VALIDATION OF GOCE GRAVITY FIELD MODELS BY ASTROGEODETIC VERTICAL DEFLECTIONS IN GERMANY

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ABSTRACT

The validation of GOCE products is one of the main objectives within the framework of the German REAL GOCE project. Within the work package "GOCE Cal/Val, Quasigeoid and Height System", global gravity field models are evaluated by various terrestrial data sets in Germany. In this contribution, the focus is on the evaluation of global gravity field models by means of astrogeodetic vertical deflections observed with the digital transportable zenith camera system TZK2-D in Germany.

The terrestrial data set comprises approx. 300 vertical deflections along a North-South and a West-East profile. Regarding accuracy (standard deviation of 0.1"), dimension (profile lengths of 500 km) and resolution (station spacings of 2.5 to 5 km), the astrogeodetic vertical deflections are well applicable for a spot check of global gravity field models. In connection with a multistage filtering process, accounting for the different spectral content of the different data sets, the comparisons are performed, meeting the accuracy requirements of the GOCE mission.

The analyses are carried out with actual global gravity field models from GOCE data, GRACE data and from combination solutions of GOCE and GRACE as well as from GRACE and terrestrial data. The first generation of GOCE models, based on an observation period of approx. 2 months, already reveal a considerable improvement within the spectral range between degree and order 150 and 180 over the GRACE models. For the second generation GOCE models, based on a longer observation period of approx. 8 months, a further significant improvement within the spectral range up to degree and order 224 can be stated. For higher degrees up to 250, however, the quality of the GOCE models still degrades with respect to the combined models from GRACE and terrestrial data.

1. INTRODUCTION

The first two generations of global gravity field models (GGM) from the GOCE mission have recently been made available, aiming at accuracies of 1 mgal for gravity and 1-2 cm for geoid heights, both at a resolution of 100 km, corresponding to a spherical harmonic expansion up to degree and order (d/o) 200. In order to reach the mission goals, various internal and external calibration and validation techniques are applied. In this contribution, the emphasis is on the external validation with astrogeodetic vertical deflections in Germany. In terms of accuracy and extent, the German terrestrial gravity field data sets are worldwide quite unique, and thus well-suited for the validation and combination with the GOCE results. Besides an extensive terrestrial gravity data set of more than 260,000 values (accuracy about 0.1 to 1.0 mgal), about 900 GPS/levelling points (accuracy about 1 to 3 cm) as well as about 300 astrogeodetic vertical deflection observations exist, the latter being observed along two profiles (see Fig. 1) with an accuracy of about 0.1**"** (for further details see Ihde et al., 2010).

The astrogeodetic vertical deflections are completely independent of any other gravity field data set and mainly serve for two purposes. Firstly, the data set is employed for the cross-validation with GPS/levelling points and gravimetric quasigeoid models, applying the method of astronomic levelling (see e.g. Voigt et al., 2009, and Ihde et al., 2010). Secondly, the astrogeodetic vertical deflections are used for the regional validation of GGMs, which is the main subject of this contribution. Although limited by the different spectral content of the GGM and vertical deflection data, mainly providing one-dimensional gravity field information, the astrogeodetic validation method can be regarded as a useful tool for a completely independent spot check of GGM data. Complementary to the other external validation methods, the astrogeodetic vertical deflections provide high-precision information about the horizontal components of the gravity field with different spectral characteristics as compared to the other gravity field functionals.

