A GOCE-ONLY GLOBAL GRAVITY FIELD MODEL BY THE SPACE-WISE APPROACH

Federica Migliaccio[(1)], Mirko Reguzzoni[(1)], Andrea Gatti[(2)], Fernando Sansò[(2)], Matija Herceg[(3)]

[(1)] Politecnico di Milano, Piazza Leonardo da Vinci, 32 – 20133 Milano (Italy),
Email: federica.migliaccio@polimi.it; Email: mirko.reguzzoni@polimi.it
[(2)] Politecnico di Milano, Polo Territoriale di Como, Via Valleggio, 11 – 22100 Como (Italy),
Email: andrea.gatti@mail.polimi.it; Email: fernando.sanso@polimi.it
[(3)] DTU Space, National Space Institute, Copenhagen (Denmark),
Email: mher@space.dtu.dk

ABSTRACT

The global gravity field model computed by the space-wise approach is one of three official solutions delivered by ESA from the analysis of the GOCE data. The model consists of a set of spherical harmonic coefficients and the corresponding error covariance matrix. The main idea behind this approach is to exploit the spatial correlation of the gravity field to estimate grids of potential and its second order radial derivatives at mean satellite altitude; from these grids, spherical harmonic coefficients are then derived by numerical integration. The filtering strategy includes also a Wiener filter along the orbit to reduce the noise variance and correlation before gridding the data.

In the first release of the space-wise approach, based on a period of about two months, some prior information coming from existing gravity field models entered into the solution especially at low degrees and low orders. In this work, the strategies adopted to remove these dependencies on external data are described. Mainly two modifications have been implemented. The first is an improved modelling of the error covariance matrix of the estimated along-track gravitational potential to remove the dependency on EGM08 at very low degrees; the second is an internally computed GOCE-only prior model to be used in place of the official quick-look model, thus removing the dependency on EIGEN5C especially in the polar gaps.

Once the procedure to obtain a GOCE-only solution has been outlined, a new global gravity field model has been computed by applying the space-wise approach to eight months of GOCE orbit and gradiometer data. The results are presented and discussed in this paper, also comparing them with respect to the previous solution.

1. THE SPACE-WISE APPROACH AND THE RATIONALE OF THIS WORK

The space-wise approach is a multi-step collocation procedure [8][16], developed in the framework of the GOCE HPF [17] data processing for the estimation of the spherical harmonic coefficients of the Earth gravitational field and their error covariance matrix. The solution is based both on the satellite-to-satellite tracking (SST) data derived from the on-board GPS receiver and on the satellite gravity gradients (SGG) observed by the on-board electrostatic gradiometer. In particular, the low frequency part of the field is estimated from kinematic orbits [11][21] by means of the energy conservation approach [4][20]. The high frequency part is then derived by combining the estimated along-track gravitational potential with the observed gravity gradients. Finally spherical harmonic coefficients are computed by integrating estimated grids of potential and of its second radial derivatives at mean satellite altitude [2]. The error covariance matrix of the estimated coefficients is derived by Monte Carlo simulations [9].

The first release of the space-wise model (SPW) [10] had been delivered during the ESA Living Planet Symposium at Bergen in July 2010. It was based on the first two months of GOCE data and it had been computed in such a way that it represented a solution in between a pure GOCE-only model (time-wise approach, TIM, [13]) and a combined model (direct approach, DIR, [1]). See also [12] for details on these delivered models.

Afterwards, it was decided to switch towards a space-wise GOCE-only model, removing dependencies on prior models based on external data.

Such dependencies can be summarized as follows:

- the EGM08 model [15] had been used to modify the estimated potential along the orbit so to reduce its error at very low frequency;
- the GOCE quick-look model [6] had been used as prior model for the space-wise solution, but this is not a GOCE-only model since in it both reduced dynamic orbits [21] and polar gaps regularization [7] come from EIGEN5C [3].

Besides removing such dependencies on data not provided by the GOCE mission, other two main features have been incorporated in the space-wise scheme:

- a semi-automatic pre-processing of the data to detect and repair outliers, data gaps, etc.
- a combination method to merge space-wise solutions based on data covering different time periods.

The latter has been made necessary because the model presented here is a GOCE-only solution derived from about eight months of data, which is definitively a too

Proc. of '4th International GOCE User Workshop', Munich, Germany 31 March – 1 April 2011 (ESA SP-696, July 2011)

large amount of data to be processed as a whole; in particular the covered time period goes from 31 October 2009 to 6 July 2010.

