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Foreword

This document will try to give user requirement for a toolbox designed around the GOCE mission. Because of the high accuracy that GOCE will hopefully attain, the data from this mission will be useful to geodesists, oceanographers, and solid earth scientists. Because geodetic data and handling of this may be new to oceanographers and solid earth scientists, this toolbox proposes to make available a simple way of dealing with the data processing that is needed to use the data from GOCE for applications within the two fields.

For an oceanographer the following would be a typical use-case. The user has hydrographic data along a cruise track providing geostrophic currents across the track. The user wishes to compare these current values with those obtained from altimeter derived geostrophic velocities for the same transect and the same time period and place in the context of the time series of surface currents in the area.

Another oceanographic application where GUT might be useful is where the user needs a gridded MDT interpolated or derived on a specific model grid. The user will then add an altimetric anomaly to the MDT to get a gridded sea level field for a particular time instant. This will then be used to assimilate into the model at that time.

For solid earth applications the main required functionalities are covered by general functionalities described above. However, on land the quantities may be needed on the surface of the earth. Hence, a few additional functionalities are required: Computation of geoid heights, gravity anomalies or deflections of the vertical at a given, user-specified, degree and order of the spherical harmonic expansion (i.e., at a given spatial resolution) at the surface of the terrain.

These use cases will be used in this document to determine which functionalities need to be included in the final toolbox, and furthermore algorithm specifications will be given for the functionalities that are to be included. These functionalities will then be collected into workflows in a later work package. However not all the functionalities included in the use cases will go into the toolbox. For some of them toolboxes already exist or the scientists know how to handle these cases.

Background

Toolbox rational

Data from the ESA GOCE mission are of fundamental importance to the oceanographic community. It is expected that in conjunction with altimetric observations, gravity data from the ESA GOCE Mission will - for the first time in history - allow access to the absolute ocean dynamic surface topography and to compute the absolute ocean surface geostrophic currents at spatial scales down to about 100 km. At the moment, only the variable part of the sea level, and thus the geostrophic currents, can be inferred from altimetric heights with sufficient accuracy.

Despite their importance for oceanographic studies, the processing and analysis of gravity mission data has proven to be complicated to the point that the lack of proper processing software is hampering progress in the use of those data. Success in the exploitation of GRACE data therefore seems to depend fundamentally on the proper knowledge of several steps of the detailed gravity data processing procedure in terms of spherical harmonics, their implicit consistent normalization factors, filtering and error data, among others. To facilitate the easy use of GOCE products for oceanographers and to support the needs of specific applications, the development of a user toolbox (GOCE user toolbox study = GUTS) was identified as an urgent step at the Second International GOCE User Workshop. Such a toolbox is required to guarantee optimal use of the existing and future gravity data acquired from GRACE and GOCE. In particular it is recognized and accepted that software packages are required that allow the gravity field data, in conjunction and consistent with any other auxiliary data sets, to be pre-processed by users not intimately familiar with gravity field processing procedure, for oceanographic and hydrologic application, regionally and globally.

From previous work, a preliminary idea about the scope of a GUTS toolbox exists already in the community, especially from experience gained through GRACE data processing. Any new effort should build directly on this insight and should expand on user needs in a flexible way. The now finished GOCINA project is another source of valuable information about needs and solutions, that will be consulted while defining GOCE user needs and toolboxes.

Applicable and Reference Documents

The applicable documents are:

- Statement of Work

Reference documents are:

- Algorithm specification standard/guideline (ENVISAT document)
- ENVISAT Product Handbook
- GOCE: Product specification for L2 Products and Auxiliary Data Products (GO-ID-HPF-GS-0041)
- GOCE: Product Data Handbook (GO-MA-HPF-GS-0110)
- GOCE: Standards (GO-TN-HPF-GS-0111)

Previous Experience from The GOCINA Project

The EU FP5 GOCINA project was undertaking a study to determine regional high accuracy gravimetric geoid and mean sea surface height fields over the Northeast Atlantic and Nordic Seas, bounded by 53°N, 73°N, 40°W and 20°E. Among the project goals was to determine a best possible mean dynamic sea surface topography and to provide specific recommendations for quality assessment of GOCE data. These recommendations will be based on the generation and dissemination of the three regional fields for mean sea surface, geoid and mean dynamic topography and their error characteristics. The input to these three fields are, respectively, satellite altimetry, high resolution marine and

airborne gravimeter data blended with GRACE long wavelength data, and output from numerical ocean models.

During the last phase of the GOCINA project these three fields and their error characteristics were evaluated and assessed. In addition, the impact of the new regional geoid for ocean circulation studies were investigated using direct current observations from an array of bottom mounted ADCP moorings between Scotland and Greenland. In doing so, the processing and analyses scheme (Figure 1) first suggested in ESA SP –1233 (1) will allow us to examine and quantify how the new GOCINA geoid, and its error covariance, combined with radar altimetry enable more reliable estimation of absolute ocean topography.

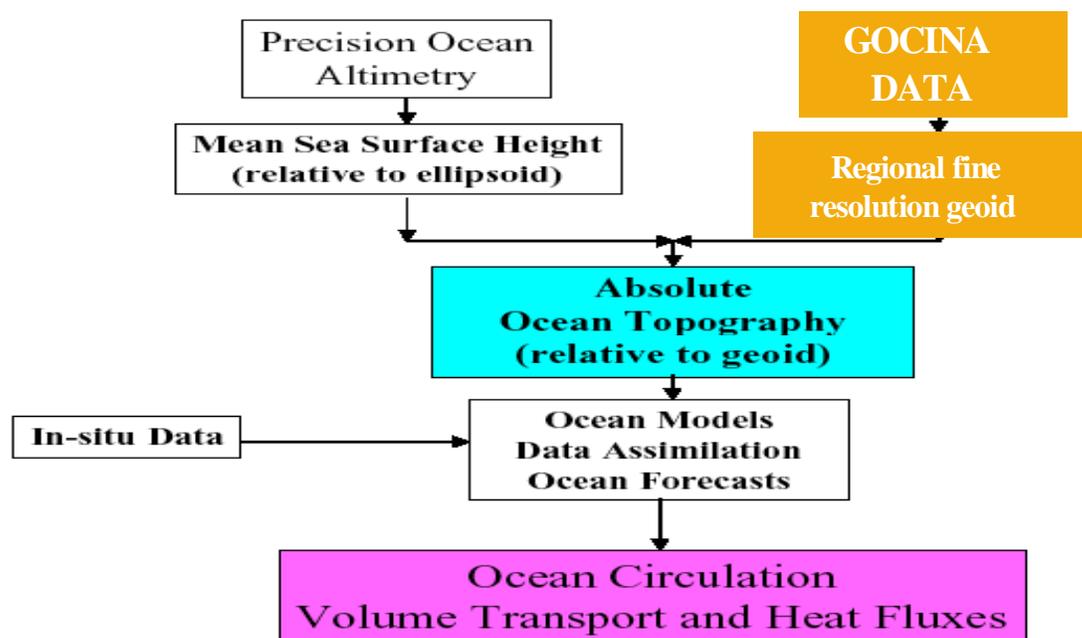


Figure 1: Schematic of the improvement in absolute ocean circulation studies from combination of the GOCE-produced marine geoid with precision altimetry (ESA-SP 1233 (1)).

Building on experience from this approach the GOCINA project delivered results that will be essential in the preparation for the GOCE mission, in the context of:

- Generation of best possible time-invariant regional gravity field and geoid model for the Northeast Atlantic and Nordic Seas that may be used for regional validation of GOCE mission products.
- Verification of the schematic analysis approach for integrated use of regional geoid and altimetry to compute the absolute dynamic topography and other relevant products such as volume and mass transport.
- Develop specific recommendations for integrating GOCE data in determination of geoid and mean dynamic topography models.

An important outcome of the GOCINA project has been that it is essential to describe proper (a-priori) statistical characteristics associated with the Mean Dynamic Topography. Those characteristics include a-priori variance and covariance functions, which in turn will be linked to statistical characteristics of the geostrophic surface currents through their associated a-priori power spectra. Those values should agree with the empirically derived values too, as the statistical characteristics of the gravity field should agree with its empirically derived values. Only by combining that information can consistent reliable error covariances of the Mean Dynamic Topography be expected.

Relation to the HPF Effort

As part of the GOCE Ground Segment, ESA is running a GOCE High-level Processing Facility (HPF) which is responsible for the generation of the final Level 2 products, and the generation of Quick-Look and External Calibration products. The final Level 2 products consist of: the precise orbit of GOCE, the gravity field in terms of spherical harmonic coefficients, the corrected, calibrated and geo-located gravity gradients, a map of the gravity anomalies and a map of the geoid. This effort will be an essential element in the entire GOCE data stream. However, no ocean circulation products are planned to be delivered as Level-2 products as part of the HPF project so that a strong need exists for GUTS to deliver oceanographers with additional information and tools that can turn the HPF products into application-dependent fields by further processing the GOCE Level-2 geoid and merging it with Radar Altimetry and other auxiliary data. Among the fields that are important for oceanographers is the geoid error covariance function. This field will be provided by HPF for spherical harmonic coefficients. For many oceanographic applications it is required in geographic coordinates or at specific locations. The toolbox therefore needs to perform this and other extra functions that are essential for using the GOCE geoid fields efficiently and for testing it in geoid validation studies.

To reach a maximum benefit from GUTS, the effort will work in close collaboration with the HPF project. This will be facilitated through joint memberships. In return, the HPF effort will be one of the first users interested in using GUT for their own geoid validation efforts. A mutual feedback and benefit and an optimum toolbox design is thereby almost automatic.

The GOCE HPF is lead by TUM. TUM participates in the GUTS consortium. Also UCPH participates in both the GOCE HPF and in GUTS. The main contact to the GOCE HPF will go through TUM who will participate actively at any stage of the work and ensure that knowledge about GOCE Level-2 products and specifications are available and utilized in GUTS and that information about the developments in GUTS are available to the HPF. To assure further consistency between GUTS and other GOCE related developments the participation of IFREMER, POL, NERSC, TUM, UCPH as members of the GOCE Mission Advisory Group is important.

Study outline

In order to fulfil the GUTS objectives, a toolbox effort was proposed which aims at consolidating the user requirements, the algorithms needed to produce the required outputs, both globally and for selected areas, and system design of a toolbox. Although the development of a user toolbox was clearly recommended, the detailed community requirements for a toolbox need to be specified through a first action of a GUTS effort. In particular, all added-value products that could potentially be output from such a tool need to be better defined.

In more detail, the aim of a GOCE gravity field toolbox is to facilitate and ease the use, viewing and post-processing of GOCE mission and useful auxiliary data products for optimal use at higher levels than Level-2 products. Such a tool would be a basis for the computation and validation of Level-3 products for oceanographers and, indirectly, for the validation of Level-2 products. One functionality of such a tool should be to provide geoid heights at any point of the ocean as well as geoid height covariances and error covariances for any pair of points on the surface of the Earth.

This user requirements and an input/output specification document (GOCE User Requirements Report), which will be a reference to identify the main users together with Products and Functionalities they require from a toolbox. The User Requirement document will modify the list above and/or complete it with additional requirements so as to contain all what the GOCE User Toolbox shall hold to help the users in fully exploiting the GOCE level 2 data.

No ocean circulation products are planned to be delivered as level-2 products as part of the GOCE project so that a strong need exists, for oceanographers, to further process the GOCE level-2 geoid and merge it with Radar Altimetry. This need, and subsequent recommendations for optimal use of GOCE data by oceanographers, was explicitly expressed during the second International GOCE Workshop, which was held in ESRIN from 8-10 March 2004. A synthesis of the main recommendations has been published in the GOCE workshop proceedings. The primary requirement of oceanographers is to have access to a geoid and its error covariance at the highest spatial resolution and accuracy possible, although required resolution depends on application. These two requirements are consistent with the conclusions reached at last ISSI workshop on Earth Gravity Field from Space (March 2002, Bern) where the need for two gridded geoid Level-2 products was identified. For effective use of the geoid data, knowledge of the error covariance is mandatory so that the two previous geoid products have to be delivered with their corresponding covariance matrices. The possibility of having such a matrix delivered by the HPF as Level 2 products with standard resolution is planned in spherical harmonic space. However, no geoid or error covariance will be delivered in geographic space. Moreover, resolution depends on the application, as does the required geographic region of interest. It is therefore necessary to provide the user community with a general tool that takes HPF Level 2 information, merges it with other auxiliary data and provides users with the possibility to obtain required fields with self-specified resolution.

Review of needs for existing and planned applications

Input information

The toolbox will have several primary input fields. Most importantly, GOCE Level 2 products are required as starting point. This will be in form of spherical harmonic (SH) coefficients of the Earth Gravity field and geoid height at maximum resolution, and the error variance and covariance matrices of those SH coefficients. In addition, auxiliary (meta) data are required for geoid fields including the following information:

- Whether the mean (zero frequency) tide was removed and the spherical harmonic expansion used.
- Which tidal models were used, including statements about which tidal frequencies are included.
- The atmospheric correction used, including a statement of whether atmospheric tides are included (separated time mean and time varying atmosphere).
- Specification of any ocean corrections that were made, including a specification of any model that was involved.
- Specification of Love number corrections made, if any.
- Any polar motion correction, including, but not limited to the Chandler period.

Compliant with the requirements stated by Jakob Flurry and Reiner Rummel in their article on *Future satellite gravimetry and Earth Dynamics*, other important auxiliary data sets are required, some of which should be included as auxiliary data in the tool box. Others, not included, should be provided through links to data servers. Among those the most important ones are:

- Altimetric data sets:
 - along-track data sets.
 - altimetric Mean Sea Surfaces Height (MSSH) field(s). Most up to date versions of those products should be included as well as the option to exchange them with newer versions once they become available.
 - Also included should be error information about the above time mean and time-varying altimetric measurements.
- In-situ data through links to Argo, tide gauge and hydrographic datasets.
- Wind field and wind stress measurements, e.g., from scatterometers or NWP centers.
- In-situ and modelled MDT. The latter would include best possible fields available from several ocean models that were constrained by ocean data and should include information about their uncertainties as well.
- In-situ gravimetric data and in-situ local geoid estimations

Some altimetric fields should be included in the tool box for instant use, but a requirement is also that the users can add their own fields easily, as long as they are in the

same format as that used for the included fields. This will allow the users to update the tool box with their own data and other readily available fields.