External validations of the first generation of GOCE GGMs have already been carried out with terrestrial gravity data, gravimetric quasigeoid models and astrogeodetic vertical deflections in Germany and Europe (Voigt et al., 2010), as well as with GPS/-levelling data, e.g. in Germany (GOCO consortium, 2010). Numerous analyses on the external validation of actual GGMs, particularly of the ultra-high degree model EGM2008 (Pavlis et al., 2008), can be found in BGI (2009). Furthermore, comparisons between astrogeodetic vertical deflections and EGM2008 data have been performed along local profiles or on single stations

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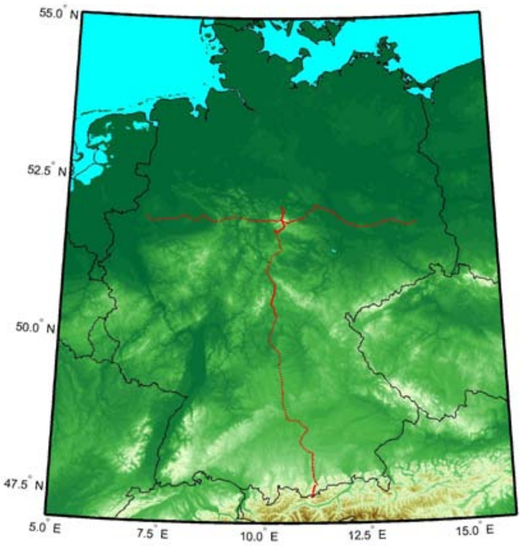

Figure 1. Astrogeodetic vertical deflection stations along a North-South and a West-East profile in Germany

in the German and Swiss Alps (Hirt, 2010) and in Europe (Hirt et al., 2010).

Section 2 describes the various GGM data sets, taking part in the validation process, as well as the astrogeodetic vertical deflections, including statistical information. In section 3, the validation method is explained, particularly with regard to the different spectral content of the GGM and terrestrial data. In sections 4 and 5, the comparisons between the astrogeodetic and the various GGM vertical deflections are carried out. The differences are displayed and evaluated.

2. DATA SETS

The first generation of GOCE GGMs, published in June 2010, are based on an observation period of approx. two months (begin of November 2009 to January 2010), using different approaches: the 1st direct approach (model complete up to d/o 240; Bruinsma et al., 2010), the space-wise approach (d/o 210; Migliaccio et al., 2010), and the 1st time-wise approach (d/o 224; Pail et al., 2010). It should be noted that the model from the 1st direct approach includes a priori gravity field information from the combined model EIGEN-5C (Förste et al., 2008). In addition, the GOCO01S model (d/o 224; GOCO consortium, 2010) combines 7 years of GRACE observations (August 2002 to August 2009) and two months of GOCE data (November and December 2009). Published in March 2011, the second generation of GOCE GGMs are based on an observation period of approx. 8 months (begin of November 2009 to July 2010), resulting in the 2nd direct approach model (d/o 240; Bruinsma et al., 2011) and the 2nd time-wise approach model (d/o 250; Pail et al., 2011).

The focus of the analyses is on the performance of the GOCE GGMs (d/o up to 250) compared to the GRACE model ITG-Grace2010s (d/o 180; Mayer-Gürr et al., 2010) and the combined models from GRACE and terrestrial data EGM2008 (d/o 2190) and EIGEN-5C (d/o 360).

Considering the total vertical deflection defined as

$$\theta = \sqrt{\xi^2 + \eta^2}, \tag{1}$$

where ξ and η are the North-South and East-West components, respectively, corresponding degree variances can be defined per degree n for $r=a=R$ (see also extended Meissl scheme; Rummel and van Gelderen, 1995) by

$$\sigma_n^2(\theta) = \frac{n(n+1)}{R^2\gamma^2}\sigma_n^2(T) \tag{2}$$

where γ is the normal gravity, and $\sigma_n^2(T)$ are the degree variances of the disturbing potential T. Based on Eq. 2, the error degree variances of the vertical deflections can be computed from the supplied standard deviations of the fully normalized spherical harmonic coefficients by