Therefore the full dataset has been divided into five subsets of different length (spanning from several days to about two months), representing both GOCE orbits and gradiometer observations. Inside each of these time frames, GOCE observations had been continuously delivered based on the same gradiometer calibration [5]. The space-wise approach shown in Figure 1 has been applied to each data subset, obtaining different solutions (i.e. data grids and spherical harmonic coefficients). Then the intermediate grids are merged together by weighting them according to their error covariance so to obtain a unique and final estimate of the gravity field model (see Figure 2).

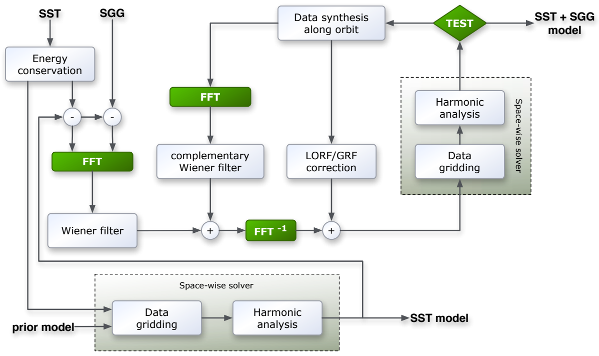

Figure 1. The scheme of the space-wise approach

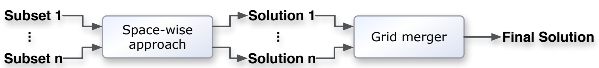

Figure 2. The scheme for the computation of the final space-wise solution

In Section 2 a short overview of the implemented pre-processing is presented. In Section 3 the modifications adopted to obtain a GOCE-only space-wise model are described. In Section 4 the impact on the signal covariance modelling due to the use of a GOCE-only prior model is discussed; a step-wise solution for the covariance modelling is presented and used. Then in Section 5 the grid merging necessary to produce a unique solution based on eight months of GOCE data is described. Finally results are shown in Section 6.

2. GOCE DATA PRE-PROCESSING IN THE SPACE-WISE APPROACH

GOCE data provided by ESA and used to compute the gravity field models are of excellent quality, however sometimes they are affected by some kind of anomalies, such as missing epochs, outliers, Kalman filter re-initialization, etc. It is therefore necessary to mark and remove them.

In the space-wise approach, detection of anomalies is automatically implemented using stochastic techniques and a first "correction" is then applied. However, a final manual check and refinement still has to be performed and, if necessary, data correction is recomputed. To this purpose, small GUIs have been developed (see Figure 3) to check and improve the outlier detection.

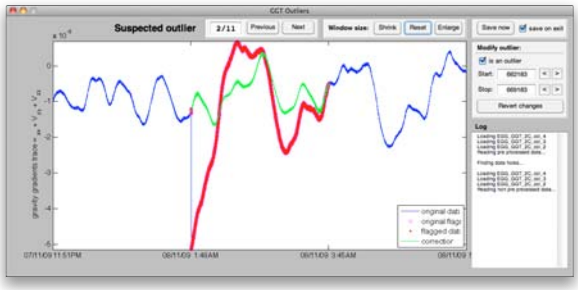

Figure 3. An example of a GUI developed to check and improve outlier detection; here the plot represents a jump followed by a long anomaly due to Kalman filter re-initialization. In blue: observations, in red: marked values, in green: corrections

Among all the GOCE datasets used for the model computation, the most critical ones from the point of view of pre-processing are represented by common mode accelerations and gravity gradients. In fact, kinematic orbits generally present much more outliers than gradiometer data, but these anomalies are easily detectable by a comparison between kinematic and reduced dynamic orbits.

Gaps and anomalies are filled with different techniques spanning from linear interpolation to least squares collocation [18]. Note that the replaced values are only used in the time-wise steps (e.g. the Wiener filter along the orbit [14]) when it is useful to have a continuous flow of data. In the core of the space-wise approach, i.e. in the gridding procedure by collocation, these replaced values are not anymore used, because input data are not required to be regularly sampled in time.

3. REMOVING DEPENDENCIES DUE TO EXTERNAL DATA

3.1. A new error modelling of the potential data

In the first GOCE space-wise model that had been released in July 2010, the EGM08 model had been used for SST data correction, i.e. the estimated potential had been "adjusted" with EGM08 synthesized data. This introduced a strong external information at very low degrees (below harmonic degree 20 ÷ 30).