In addition to the data sets or links to them, tutorials in how to use the toolbox should provide methods of including these auxiliary datasets. In addition – links to other appropriate data service, such as the IAG's ACGEM should be specified, where a list of global earth models (<http://icgem.gzf-potsdam.de/ICGEM/ICGEM.html>) is being maintained.

Toolbox Functionality

Toolbox functionalities will be described qualitatively here and quantitatively later. As specified later, main functionalities of the toolbox would be to select data, make global, regional and local approximations of the gravity and geoid fields over the ocean, translate and adapt the spherical harmonic information into gridded information for any location in world, and to translate the errors associated with the scales of interest. The main aim, however, is to produce a geoid and geoid slopes that can be used for oceanographic purposes of ocean modeling, assimilation, and current estimation. This will require solutions valid over whole ocean basins starting from level 2 global solutions. Moreover, errors should be delivered in a form that they can be used for oceanography. As an example, nearly all the GRACE geoid products have correlated errors along the orbits in the meridional direction. These usually get removed by smoothing the entire geoid. An error description should capture the anisotropy in the errors and should allow the data to be combined with an a priori solution to allow the final solution to be isotropic and not display the banding.

Accordingly, toolbox requirements can be split into three main aspects, related to their use in geodetic, oceanographic and the solid Earth applications.

Geoid and gravity fields

In a variety of geophysical studies information associated with the Earth's gravity are used in form of geoid heights, gravity anomalies or deflections of the vertical. Those quantities may be represented in the nodes along a profile, in a grid or in discretely located points. Associated with such quantities error covariance information may be needed. Hence, the following functionalities are required:

1. Computation of global, gridded geoid heights or gravity anomalies at a given, user-specified, degree and order of the spherical harmonic expansion (i.e., at a given spatial resolution. The maximum resolution/degree is defined by the final GOCE products). User input would be the spatial resolution or the grid and the maximum degree order of the expansion. If several different geoid fields are available, input would also be the choice of geoid model or gravity field, if necessary). Output would be a gridded geoid. The gridded field should be available as both a single point representation and as area averages.
2. Computation of geoid heights at a given spatial resolution (as point-wise or area averages) and a given point or list of points (e.g. unstructured grid, transect).

- Input would be a list of user-specified geographic position parameters and the maximum degree order of the expansion. If several different geoid fields are available, input would also be the choice of geoid model.
3. Option to replace geoid heights by geoid slopes (deflection from the vertical) for either the gridded field or the unstructured positions.
 4. Computation of geoid heights covariance for any pair of points on the sphere or the computation of a full covariance matrix for a given maximum degree and order of the spherical harmonic expansion.
 5. Computation of geoid cumulative height errors and error covariances at a given spatial resolution on a global regular grid or for a list of points.
 6. Covariance error matrix within chosen degree/order range for commission and omission error. The computation of omission errors maybe based on the original gravity gradients.
 7. Option to include the omission errors for the GOCE gravity field.
 8. Regional geoid solutions starting from the global products, This is not ideal for regional geoid implementation. At a later stage, Level 1 data might be required as the basis for an extended GOCE geoid, Concerning regional geoids, a combination of model solutions using both GOCE and in situ gravity data (see Haines et al., 2003, space science reviews, 108, p 205-217) could be useful and including such regional models for the European Seas would be a useful element of GUTS.

Additional functionalities would be required for geoid validation, to:

- Handle appropriate ancillary data, e.g., external MDT from in-situ/modelled, as well as local geoids, combined CHAMP/GRACE/GOCE geoids, among others.
- Compute differences / Root Mean Square differences / correlation coefficient / regression slopes between GOCE geoid and external geoids / 'GOCE' MDT and external MDT / between absolute altimetric dynamic topography and in-situ absolute dynamic topography / between absolute altimetric geostrophic velocities and in situ geostrophic velocities.
- Determine the parameters in a priori degree variance model for the modeling of the gravity field a priori spectrum (global and/or regional models).
- Derive a degree variance model for the MDT and determine statistical properties of the MDT and its associated geostrophic surface currents.
- Derive an optimal filter for the low pass filtering of the altimetric MSSH and/or the MDT derived from the altimetric MSSH and the GOCE geoid.

Dynamic Surface Topography

The primary oceanography variable of interest to be provided by the tool box is the dynamic topography resulting from the difference between altimetric measurements and the geoid model. Altimetric MSSH fields or time-varying SSH fields would be auxiliary input data set fields from which a consistently filtered mean dynamic topography (and associated mean geostrophic circulation) need to be computed by the toolbox. This

requires that a consistent reference system be chosen for both the geoid and the (M)SSH (both surfaces expressed relative to the same reference ellipsoid) as well as a consistent permanent tide system (tide mean, zero tide or tide free system) or that supporting tools exist with which to move from one system to another. In this context, it would be useful to compute the magnitude of the aliasing parts of the MDT that are not resolved by the model/grid spacing. The three different tide systems will be briefly explained here. The mean tide contains the mean of all the different tide constituents (the permanent tide) in addition to the indirect effects on the Earth due to the elasticity of the earth. The zero tide system has the indirect effects but the permanent direct tide effects are removed. Finally the tide free system has neither indirect nor direct tide effects. Ideally the same tidal models or other corrections should have been used in the geoid and the altimetric fields. But as a requirement this would be too stringent. Instead a frequent update of both input field would be more useful. In addition enough information is required about the geophysical and environmental corrections applied to the altimetric data, such as atmospheric tide correction.

The possibility of defining the geoid in alternative “over the ocean-only” basis functions may allow the user to avoid problems with continental gaps that arise when using global spherical harmonics. The development of alternative basis functions is still a research topic and in that sense out of the scope of the toolbox. Nevertheless, it is a user requirement that eventually needs to be approached: Because the ocean is a not simply connected domain, the use of global basis functions is therefore not trivial or desirable. To avoid problems with continental gaps (Gibbs effects) a representation of the geoid and MSSH in terms of ocean basis functions is desirable. The possibility of including ocean basis functions into the toolbox software would therefore be a worthwhile effort. Such functionality would be an alternative to sophisticated filter programs designed to cope with Gibbs effects at continental boundaries that arise when using global basic functions (spherical harmonics) to develop and smooth MDT fields. Concerning regional geoids, a combination of model solutions using both GOCE and in situ gravity data (see Haines et al., 2003, space science reviews, 108, p 205-217) could be useful and including such regional models for the European Seas would be a useful element of GUTS. Alternatively, it may be possible to work in spherical harmonic space and do MSSH-Geoid residuals in spherical harmonics. The field of MSSH-MDT which exists over the ocean only has a natural continuation over land as the Geoid itself. The use of a priori estimates of both MSSH and MDT could lead to a complete global a priori field of MSSH-MDT over the ocean which matches to a geoid over land. The goal then is to solve for corrections to this global field. It would be assumed that this correction over the ocean represents a correction to the a priori MDT rather than to the MSSH. This approach could avoid problems near the coasts.

Functionalities related to the dynamic sea surface topography aspects of the tool box would include:

- Interpolation of external MSSH on any regular grid or at given points (unstructured grid): This functionality should include procedures to translate and adapt the spherical harmonic information into geoid height information for any location in world (many of the oceanographic applications require geoid

- information on a global or regional geographical – often non-structured – grid), and to translate the errors associated with the scales of interest.
- Spatial filtering of MSSH consistent with a specific harmonic geoid height field expansion.
 - Change of reference system for the geoid and/or MSSH (reference ellipsoid, tide system)
 - Computation of a ‘GOCE’ MDT (MSSH-GOCE geoid) at a given spatial resolution and on a given structured or unstructured grid as the difference between the MSSH and the geoid.
 - Computation of altimetric time-varying absolute dynamic topography as the difference between altimetric fields and a geoid model.
 - Provision of a priori MSSH, MDT and Geoid data on a grid
 - Provision of tools to produce a global description of a combination of these a priori gridded fields in spherical harmonics
 - Computation of altimetric absolute geostrophic velocities from the spatial gradients of the geoid field.
 - Covariance error matrix within chosen degree/order range for commission and omission error for mean dynamic surface topographies. This needs to include the auxiliary altimetric error covariances on top of the geoid model error covariance.
 - Option to include the omission errors for the MDT and the associated geostrophic surface currents.

In addition, the computation of gridded time-mean geostrophic velocities with arbitrary resolution should be possible. Those computations should be possible from either pure data products (MDT) or from assimilation results. The latter results should be provided as auxiliary input from assimilation models.

Solid Earth Applications

In studies of the solid Earth the relations between different physical properties of different geological materials are utilized together with a series of observations that reflect those different physical properties. The relevant property in this perspective is the density of geological materials which may vary between significantly from materials such as rock salt, sediments, and bedrock. Hence, geological structures may be associated with changes in the density which are mirrored into the gravity field. By mapping the gravity field on the surface of the Earth information about the interior of the Earth is obtained. Low gravity reflects light materials and high gravity reflects heavy materials in the subsurface.

The commonly used quantity in solid earth studies is gravity anomalies and associated error information. Hence, the main required functionalities are covered by general functionalities described above. However, on land the quantities may be needed on the surface of the earth. Hence, a few additional functionalities are required:

- Computation of geoid heights, gravity anomalies or deflections of the vertical at a given, user-specified, degree and order of the spherical harmonic expansion (i.e., at a

given spatial resolution) at the surface of the terrain. Output would be a grid or at single point representation and as area averages.

- Computation of geoid cumulative errors and error covariances associated with the above at a given spatial resolution on a global regular grid or for a list of points.
- Covariance error matrix within chosen degree/order range for commission and with the option of including the omission errors for the GOCE gravity field.

Pre-viewing Function

The toolbox functionalities should include

- pre-viewing capabilities for results from the toolbox. This is not to be a full-fledged graphics representation of the data, but should rather be a way of easily checking the veracity of the processed data immediately after the processing is done.
- The previewing function should include the possibility of adding a latitude-longitude grid to the previewed data and should present rendered graphics.
- data access and the possibility to transfer appropriate data from remote location to local machine.

Comments on the functionality

The previous sections have talked at some length about the functionalities that optimally would be included in the toolbox, mostly in a qualitative way. Some of the functionalities are essential, some are highly desirable, and others less so.

The following section will try to give a quantitative description of the functionalities that will possibly be incorporated in the toolbox, a first thinning of the functionalities.

There are a few exceptions here, functionalities that will need to be included but that will not be described in the following section. The ability for the user to provide external MSSH, geoid, MDT and other fields is vital to the toolbox, but this functionality will not be described in the following except for the most general handling of the fields.

Additionally some fields will be included in the toolbox, however these will be determined in a later work package together with the workflows that describe exactly how the functionalities described in the following section will be implemented. Note that as mentioned not all the functionalities required for the use-cases from the foreword will be implemented. Some of these will have to come from other sources.

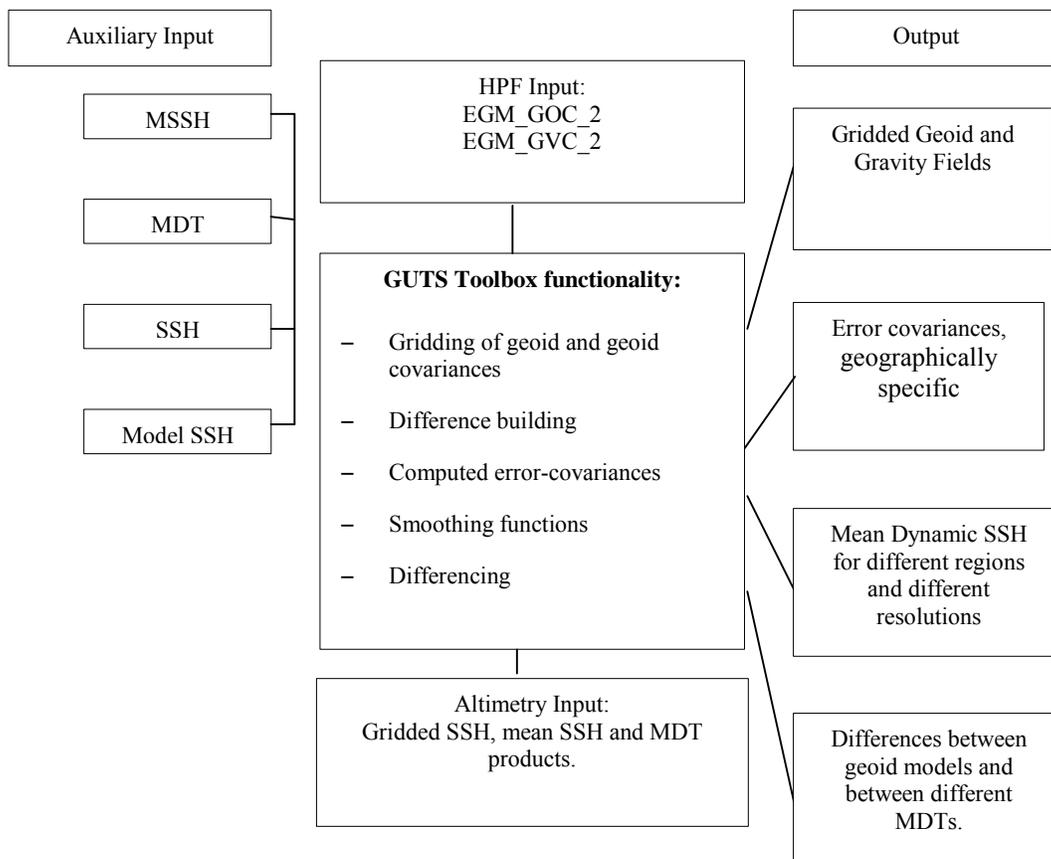


Figure. 2 Schematic of a preliminary input/output lists and toolbox functionalities.

Required functionality, input and output parameters

Introduction

The task here is to compile an input/output definition document and an Algorithm Specification document. This will provide the source algorithms for providing the range of user requirements. The application of these algorithms will be defined in WP3000, together with the specification of the toolbox functionality. The input and output for each algorithm is given in terms of the required parameters. The source file and output format definitions will also be supplied in WP3000.

