A GLOBAL GRAVITATIONAL FIELD MODEL FROM GOCE GRADIOMETER OBSERVATIONS

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ABSTRACT

In this contribution, the focus is set on the global gravity field determination from GOCE gradiometer observations using a spherical harmonic representation. The key characteristic of the processing strategy is the use of short arcs of the satellite orbit. The gradiometer observations within each arc are decorrelated using an empirical covariance function. By introducing additional empirical parameters into the functional model, the long-periodic gradiometer error behaviour is taken into account. An adequate separation of the gravity signal from the observation noise is crucial in order to exploit the strength of the satellite mission GOCE in recovering the static Earth's gravity field with unprecedented accuracy and resolution. The developed gradiometry gravity field model is validated by comparison with the official ESA models. The validation results are highly encouraging.

Key words: GOCE; SGG; gravity field analysis; short arc approach.

1. INTRODUCTION

The Gravity field and steady-state Ocean Circulation Explorer (GOCE) mission [3] features the concept of satellite gravity gradiometry (SGG). Measurements of the gravity gradiometer, supported by additional information, yield the gravitational tensor. Its components, the gravity gradients, are the second derivatives of the gravitational potential. In principle, determining the unknown coefficients of a spherical harmonic gravity field expansion from the linearly related gravity gradients is a straight forward procedure. However, gravity field analysis techniques have to deal with problems such as data gaps, outliers, coloured observation noise or signal attenuation. In case of a spherical harmonic representation the analysis technique has to cope with a huge amount of observations and parameters. In this study the short arc approach is applied and will be discussed in the following section.

The results that will be shown here have been achieved within the project GLOREGOCE which is part of the German research programme REAL GOCE (supported by BMBF, Bundesministrium für Bildung und Forschung). GLOREGOCE (GLObal gravity field determination with REgional refinements by the analysis of GOCE-level-1b data) aims at providing GOCE only models, represented by spherical harmonics on the one hand and regionally refined by means of space localising base functions on the other hand. A further objective is to provide combined solutions based on additional satellite information and terrestrial data.
The power of regional analysis techniques for GOCE has been demonstrated in several simulation studies (e.g. Eicker et al. [4]). In Shabanloui et al. [11] first regional solutions from real GOCE satellite-to-satellite tracking (SST) and SGG data have been presented. To evaluate the influence of the regional refinement procedure, the comparison to a spherical harmonic model from GOCE data that is based on exactly the same processing strategy and standards is necessary. It is the intention of this study to provide this reference model. Results will be shown and validated in the third section of this article.

2. PROCESSING STRATEGY

The measuring concept of satellite gravity gradiometry compensates to some extend for the attenuation of the gravity field signal with orbit height, due to measuring second-order derivatives. This fact enables GOCE to observe detailed gravity field structures. The large number of parameters that is needed for the representation of a high resolution gravity field has to be derived from an even larger number of correlated observations. Estimating the unknown gravity field parameters by a standard least squares adjustment results in very large equation systems, whose accumulation would require more storage than generally available. Also for runtime reasons it is advised to reduce the problem.
The analysis procedure applied here works with short arcs of the satellite orbit. Thus the system of normal equations can be accumulated by means of

$$\mathbf{N} = \sum_i \mathbf{A}_i^T \mathbf{P}_i \mathbf{A}_i \text{ and } \mathbf{n} = \sum_i \mathbf{A}_i^T \mathbf{P}_i \mathbf{l}_i \,. \qquad (1)$$

Here $\mathbf{A}_i$, $\mathbf{P}_i$ and $\mathbf{l}_i$ denote the individual blocks of the

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design matrix, weight matrix and the observation vector given for each arc i. In order to accelerate this process, an iterative solution strategy has been used, which avoids the expensive matrix multiplications. We use a least squares preconditioned conjugate gradient method [1], details of the implementation can be found in Mayer-Gürr [6].
The formulation (1) implies that the observation weight matrix is block diagonal and neighbouring arcs are independent, which is not the case in reality. Variance covariance information per short arc is derived by estimating an empirical covariance function from the observation residuals referred to a reference model. To be independent of any apriori information, the gravity field analysis procedure is iterated. Since the gravity gradients are correlated over long periods of time, the assumption of uncorrelated short arcs is not strictly valid. Therefore, further empirical parameters are introduced into the noise model. These additional parameters account for the long term error behaviour and should decorrelate subsequent orbit arcs. For the time being, an unknown constant per arc and gradient tensor element is estimated.