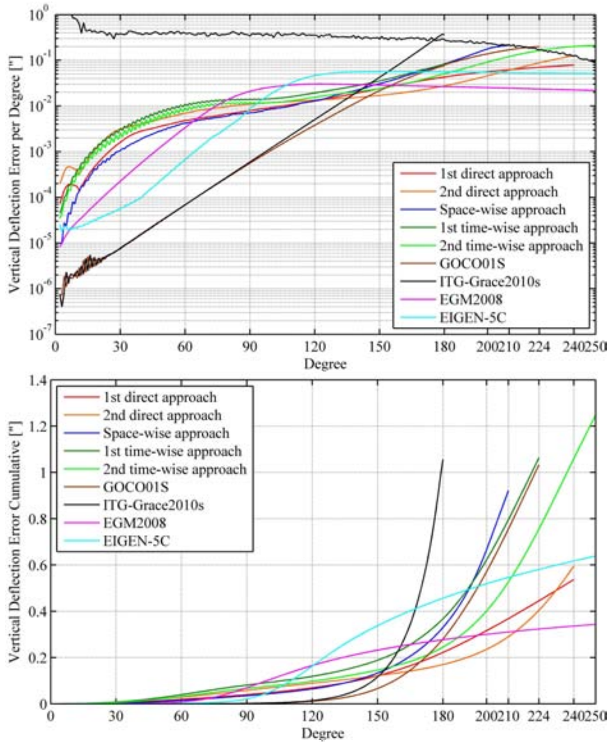

Figure 2. Vertical deflection error ["] per degree (top) and cumulative (bottom) of actual global gravity field models

$$\sigma_n^2(\varepsilon_\theta) = n(n+1) \sum_{m=0}^{n} \left(\sigma^2_{\Delta \overline{C}_{nm}} + \sigma^2_{\Delta \overline{S}_{nm}} \right). \quad (3)$$

In Fig. 2, the vertical deflection errors of the various GGMs are illustrated, using the square roots of the error degree variances from Eq. 3. In the spectral range up to d/o 90, the GOCE GGMs show weak characteristics compared to the GRACE and the combined GGMs. For d/o beyond 100, the impact of the GOCE gradiometer observations becomes visible. Between d/o 120 and 150, the GOCE and GRACE GGMs show similar accuracies. On the other hand, the GRACE accuracy decreases rapidly up to d/o 180, while the GOCE GGMs show improved characteristics up to d/o 250. Then at about d/o 210, the GOCE GGMs exceed the accuracy levels of the combined GGMs. However, it should be noted that the supplied errors are formal for the GOCE and calibrated for the combined GGMs.

From Fig. 2, the requirements on an adequate terrestrial data set for a thorough validation of the GOCE GGMs become apparent. The accuracy of the vertical deflections θ should be at the level of at least 0.2", corresponding to 0.14" for the components ξ and η. In order to perform accuracy assessments in the spectral range between d/o 180 and 250, corresponding to wavelengths of 220 to 160 km, the dimension of the terrestrial data set should be at least a few 100 kilometres. These requirements are fulfilled by the high-precision (0.1") astrogeodetic data set comprising 161 and 134 vertical deflections (ξ,η) along the North-South and the West-East profile, respectively, both having a length of about 500 km with a spacing of 2.5 to 5 km. For further details on the deployed digital transportable zenith camera system TZK2-D see Hirt (2004).

3. METHOD

While the GGM data are spectrally limited due to the maximum d/o of the spherical harmonic expansion, the astrogeodetic vertical deflections include the complete spectrum. The signal not included in the GGMs due to the limited spherical harmonic expansion is called the omission error. The spectral characteristics of the vertical deflections and height anomalies are shown in Tab. 1. In comparison to the height anomalies, vertical deflections contain a significantly larger portion of the signal in the spectral range beyond d/o 180, i.e. almost half of the total signal (43%). Hence, the high-frequency signals, not included in the GGM data, have to be taken into account appropriately within the validation process.