The following provides a general overview of the input data, ancillary data and output data:

Input data

- GOCE Level-2 products:

EGM_GOC_2: Final GOCE gravity field model with error estimates and quality report.

This product has the following sub product files:

EGM_GCF_2: Spherical harmonic series in ICGEM format
EGM_GEO_2: Grid with geoid heights in Grid format
EGM_GAN_2: Grid with gravity anomalies in Grid format
EGM_GVE_2: Grid with east-west vertical deflections in Grid format
EGM_GVN_2: Grid with north-south vertical deflections in Grid format
EGM_GER_2: Grid with geoid height errors in Grid format
EGM_GRP_2: Quality report in PDF format

and

EGM_GVC_2: Variance-covariance matrix file of the spherical harmonics coefficients

- Altimetric Mean Sea Surfaces
- Error information about time mean altimetric measurements.
- Digital terrain model

Software

Several software packages are available that will be collected here as the basis of an open source software package and that will be used as a base line in the trade-off studies of

WP3000. Among those is the GRAVSOF program GEOCOL that is being used in the ARCGICE effort. Another example is the spherical harmonic MATLAB toolbox (SHBUNDLE). The MATLAB toolbox can compute all relevant quantities from a spherical harmonic model such as geoid undulations, gravity anomalies, gravity disturbances, and the entire tensor of second order derivatives. This can be done in a grid or on given data points. The software collected in this WP will be used and tested later as prototype toolbox. The software will be bundled later with a tutorial document to become a first prototype of toolbox.

A note on algorithms

It has been decided in the GUT consortium to use the full expressions for the wanted quantities instead of the spherical approximations used in the products obtained from the HPF since using the non-approximated algorithms will give a higher accuracy.

The HPF grids will also be supplied in the GUT and tools will be supplied to make interpolation to other user-defined grids possible.

A note on units

The units for the products will be as follows:

- Geoid height/height anomaly will be given in meters (m)
- Gravity anomalies will be given in milligal (mgal)
- Deflections of the vertical will be given in arcseconds.
- Dynamic topography will be in meters (m)
- Geostrophic currents will be in m/s

Handling of data

The requirements on the data input from the toolbox should be as low as possible to make it easier to include data supplied by the users. As such there should be only a demand on the format that this data should be in, and not on the naming of input data. Rather this name should be an input to the toolbox from the user. The fields that will be included in the toolbox should however, be named in a way that makes it easy to recognise what quantity they contain and which model the data comes from.

Regarding the output, this could easily be named GUT_XXX according to the functionalities described in the next section. With this document in hand the user could then easily see which product comes out.

Products, Input and Output Parameters

EGM_GOC_2: Final GOCE gravity field model

This product contains several subproducts:

- EGM_GCF_2: Spherical harmonic series in ICGEM format
- EGM_GEO_2: Grid with geoid heights in Grid format
- EGM_GAN_2: Grid with gravity anomalies in Grid format
- EGM_GVE_2: Grid with east-west vertical deflections in Grid format
- EGM_GVN_2: Grid with north-south vertical deflections in Grid format
- EGM_GER_2: Grid with geoid height errors in Grid format

All of these will be included in the GUT and the application of these will be described later. The two products: EGM_GER_2 and EGM_GCF_2 will be especially useful and will therefore be described in greater detail.

EGM_GER_2: Grid with geoid height errors in Grid format

This contains the geoid errors of the delivered gridded geoid heights, EGM_GEO_2. These will be an easy way of getting a priori errors for some of the products that GUT will deliver.

This product will, as mentioned, be delivered from the HPF in Grid format. Both a header and a data section will be included. The information contained in the header is:

- Data_set_name
- Northern_latitude. The northern border of the grid in geographical coordinates given in sdd.mmss. Latitudes on the southern hemisphere is given as e.g. -70.
- Southern_latitude. Southern border of the grid
- Western_latitude. Western border of the grid.
- Eastern_latitude. Eastern border of the grid.
- Latitude_cell_size. Grid cell size in latitude direction, given in dd.mmss.
- Longitude_cell_size. Grid cell size in longitude direction.
- Number_of_cells_latitude_dir.
- Number_of_cells_longitude_dir.
- Mean_(0)_or_point_(1)_values. 0=mean-values, the most north west corner of one grid identifies this grid. 1=point values
- Geocentric(0)_geodetic(0)_lat. 0=geocentric latitudes and 1=geodetic latitudes.
- Reference_ellipsoid
- Format_of_data. The data format given in FORTRAN notation.
- gap_value. The value used for specifying an unknown value in the grid. This number must correspond to the format_of_data format.
- Description_of_data. Description of data records.
- Unit

- Any_comments
- End_of_header

The data section is cut into two sections: The Row Leader Record and the Data Record. The Row Leader contains the following:

- Latitude. The latitude of the current row
- Number_of_data_values.

The Data Record consists of the values written from west to east, with rows going north to south.

EGM_GCF_2: Spherical harmonic series in ICGEM format

This is given in the form of a spherical harmonic expansion of the full gravitational potential, V . The full expression for the expansion in fully normalized spherical harmonics is

$$V(r, \theta, \lambda) = \frac{GM}{r} \sum_{n=0}^{N_{\max}} \left(\frac{a}{r}\right)^n \sum_{m=0}^n (\bar{C}_{nm} \cos m\lambda + \bar{S}_{nm} \sin m\lambda) \bar{P}_{nm}(\cos \theta)$$

$$\bar{C}_{n0} = \frac{1}{\sqrt{2n+1}} C_{n0}, \quad \bar{P}_{n0}(\cos \theta) = \sqrt{2n+1} P_{n0}(\cos \theta)$$

$$\begin{cases} \bar{C}_{nm} \\ \bar{S}_{nm} \end{cases} = \sqrt{\frac{(n+m)!}{2(2n+1)(n-m)!}} \begin{cases} C_{nm} \\ S_{nm} \end{cases}, \quad \bar{P}_{nm}(\cos \theta) = \sqrt{\frac{2(2n+1)(n-m)!}{(n+m)!}} P_{nm}(\cos \theta)$$

where θ is the co-latitude, λ is the longitude, a is the semimajor axis of the ellipsoid used in the determination of the coefficients, r is the height at which this is calculated and the C_{nm} and S_{nm} are the spherical harmonic coefficients and the barred versions are the normalized coefficients. What will be provided from this expansion are the coefficients up to degree and order 250. This expansion forms the basis for the other level-2 products that will be used as input. The provided files will contain a header and a data section. The header will contain the information that does not depend on degree and order. The information given in this is:

- Product type, i.e. “gravity_field”.
- Name of the model.
- The value used for GM , which in GOCE is $3.986004415 \cdot 10^{14} \text{ m}^3/\text{s}^2$
- The radius, a . The value used in GOCE is 6378136.3 m
- The maximum degree of the expansion, i.e. 250.
- The errors, this will be either “no”, “calibrated”, “formal”, or “calibrated_and_formal”.
- The normalization of the expansion. Either “fully normalized” or “unnormalized”, as default this is “fully normalized”.
- The tide system used. Either “zero_tide”, “tide_free”, or “unknown”.
- Finally an end_of_head to indicate the end of the header.

Depending on the kind of error specified in the header, the data section will contain different data. What kind of data is given is specified by a parameter at the start of each line. This is followed by the data. The parameters are

- gfc: This gives the degree, order, C_{nm} , S_{nm} , and possibly the errors of these.

- gfct: The same as the above and in addition the time is provided.
- dot: This gives the degree, order, the time derivatives of the coefficients, and possibly the errors on these.
- For more in depth information on these parameters look at the *Product Specification for Level-2 products and Auxiliary Data Products* from the GOCE High Level Processing Facility. In general these are just a way of specifying which information is included.

The rest of the products described herein, except for the variance-covariance matrix for the coefficients and the gridded geoid height errors, derive from these coefficients.

EGM_GVC_2: Variance-covariance matrix file of the spherical harmonics coefficients

This product contains the variance-covariance matrix of the coefficients of the spherical harmonic expansion of the gravity potential. The variance-covariance matrix is the result of a least squares estimation of the spherical harmonic coefficients.

As the expansion goes to degree and order 250 this matrix is somewhat large. This means that the matrix is split over several files with the matrix entries of each order m in a separate file. This set of files containing the matrix entries are accompanied by a meta-data file describing the data. The content of this meta-data file is:

- The product type which is a variance-covariance matrix
- The model name
- The value of GM used.
- The value of a used.
- The maximum degree of the spherical harmonic expansion.
- Whether the errors are “formal”, “calibrated”, or “unknown”.
- Whether the covariance is a full system or a block-diagonal system.
- The number of coefficients in the matrix, and next the sequence of the coefficients, with each coefficient written on its own line.
- The number of files used to store the variance-covariance matrix, followed by the names. These are ordered by harmonic order.

The data files contains

- The name of the meta-data file.
- The harmonic order of the file.
- The number of entries in the file.
- A `begin_data` and an `end_data`

GUT_001: Grid with geoid height, sph. approx.

This product will take the geoid height grid delivered as a level 2 product from the HPF and by use of linear interpolation (GUT_109) generate a user-specified subgrid. This means that the degree and order of the gridded geoid height are predetermined by the HPF.

Input:

- **EGM_GEO_2.**
- Wanted grid specifications:
 - Starting latitude, φ_1
 - Ending latitude, φ_2
 - Starting longitude, λ_1
 - Ending longitude, λ_2

Output:

- A sub-grid of the input grid defined by the grid specifications. This grid will be in GRAVSOFTE grid format. This format contains a line at the start of the file containing in order and in decimal degree notation
 - Starting latitude, φ_1
 - Ending latitude, φ_2
 - Starting longitude, λ_1
 - Ending longitude, λ_2
 - Grid spacing in latitude direction, $\Delta\varphi$
 - Grid spacing in longitudinal direction, $\Delta\lambda$

GUT_002: Grid with gravity anomalies, sph. approx.

This product will take the gravity anomaly grid delivered as a level 2 product from the HPF and by use of linear interpolation (GUT_109) generate a user-specified subgrid. This means that the degree and order of the gridded gravity anomalies are predetermined by the HPF.

Input:

- **EGM_GAN_2.**
- Wanted grid specifications:
 - Starting latitude, φ_1
 - Ending latitude, φ_2
 - Starting longitude, λ_1
 - Ending longitude, λ_2

Output:

- A sub-grid of the input grid defined by the grid specifications. This grid will be in gravsoft grid format. This format contains a line at the start of the file containing in order and in decimal degree notation
 - Starting latitude, φ_1
 - Ending latitude, φ_2
 - Starting longitude, λ_1
 - Ending longitude, λ_2
 - Grid spacing in latitude direction, $\Delta\varphi$
 - Grid spacing in longitudinal direction, $\Delta\lambda$

GUT_003: Grid with east-west vertical deflections, sph. approx.

This product will take the E-W vertical deflections grid delivered as a level 2 product from the HPF and by use of interpolation (GUT_109) generate a user-specified subgrid. This means that the degree and order of the gridded deflections are predetermined by the HPF.

Input:

- **EGM_GVE_2.**
- Wanted grid specifications:
 - Starting latitude, φ_1
 - Ending latitude, φ_2
 - Starting longitude, λ_1
 - Ending longitude, λ_2

Output:

- A sub-grid of the input grid defined by the grid specifications. This grid will be in gravsoft grid format. This format contains a line at the start of the file containing in order and in decimal degree notation
 - Starting latitude, φ_1
 - Ending latitude, φ_2
 - Starting longitude, λ_1
 - Ending longitude, λ_2
 - Grid spacing in latitude direction, $\Delta\varphi$
 - Grid spacing in longitudinal direction, $\Delta\lambda$

GUT_004: Grid with north-south vertical deflections, sph. approx.

This product will take the N-S vertical deflections grid delivered as a level 2 product from the HPF and by use of interpolation (GUT_109) generate a user-specified subgrid. This means that the degree and order of the gridded deflections are predetermined by the HPF.

Input:

- **EGM_GVN_2.**
- Wanted grid specifications:
 - Starting latitude, φ_1
 - Ending latitude, φ_2
 - Starting longitude, λ_1
 - Ending longitude, λ_2

Output:

- A sub-grid of the input grid defined by the grid specifications. This grid will be in gravsoft grid format. This format contains a line at the start of the file containing in order and in decimal degree notation
 - Starting latitude, φ_1
 - Ending latitude, φ_2
 - Starting longitude, λ_1
 - Ending longitude, λ_2
 - Grid spacing in latitude direction, $\Delta\varphi$
 - Grid spacing in longitudinal direction, $\Delta\lambda$

GUT_005: Height anomaly in grid format, full

The height anomaly – which is usable over land as well as over ocean – is found through the following formula

$$\zeta = \frac{(W - U)_P}{\gamma}$$

$$W = V + \frac{\omega^2}{2}(r \cos \theta)^2$$

$$\gamma = |\nabla U| = (U_x^2 + U_y^2 + U_z^2)^{1/2}$$

$$U_x = \frac{1}{r \cos \theta} \frac{\partial U}{\partial \lambda}, U_y = \frac{1}{r} \frac{\partial U}{\partial \theta}, U_z = \frac{\partial U}{\partial r}$$

γ is the normal gravity at the point Q on the equipotential surface that has $U_Q = W_P$ at all points. The potentials will be found from the given coefficients and parameters and then subtracted. This is done in a grid.

The “full” means that this product does not make the spherical approximations that are made in the grids coming from the HPF.