In order to understand why this "correction" had been applied, one has to recall how the error covariance matrix of the estimated potential is computed:

- error variances of kinematic positions from PCV input files are first considered;

- velocity error covariances (correlated up to 30 s) are computed by propagating position error variances through the used least-squares prediction moving window;
- potential error covariances (correlated up to 30 s) are computed by propagating velocity errors through the linearized energy conservation formula.

According to tests based on simulated data [11], the accelerometer noise is not propagated to potential error. Note that the potential error is not stationary, because initial error variances of kinematic positions are dependent on latitude. All in all, the resulting error covariance matrix is band diagonal but it is not Toeplitz.
In order to evaluate the accuracy of this error covariance estimation, one can assume that the potential error is stationary and approximate the error covariance matrix accordingly. The resulting error covariance function is transformed into a power spectral density (PSD) and compared with the empirical PSD of the difference between estimated potential and the one synthesized from EGM08 (see Figure 4).

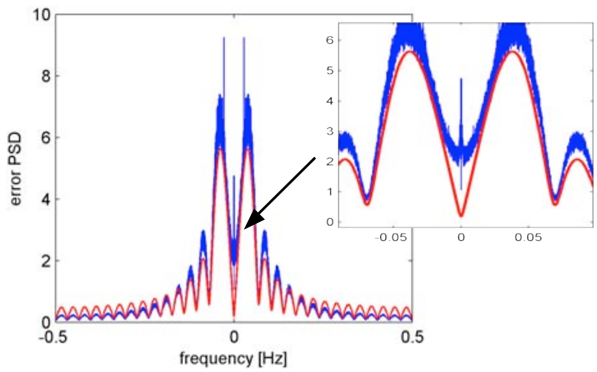

Figure 4. Error PSD of the estimated potential. In blue: empirical error PSD w.r.t. EGM08; in red: estimated error PSD from PCV assuming stationarity

In the first release of the space-wise model, the discrepancies at low frequencies had been adjusted by "correcting" the estimated potential with synthesized data from EGM08. Now data are unchanged, but the error covariance modelling is corrected in such a way that the empirical and the estimated PSDs are consistent with each other. In particular a Toeplitz matrix describing the corrections for the low frequencies is added to the non-stationary covariance matrix coming from the position error propagation (see Figure 5).

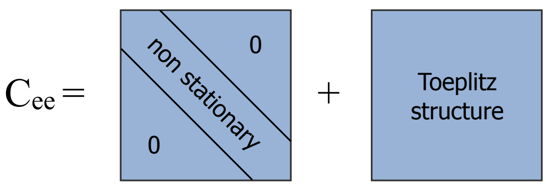

Figure 5. The form of the new error covariance matrix

3.2. A new prior model for the space-wise approach

As already mentioned, the quick-look model introduced some unwanted dependencies on external data. To avoid this effect, a new GOCE-only prior model has been developed for the space-wise approach.
This prior model is based on observations coming from the first two months of GOCE data and it has been used for all the five intermediate solutions. It is computed by global collocation, i.e. with data covering all the Earth apart from polar gaps, directly mapping observations into spherical harmonic coefficients without passing through gridded data.
Note that global collocation can work on a full signal, but it requires a strong under-sampling (about 1:800) for computational reasons. In this way a first sufficient solution has been obtained but its accuracy should be improved, especially in the polar areas.
More important than the model accuracy, it is the estimate of a reliable error covariance which will be afterwards used for the Monte Carlo simulation. This error covariance can be safely estimated by collocation because degree variances are here appropriate for the full signal modelling.
Anyway, in order to improve the prior model accuracy a step-wise collocation procedure has been implemented, considering the error covariance of the (i-1)th step as the signal covariance of the ith step (see Figure 6). The procedure can be summarized as follows:

- eight global collocations with data under-sampling at 1:800, each working on data shifted by 100 epochs;
- two collocations with data under-sampling at 1:33 (data shifted by 33 epochs), but considering only data close to the polar caps ("polar doughnut"), thus improving the polar gap extrapolation.

Once the prior model has been computed, a patch-wise collocation gridding has been applied as in the baseline of the space-wise approach to produce the SST model.

