resolution and satellite-only global gravity models. Submitted to *Journal of Geodesy*.
21. Rummel R., Gruber T. & Koop R. (2004). High Level Processing Facility for GOCE: Products and Processing Strategy. In: *Proc. of the 2nd International GOCE User Workshop*, ESA SP-569.
22. Sansò F. (1986). Statistical methods in physical geodesy. In: *Mathematical and Numerical Techniques in Physical Geodesy*, Lecture Notes in Earth Sciences, Sünkel H. (ed), Vol. 7, Springer-Verlag, Berlin, pp. 49-155.
23. Tapley B.D., Bettadpur S., Watkins M., Reigber C. (2004). The gravity recovery and climate experiment: Mission overview and early results. *Geophys. Res. Lett.*, 31(9), L09607, American Geophysical Union.
24. Visser P.N.A.M., Sneeuw N. & Gerlach C. (2003). Energy integral method for gravity field determination from satellite orbit coordinates. *Journal of Geodesy*, Vol. 77, N. 3-4, pp. 207-216.
25. Visser P.N.A.M., van den Ijseel J., van Helleputte T., Bock H., Jäggi A., Beutler G. & Heinze M. (2010). Rapid and precise orbit determination for the GOCE satellite. In: *Proc. of the ESA Living Planet Symposium*, Bergen (Norway), ESA SP-686, ISBN 978-92-9221-250-6, ISSN 1609-042X.