Input:

- A set of spherical harmonic coefficients.
- The specifications for the reference ellipsoid, this input can be in several forms. The choices are
 - GM, a, J_2 , ω
 - GM, a, f, ω
 - γ_a , a, f, ω
- Maximum harmonic degree and order, N_{\max}
- Altitude of the grid points. This altitude should be above the terrain.
- Wanted grid specifications:
 - Starting latitude, ϕ_1
 - Ending latitude, ϕ_2
 - Starting longitude, λ_1
 - Ending longitude, λ_2
 - Grid spacing in latitude direction, $\Delta\phi$
 - Grid spacing in longitudinal direction, $\Delta\lambda$

Output:

- A sub-grid of the input grid defined by the grid specifications. This grid will be in gravsoft grid format. This format contains a line at the start of the file containing in order and in decimal degree notation
 - Starting latitude, ϕ_1
 - Ending latitude, ϕ_2

- Starting longitude, λ_1
- Ending longitude, λ_2
- Grid spacing in latitude direction, $\Delta\varphi$
- Grid spacing in longitudinal direction, $\Delta\lambda$

GUT_006: Height anomaly in points, full

The height anomaly – which is usable over land as well as over ocean – is found through the following formula

$$\zeta = \frac{(W - U)_P}{\gamma}$$

$$W = V + \frac{\omega^2}{2} (r \cos \theta)^2$$

$$\gamma = |\nabla U| = (U_x^2 + U_y^2 + U_z^2)^{1/2}$$

$$U_x = \frac{1}{r \cos \theta} \frac{\partial U}{\partial \lambda}, U_y = \frac{1}{r} \frac{\partial U}{\partial \theta}, U_z = \frac{\partial U}{\partial r}$$

γ is the normal gravity at the point Q on the equipotential surface that has $U_Q = W_P$ at all points. The potentials will be found from the given coefficients and parameters and then subtracted. This is done in a set of points.

The “full” means that this product does not make the spherical approximations that are made in the grids coming from the HPF.

Input:

- A set of spherical harmonic coefficients.
- The specifications for the reference ellipsoid, this input can be in several forms. The choices are
 - GM, a, J_2 , ω
 - GM, a, f, ω
 - γ_a , a, f, ω
- Maximum harmonic degree and order, N_{\max}
- Name of input point file
- Name of output file

Output:

- Values in points in the output file in gravsoft point list format.

GUT_007: Gravity anomaly in grid format, full

The gravity anomalies are found from the norm of the gravity vector and the normal gravity vector derived from the reference ellipsoid. It will be done in the following way to avoid using spherical approximations. The W here is the potential V plus the potential stemming from the rotation of the Earth, all in all this gives

$$W = V + \frac{\omega^2}{2} (r \cos \theta)^2$$

$$g = |\nabla W| = (W_x^2 + W_y^2 + W_z^2)^{1/2}$$

$$W_x = \frac{1}{r \cos \theta} \frac{\partial W}{\partial \lambda}, W_y = \frac{1}{r} \frac{\partial W}{\partial \theta}, W_z = \frac{\partial W}{\partial r}$$

$$\gamma = |\nabla U| = (U_x^2 + U_y^2 + U_z^2)^{1/2}$$

$$U_x = \frac{1}{r \cos \theta} \frac{\partial U}{\partial \lambda}, U_y = \frac{1}{r} \frac{\partial U}{\partial \theta}, U_z = \frac{\partial U}{\partial r}$$

$$\Delta g = g - \gamma$$

Note here that the potential W needs to be evaluated at the surface of the Earth while the reference potential U needs to be evaluated at the point Q determined by $U_Q = W_P$.

Depending on which height is given, the ellipsoidal height of the point P , h , or the normal orthometric height of the point Q , H , the height anomaly needs to be taken into account. For a given ellipsoidal height, h , of the point P the height of Q is found as $H = h - \zeta$ where U then needs to be evaluated to find the gravity anomaly. If the orthometric height, H , is given the height of P will be $h = H + \zeta$ and W then has to be evaluated at this height to get the anomaly.

A way of circumventing this, is using the formula

$$\Delta g = g - \gamma - \frac{2}{r\gamma} \frac{\partial \gamma}{\partial n} (W - U)$$

The last term is a correction term where the differentiation is along the normal. Now this can be evaluated at both points P and Q but it will give a small error in the gravity anomaly.

The “full” means that this product does not make the spherical approximations that are made in the grids coming from the HPF.

Input:

- A set of spherical harmonic coefficients.
- The specifications for the reference ellipsoid, this input can be in several forms. The choices are
 - GM, a, J_2 , ω
 - GM, a, f, ω

- γ_a, a, f, ω
- Maximum harmonic degree and order, N_{\max}
- Altitude of the grid points. This altitude should be above the terrain.
- Wanted grid specifications:
 - Starting latitude, φ_1
 - Ending latitude, φ_2
 - Starting longitude, λ_1
 - Ending longitude, λ_2
 - Grid spacing in latitude direction, $\Delta\varphi$
 - Grid spacing in longitudinal direction, $\Delta\lambda$

Output:

- A sub-grid of the input grid defined by the grid specifications. This grid will be in gravsoft grid format. This format contains a line at the start of the file containing in order and in decimal degree notation
 - Starting latitude, φ_1
 - Ending latitude, φ_2
 - Starting longitude, λ_1
 - Ending longitude, λ_2
 - Grid spacing in latitude direction, $\Delta\varphi$
 - Grid spacing in longitudinal direction, $\Delta\lambda$

GUT_008: Gravity anomaly in points, full

The gravity anomalies are found from the norm of the gravity vector and the normal gravity vector derived from the reference ellipsoid. It will be done in the following way to avoid using spherical approximations. The W here is the potential V plus the potential stemming from the rotation of the Earth, all in all this gives

$$W = V + \frac{\omega^2}{2} (r \cos \theta)^2$$

$$g = |\nabla W| = (W_x^2 + W_y^2 + W_z^2)^{1/2}$$

$$W_x = \frac{1}{r \cos \theta} \frac{\partial W}{\partial \lambda}, W_y = \frac{1}{r} \frac{\partial W}{\partial \theta}, W_z = \frac{\partial W}{\partial r}$$

$$\gamma = |\nabla U| = (U_x^2 + U_y^2 + U_z^2)^{1/2}$$

$$U_x = \frac{1}{r \cos \theta} \frac{\partial U}{\partial \lambda}, U_y = \frac{1}{r} \frac{\partial U}{\partial \theta}, U_z = \frac{\partial U}{\partial r}$$

$$\Delta g = g - \gamma$$

Note here that the potential W needs to be evaluated at the surface of the Earth while the reference potential U needs to be evaluated at the point Q determined by $U_Q = W_P$.

Depending on which height is given, the ellipsoidal height of the point P , h , or the normal orthometric height of the point Q , H , the height anomaly needs to be taken into account. For a given ellipsoidal height, h , of the point P the height of Q is found as $H = h - \zeta$ where U then needs to be evaluated to find the gravity anomaly. If the orthometric height, H , is given the height of P will be $h = H + \zeta$ and W then has to be evaluated at this height to get the anomaly.

A way of circumventing this, is using the formula

$$\Delta g = g - \gamma - \frac{2}{r\gamma} \frac{\partial \gamma}{\partial n} (W - U)$$

The last term is a correction term where the differentiation is along the normal. Now this can be evaluated at both points P and Q but it will give a small error in the gravity anomaly.

The “full” means that this product does not make the spherical approximations that are made in the grids coming from the HPF.

Input:

- A set of spherical harmonic coefficients.
- The specifications for the reference ellipsoid, this input can be in several forms. The choices are
 - GM, a, J_2 , ω
 - GM, a, f, ω

- γ_a, a, f, ω
- Maximum harmonic degree and order, N_{\max}
- Name of input point file, containing heights of the points
- Name of output file

Output:

- Values in points in the output file in gravsoft point list format.

GUT_009: E-W deflections in grid format, full

The vertical deflections are the differences in direction of the true gravity vector and the normal gravity vector. The formula for the east-west deflections without spherical approximation is

$$\eta = \frac{-W_x}{g}$$

This expression needs to be evaluated above the terrain.

The “full” means that this product does not make the spherical approximations that are made in the grids coming from the HPF.

Input:

- A set of spherical harmonic coefficients.
- The specifications for the reference ellipsoid, this input can be in several forms. The choices are
 - GM, a, J_2 , ω
 - GM, a, f, ω
 - γ_a , a, f, ω
- Maximum harmonic degree and order, N_{\max}
- Altitude of the grid points. This altitude should be above the terrain.
- Wanted grid specifications:
 - Starting latitude, φ_1
 - Ending latitude, φ_2
 - Starting longitude, λ_1
 - Ending longitude, λ_2
 - Grid spacing in latitude direction, $\Delta\varphi$
 - Grid spacing in longitudinal direction, $\Delta\lambda$

Output:

- A sub-grid of the input grid defined by the grid specifications. This grid will be in gravsoft grid format. This format contains a line at the start of the file containing in order and in decimal degree notation
 - Starting latitude, φ_1
 - Ending latitude, φ_2
 - Starting longitude, λ_1
 - Ending longitude, λ_2
 - Grid spacing in latitude direction, $\Delta\varphi$
 - Grid spacing in longitudinal direction, $\Delta\lambda$

GUT_010: E-W deflections in points, full

The vertical deflections are the differences in direction of the true gravity vector and the normal gravity vector. The formula for the east-west deflections without spherical approximation is

$$\eta = \frac{-W_x}{g}$$

This expression needs to be evaluated at the terrain.

The “full” means that this product does not make the spherical approximations that are made in the grids coming from the HPF.

Input:

- A set of spherical harmonic coefficients.
- The specifications for the reference ellipsoid, this input can be in several forms. The choices are
 - GM, a, J₂, ω
 - GM, a, f, ω
 - γ_a, a, f, ω
- Maximum harmonic degree and order, N_{max}
- Name of input point file
- Name of output file

Output:

- Values in points in the output file in gravsoft point list format.

GUT_011: N-S deflections in grid format, full

The north-south deflections are given by

$$\xi = \frac{(U_y - W_y) \cos(\theta - \varphi) + (U_z - W_z) \sin(\theta - \varphi)}{g}$$

where φ is the ellipsoidal latitude. This expression needs to be evaluated above the terrain.

The “full” means that this product does not make the spherical approximations that are made in the grids coming from the HPF.

Input:

- A set of spherical harmonic coefficients.
- The specifications for the reference ellipsoid, this input can be in several forms. The choices are
 - GM, a, J_2 , ω
 - GM, a, f, ω
 - γ_a , a, f, ω
- Maximum harmonic degree and order, N_{\max}
- Altitude of the grid points. This altitude should be above the terrain.
- Wanted grid specifications:
 - Starting latitude, φ_1
 - Ending latitude, φ_2
 - Starting longitude, λ_1
 - Ending longitude, λ_2
 - Grid spacing in latitude direction, $\Delta\varphi$
 - Grid spacing in longitudinal direction, $\Delta\lambda$

Output:

- A sub-grid of the input grid defined by the grid specifications. This grid will be in gravsoft grid format. This format contains a line at the start of the file containing in order and in decimal degree notation
 - Starting latitude, φ_1
 - Ending latitude, φ_2
 - Starting longitude, λ_1
 - Ending longitude, λ_2
 - Grid spacing in latitude direction, $\Delta\varphi$
 - Grid spacing in longitudinal direction, $\Delta\lambda$

GUT_012: N-S deflections in points, full

The north-south deflections are given by

$$\xi = \frac{(U_y - W_y) \cos(\theta - \varphi) + (U_z - W_z) \sin(\theta - \varphi)}{g}$$

where φ is the ellipsoidal latitude. This expression needs to be evaluated at the terrain.

The “full” means that this product does not make the spherical approximations that are made in the grids coming from the HPF.

Input:

- A set of spherical harmonic coefficients.
- The specifications for the reference ellipsoid, this input can be in several forms. The choices are
 - GM, a, J_2 , ω
 - GM, a, f, ω
 - γ_a , a, f, ω
- Maximum harmonic degree and order, N_{\max}
- Name of input point file
- Name of output file

Output:

- Values in points in the output file in gravsoft point list format.

GUT_013: Geostrophic velocities in grid

The geostrophic velocities are calculated from an ocean topography. The algorithms to do this is

$$u = \frac{-\gamma}{fR} \frac{\partial \zeta}{\partial \theta}$$

$$v = \frac{\gamma}{fR \cos \theta} \frac{\partial \zeta}{\partial \lambda}$$

$$f = 2\omega \sin \theta$$

where ζ is the topography input.

Input:

- The ocean topography from which the velocities are to be calculated. This must be in the form of a grid with the same specifications as the wanted grid.
- Altitude of the grid points. This altitude should be above the terrain.
- Wanted grid specifications:
 - Starting latitude, φ_1
 - Ending latitude, φ_2
 - Starting longitude, λ_1
 - Ending longitude, λ_2
 - Grid spacing in latitude direction, $\Delta\varphi$
 - Grid spacing in longitudinal direction, $\Delta\lambda$

Output:

- A grid defined by the grid specifications. This grid will be in gravsoft grid format. This format contains a line at the start of the file containing in order and in decimal degree notation
 - Starting latitude, φ_1
 - Ending latitude, φ_2
 - Starting longitude, λ_1
 - Ending longitude, λ_2
 - Grid spacing in latitude direction, $\Delta\varphi$
 - Grid spacing in longitudinal direction, $\Delta\lambda$

GUT_014: Geostrophic velocities in points

The geostrophic velocities are calculated from an ocean topography. The algorithms to do this is

$$u = \frac{-\gamma}{fR} \frac{\partial \zeta}{\partial \theta}$$

$$v = \frac{\gamma}{fR \cos \theta} \frac{\partial \zeta}{\partial \lambda}$$

$$f = 2\omega \sin \theta$$

where ζ is the topography input.

Input:

- Name of input point file containing the values of the topography at the wanted points
- Name of output file

Output:

- Values in points in the output file in gravsoft point list format.

GUT_015: Grid with geoid height errors, Pre-computed.

The pre-computed geoid height error variances have been found using covariance propagation on the coefficient variance-covariance matrix associated with the spherical harmonic coefficients (see GOCE Level 2 product EGM_GVC_2), i.e.

$$\sigma_x^2 = \mathbf{a}^T \mathbf{E}_x \mathbf{a}.$$

where E_x is the variance –covariance matrix containing and the vector \mathbf{a} contain the evaluation functional for each coefficient associated with geoid heights in the point P where the geoid height error covariance is computed.

$$\mathbf{a}_j = \left\{ \frac{GM}{r\gamma_Q} \left(\frac{a}{r} \right)^n \begin{pmatrix} \cos(m\lambda) \\ \sin(m\lambda) \end{pmatrix} P_{nm}(\cos\theta) \right\}_{j=\begin{pmatrix} n^2+2m \\ n^2+2m+1 \end{pmatrix}}$$

The geoid errors are calculated through use of this and given in a grid file.