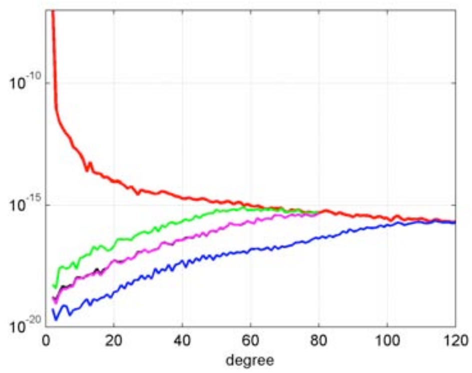

Figure 6. Error degree variances of the step-wise collocation procedure for the SST data analysis.
1) green: 1st step of the global collocation (1:800);
2) black: 8th step of the global collocation (1:100);
3) purple: "polar doughnut" collocation (1:33);
4) blue: patch-wise collocation (1:3, 20°×20°)

In Figure 7, it is possible to see the geoid differences from degree 2 to 20, in the polar area and surroundings, between EGM08 and the estimated models at different steps of the step-wise collocation procedure. Note the change of the error scale bar at the different steps. In the end, the error in the polar areas results smaller than the corresponding one of the time-wise solution based on the first two months of SST data.

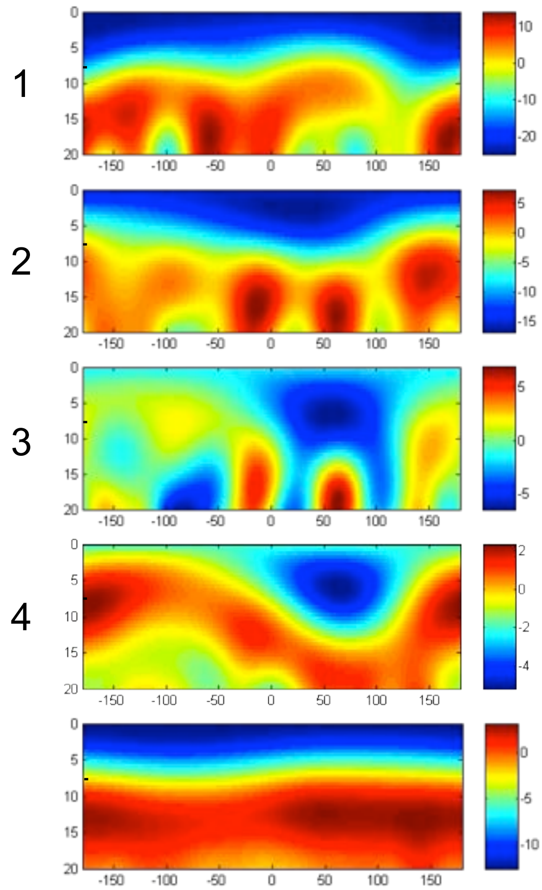

Figure 7. Geoid differences [cm] in the northern polar cap between EGM08 and the estimated model at different steps of the collocation procedure (same model sequence as in Figure 6). The lower panel represents the TIM solution. The black line represents the GOCE orbit limit.

4. SIGNAL COVARIANCE MODELLING

In order to make collocation more efficient, a prior model is always removed from the data before gridding. For this reason, the covariance of the residual signal has to be modelled.

Differently from the quick-look model that had been used in the first release of the space-wise solution, the new prior model coming from the SST data analysis produces a residual signal with strong anisotropies especially due to the effect of polar gaps [19]. Note that the residual signal is just the estimation error of the adopted prior model.

In principle one has to propagate the estimated full error covariance matrix of the SST model to the different functionals (potential and gravity gradients) of the gravity field. In practice the following approximations are used for the SST coefficients covariance (with decreasing accuracy):

- block diagonal covariance matrix (order by order);
- diagonal covariance matrix with different variances $\sigma_{\ell m}^2$ for each coefficient;
- diagonal covariance matrix with degree variances σ_ℓ^2.