The short arc approach provides numerous advantages: Firstly, discontinuities and gaps in the observation series can easily be dealt with by starting a new arc after each data gap. By arc-wise re-weighting of the observations the influence of outliers can be kept low, as explained in Kusche [5] and applied by Mayer-Gürr [6]. It should be mentioned that the arc-wise re-weighting strategy has not been switched on in our analysis software. Major outliers in the observation time series are flagged using a threshold value procedure. Observation epochs affected by outliers are removed previous to the gravity field analysis.

3. PROCESSING RESULTS

3.1. Model Characteristics

The model presented is based on about 71 days of GOCE observations (precise data period: 01/11/2009, 00:49:15 to 11/01/2010, 07:38:15). The input data are specified in the following:

Gradients from EGG_NOM_1b product, EGG_GGT measurement data set

Orbits from SST_PSO_2 product, SST_PRD_2 subproduct (reduced dynamic precise science orbits)

Attitude from EGG_NOM_1b, EGG_IAQ data set (inertial attitude quaternions)

The three main diagonal components of the gravitational tensor (V_{xx}, V_{yy} and V_{zz}) serve as observations in the gravity field determination process. The gradiometer observations are interpolated on integer seconds and, in order to reduce the amount of data, resampled to a 5 sec sampling rate using low pass filtering [6]. Next, the gradiometer observations are synchronised with the other observation groups. Orbit information is required in order to geolocate the gravity gradients. Attitude observations, supported by an earth rotation model, are used to transform the observation equations, which are established in an earth fixed frame, to the gradiometer reference frame. Additionally, models for direct tides and earth tides are applied to reduce time variable gravitational effects.

The gravity model presented here is based on the short arc approach, as introduced above. Short arcs of 15 min arc length have been used. The model is represented by a spherical harmonic expansion up to degree and order 224. No regularisation has been applied.

3.2. Results

Figure 1 shows our short arc gravity field model (in the following labelled as 'GOCE SGG short arc') compared to the official ESA products, which are based on the direct method [2], the time-wise method [10] or the space-wise approach [9]. All models include almost the same data period of GOCE observations. The official GOCE models provided by ESA are combined solutions based on SST and SGG data and involve regularisation. Therefore, a comparison is valid only for a limited frequency band in the spectral domain. Regarding a frequency range from about degree 120 up to degree 180 the GOCE SGG short arc model is in remarkably good agreement with the gravity solutions using either the time-wise or the space-wise processing strategy. Beyond, differences become visible that gradually increase with growing degree. That is because no regularisation has been applied to constrain the energy of our solution. The model based on the direct approach shows large differences to any model presented.
Figure 4 illustrates the differences between our GOCE SGG short arc solution and the combination model EGM2008 in the spatial domain. Clearly visible is a long-periodic oscillation. This is due to the fact that coefficients of low degree can only poorly be determined on basis of gradiometer observations only. Besides that, deficiencies in the EGM2008 combined gravity field model can be detected in parts of South America, Africa, Asia and in Antarctica. The oscillating structures in the Southern Ocean south of Australia are not fully understood yet. Those noisy structures have already been detected in other GOCE gradiometry models (e.g. Mayrhofer et al. [8]). They will not be part of the further discussion.

In the following our new gravitational field model will be compared to the unconstrained SGG-only part of the time-wise solution (termed as 'GOCE SGG tim'), which has been provided for validation purposes courtesy of Jan Martin Brockmann.
In figure 2, differences to the high resolution combination model ITG-Grace2010c (compare Mayer-Gürr et al. [7]) are displayed by means of degree variances. These difference degree variances (solid red line) provide a good approximation for the model error. Obviously, the variance covariance information (marked by the dotted red line)