Input:

- **EGM_GER_2.**
- Wanted grid specifications:
 - Starting latitude, φ_1
 - Ending latitude, φ_2
 - Starting longitude, λ_1
 - Ending longitude, λ_2
 - Grid spacing in latitude direction, $\Delta\varphi$
 - Grid spacing in longitudinal direction, $\Delta\lambda$

Output:

- A sub-grid of the input grid defined by the grid specifications. This grid will be in gravsoft grid format. This format contains a line at the start of the file containing in order and in decimal degree notation
 - Starting latitude, φ_1
 - Ending latitude, φ_2
 - Starting longitude, λ_1
 - Ending longitude, λ_2
 - Grid spacing in latitude direction, $\Delta\varphi$
 - Grid spacing in longitudinal direction, $\Delta\lambda$

GUT_016: Geoid height errors in grid format.

The geoid height error variances are found using covariance propagation on the coefficient variance-covariance matrix associated with the spherical harmonic coefficients (GOCE Level 2 product EGM_GVC_2) where the vector \mathbf{a} contain the evaluation functional for each coefficient associated with geoid heights in the point P where the geoid height error covariance is computed, i.e.

$$\sigma_x^2 = \mathbf{a}^T \mathbf{E}_{\hat{\mathbf{x}}} \mathbf{a} .$$

where $\mathbf{E}_{\hat{\mathbf{x}}}$ is the variance –covariance matrix containing and \mathbf{a} is

$$\mathbf{a}_j = \left\{ \frac{GM}{r\gamma_Q} \left(\frac{a}{r} \right)^n \begin{pmatrix} \cos(m\lambda) \\ \sin(m\lambda) \end{pmatrix} P_{nm}(\cos\theta) \right\}_{j=\binom{n^2+2m}{n^2+2m+1}}$$

The geoid errors are calculated through use of this and given in a grid file.

Input:

- The variance-covariance matrix of the spherical harmonic coefficients.
- The specifications for the reference ellipsoid, this input can be in several forms. The choices are
 - GM, a, J_2 , ω
 - GM, a, f, ω
 - γ_a , a, f, ω
- Maximum harmonic degree and order, N_{\max}
- Altitude of the grid points. This altitude should be above the terrain.
- Wanted grid specifications:
 - Starting latitude, φ_1
 - Ending latitude, φ_2
 - Starting longitude, λ_1
 - Ending longitude, λ_2
 - Grid spacing in latitude direction, $\Delta\varphi$
 - Grid spacing in longitudinal direction, $\Delta\lambda$

Output:

- A sub-grid of the input grid defined by the grid specifications. This grid will be in gravsoft grid format. This format contains a line at the start of the file containing in order and in decimal degree notation
 - Starting latitude, φ_1
 - Ending latitude, φ_2
 - Starting longitude, λ_1

- Ending longitude, λ_2
- Grid spacing in latitude direction, $\Delta\phi$
- Grid spacing in longitudinal direction, $\Delta\lambda$

GUT_017: Geoid height errors in points.

The geoid height error variances are found using covariance propagation on the coefficient variance-covariance matrix associated with the spherical harmonic coefficients (GOCE Level 2 product EGM_GVC_2) where the vector \mathbf{a} contain the evaluation functional for each coefficient associated with geoid heights in the point P where the geoid height error covariance is computed, i.e.

$$\sigma_x^2 = \mathbf{a}^T \mathbf{E}_x \mathbf{a}.$$

where \mathbf{E}_x is the variance –covariance matrix containing and \mathbf{a} is

$$\mathbf{a}_j = \left\{ \frac{GM}{r\gamma_Q} \left(\frac{a}{r} \right)^n \begin{pmatrix} \cos(m\lambda) \\ \sin(m\lambda) \end{pmatrix} P_{nm}(\cos\theta) \right\}_{j=\begin{pmatrix} n^2+2m \\ n^2+2m+1 \end{pmatrix}}$$

The geoid errors are calculated through use of this and given in a grid file.

Input:

- The variance-covariance matrix of the spherical harmonic coefficients.
- The specifications for the reference ellipsoid, this input can be in several forms. The choices are
 - GM, a, J_2 , ω
 - GM, a, f, ω
 - γ_a , a, f, ω
- Maximum harmonic degree and order, N_{\max}
- Name of input point file
- Name of output file

Output:

- Values in points in the output file in gravsoft point list format.

GUT_018: Gravity anomaly errors in grid format.

The error variances are found using covariance propagation on the coefficient variance-covariance matrix associated with the spherical harmonic coefficients (GOCE Level 2 product EGM_GVC_2) where the vector \mathbf{a} contain the evaluation functional for each coefficient associated with gravity anomalies in the point P where the geoid height error covariance is computed, i.e.

$$\sigma_x^2 = \mathbf{a}^T \mathbf{E}_{\hat{x}} \mathbf{a}.$$

where $\mathbf{E}_{\hat{x}}$ is the variance –covariance matrix containing and \mathbf{a} is

$$\mathbf{a}_j = \left\{ \frac{GM}{r} \left(\frac{a}{r} \right)^n \frac{(n-1)}{r} \begin{pmatrix} \cos(m\lambda) \\ \sin(m\lambda) \end{pmatrix} P_{nm}(\cos\theta) \right\}_{j=\begin{pmatrix} n^2+2m \\ n^2+2m+1 \end{pmatrix}}$$

The geoid errors are calculated through use of this and given in a grid file.

Input:

- The variance-covariance matrix of the spherical harmonic coefficients.
- The specifications for the reference ellipsoid, this input can be in several forms. The choices are
 - GM, a, J_2 , ω
 - GM, a, f, ω
 - γ_a , a, f, ω
- Maximum harmonic degree and order, N_{\max}
- Altitude of the grid points. This altitude should be above the terrain.
- Wanted grid specifications:
 - Starting latitude, ϕ_1
 - Ending latitude, ϕ_2
 - Starting longitude, λ_1
 - Ending longitude, λ_2
 - Grid spacing in latitude direction, $\Delta\phi$
 - Grid spacing in longitudinal direction, $\Delta\lambda$

Output:

- A sub-grid of the input grid defined by the grid specifications. This grid will be in gravsoft grid format. This format contains a line at the start of the file containing in order and in decimal degree notation
 - Starting latitude, ϕ_1
 - Ending latitude, ϕ_2

- Starting longitude, λ_1
- Ending longitude, λ_2
- Grid spacing in latitude direction, $\Delta\phi$
- Grid spacing in longitudinal direction, $\Delta\lambda$

GUT_019: Gravity anomaly errors in points.

The error variances are found using covariance propagation on the coefficient variance-covariance matrix associated with the spherical harmonic coefficients (GOCE Level 2 product EGM_GVC_2) where the vector \mathbf{a} contain the evaluation functional for each coefficient associated with gravity anomalies in the point P where the geoid height error covariance is computed, i.e.

$$\sigma_x^2 = \mathbf{a}^T \mathbf{E}_x \mathbf{a} .$$

where \mathbf{E}_x is the variance –covariance matrix containing and \mathbf{a} is

$$\mathbf{a}_j = \left\{ \frac{GM}{r} \left(\frac{a}{r} \right)^n \frac{(n-1)}{r} \begin{pmatrix} \cos(m\lambda) \\ \sin(m\lambda) \end{pmatrix} P_{nm}(\cos\theta) \right\}_{j=\begin{pmatrix} n^2+2m \\ n^2+2m+1 \end{pmatrix}}$$

The geoid errors are calculated through use of this and given in a grid file.

Input:

- The variance-covariance matrix of the spherical harmonic coefficients.
- The specifications for the reference ellipsoid, this input can be in several forms. The choices are
 - GM, a, J_2 , ω
 - GM, a, f, ω
 - γ_a , a, f, ω
- Maximum harmonic degree and order, N_{\max}
- Name of input point file
- Name of output file

Output:

- Values in points in the output file in gravsoft point list format.

GUT_020: E-W deflection errors in grid format.

The error variances are found using covariance propagation on the coefficient variance-covariance matrix associated with the spherical harmonic coefficients (GOCE Level 2 product EGM_GVC_2) where the vector \mathbf{a} contain the evaluation functional for each coefficient associated with E-W deflections in the point P where the geoid height error covariance is computed, i.e.

$$\sigma_x^2 = \mathbf{a}^T \mathbf{E}_{\hat{\mathbf{x}}} \mathbf{a} .$$

where $\mathbf{E}_{\hat{\mathbf{x}}}$ is the variance –covariance matrix containing and \mathbf{a} is

$$\mathbf{a}_j = \left\{ \frac{GM}{r\gamma_Q a \sin(\theta)} \left(\frac{a}{r}\right)^n \begin{pmatrix} -m \sin(m\lambda) \\ m \cos(m\lambda) \end{pmatrix} P_{nm}(\cos \theta) \right\}_{j=\begin{pmatrix} n^2+2m \\ n^2+2m+1 \end{pmatrix}}$$

The geoid errors are calculated through use of this and given in a grid file.

Input:

- The variance-covariance matrix of the spherical harmonic coefficients.
- The specifications for the reference ellipsoid, this input can be in several forms. The choices are
 - GM, a, J_2 , ω
 - GM, a, f, ω
 - γ_a , a, f, ω
- Maximum harmonic degree and order, N_{\max}
- Altitude of the grid points. This altitude should be above the terrain.
- Wanted grid specifications:
 - Starting latitude, ϕ_1
 - Ending latitude, ϕ_2
 - Starting longitude, λ_1
 - Ending longitude, λ_2
 - Grid spacing in latitude direction, $\Delta\phi$
 - Grid spacing in longitudinal direction, $\Delta\lambda$

Output:

- A sub-grid of the input grid defined by the grid specifications. This grid will be in gravsoft grid format. This format contains a line at the start of the file containing in order and in decimal degree notation
 - Starting latitude, ϕ_1
 - Ending latitude, ϕ_2

- Starting longitude, λ_1
- Ending longitude, λ_2
- Grid spacing in latitude direction, $\Delta\varphi$
- Grid spacing in longitudinal direction, $\Delta\lambda$

GUT_021: E-W deflection errors in points.

The error variances are found using covariance propagation on the coefficient variance-covariance matrix associated with the spherical harmonic coefficients (GOCE Level 2 product EGM_GVC_2) where the vector \mathbf{a} contain the evaluation functional for each coefficient associated with E-W deflections in the point P where the geoid height error covariance is computed, i.e.

$$\sigma_x^2 = \mathbf{a}^T \mathbf{E}_x \mathbf{a} .$$

where E_x is the variance –covariance matrix containing and \mathbf{a} is

$$\mathbf{a}_j = \left\{ \frac{GM}{r \gamma_Q a \sin(\theta)} \left(\frac{a}{r} \right)^n \begin{pmatrix} -m \sin(m\lambda) \\ m \cos(m\lambda) \end{pmatrix} P_{nm}(\cos \theta) \right\}_{j=\begin{pmatrix} n^2+2m \\ n^2+2m+1 \end{pmatrix}}$$

The geoid errors are calculated through use of this and given in a grid file.

Input:

- The variance-covariance matrix of the spherical harmonic coefficients.
- The specifications for the reference ellipsoid, this input can be in several forms. The choices are
 - GM, a, J_2 , ω
 - GM, a, f, ω
 - γ_a , a, f, ω
- Maximum harmonic degree and order, N_{\max}
- Name of input point file
- Name of output file

Output:

- Values in points in the output file in gravsoft point list format.

GUT_022: N-S deflection errors in grid format.

The error variances are found using covariance propagation on the coefficient variance-covariance matrix associated with the spherical harmonic coefficients (GOCE Level 2 product EGM_GVC_2) where the vector \mathbf{a} contain the evaluation functional for each coefficient associated with N-S deflections in the point P where the geoid height error covariance is computed, i.e.

$$\sigma_x^2 = \mathbf{a}^T \mathbf{E}_{\hat{\mathbf{x}}} \mathbf{a} .$$

where $\mathbf{E}_{\hat{\mathbf{x}}}$ is the variance –covariance matrix containing and \mathbf{a} is

$$\mathbf{a}_j = \left\{ \frac{GM}{r\gamma_Q a} \left(\frac{a}{r} \right)^n \begin{pmatrix} \cos(m\lambda) \\ \sin(m\lambda) \end{pmatrix} \frac{\partial P_{nm}(\cos\theta)}{\partial \theta} \right\}_{j=\begin{pmatrix} n^2+2m \\ n^2+2m+1 \end{pmatrix}}$$

The geoid errors are calculated through use of this and given in a grid file.

Input:

- The variance-covariance matrix of the spherical harmonic coefficients.
- The specifications for the reference ellipsoid, this input can be in several forms. The choices are
 - GM, a, J_2 , ω
 - GM, a, f, ω
 - γ_a , a, f, ω
- Maximum harmonic degree and order, N_{\max}
- Altitude of the grid points. This altitude should be above the terrain.
- Wanted grid specifications:
 - Starting latitude, φ_1
 - Ending latitude, φ_2
 - Starting longitude, λ_1
 - Ending longitude, λ_2
 - Grid spacing in latitude direction, $\Delta\varphi$
 - Grid spacing in longitudinal direction, $\Delta\lambda$

Output:

- A sub-grid of the input grid defined by the grid specifications. This grid will be in gravsoft grid format. This format contains a line at the start of the file containing in order and in decimal degree notation
 - Starting latitude, φ_1
 - Ending latitude, φ_2

- Starting longitude, λ_1
- Ending longitude, λ_2
- Grid spacing in latitude direction, $\Delta\varphi$
- Grid spacing in longitudinal direction, $\Delta\lambda$

GUT_023: N-S deflection errors in points.

The error variances are found using covariance propagation on the coefficient variance-covariance matrix associated with the spherical harmonic coefficients (GOCE Level 2 product EGM_GVC_2) where the vector \mathbf{a} contain the evaluation functional for each coefficient associated with N-S deflections in the point P where the geoid height error covariance is computed, i.e.

$$\sigma_x^2 = \mathbf{a}^T \mathbf{E}_{\hat{x}} \mathbf{a}.$$

where E_x is the variance –covariance matrix containing and \mathbf{a} is

$$\mathbf{a}_j = \left\{ \frac{GM}{r\gamma_Q a} \left(\frac{a}{r} \right)^n \begin{pmatrix} \cos(m\lambda) \\ \sin(m\lambda) \end{pmatrix} \frac{\partial P_{nm}(\cos\theta)}{\partial \theta} \right\}_{j=\begin{pmatrix} n^2+2m \\ n^2+2m+1 \end{pmatrix}}$$

The geoid errors are calculated through use of this and given in a grid file.