In the implemented collocation gridding, only degree variances have been taken into account. This means that the covariance function of the gravitational potential depends on the spherical distance ψ, i.e.

$$C_T(\psi,r,r') = \left(\frac{GM}{R}\right)^2 \sum_\ell \left(\frac{R^2}{rr'}\right)^{\ell+1} (2\ell+1)\sigma_\ell^2 P_\ell(\psi) , \qquad (1)$$

which also means that points at the same altitude but different latitudes have the same variance. Obviously this is a strong approximation in case of polar gaps. Note that if single coefficient variances (assuming that cosine and sine coefficients are independent and with the same variances) are considered, the corresponding potential covariance function would be

$$C_T(\vartheta,\vartheta',\Delta\lambda,r,r') = \\ = \left(\frac{GM}{R}\right)^2 \sum_{\ell,m} \left(\frac{R^2}{rr'}\right)^{\ell+1} \sigma_{\ell m}^2 \bar{P}_{\ell m}(\vartheta)\bar{P}_{\ell m}(\vartheta')\cos(m\Delta\lambda) \qquad (2)$$

which can better describe observations acquired from satellite and therefore with a spatial density depending on the latitude.

Therefore the choice of using only degree variances for the residual signal modelling in the collocation gridding has the main advantage of reducing the computational burden, but requires some adaptations to take into account the anisotropic spectrum of the SST coefficients error (see Figures 8 and 9).

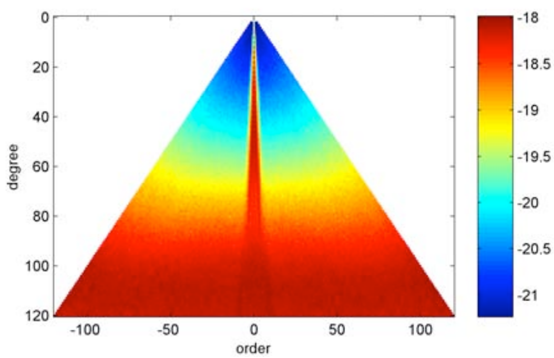

Figure 8. Error variances of the estimated coefficients of the SST model (log_{10} scale)

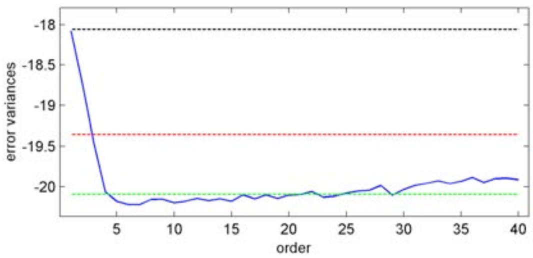

Figure 9. Error variances of the estimated cosine coefficients of the SST model for the degree 40; blue: variances; black: maximum value of variances; red: degree variance; green: square degree median

In particular, two iterations of the space-wise scheme have been implemented for the new release of the space-wise model:

- in the first one degree variances have been over-estimated to the maximum variance for each degree, so to allow data to better estimate low orders, i.e. make a good extrapolation in polar gaps and reduce border effects;
- in the second one degree variances have been replaced with degree medians, so to better weight coefficients not affected by polar gaps.

This step-wise approach is however an approximate solution. The most reasonable approach would probably be to consider block covariances for low orders and coefficient variances for others.

5. MERGING INTERMEDIATE SPACE-WISE SOLUTIONS

As already mentioned above, the eight months of GOCE data used to compute the new solution have been divided into subsets of continuous observations with similar behavior, then the subsets have been pre-processed in such a way to detect and remove outliers and fill small data gaps. Datasets with not enough valid data have been disregarded.

Five subsets have thus been selected to produce the solution presented here; from about eight months of data, only 80% of them have been finally used (see Figure 10).

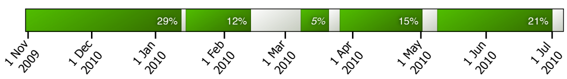

Figure 10. Subsets of data used for the new space-wise model: in grey, discarded subsets of data; in green, subsets of data used in the model computation

Different steps have then been followed in order to obtain a unique solution (see Figure 11):

- each subset has been processed following the usual space-wise approach (see Figure 1) producing grids of potential and second order radial derivative, plus Monte Carlo (MC) sample grids describing the error;
- merged grids of the two functionals have been obtained by using a moving window and weighting data on the basis of MC error covariance matrices;
- harmonic analysis has been applied to these grids, obtaining two sets of coefficients that have been finally merged by collocation based on the error covariances propagated from the MC sample grids.

