

# DYNAMIC ORBIT DETERMINATION FOR AND CALIBRATION OF GOCE

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## ABSTRACT

We have used the IERS 2010 gravity field and the GOCE accelerometers to compute the satellite's orbit. A comparison of these model trajectories with the GOCE Precise Science Orbits indicates that the adopted accelerometer model is adequate for orbit computation purposes. However, the IERS 2010 field alone does not seem to describe the GOCE gravitational acceleration perfectly at the satellite's altitude of 250 km.

## 1. INTRODUCTION

The Norwegian Mapping Authority (NMA) intends to use data from GOCE to determine a GRAVity field over NORwegian (GRAVNOR) territories with the use of space geodesy software GEOSAT. A field determination will enable NMA to improve the Norwegian geoid and the monitoring of local ocean currents.

As a first step (GRAVNOR I) we will use the IERS 2010 gravity field [1] and GOCE accelerometers to perform a GOCE orbit determination without so-called empirical parameters. A least-squares method will then be used to map the minimum difference between this dynamic model orbit and the GOCE Precise Science Orbit. The goal is to test if the gravitational acceleration experienced by GOCE is appropriately described by the IERS 2010 gravity field. Also, it will be possible to test the adopted model for the GOCE accelerometers.

## 2. DATA & TOOLS

For this task we will use the space geodesy software GEOSAT developed by P.H. Andersen at the Norwegian Defence Research Establishment. The software has previously been used to process VLBI, GPS and SLR data, and has now been extended by the Norwegian Mapping Authority (NMA) to include the use of accelerometer observations. In GRAVNOR I, however, the observations are given by an arc of Precise Science Orbit (SST\_PSO\_2) positions (1 second sampling, reduced-dynamic method,

Terrestrial Reference Frame), stretching from 18:00:00 to 17:59:59 (UTC) the next day.

Calibrated single accelerometer observations in the Gradiometer Reference Frame (GRF), reconstructed from the calibrated common and differential mode data (EGG\_NOM\_1B: EGG\_CCD), will be used to calculate the non-gravitational acceleration of GOCE. Usually, common mode data is adopted for this purpose, but the use of single observations will enable us to test the accelerometer model in more detail. Finally, satellite attitude quaternions (EGG\_NOM\_1B: EGG\_IAQ) are needed for the calculation of the non-gravitational acceleration in both the GRF and the Celestial Reference Frame (CRF) from the single accelerometer observations.

## 3. ACCELEROMETER MODEL

The output vector  $A_j$  of accelerometer  $j$  in the GOCE fixed Gradiometer Reference Frame (GRF) is assumed connected to the true acceleration  $a_j$  at the location of accelerometer  $j$  through

$$A_j = M_j a_j + B_j + \dot{B}_j \Delta t. \quad (1)$$

Above,  $B_j$  and  $\dot{B}_j$  represent a bias and a drift, and  $\Delta t$  is the time elapsed relative to some reference epoch. The calibration matrix  $M_j$  is set to identity since the accelerometer data has been very well calibrated (EGG\_CCD). Therefore, by calibration we here mean the determination of the biases  $B_j$ .

In our work we have assumed that the calibrated single accelerometer observations can be reconstructed by taking appropriate sums and differences of calibrated common and differential mode data (EGG\_CCD).

The true acceleration at the location of accelerometer  $j$  can be described by (see for instance [2])

$$a_j = \left( R - \Omega^t T \nabla^2 V T^t \Omega \right) (L_j + O) + \Omega^t D. \quad (2)$$

Above, the  $3 \times 3$  matrix  $R$  is dependent on the satellite's angular velocity and its time derivative. For the purpose of GOCE orbit computation only,  $R$  is derived solely from interpolation of the attitude quaternions

(EGG-IAQ). The method of interpolation does not seem to have an effect on the presented results. Likewise, the rotation  $\Omega$  from the GRF to the Celestial Reference Frame (CRF) is determined by the quaternions. The matrix  $T$  then rotates from the Terrestrial Reference Frame (TRF), in which the IERS 2010 gravity tensor  $\nabla^2 V$  is given, to the CRF. The coordinates of accelerometer  $j$  in the GRF are given by  $L_j$ , and the coordinates of the GRF relative to the GOCE center of mass are described by the vector  $O$ . In Eq. (2) the superscript  $t$  means that the matrix is transposed.

#### 4. DYNAMIC ORBIT DETERMINATION

Equations (1) and (2) can be inverted to yield  $D$ , the non-gravitational acceleration of GOCE in the Celestial Reference Frame (CRF). Given a set of guesses for orbit parameter values, like the satellite epoch position  $\vec{r}_0$  and velocity  $\dot{\vec{r}}_0$ , the GOCE CRF positions

$$\vec{r} = \vec{r} \left[ \vec{r}_0, \dot{\vec{r}}_0, D(B_j, L_j, \dots), \dots \right]. \quad (3)$$

can be compared to the Precise Science Orbit (PSO). The difference between the PSO positions and the GEOSAT dynamic model orbit are mapped by a least-squares method into parameter corrections. These corrected parameters are then used to produce an improved model orbit which through a comparison with the PSO again lead to improved parameter values.

To ensure convergence we perform four iterations like this. No constraints are imposed on the parameters.

### 5. RESULTS

#### 5.1. Adequacy of models

Figure 1 shows the difference between the observations (PSO) and the model (GEOSAT dynamic orbit) when the epoch position and velocity of the satellite (six parameters) and three biases are solved for through the iterative scheme previously described. Accelerometer number 1, located at +0.25 m on the Gradiometer Reference Frame (GRF) x-axis (along-track), is used for the calculation of non-gravitational forces. The Root-Mean-Square (RMS) is 0.16 m, but should have been around 1-3 cm from GPS analysis [3].