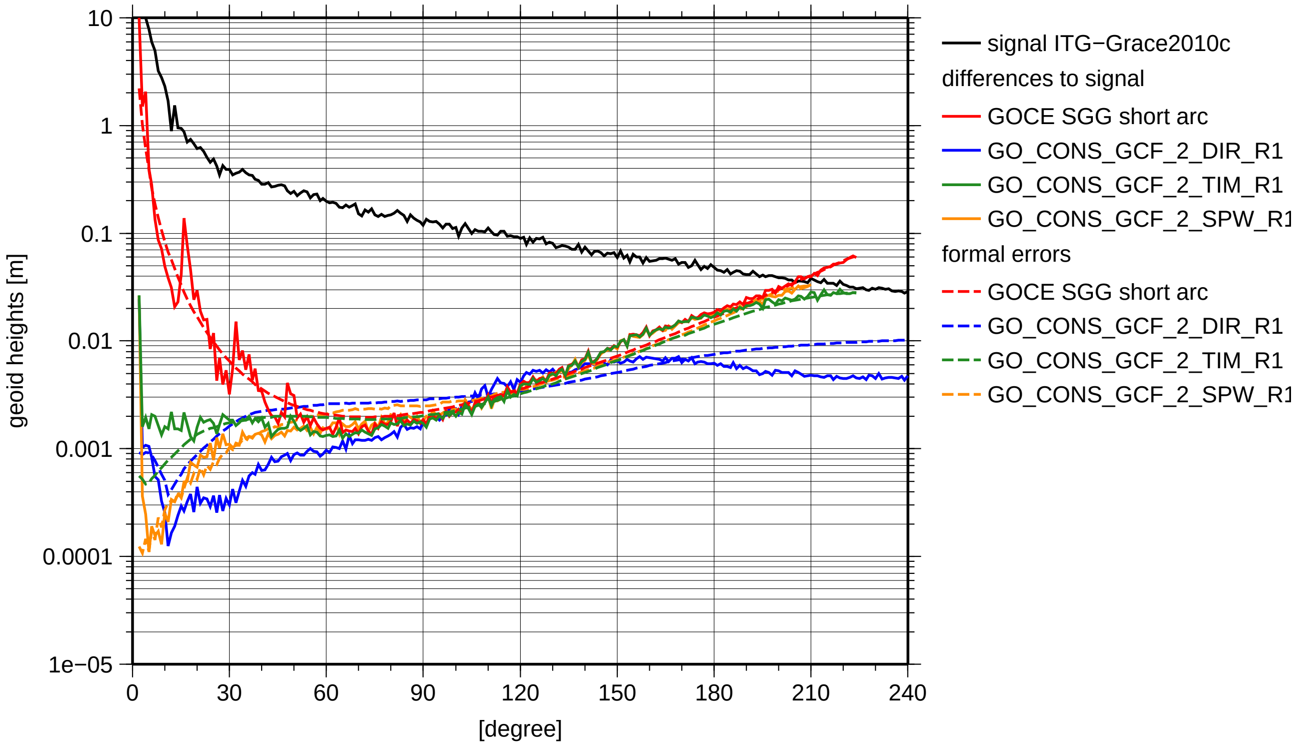

Figure 1. Comparison between our new model (GOCE SGG short arc) and the official ESA products by means of degree variances. Because of the polar gap problem the rms is calculated excluding the low order part of the spherical harmonic coefficients. The same is valid for the following illustrations.

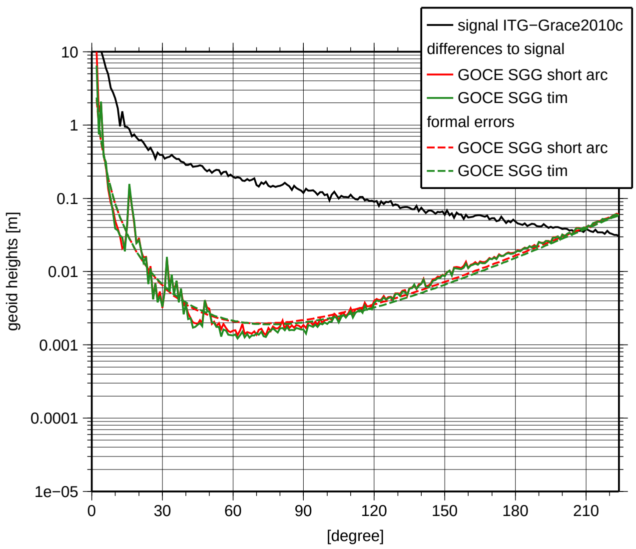

Figure 2. Differences of two unconstrained SGG-only models (GOCE SGG tim, GOCE SGG short arc) to the combined model ITG-Grace2010c