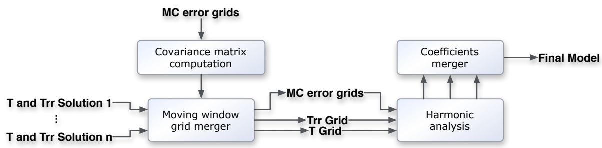

Figure 11. The scheme for the computation of the final model from intermediate space-wise solutions

A simpler alternative strategy would have been to directly combine the intermediate sets of harmonic coefficients on the basis of their error covariance matrix. In this case, a block diagonal approximation would have been considered to make the covariance matrices inversion more stable and faster. However, since some of the data subsets span a very short data period, the error distribution of the corresponding solutions is not latitude dependent or, in other words, the block diagonal approximation of the coefficients error covariance matrix is not very reliable. Vice versa, a grid combination based on a moving window can better describe the local error distribution of the different data subsets.

6. RESULTS

As a result of the upgrades implemented in the space-wise processing chain, it can be stated that this approach is now able to produce GOCE-only solutions.

In order to highlight this new characteristic, a space-wise model based on the updated processing chain and computed from only two months of data has been compared with the “Bergen” solution (which was also obtained from two months of GOCE data, but it was not GOCE-only). Besides, this new space-wise solution has been compared with the “Bergen” time-wise model (which was already GOCE-only).

The results of these comparisons can be seen in Figure 12. Obviously the new space-wise solution is weaker at low degrees, because the old one made use of GRACE data (through EGM08 and EIGEN5C) which are known to be superior to GOCE at low degrees. Regarding the comparison with the time-wise solution, the slightly better behaviour below degrees 70 ÷ 80 (upper panel of Figure 12) can be attributed to a good estimation of the low order coefficients, i.e. to a good data extrapolation in the polar gaps (see also Figure 7). On the other hand, the space-wise solution seems to be over-regularized at the highest degrees (which can be better visualized in the lower panel of Figure 12); this is very likely due to the approximate covariance modelling of the residual signal in the collocation gridding (see Section 4).

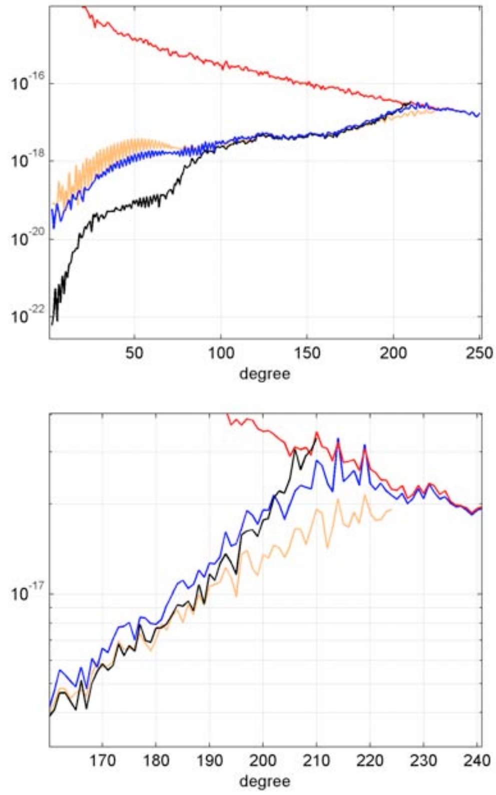

Figure 12. Error degree variances of the new SPW solution (in blue), as compared with the first SPW solution (in black) and with the TIM solution (in orange). The reference model is EGM08. In the lower panel the figure is enlarged from degree 160 on

The main outcome of this work is the computation of the space-wise solution based on the first eight months of GOCE data.

As a first obvious consideration, this space-wise GOCE-only model improves the accuracy of the estimation by exploiting three times the amount of data available for the first release. This can be seen, for example, from the plots of the error degree medians with respect to EGM08 (Figure 13). Error degree medians are preferred to error degree variances because they are more robust against the effect of polar gaps and therefore can better emphasize the obtained improvement. Apart from the medium degrees where the EGM08 error is dominant, the improvement is clearly visible.

The error degree variances of the final space-wise model, computed with respect to EGM08 and estimated with the Monte Carlo method, are plotted in Figure 14 with the aim of showing the good reliability of the error estimates, which are so to say "calibrated". Error degree variances of the corresponding time-wise model are also shown in Figure 14 for comparison.

A more detailed representation on the error structure can be seen in Figure 15, where the improvement in terms of the error standard deviation of the single coefficients from the first two-month solution to the final one is shown.