Input:

- The variance-covariance matrix of the spherical harmonic coefficients.
- The specifications for the reference ellipsoid, this input can be in several forms. The choices are
 - GM, a, J_2 , ω
 - GM, a, f, ω
 - γ_a , a, f, ω
- Maximum harmonic degree and order, N_{\max}
- Name of input point file
- Name of output file

Output:

- Values in points in the output file in gravsoft point list format.

GUT_024: Geoid height error covariances on a grid.

The error covariances are found using covariance propagation on the coefficient variance-covariance matrix associated with the spherical harmonic coefficients (GOCE Level 2 product EGM_GVC_2)

$$\sigma_x^2 = \mathbf{a}^T \mathbf{E}_x \mathbf{b}$$

where the vectors \mathbf{a} and \mathbf{b} contain the evaluation functional for each coefficient associated with geoid heights in the points P and Q respectively where the geoid height error covariance is computed between, i.e.

$$\left\{ \frac{GM}{r\gamma_Q} \left(\frac{a}{r} \right)^n \begin{pmatrix} \cos(m\lambda) \\ \sin(m\lambda) \end{pmatrix} P_{nm}(\cos\theta) \right\}_{j=\begin{pmatrix} n^2+2m \\ n^2+2m+1 \end{pmatrix}}$$

The error covariances are calculated between points given in a grid file. This gives a 4-D output array and should not be attempted with big arrays as the calculation time would be enormous.

Input:

- The variance-covariance matrix of the spherical harmonic coefficients.
- The specifications for the reference ellipsoid, this input can be in several forms. The choices are
 - GM, a, J_2 , ω
 - GM, a, f, ω
 - γ_a , a, f, ω
- Maximum harmonic degree and order, N_{\max}
- Altitude of the grid points. This altitude should be above the terrain.
- Wanted grid specifications:
 - Starting latitude, φ_1
 - Ending latitude, φ_2
 - Starting longitude, λ_1
 - Ending longitude, λ_2
 - Grid spacing in latitude direction, $\Delta\varphi$
 - Grid spacing in longitudinal direction, $\Delta\lambda$

Output:

- The output will be in the form of a 4-D array with two of the axes defined by the latitude limits in the input grid specification and two of the axes defined by the longitude limits in the input grid specification. Thus the header will be identical to

the header from the Gravsoft grid format, but the array will be defined as $F(\varphi_{11}, \varphi_{12}, \varphi_{21}, \varphi_{22}, \lambda_{11}, \lambda_{12}, \lambda_{21}, \lambda_{22})$ with

- Starting latitude axis 1, φ_{11}
- Ending latitude axis 1, φ_{12}
- Starting latitude axis 2, φ_{21}
- Ending latitude axis 2, φ_{22}
- Starting longitude axis 3, λ_{11}
- Ending longitude axis 3, λ_{12}
- Starting longitude axis 3, λ_{21}
- Ending longitude axis 3, λ_{22}
- Grid spacing in latitude direction, $\Delta\varphi$
- Grid spacing in longitudinal direction, $\Delta\lambda$

GUT_025: Geoid height error covariances in points.

The error covariances are found using covariance propagation on the coefficient variance-covariance matrix associated with the spherical harmonic coefficients (GOCE Level 2 product EGM_GVC_2)

$$\sigma_x^2 = \mathbf{a}^T \mathbf{E}_x \mathbf{b}$$

where the vectors \mathbf{a} and \mathbf{b} contain the evaluation functional for each coefficient associated with geoid heights in the points P and Q respectively where the geoid height error covariance is computed between, i.e.

$$\left\{ \frac{GM}{r\gamma_Q} \left(\frac{a}{r} \right)^n \begin{pmatrix} \cos(m\lambda) \\ \sin(m\lambda) \end{pmatrix} P_{nm}(\cos\theta) \right\}_{j=\begin{pmatrix} n^2+2m \\ n^2+2m+1 \end{pmatrix}}$$

The error covariances are calculated between two points.

Input:

- The variance-covariance matrix of the spherical harmonic coefficients.
- The specifications for the reference ellipsoid, this input can be in several forms. The choices are
 - GM, a, J_2 , ω
 - GM, a, f, ω
 - γ_a , a, f, ω
- Maximum harmonic degree and order, N_{\max}
- Specification of the two points for which the covariance is to be calculated.

Output:

- The covariance of the two input points.

GUT_026: Gravity anomaly error covariances on a grid.

The error covariances are found using covariance propagation on the coefficient variance-covariance matrix associated with the spherical harmonic coefficients (GOCE Level 2 product EGM_GVC_2)

$$\sigma_x^2 = \mathbf{a}^T \mathbf{E}_x \mathbf{b}$$

where the vectors \mathbf{a} and \mathbf{b} contain the evaluation functional for each coefficient associated with gravity anomalies in the points P and Q respectively where the error covariance is computed between, i.e.

$$\left\{ \frac{GM}{r} \left(\frac{a}{r} \right)^n \frac{(n-1)}{r} \begin{pmatrix} \cos(m\lambda) \\ \sin(m\lambda) \end{pmatrix} P_{nm}(\cos\theta) \right\}_{j=\begin{pmatrix} n^2+2m \\ n^2+2m+1 \end{pmatrix}}$$

The error covariances are calculated between points given in a grid file. This gives a 4-D output array and should not be attempted with big arrays as the calculation time would be enormous.

Input:

- The variance-covariance matrix of the spherical harmonic coefficients.
- The specifications for the reference ellipsoid, this input can be in several forms. The choices are
 - GM, a, J_2 , ω
 - GM, a, f, ω
 - γ_a , a, f, ω
- Maximum harmonic degree and order, N_{\max}
- Altitude of the grid points. This altitude should be above the terrain.
- Wanted grid specifications:
 - Starting latitude, φ_1
 - Ending latitude, φ_2
 - Starting longitude, λ_1
 - Ending longitude, λ_2
 - Grid spacing in latitude direction, $\Delta\varphi$
 - Grid spacing in longitudinal direction, $\Delta\lambda$

Output:

- The output will be in the form of a 4-D array with two of the axes defined by the latitude limits in the input grid specification and two of the axes defined by the longitude limits in the input grid specification. Thus the header will be identical to

the header from the Gravsoft grid format, but the array will be defined as $F(\varphi_{11}, \varphi_{12}, \varphi_{21}, \varphi_{22}, \lambda_{11}, \lambda_{12}, \lambda_{21}, \lambda_{22})$ with

- Starting latitude axis 1, φ_{11}
- Ending latitude axis 1, φ_{12}
- Starting latitude axis 2, φ_{21}
- Ending latitude axis 2, φ_{22}
- Starting longitude axis 3, λ_{11}
- Ending longitude axis 3, λ_{12}
- Starting longitude axis 3, λ_{21}
- Ending longitude axis 3, λ_{22}
- Grid spacing in latitude direction, $\Delta\varphi$
- Grid spacing in longitudinal direction, $\Delta\lambda$

GUT_027: Gravity anomaly error covariances in points.

The error covariances are found using covariance propagation on the coefficient variance-covariance matrix associated with the spherical harmonic coefficients (GOCE Level 2 product EGM_GVC_2)

$$\sigma_x^2 = \mathbf{a}^T \mathbf{E}_{\hat{x}} \mathbf{b}$$

where the vectors \mathbf{a} and \mathbf{b} contain the evaluation functional for each coefficient associated with gravity anomalies in the points P and Q respectively where the error covariance is computed between, i.e.

$$\left\{ \frac{GM}{r} \left(\frac{a}{r} \right)^n \frac{(n-1)}{r} \begin{pmatrix} \cos(m\lambda) \\ \sin(m\lambda) \end{pmatrix} P_{nm}(\cos\theta) \right\}_{j=\begin{pmatrix} n^2+2m \\ n^2+2m+1 \end{pmatrix}}$$

The error covariances are calculated between two points given in a point file.

Input:

- The variance-covariance matrix of the spherical harmonic coefficients.
- The specifications for the reference ellipsoid, this input can be in several forms. The choices are
 - GM, a, J_2 , ω
 - GM, a, f, ω
 - γ_a , a, f, ω
- Maximum harmonic degree and order, N_{\max}
- Specification of the two points for which the covariance is to be calculated.

Output:

- The covariance of the two input points.

GUT_028: E-W deflection error covariances on a in grid.

The error covariances are found using covariance propagation on the coefficient variance-covariance matrix associated with the spherical harmonic coefficients (GOCE Level 2 product EGM_GVC_2)

$$\sigma_x^2 = \mathbf{a}^T \mathbf{E}_x \mathbf{b}$$

where the vectors \mathbf{a} and \mathbf{b} contain the evaluation functional for each coefficient associated with E-W deflections in the points P and Q respectively where the error covariance is computed between, i.e.

$$\left\{ \frac{GM}{r\gamma_Q a \sin(\theta)} \left(\frac{a}{r}\right)^n \begin{pmatrix} -m \sin(m\lambda) \\ m \cos(m\lambda) \end{pmatrix} P_{nm}(\cos\theta) \right\}_{j=\begin{pmatrix} n^2+2m \\ n^2+2m+1 \end{pmatrix}}$$

The error covariances are calculated between points given in a grid file. This gives a 4-D output array and should not be attempted with big arrays as the calculation time would be enormous.

Input:

- The variance-covariance matrix of the spherical harmonic coefficients.
- The specifications for the reference ellipsoid, this input can be in several forms. The choices are
 - GM, a, J_2 , ω
 - GM, a, f, ω
 - γ_a , a, f, ω
- Maximum harmonic degree and order, N_{\max}
- Altitude of the grid points. This altitude should be above the terrain.
- Wanted grid specifications:
 - Starting latitude, φ_1
 - Ending latitude, φ_2
 - Starting longitude, λ_1
 - Ending longitude, λ_2
 - Grid spacing in latitude direction, $\Delta\varphi$
 - Grid spacing in longitudinal direction, $\Delta\lambda$

Output:

- The output will be in the form of a 4-D array with two of the axes defined by the latitude limits in the input grid specification and two of the axes defined by the longitude limits in the input grid specification. Thus the header will be identical to

the header from the Gravsoft grid format, but the array will be defined as $F(\varphi_{11}, \varphi_{12}, \varphi_{21}, \varphi_{22}, \lambda_{11}, \lambda_{12}, \lambda_{21}, \lambda_{22})$ with

- Starting latitude axis 1, φ_{11}
- Ending latitude axis 1, φ_{12}
- Starting latitude axis 2, φ_{21}
- Ending latitude axis 2, φ_{22}
- Starting longitude axis 3, λ_{11}
- Ending longitude axis 3, λ_{12}
- Starting longitude axis 4, λ_{21}
- Ending longitude axis 4, λ_{22}
- Grid spacing in latitude direction, $\Delta\varphi$
- Grid spacing in longitudinal direction, $\Delta\lambda$

GUT_029: E-W deflection error covariances in points.

The error covariances are found using covariance propagation on the coefficient variance-covariance matrix associated with the spherical harmonic coefficients (GOCE Level 2 product EGM_GVC_2)

$$\sigma_x^2 = \mathbf{a}^T \mathbf{E}_{\hat{\mathbf{x}}} \mathbf{b}$$

where the vectors \mathbf{a} and \mathbf{b} contain the evaluation functional for each coefficient associated with E-W deflections in the points P and Q respectively where the error covariance is computed between, i.e.

$$\left\{ \frac{GM}{r\gamma_Q a \sin(\theta)} \left(\frac{a}{r}\right)^n \begin{pmatrix} -m \sin(m\lambda) \\ m \cos(m\lambda) \end{pmatrix} P_{nm}(\cos\theta) \right\}_{j=\begin{pmatrix} n^2+2m \\ n^2+2m+1 \end{pmatrix}}$$

The error covariances are calculated between two points given in point file.

Input:

- The variance-covariance matrix of the spherical harmonic coefficients.
- The specifications for the reference ellipsoid, this input can be in several forms. The choices are
 - GM, a, J_2 , ω
 - GM, a, f, ω
 - γ_a , a, f, ω
- Maximum harmonic degree and order, N_{\max}
- Specification of the two points for which the covariance is to be calculated.

Output:

- The covariance of the two input points.

GUT_030: N-S deflection error covariances on a grid.

The error covariances are found using covariance propagation on the coefficient variance-covariance matrix associated with the spherical harmonic coefficients (GOCE Level 2 product EGM_GVC_2)

$$\sigma_x^2 = \mathbf{a}^T \mathbf{E}_x \mathbf{b}$$

where the vectors \mathbf{a} and \mathbf{b} contain the evaluation functional for each coefficient associated with N-S deflections in the points P and Q respectively where the error covariance is computed between, i.e.

$$\left\{ \frac{GM}{r\gamma_Q a} \left(\frac{a}{r} \right)^n \begin{pmatrix} \cos(m\lambda) \\ \sin(m\lambda) \end{pmatrix} \frac{\partial P_{nm}(\cos\theta)}{\partial \theta} \right\}_{j=\begin{pmatrix} n^2+2m \\ n^2+2m+1 \end{pmatrix}}$$

The error covariances are calculated between points given in a grid file. This gives a 4-D output array and should not be attempted with big arrays as the calculation time would be enormous.

Input:

- The variance-covariance matrix of the spherical harmonic coefficients.
- The specifications for the reference ellipsoid, this input can be in several forms. The choices are
 - GM, a, J₂, ω
 - GM, a, f, ω
 - γ_a, a, f, ω
- Maximum harmonic degree and order, N_{max}
- Altitude of the grid points. This altitude should be above the terrain.
- Wanted grid specifications:
 - Starting latitude, φ₁
 - Ending latitude, φ₂
 - Starting longitude, λ₁
 - Ending longitude, λ₂
 - Grid spacing in latitude direction, Δφ
 - Grid spacing in longitudinal direction, Δλ

Output:

- The output will be in the form of a 4-D array with two of the axes defined by the latitude limits in the input grid specification and two of the axes defined by the longitude limits in the input grid specification. Thus the header will be identical to

the header from the Gravsoft grid format, but the array will be defined as $F(\varphi_{11}, \varphi_{12}, \varphi_{21}, \varphi_{22}, \lambda_{11}, \lambda_{12}, \lambda_{21}, \lambda_{22})$ with

- Starting latitude axis 1, φ_{11}
- Ending latitude axis 1, φ_{12}
- Starting latitude axis 2, φ_{21}
- Ending latitude axis 2, φ_{22}
- Starting longitude axis 3, λ_{11}
- Ending longitude axis 3, λ_{12}
- Starting longitude axis 4, λ_{21}
- Ending longitude axis 4, λ_{22}
- Grid spacing in latitude direction, $\Delta\varphi$
- Grid spacing in longitudinal direction, $\Delta\lambda$

GUT_031: N-S deflection error covariances in points.