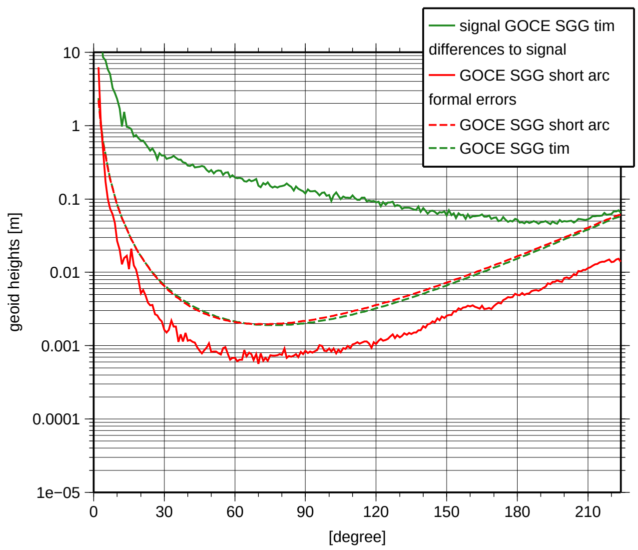

Figure 3. Direct comparison between GOCE SGG short arc and GOCE SGG tim

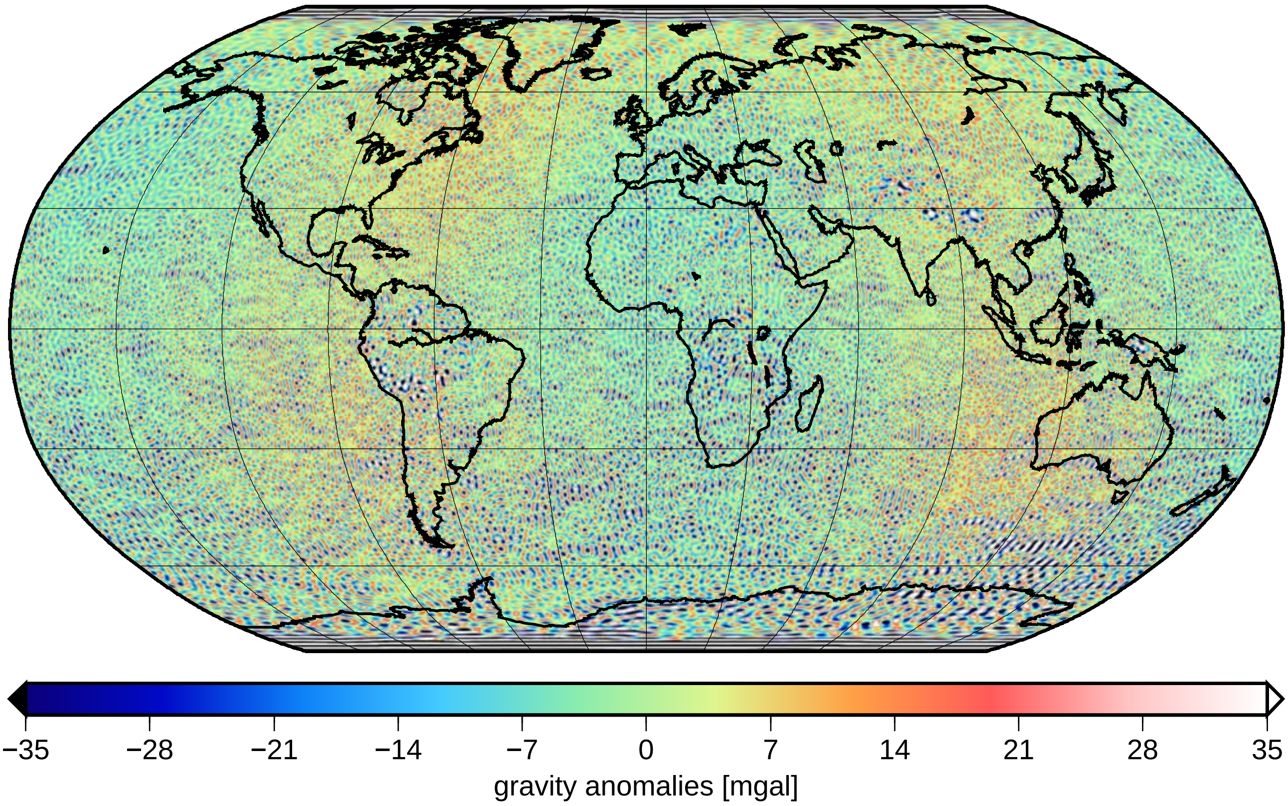

Figure 4. Gravity anomaly deviations up to d/o 224 of the developed GOCE short arc model from EGM2008