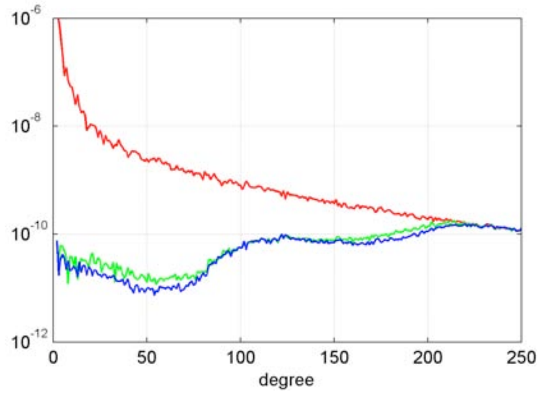

Figure 13. Error degree medians of the new SPW solution (in blue), as compared with the first SPW solution (in green). The reference model is EGM08

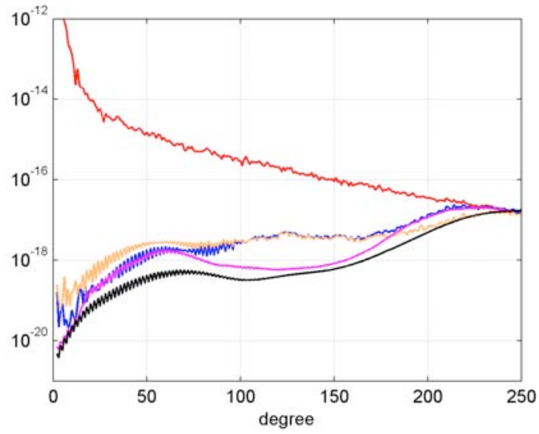

Figure 14. Error degree variances of the new SPW solution, computed with respect to EGM08 (in blue) and estimated from Monte Carlo simulation (in magenta). Error degree variances of the TIM solution computed with respect to EGM08 (in orange) and derived from the estimated error covariance matrix (in black)

In the course of the space-wise processing chain, grids of GOCE observables (potential T and second radial derivatives T_{rr}) are computed at satellite altitude with a resolution of 0.5°×0.5° and their error covariances are used for merging intermediate solutions. An important point is that such grids could be made available to the scientific users of GOCE data and could be exploited for geophysical applications too.

In Figure 16, the improvement in terms of T_{rr} grid error standard deviation from the first two-month solution to the final one is shown. Furthermore, the differences between the final GOCE space-wise T_{rr} grid and the one computed from EGM08 are displayed in Figure 16, emphasizing the areas where EGM08 could benefit more from GOCE data, i.e. part of South America, Central Africa, Himalayas, etc.

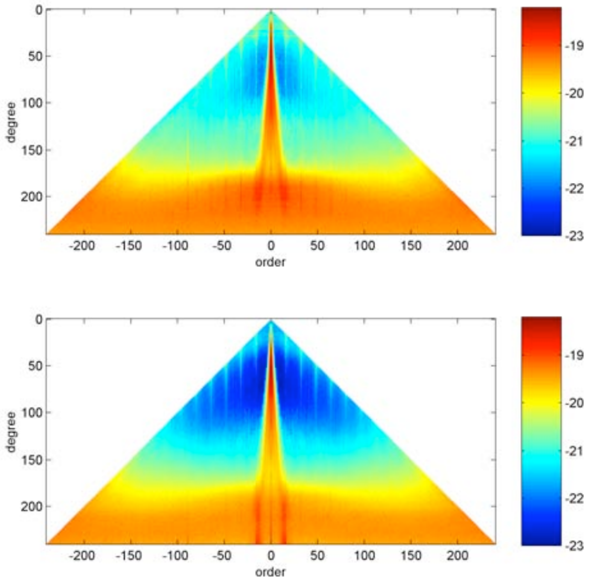

Figure 15. Coefficients error variances (log_{10} scale) for the two-month SPW solution (upper panel) and for the final SPW solution (lower panel)