Over larger areas, e.g., regarding height or gravity anomaly grids covering Germany or Europe, the high-frequency signals beyond the maximum d/o of the GGMs can be filtered out by applying e.g. a Gaussian filter with filter widths of 100 and 200 km (Voigt et al., 2010). However, this method is not well suited in connection with the astrogeodetic vertical deflection profiles, mainly due to the occurring edge effects, but also because the signal contents within the spectral range of interest between d/o 180 and 250 is quite small (a few arc seconds; 2.3" in Tab. 2), and therefore should be affected as little as possible by the filtering procedure.

Degree n_i - n_j	Wavelengths	$\left[\sum_{n=n_i}^{n_j}\sigma_n(\theta)\right]^{\frac{1}{2}}$		$\left[\sum_{n=n_i}^{n_j}\sigma_n(\zeta)\right]^{\frac{1}{2}}$	
	[km]	["]	[%]	[m]	[%]
2 - 180	20000 – 220	7.18	56.85	30.391	99.98
181 - 250	220 – 160	2.28	5.74	0.333	0.01
251 - 2190	160 – 18	5.34	31.47	0.335	0.01
2191 - ∞	18 – 0	2.32	5.94	0.022	0.00
2 - ∞	20000 – 0	9.52	100	30.395	100

Table 1. Spectral division of the vertical deflections and height anomalies based on the anomaly degree variance model of Tscherning and Rapp (1974)

Hence, in this contribution, the different spectral content is considered in a stepwise procedure. First, the GGM to be evaluated is taken up to some maximum d/o (in steps of 180, 200, 210, 224, 240 and 250), augmented by the coefficients of EGM2008 to d/o 2190; then vertical deflections are computed from the GGM and compared with the astrogeodetic observations. This method was also applied by Gruber (2009) for a validation with GPS/levelling data; as the height anomaly omission error beyond d/o 2190 does not exceed the level of a few centimetres (2.2 cm in Tab. 1), no additional modelling was applied in that investigation. However, the astrogeodetic vertical deflections are strongly correlated with local terrain effects, easily reaching more than a few arc seconds (2.3" in Tab. 1). Therefore, the high frequencies beyond d/o 2190 are considered here by the so-called residual terrain model (RTM) approach (Forsberg, 1984), using a reference topography with a resolution of 5'x5'. This method was also applied in Hirt (2010) and Hirt et al. (2010).

Moreover, the station spacing of a few kilometres along the profiles allows for the application of a low-pass filter in the space domain. Thus an additional Gaussian filtering with a radius of 10 km was applied in order to further reduce remaining high-frequency signals, which are not the subject of this analysis. The Gaussian filtering was done for both the GGM and the astrogeodetic deflection data. Hence, a comparison is carried out between the terrestrial quantities

$$gauss\left[(\xi,\eta)_{\text{astro}} - (\xi,\eta)_{\text{RTM}}\right] \quad (4)$$