At this relatively large RMS we experience that a simple model for the accelerometer output is sufficient (adjusting only three parameters as above). We are therefore lead to believe that the gravity experienced by GOCE at an altitude of 250 km is not perfectly described by the IERS 2010 conventions. When we introduce so-called 1-cycle-per-revolution (reset for each orbit revolution) and 2-cycles-per-revolution (reset for each day) parameters, we obtain sub-cm residuals.

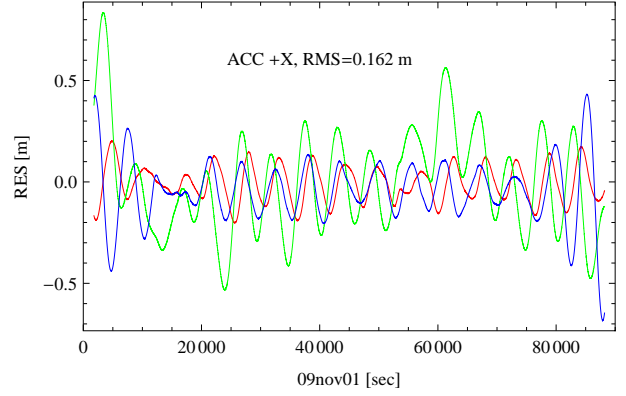


Figure 1. Converged differences between the PSO and GEOSAT dynamic orbit for the time interval 18:00:00 (Nov 1)-17:59:59 (Nov 2), 2009. The red curve is the difference in the radial direction, while green is along- and blue represents cross-track residuals.

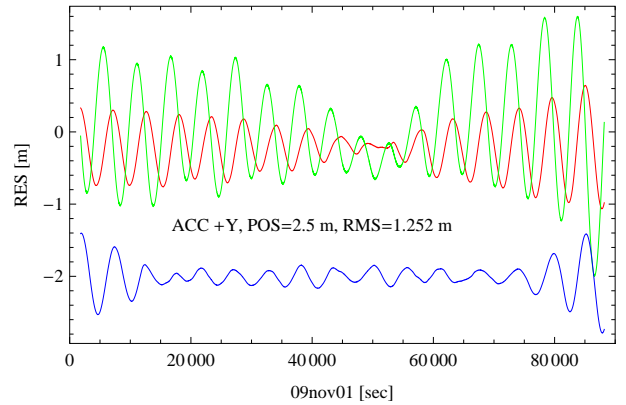


Figure 2. Converged differences between observed (PSO) and model (GEOSAT dynamic) orbit. The residual in the radial direction is represented by the red curve while the green and blue curves yield the along- and cross-track differences, respectively.

#### 5.2. Error sensitivity

Figure 2 shows the converged differences between the PSO and the GEOSAT orbit when three accelerometer biases and the initial position and velocity of the satellite are solved for. Accelerometer number 2, located at +0.25 m on the GRF y-axis (cross-track), is used to model the non-gravitational accelerations, but now its position is set to an erroneous +2.5 m on the GRF y-axis. Clearly, there is information on the accelerometer position. Indeed, if we include the accelerometer position as a solve-for parameter, the y-axis coordinate is adjusted from +2.5 m (RMS=1.252 m) to a more correct +0.204 m (RMS=0.160 m).

Included as Fig. 3 is a plot of the converged residuals when accelerometer number 3 is used to calculate the non-gravitational acceleration of GOCE. However, the model orbit is generated with an accelerometer position

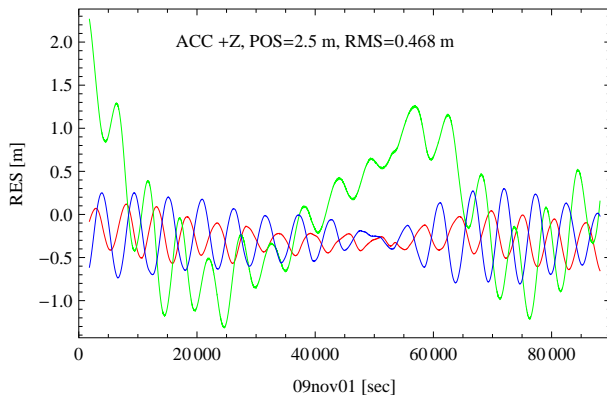


Figure 3. Residuals in the radial (red), along- (green) and cross-track (blue) direction during a data arc stretching from Nov 1 to Nov 2, 2009.

+2.5 m along the GRF z-axis (radial direction) as contrary to the correct +0.25 m. The increased RMS indicates that there is information on the position of the accelerometers on the GRF z-axis. Furthermore, the nature of the residuals implies that this type of error is different from the one apparent from Fig. 2. If we include the arm (GRF z-axis coordinate) of the used accelerometer as a solve-for parameter, the arm is adjusted to a more correct value of +0.232 m (RMS=0.163 m).

Similar calculations show that there is no or little information on the location of the accelerometers on the GRF x-axis (along-track) from the PSO.

## 6. GRAVNOR II

We have used the GOCE Level 1b and Level 2 products to quantify the limitations of the IERS 2010 gravity field, and to some extent test the adopted accelerometer model. As the next step, the outputs of the six GOCE accelerometers will be used as observations to calculate local mascon corrections to a background gravity field (IERS 2010). It is anticipated that this mascon correction field will yield an improved Norwegian geoid.

## REFERENCES

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- [3] Visser P.N.A.M., et al., 2009, Adv. Sp. Res. 43, 760