The error covariances are found using covariance propagation on the coefficient variance-covariance matrix associated with the spherical harmonic coefficients (GOCE Level 2 product EGM_GVC_2)

$$\sigma_x^2 = \mathbf{a}^T \mathbf{E}_{\hat{x}} \mathbf{b}$$

where the vectors \mathbf{a} and \mathbf{b} contain the evaluation functional for each coefficient associated with N-S deflections in the points P and Q respectively where the error covariance is computed between, i.e.

$$\left\{ \frac{GM}{r\gamma_Q a} \left(\frac{a}{r}\right)^n \begin{pmatrix} \cos(m\lambda) \\ \sin(m\lambda) \end{pmatrix} \frac{\partial P_{nm}(\cos\theta)}{\partial\theta} \right\}_{j=\begin{pmatrix} n^2+2m \\ n^2+2m+1 \end{pmatrix}}$$

The error covariances are calculated between two points given in a point file.

Input:

- The variance-covariance matrix of the spherical harmonic coefficients.
- The specifications for the reference ellipsoid, this input can be in several forms. The choices are
 - GM, a, J_2 , ω
 - GM, a, f, ω
 - γ_a , a, f, ω
- Maximum harmonic degree and order, N_{\max}
- Specification of the two points for which the covariance is to be calculated.

Output:

- The covariance of the two input points.

GUT_032: Geoid height omission error covariances on a grid.

Using spherical harmonic functions, the omission error covariances associated with the gravity field between points P and Q may be expressed as a sum of Legendre's polynomials multiplied by degree variances as

$$K(P, Q) = \sum_{i = N_{\max} + 1}^{\infty} \sigma_i^{TT} P_i(\cos \psi)$$

where σ_i^{TT} are degree variances associated with the anomalous gravity potential field and ψ is the spherical distance between P and Q. Expressions associated with geoid heights and gravity anomalies are obtained by applying the respective functionals on $K(P, Q)$, e.g. $C_{MN} = L_N(L_M(K(P, Q)))$. These functionals are applied to get e.g. from the covariance of the potential to the covariance of the geoid. The functional is the same that generates the transition from the spherical harmonic sum of the potential to the spherical harmonic sum of the geoid.

The degree variances are expressed as

$$\sigma_i^{TT} = \frac{A}{(i-1)(i-2)(i+4)} \left(\frac{R_B^2}{R^2} \right)^{i+1}$$

where $A = 1544850 \text{ m}^4/\text{s}^4$, $R_B = R - 6.823 \text{ m}$.

The omission error covariances are calculated between points given in a grid file. This gives a 4-D output array and should not be attempted with big arrays as the calculation time would be enormous.

Input:

- The variance-covariance matrix of the spherical harmonic coefficients.
- The specifications for the reference ellipsoid, this input can be in several forms. The choices are
 - GM, a, J_2 , ω
 - GM, a, f, ω
 - γ_a , a, f, ω
- Maximum harmonic degree and order, N_{\max}
- Altitude of the grid points. This altitude should be above the terrain.
- Wanted grid specifications:
 - Starting latitude, φ_1
 - Ending latitude, φ_2
 - Starting longitude, λ_1
 - Ending longitude, λ_2
 - Grid spacing in latitude direction, $\Delta\varphi$
 - Grid spacing in longitudinal direction, $\Delta\lambda$

Output:

- The output will be in the form of a 4-D array with two of the axes defined by the latitude limits in the input grid specification and two of the axes defined by the longitude limits in the input grid specification. Thus the header will be identical to the header from the Gravsoft grid format, but the array will be defined as $F(\varphi_{11}, \varphi_{12}, \varphi_{21}, \varphi_{22}, \lambda_{11}, \lambda_{12}, \lambda_{21}, \lambda_{22})$ with
 - Starting latitude axis 1, φ_{11}
 - Ending latitude axis 1, φ_{12}
 - Starting latitude axis 2, φ_{21}
 - Ending latitude axis 2, φ_{22}
 - Starting longitude axis 3, λ_{11}
 - Ending longitude axis 3, λ_{12}
 - Starting longitude axis 4, λ_{21}
 - Ending longitude axis 4, λ_{22}
 - Grid spacing in latitude direction, $\Delta\varphi$
 - Grid spacing in longitudinal direction, $\Delta\lambda$

GUT_033: Geoid height omission error covariances in points.

Using spherical harmonic functions, the omission error covariances associated with the gravity field between points P and Q may be expressed as a sum of Legendre's polynomials multiplied by degree variances as

$$K(P, Q) = \sum_{i = N_{\max} + 1}^{\infty} \sigma_i^{TT} P_i(\cos \psi)$$

where σ_i^{TT} are degree variances associated with the anomalous gravity potential field and ψ is the spherical distance between P and Q. Expressions associated with geoid heights and gravity anomalies are obtained by applying the respective functionals on $K(P, Q)$, e.g. $C_{MN} = L_N(L_M(K(P, Q)))$. These functionals are applied to get e.g. from the covariance of the potential to the covariance of the geoid. The functional is the same that generates the transition from the spherical harmonic sum of the potential to the spherical harmonic sum of the geoid.

The degree variances are expressed as

$$\sigma_i^{TT} = \frac{A}{(i-1)(i-2)(i+4)} \left(\frac{R_B^2}{R^2} \right)^{i+1}$$

where $A = 1544850 \text{ m}^4/\text{s}^4$, $R_B = R - 6.823 \text{ m}$.

The omission error covariances are calculated between two points given in a point file.

Input:

- Parameters A , R_B and R .
- Maximum harmonic degree and order, N_{\max}
- Specification of the two points for which the covariance is to be calculated.

Output:

- The covariance of the two input points.

GUT_034: Gravity anomaly omission error covariances on a grid.

Using spherical harmonic functions, the omission error covariances associated with the gravity field between points P and Q may be expressed as a sum of Legendre's polynomials multiplied by degree variances as

$$K(P, Q) = \sum_{i = N_{\max} + 1}^{\infty} \sigma_i^{TT} P_i(\cos \psi)$$

where σ_i^{TT} are degree variances associated with the anomalous gravity potential field and ψ is the spherical distance between P and Q. Expressions associated with geoid heights and gravity anomalies are obtained by applying the respective functionals on $K(P, Q)$, e.g. $C_{MN} = L_N(L_M(K(P, Q)))$. These functionals are applied to get e.g. from the covariance of the potential to the covariance of the geoid. The functional is the same that generates the transition from the spherical harmonic sum of the potential to the spherical harmonic sum of the geoid.

The degree variances are expressed as

$$\sigma_i^{TT} = \frac{A}{(i-1)(i-2)(i+4)} \left(\frac{R_B}{R} \right)^{i+1}$$

where $A = 1544850 \text{ m}^4/\text{s}^4$, $R_B = R - 6.823 \text{ m}$.

The omission error covariances are calculated between points given in a grid file. This gives a 4-D output array and should not be attempted with big arrays as the calculation time would be enormous.

Input:

- Parameters A , R_B and R .
- Maximum harmonic degree and order, N_{\max}
- Altitude of the grid points. This altitude should be above the terrain.
- Wanted grid specifications:
 - Starting latitude, φ_1
 - Ending latitude, φ_2
 - Starting longitude, λ_1
 - Ending longitude, λ_2
 - Grid spacing in latitude direction, $\Delta\varphi$
 - Grid spacing in longitudinal direction, $\Delta\lambda$

Output:

- The output will be in the form of a 4-D array with two of the axes defined by the latitude limits in the input grid specification and two of the axes defined by the longitude limits in the input grid specification. Thus the header will be identical to

the header from the Gravsoft grid format, but the array will be defined as $F(\varphi_{11}, \varphi_{12}, \varphi_{21}, \varphi_{22}, \lambda_{11}, \lambda_{12}, \lambda_{21}, \lambda_{22})$ with

- Starting latitude axis 1, φ_{11}
- Ending latitude axis 1, φ_{12}
- Starting latitude axis 2, φ_{21}
- Ending latitude axis 2, φ_{22}
- Starting longitude axis 3, λ_{11}
- Ending longitude axis 3, λ_{12}
- Starting longitude axis 4, λ_{21}
- Ending longitude axis 4, λ_{22}
- Grid spacing in latitude direction, $\Delta\varphi$
- Grid spacing in longitudinal direction, $\Delta\lambda$

GUT_035: Gravity anomaly omission error covariances in points.

Using spherical harmonic functions, the omission error covariances associated with the gravity field between points P and Q may be expressed as a sum of Legendre's polynomials multiplied by degree variances as

$$K(P, Q) = \sum_{i = N_{\max} + 1}^{\infty} \sigma_i^{TT} P_i(\cos \psi)$$

where σ_i^{TT} are degree variances associated with the anomalous gravity potential field and ψ is the spherical distance between P and Q. Expressions associated with geoid heights and gravity anomalies are obtained by applying the respective functionals on $K(P, Q)$, e.g. $C_{MN} = L_N(L_M(K(P, Q)))$. These functionals are applied to get e.g. from the covariance of the potential to the covariance of the geoid. The functional is the same that generates the transition from the spherical harmonic sum of the potential to the spherical harmonic sum of the geoid.

The degree variances are expressed as

$$\sigma_i^{TT} = \frac{A}{(i-1)(i-2)(i+4)} \left(\frac{R_B^2}{R^2} \right)^{i+1}$$

where $A = 1544850 \text{ m}^4/\text{s}^4$, $R_B = R - 6.823 \text{ m}$.

The omission error covariances are calculated between two points given in a point file.

Input:

- Parameters A , R_B and R .
- Maximum harmonic degree and order, N_{\max}
- Specification of the two points for which the covariance is to be calculated.

Output:

- The covariance of the two input points.

GUT_036: E-W deflection omission error covariances on a grid.

Using spherical harmonic functions, the omission error covariances associated with the gravity field between points P and Q may be expressed as a sum of Legendre's polynomials multiplied by degree variances as

$$K(P, Q) = \sum_{i = N_{\max} + 1}^{\infty} \sigma_i^{TT} P_i(\cos \psi)$$

where σ_i^{TT} are degree variances associated with the anomalous gravity potential field and ψ is the spherical distance between P and Q. Expressions associated with geoid heights and gravity anomalies are obtained by applying the respective functionals on $K(P, Q)$, e.g. $C_{MN} = L_N(L_M(K(P, Q)))$. These functionals are applied to get e.g. from the covariance of the potential to the covariance of the geoid. The functional is the same that generates the transition from the spherical harmonic sum of the potential to the spherical harmonic sum of the geoid.

The degree variances are expressed as

$$\sigma_i^{TT} = \frac{A}{(i-1)(i-2)(i+4)} \left(\frac{R_B}{R} \right)^{i+1}$$

where $A = 1544850 \text{ m}^4/\text{s}^4$, $R_B = R - 6.823 \text{ m}$.

The omission error covariances are calculated between points given in a grid file. This gives a 4-D output array and should not be attempted with big arrays as the calculation time would be enormous.

Input:

- Parameters A , R_B and R .
- Maximum harmonic degree and order, N_{\max}
- Altitude of the grid points. This altitude should be above the terrain.
- Wanted grid specifications:
 - Starting latitude, φ_1
 - Ending latitude, φ_2
 - Starting longitude, λ_1
 - Ending longitude, λ_2
 - Grid spacing in latitude direction, $\Delta\varphi$
 - Grid spacing in longitudinal direction, $\Delta\lambda$

Output:

- The output will be in the form of a 4-D array with two of the axes defined by the latitude limits in the input grid specification and two of the axes defined by the longitude limits in the input grid specification. Thus the header will be identical to

the header from the Gravsoft grid format, but the array will be defined as $F(\varphi_{11}, \varphi_{12}, \varphi_{21}, \varphi_{22}, \lambda_{11}, \lambda_{12}, \lambda_{21}, \lambda_{22})$ with

- Starting latitude axis 1, φ_{11}
- Ending latitude axis 1, φ_{12}
- Starting latitude axis 2, φ_{21}
- Ending latitude axis 2, φ_{22}
- Starting longitude axis 3, λ_{11}
- Ending longitude axis 3, λ_{12}
- Starting longitude axis 4, λ_{21}
- Ending longitude axis 4, λ_{22}
- Grid spacing in latitude direction, $\Delta\varphi$
- Grid spacing in longitudinal direction, $\Delta\lambda$

GUT_037: E-W deflection omission error covariances in points.

Using spherical harmonic functions, the omission error covariances associated with the gravity field between points P and Q may be expressed as a sum of Legendre's polynomials multiplied by degree variances as

$$K(P, Q) = \sum_{i = N_{\max} + 1}^{\infty} \sigma_i^{TT} P_i(\cos \psi)$$

where σ_i^{TT} are degree variances associated with the anomalous gravity potential field and ψ is the spherical distance between P and Q. Expressions associated with geoid heights and gravity anomalies are obtained by applying the respective functionals on $K(P, Q)$, e.g. $C_{MN} = L_N(L_M(K(P, Q)))$. These functionals are applied to get e.g. from the covariance of the potential to the covariance of the geoid. The functional is the same that generates the transition from the spherical harmonic sum of the potential to the spherical harmonic sum of the geoid.

The degree variances are expressed as

$$\sigma_i^{TT} = \frac{A}{(i-1)(i-2)(i+4)} \left(\frac{R_B^2}{R^2} \right)^{i+1}$$

where $A = 1544850 \text{ m}^4/\text{s}^4$, $R_B = R - 6.823 \text{ m}$.

The omission error covariances are calculated between two points given in point file.