Gravity Field Model	Truncation	North-South Profile RMS Δξ	North-South Profile RMS Δη	West-East Profile RMS Δξ	West-East Profile RMS Δη
1st direct approach	180	0.53	0.55	0.33	0.22
2nd direct approach	180	0.51	0.54	0.38	0.24
Space-wise approach	180	0.52	0.68	0.34	0.39
1st time-wise approach	180	0.53	0.62	0.36	0.34
2nd time-wise approach	180	0.50	0.53	0.33	0.21
GOCO01S	180	0.51	0.60	0.35	0.28
ITG-Grace2010s	180	0.92	0.88	1.11	0.56
EGM2008	180	0.49	0.48	0.33	0.21
EIGEN-5C	180	0.52	0.51	0.35	0.23
1st direct approach	200	0.63	0.63	0.35	0.27
2nd direct approach	200	0.54	0.57	0.34	0.35
Space-wise approach	200	0.84	1.08	0.48	0.43
1st time-wise approach	200	0.85	0.88	0.50	0.46
2nd time-wise approach	200	0.53	0.62	0.33	0.28
GOCO01S	200	0.80	0.85	0.47	0.48
EGM2008	200	0.49	0.48	0.33	0.21
EIGEN-5C	200	0.51	0.50	0.42	0.23
1st direct approach	210	0.62	0.66	0.33	0.28
2nd direct approach	210	0.60	0.51	0.40	0.59
Space-wise approach	210	0.94	0.98	0.56	0.56
1st time-wise approach	210	0.66	0.81	0.50	0.60
2nd time-wise approach	210	0.56	0.50	0.38	0.48
GOCO01S	210	0.61	0.79	0.47	0.71
EGM2008	210	0.49	0.48	0.33	0.21
EIGEN-5C	210	0.57	0.56	0.36	0.25
1st direct approach	224	0.55	0.63	0.36	0.29
2nd direct approach	224	0.72	0.86	0.79	0.83
1st time-wise approach	224	0.63	1.19	0.80	0.78
2nd time-wise approach	224	0.65	0.78	0.66	0.52
GOCO01S	224	0.58	1.21	0.77	0.72
EGM2008	224	0.49	0.48	0.33	0.21
EIGEN-5C	224	0.55	0.54	0.36	0.22
1st direct approach	240	0.57	0.65	0.41	0.33
2nd direct approach	240	0.86	1.53	0.96	1.33
2nd time-wise approach	240	0.78	1.08	0.77	0.86
EGM2008	240	0.49	0.48	0.33	0.21
EIGEN-5C	240	0.58	0.59	0.39	0.24
2nd time-wise approach	250	0.87	0.97	1.21	0.86
EGM2008	250	0.49	0.48	0.33	0.21
EIGEN-5C	250	0.57	0.54	0.38	0.24

Table 2. RMS differences $\Delta\xi$ and $\Delta\eta$ [″] between the filtered astrogeodetic vertical deflections and corresponding values from actual global gravity field models

and the GGM quantities

$$gauss\left[\left(\xi,\eta\right)_{n_{\max}}^{\mathrm{GGM}}+\left(\xi,\eta\right)_{1+n_{\max},2190}^{\mathrm{EGM2008}}\right]. \qquad (5)$$

In addition to this, some systematic corrections have to be considered in the comparisons. This is mainly the curvature of the normal plumb line in North-South direction (see e.g. Heiskanen and Moritz, 1967, p. 196). Furthermore, differences in the reference system and the permanent tide system have to be considered; they do not exceed mean values of 0.02" in our case. Approximations within the spherical-harmonic synthesis are described in detail in Jekeli (1999), also not exceeding maximum values of a few 0.01".

4. ANALYSES

Tab. 2 shows the RMS differences $\Delta\xi$ and $\Delta\eta$ between the filtered astrogeodetic and the GGM vertical deflection components according to Eqs. 4 and 5 with respect to different maximum d/o of the GGMs in the spectral range between 180 and 250. The RMS differences between the astrogeodetic and the EGM2008 vertical deflections remain unchanged due to the modelling of the high-frequency spectrum by EGM2008 itself. They are at the level of 0.2 to 0.5", showing the high accuracy of this analysis.

For maximum d/o 100 and 150 (not displayed here) the RMS differences do not exceed 0.04". Up to d/o 180, the RMS differences between the astrogeodetic and the GRACE model ITG-Grace2010s are massively increasing in contrast to all other models, which means a significant improvement in the spectral range between d/o 150 and 180, when utilizing the GOCE GGMs. Beyond d/o 180, a significant improvement of the second generation GOCE models with respect to the first generation models is obvious. The improvement between the 1st and the 2nd time-wise approach model is almost 40% (from 0.46 to 0.28") for maximum d/o 200, at the level of the combined models, and still up to 35% (from 1.19 to 0.78") for maximum d/o 224. Up to d/o 250, the differences are steadily increasing compared to the combined models.