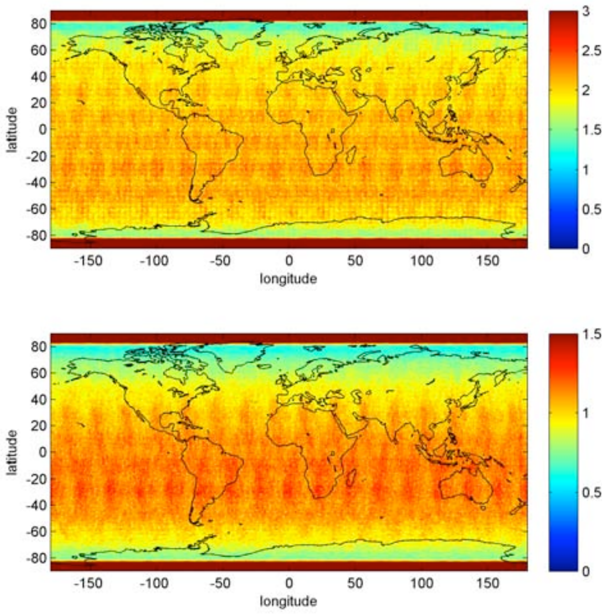

Figure 16. Error standard deviations [mE] of the T_{rr} grid for the two-month SPW solution (upper panel) and for the final SPW merged solution (lower panel). Note the different colour scale

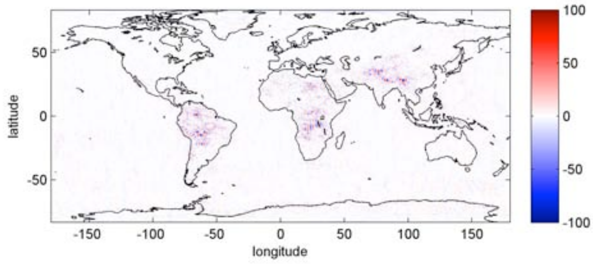

Figure 17. Differences [mE] between the GOCE SPW T_{rr} grid and the T_{rr} grid computed from EGM08

Finally the commission error of the geoid undulations in the latitude interval -80° < φ < 80° and as a function of the maximum harmonic degree, is shown in Figure 18. The same plot for the gravity anomalies is displayed in Figure 19. It comes out that the estimated accuracy of the space wise model up to degree and order 200 is about 8.5 cm in terms of geoid undulations, and about 2.5 mgal in terms of gravity anomalies. Concerning the model resolution, the chosen maximum harmonic degree is 240.

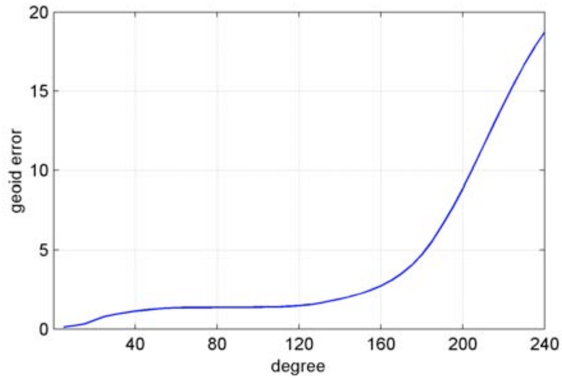

Figure 18. Commission error in terms of geoid undulations [cm] in the latitude interval -80° < φ < 80°

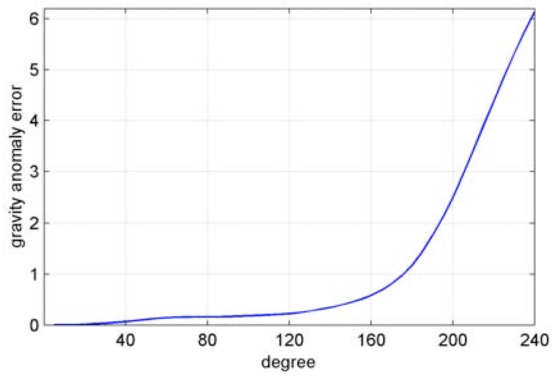

Figure 19. Commission error in terms of gravity anomalies [mgal] in the latitude interval -80° < φ < 80°

7. CONCLUSIONS

The analysis presented in this paper shows that the space-wise approach is able to produce a GOCE-only model based on several months of GOCE data.

At the moment the main weakness of the solution seems to be an over-regularization at high degrees. This can be overcome by improving the residual signal covariance modelling. In addition, the fact that all the intermediate solutions are regularized (they are all based on a collocation gridding) also contributes to the over-regularization of the final model. To avoid this drawback, instead of computing many independent intermediate solutions, one can think of updating the last solution with the new available data. This can be done by a step-wise collocation procedure, where the residual signal covariance modelling is again the key issue.

ACKNOWLEDGEMENTS

This work has been performed under ESA contract 18308/04/NL/NM (GOCE HPF).

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