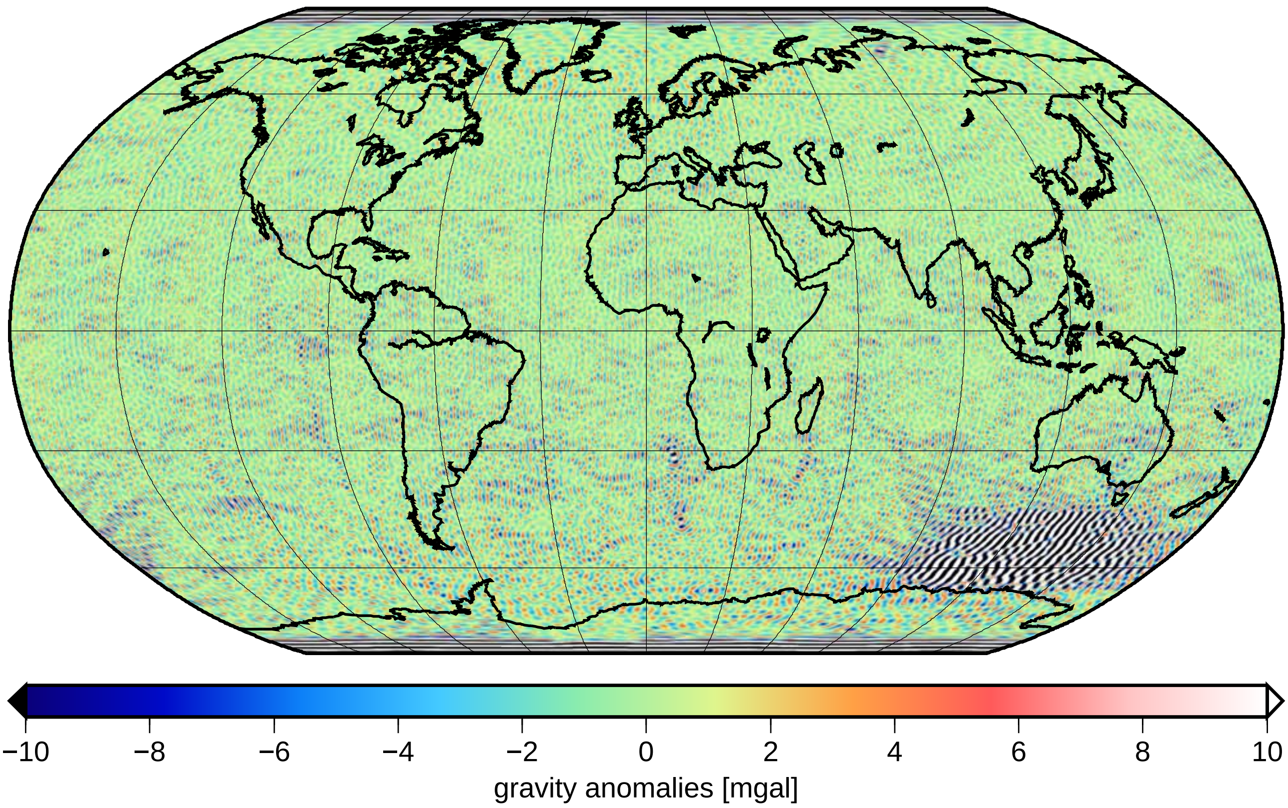

Figure 5. Gravity anomaly deviations (d/o 10-224) of our new model from the unconstrained SGG-only part of the time-wise solution

represents quite well the true error of the coefficients. This indicates that the observation noise has been described adequately in our stochastic model. It should also be recognised that the GOCE SGG tim model slightly better agrees with the GRACE solution in the medium frequency range. However, according to figure 3 the coefficients of both models are identical within their accuracies.
A final comparison is presented between GOCE SGG short arc and GOCE SGG tim in the spatial domain (figure 5). Apart from large differences in the ocean south of Australia there are geographical structures that seem rather systematical, i.e. peaks in the South Atlantic Ocean west of South Africa or some kind of orbit track parallel to the South American coast line. An interpretation of these structures is difficult since there is no independent gravity field solution available. The peaks mentioned above might be explained by smaller outliers in the observation time series. Up to now, large outliers have been identified using a threshold value procedure. Whether a more sophisticated handling of outliers and arc re-weighting is able to reduce these differences, will be investigated in future work.