Fig. 3 shows exemplarily the differences along the West-East profile between the filtered astrogeodetic East-West vertical deflection components and corresponding values from various GGMs, truncated at d/o 180 (top) and 200 (bottom). In the upper part, the weak characteristics of the GRACE model ITG-Grace2010s (black) compared to all other models are obvious. In the lower part, the considerable improvement of the second generation GOCE models, at the level of the combined models for maximum d/o 200, is evident (the time-wise approach; light green and dark green, respectively). But from d/o 210 onwards, the GOCE models are not yet at the accuracy level of the combined models based on GRACE and (the very good German) terrestrial gravity data.

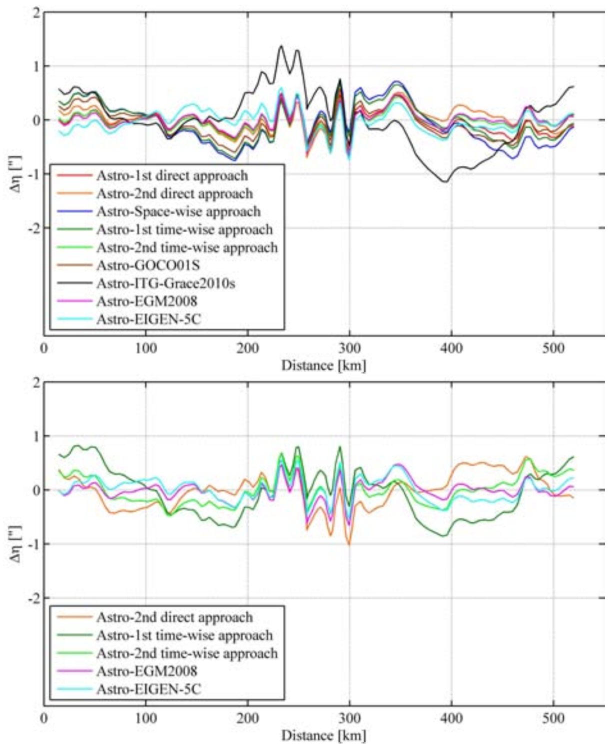

Figure 3. Differences $\Delta\eta$ ["] between the East-West astrogeodetic vertical deflection components and corresponding values from actual global gravity field models up to a maximum (d/o) of 180 (top) and 200 (bottom) along the West-East profile after a multistage filtering process

The supplied spectral errors (Fig. 2) and the spatial RMS differences of the vertical deflections along the profiles (Tab. 2) show quite good agreements, although the dimension of the astrogeodetic data set is very limited. The calibrated errors for the EGM2008 vertical deflections θ are 0.3 to 0.35" for the spectral range up to d/o 250, corresponding to 0.2 to 0.25" for the components ξ and η, whereas the RMS differences are at the level of 0.2 to 0.5". For the 2nd time-wise approach model, the formal error of θ is 0.4" for d/o 200, corresponding to 0.3" at a resolution of 100 km for ξ and η. The RMS differences of 0.3 to 0.6", including the commission error of the EGM2008 between d/o 201 and 2190 as well as the astrogeodetic vertical deflection error and modelling uncertainties, confirm the quite realistic estimation of the supplied formal errors.

5. SUMMARY AND CONCLUSIONS

The data set of astrogeodetic vertical deflections in Germany has been applied for an independent spot-check of actual global gravity field models with a focus on the impact of the recently published GOCE GGMs. The GOCE models contain more information than the GRACE models between the spherical harmonic d/o 150 and 180. For d/o between 180 and 224, the enhanced quality of the second generation GOCE models is evident with improvements up to 40 %. Among the GOCE models, the 2nd time-wise approach model shows the best performance except for the 1st direct approach model (due to the inclusion of a priori information). Up to d/o 200, the second generation GOCE models almost reach the accuracy level of the combined models from GRACE and terrestrial data, while for the higher spectral range up to d/o 250, the accuracy still degrades. However, the very high quality of the German terrestrial gravity data, included in the combined models, has to be emphasized. Regarding other regions with no or less high quality terrestrial gravity data, the GOCE models provide significant improvements as compared to the GRACE models.

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