4. CONCLUSIONS

In this contribution, an unconstrained spherical harmonic gravity field model from 71 days of GOCE gradiometer observations has been presented, which is based on the short arc processing strategy. Our new model has been compared e.g. to GOCE models calculated by the official ESA processing groups. The validation results are very encouraging. While the comparison in the spectral domain has demonstrated the competitive accuracy of the developed model, a comparison in the spatial domain has revealed systematic differences that are not understood yet. However, it can be concluded that the short arc approach is suitable to invert GOCE gravity gradients into spherical harmonic coefficients. The calculation of a combined gravity field model based on SST and SGG data will be the subject of near future work. Next, we will focus on regional gravity field analysis.

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REFERENCES

[1] Björck, Å. (1996). *Numerical methods for least squares problems*. Society for Industrial Mathematics

[2] Bruinsma, S.L., Marty, J.C., Balmino, G., Biancale, R., Foerste, C., Abrikosov, O. & Neumayer, H. (2010). GOCE Gravity Field Recovery by Means of the Direct Numerical Method. Presented at the ESA Living Planet Symposium, Bergen, Norway

[3] Drinkwater, M.R., Floberghagen, R., Haagmans, R., Muzi, D. & Popescu, A. (2003). GOCE: ESA's first Earth Explorer Core mission. In G.B. Beutler, M.R. Drinkwater, R. Rummel & R. von Steiger (Eds.), *Earth Gravity Field from Space - from Sensors to Earth Sciences*, vol. 18 of *Space Sciences Series of ISSI*. Kluwer Academic Publishers

[4] Eicker, A., Mayer-Gürr, T. & Ilk, K.-H. (2006). An integrated global/regional gravity field determination approach based on GOCE observations. In R. Rummel, C. Reigber, M. Rothacher, G. Boedecker, U. Schreiber & J. Flury (Eds.), *Observation of the Earth System from Space*. Springer Berlin - Heidelberg, pp. 225–237

[5] Kusche, J. (2003). A Monte-Carlo technique for weight estimation in satellite geodesy. *Journal of Geodesy*, **76**(11), 641–652

[6] Mayer-Gürr, T. (2006). Gravitationsfeldbestimmung aus der Analyse kurzer Bahnbögen am Beispiel der Satellitenmissionen CHAMP und GRACE. Ph.d. thesis, University of Bonn, Bonn, Germany

[7] Mayer-Gürr, T., Kurtenbach, E. & Eicker, A. (2010). ITG-Grace2010 gravity field model. `http://www.igg.uni-bonn.de/apmg/index.php?id=itg-grace2010`

[8] Mayrhofer, R., Pail, R. & Fecher, T. (2010). Quicklook Gravity Field Solutions as part of the GOCE Quality Assessment. In *Proceedings of the ESA Living Planet Symposium, Bergen, Norway*. ESA SP-686 (CD-ROM), ESA Communications, ESTEC, Noordwijk, The Netherlands

[9] Migliaccio, F., Reguzzoni, M., Sanso, F., Tscherning, C.C. & Veicherts, M. (2010). GOCE data analysis: the space-wise approach and the first space-wise gravity field model. In *Proceedings of the ESA Living Planet Symposium, Bergen, Norway*. ESA SP-686 (CD-ROM), ESA Communications, ESTEC, Noordwijk, The Netherlands

[10] Pail, R., Goiginger, H., Mayrhofer, R., Schuh, W.-D., Brockmann, J.M., Krasbutter, I., Hoeck, E. & Fecher, T. (2010). GOCE gravity field model derived from orbit and gradiometry data applying the time-wise method. In *Proceedings of the ESA Living Planet Symposium, Bergen, Norway*. ESA SP-686 (CD-ROM), ESA Communications, ESTEC, Noordwijk, The Netherlands

[11] Shabanloui, A., Schall, J., Mayer-Gürr, T., Eicker, A., Kusche, J. & Kurtenbach, E. (2010). Global gravity field determination with Regional refinements by the analysis of GOCE level-1b data (GLO-REGOCE). In *Observation of the System Earth from Space*, no. 17 in Geotechnologien Science Report. Koordinierungsbüro GEOTECHNOLOGIEN, pp. 77–83