Input:

- Parameters A , R_B and R .
- Maximum harmonic degree and order, N_{\max}
- Specification of the two points for which the covariance is to be calculated.

Output:

- The covariance of the two input points.

GUT_038: N-S deflection omission error covariances on a grid.

Using spherical harmonic functions, the omission error covariances associated with the gravity field between points P and Q may be expressed as a sum of Legendre's polynomials multiplied by degree variances as

$$K(P, Q) = \sum_{i = N_{\max} + 1}^{\infty} \sigma_i^{TT} P_i(\cos \psi)$$

where σ_i^{TT} are degree variances associated with the anomalous gravity potential field and ψ is the spherical distance between P and Q. Expressions associated with geoid heights and gravity anomalies are obtained by applying the respective functionals on $K(P, Q)$, e.g. $C_{MN} = L_N(L_M(K(P, Q)))$. These functionals are applied to get e.g. from the covariance of the potential to the covariance of the geoid. The functional is the same that generates the transition from the spherical harmonic sum of the potential to the spherical harmonic sum of the geoid.

The degree variances are expressed as

$$\sigma_i^{TT} = \frac{A}{(i-1)(i-2)(i+4)} \left(\frac{R_B}{R} \right)^{i+1}$$

where $A = 1544850 \text{ m}^4/\text{s}^4$, $R_B = R - 6.823 \text{ m}$.

The omission error covariances are calculated between points given in a grid file. This gives a 4-D output array and should not be attempted with big arrays as the calculation time would be enormous.

Input:

- Parameters A , R_B and R .
- Maximum harmonic degree and order, N_{\max}
- Altitude of the grid points. This altitude should be above the terrain.
- Wanted grid specifications:
 - Starting latitude, φ_1
 - Ending latitude, φ_2
 - Starting longitude, λ_1
 - Ending longitude, λ_2
 - Grid spacing in latitude direction, $\Delta\varphi$
 - Grid spacing in longitudinal direction, $\Delta\lambda$

Output:

- The output will be in the form of a 4-D array with two of the axes defined by the latitude limits in the input grid specification and two of the axes defined by the longitude limits in the input grid specification. Thus the header will be identical to

the header from the Gravsoft grid format, but the array will be defined as $F(\varphi_{11}, \varphi_{12}, \varphi_{21}, \varphi_{22}, \lambda_{11}, \lambda_{12}, \lambda_{21}, \lambda_{22})$ with

- Starting latitude axis 1, φ_{11}
- Ending latitude axis 1, φ_{12}
- Starting latitude axis 2, φ_{21}
- Ending latitude axis 2, φ_{22}
- Starting longitude axis 3, λ_{11}
- Ending longitude axis 3, λ_{12}
- Starting longitude axis 4, λ_{21}
- Ending longitude axis 4, λ_{22}
- Grid spacing in latitude direction, $\Delta\varphi$
- Grid spacing in longitudinal direction, $\Delta\lambda$

GUT_039: N-S deflection omission error covariances in points.

Using spherical harmonic functions, the omission error covariances associated with the gravity field between points P and Q may be expressed as a sum of Legendre's polynomials multiplied by degree variances as

$$K(P, Q) = \sum_{i = N_{\max} + 1}^{\infty} \sigma_i^{TT} P_i(\cos \psi)$$

where σ_i^{TT} are degree variances associated with the anomalous gravity potential field and ψ is the spherical distance between P and Q. Expressions associated with geoid heights and gravity anomalies are obtained by applying the respective functionals on $K(P, Q)$, e.g. $C_{MN} = L_N(L_M(K(P, Q)))$. These functionals are applied to get e.g. from the covariance of the potential to the covariance of the geoid. The functional is the same that generates the transition from the spherical harmonic sum of the potential to the spherical harmonic sum of the geoid.

The degree variances are expressed as

$$\sigma_i^{TT} = \frac{A}{(i-1)(i-2)(i+4)} \left(\frac{R_B^2}{R^2} \right)^{i+1}$$

where $A = 1544850 \text{ m}^4/\text{s}^4$, $R_B = R - 6.823 \text{ m}$.

The omission error covariances are calculated between two points given in a point file.

Input:

- Parameters A , R_B and R .
- Maximum harmonic degree and order, N_{\max}
- Specification of the two points for which the covariance is to be calculated.

Output:

- The covariance of the two input points.

GUT_040: MDT a-priori covariances on a grid.

Using spherical harmonic functions, the a-priori error covariances associated with the MDT between points P and Q may be expressed as a sum of Legendre's polynomials multiplied by degree variances as

$$C_{\zeta\zeta} = \sum_{i=N_{Min}}^{N_{Max}} \sigma_i^{\zeta\zeta} P_i(\cos\psi)$$

where the degree variances in this expression are associated with the MDT.

$$\sigma_i^{ss} = b \cdot \left(\frac{k_2^3}{k_2^3 + i^3} - \frac{k_1^3}{k_1^3 + i^3} \right) \cdot s^{i+1}$$

where b , k_1 , k_2 , and s are determined so that the variance and the correlation length agree with empirically derived characteristics, $b = 6.3 \cdot 10^{-4} \text{ m}^2$, $k_1 = 1$, $k_2 = 90$, $s = ((R - 5000.0)^2 / R^2)^2$.

The omission error covariances are calculated between points given in a grid file. This gives a 4-D output array and should not be attempted with big arrays as the calculation time would be enormous.

Input:

- Parameters b , k_1 , k_2 and s .
- Minimum and maximum harmonic degree and order, N_{min} and N_{max}
- Altitude of the grid points. This altitude should be above the terrain.
- Wanted grid specifications:
 - Starting latitude, φ_1
 - Ending latitude, φ_2
 - Starting longitude, λ_1
 - Ending longitude, λ_2
 - Grid spacing in latitude direction, $\Delta\varphi$
 - Grid spacing in longitudinal direction, $\Delta\lambda$

Output:

- The output will be in the form of a 4-D array with two of the axes defined by the latitude limits in the input grid specification and two of the axes defined by the longitude limits in the input grid specification. Thus the header will be identical to the header from the Gravsoft grid format, but the array will be defined as $F(\varphi_{11}, \varphi_{12}, \varphi_{21}, \varphi_{22}, \lambda_{11}, \lambda_{12}, \lambda_{21}, \lambda_{22})$ with
 - Starting latitude axis 1, φ_{11}
 - Ending latitude axis 1, φ_{12}
 - Starting latitude axis 2, φ_{21}
 - Ending latitude axis 2, φ_{22}

- Starting longitude axis 3, λ_{11}
- Ending longitude axis 3, λ_{12}
- Starting longitude axis 4, λ_{21}
- Ending longitude axis 4, λ_{22}
- Grid spacing in latitude direction, $\Delta\varphi$
- Grid spacing in longitudinal direction, $\Delta\lambda$

GUT_041: MDT a-priori covariances in points.

Using spherical harmonic functions, the a-priori error covariances associated with the MDT between points P and Q may be expressed as a sum of Legendre's polynomials multiplied by degree variances as

$$C_{\zeta\zeta} = \sum_{i=N_{Min}}^{N_{Max}} \sigma_i^{\zeta\zeta} P_i(\cos\psi)$$

where the degree variances in this expression are associated with the MDT.

$$\sigma_i^{ss} = b \cdot \left(\frac{k_2^3}{k_2^3 + i^3} - \frac{k_1^3}{k_1^3 + i^3} \right) \cdot s^{i+1}$$

where b , k_1 , k_2 , and s are determined so that the variance and the correlation length agree with empirically derived characteristics, $b = 6.3 \cdot 10^{-4} \text{ m}^2$, $k_1 = 1$, $k_2 = 90$, $s = ((R - 5000.0)^2 / R^2)^2$.

The omission error covariances are calculated between two points given in a point file.

Input:

- Parameters b , k_1 , k_2 and s .
- Minimum and maximum harmonic degree and order, N_{min} and N_{max}
- Specification of the two points for which the covariance is to be calculated.

Output:

- The covariance of the two input points.

GUT_042: MDT a-posteriori covariances on a grid.

Using spherical harmonic functions, the a-posteriori error covariances associated with the MDT between points P and Q may be expressed as a sum of Legendre's polynomials multiplied by degree variances as

$$C_{\zeta\zeta} = \sum_{i=N_{Min}}^{N_{Max}} \sigma_i P_i(\cos\psi)$$

where the a-posteriori degree variances in this expression are associated with the MDT have been estimated externally.

The omission error covariances are calculated between points given in a grid file. This gives a 4-D output array and should not be attempted with big arrays as the calculation time would be enormous.

Input:

- A-posteriori degree variances σ_i , as an array[$N_{min}:N_{max}$]
- Minimum and maximum harmonic degree and order, N_{min} and N_{max}
- Altitude of the grid points. This altitude should be above the terrain.
- Wanted grid specifications:
 - Starting latitude, φ_1
 - Ending latitude, φ_2
 - Starting longitude, λ_1
 - Ending longitude, λ_2
 - Grid spacing in latitude direction, $\Delta\varphi$
 - Grid spacing in longitudinal direction, $\Delta\lambda$

Output:

- The output will be in the form of a 4-D array with two of the axes defined by the latitude limits in the input grid specification and two of the axes defined by the longitude limits in the input grid specification. Thus the header will be identical to the header from the Gravsoft grid format, but the array will be defined as $F(\varphi_{11}, \varphi_{12}, \varphi_{21}, \varphi_{22}, \lambda_{11}, \lambda_{12}, \lambda_{21}, \lambda_{22})$ with
 - Starting latitude axis 1, φ_{11}
 - Ending latitude axis 1, φ_{12}
 - Starting latitude axis 2, φ_{21}
 - Ending latitude axis 2, φ_{22}
 - Starting longitude axis 3, λ_{11}
 - Ending longitude axis 3, λ_{12}
 - Starting longitude axis 4, λ_{21}
 - Ending longitude axis 4, λ_{22}
 - Grid spacing in latitude direction, $\Delta\varphi$

- Grid spacing in longitudinal direction, $\Delta\lambda$

GUT_043: MDT a-posteriori covariances in points.

Using spherical harmonic functions, the a-posteriori error covariances associated with the MDT between points P and Q may be expressed as a sum of Legendre's polynomials multiplied by degree variances as

$$C_{\zeta\zeta} = \sum_{i=N_{Min}}^{N_{Max}} \sigma_i P_i(\cos\psi)$$

where the a-posteriori degree variances in this expression are associated with the MDT have been estimated externally.

The omission error covariances are calculated between two points given in a point file.

Input:

- A-posteriori degree variances σ_i , as an array[$N_{min}:N_{max}$]
- Minimum and maximum harmonic degree and order, N_{min} and N_{max}
- Specification of the two points for which the covariance is to be calculated.

Output:

- The covariance of the two input points.

GUT_044: Geostrophic velocities covariances on a grid.

The covariance functions associated with the geostrophic velocities are given by the formulas

$$C_{uu} = \frac{\gamma^2}{f_P f_Q} \left(-\cos \alpha_{PQ} \cos \alpha_{QP} C_{ll} - \sin \alpha_{PQ} \sin \alpha_{QP} C_{qq} \right)$$

$$C_{vv} = \frac{\gamma^2}{f_P f_Q} \left(-\sin \alpha_{PQ} \sin \alpha_{QP} C_{ll} - \cos \alpha_{PQ} \cos \alpha_{QP} C_{qq} \right)$$

$$C_{u\zeta} = \frac{-\gamma}{f_P} \cos \alpha_{PQ} C_{l\zeta}$$

$$C_{v\zeta} = \frac{\gamma}{f_P} \sin \alpha_{PQ} C_{l\zeta}$$

$$C_{ll} = \frac{1}{R^2} \left(\cos \psi C'_{\zeta\zeta} - \sin^2 \psi C''_{\zeta\zeta} \right)$$

$$C_{qq} = \frac{1}{R^2} C'_{\zeta\zeta}$$

$$C_{l\zeta} = \frac{-1}{R} \sin \psi C'_{\zeta\zeta}$$

Where the α_{PQ} is the azimuth between the two points and ψ is the spherical distance between them.

The error covariances are calculated between points given in a grid file. This gives a 4-D output array and should not be attempted with big arrays as the calculation time would be enormous.

Input:

- Altitude of the grid points. This altitude should be above the terrain.
- Wanted grid specifications:
 - Starting latitude, φ_1
 - Ending latitude, φ_2
 - Starting longitude, λ_1
 - Ending longitude, λ_2
 - Grid spacing in latitude direction, $\Delta\varphi$
 - Grid spacing in longitudinal direction, $\Delta\lambda$

Output:

- The output will be in the form of a 4-D array with two of the axes defined by the latitude limits in the input grid specification and two of the axes defined by the longitude limits in the input grid specification. Thus the header will be identical to the header from the Gravsoft grid format, but the array will be defined as

$F(\varphi_{11}, \varphi_{12}, \varphi_{21}, \varphi_{22}, \lambda_{11}, \lambda_{12}, \lambda_{21}, \lambda_{22})$ with

- Starting latitude axis 1, φ_{11}
- Ending latitude axis 1, φ_{12}
- Starting latitude axis 2, φ_{21}
- Ending latitude axis 2, φ_{22}
- Starting longitude axis 3, λ_{11}
- Ending longitude axis 3, λ_{12}
- Starting longitude axis 4, λ_{21}
- Ending longitude axis 4, λ_{22}
- Grid spacing in latitude direction, $\Delta\varphi$
- Grid spacing in longitudinal direction, $\Delta\lambda$

GUT_045: Geostrophic velocities covariances in points

The covariance functions associated with the geostrophic velocities are given by the formulas

$$C_{uu} = \frac{\gamma^2}{f_P f_Q} \left(-\cos \alpha_{PQ} \cos \alpha_{QP} C_{ll} - \sin \alpha_{PQ} \sin \alpha_{QP} C_{qq} \right)$$

$$C_{vv} = \frac{\gamma^2}{f_P f_Q} \left(-\sin \alpha_{PQ} \sin \alpha_{QP} C_{ll} - \cos \alpha_{PQ} \cos \alpha_{QP} C_{qq} \right)$$

$$C_{u\zeta} = \frac{-\gamma}{f_P} \cos \alpha_{PQ} C_{l\zeta}$$

$$C_{v\zeta} = \frac{\gamma}{f_P} \sin \alpha_{PQ} C_{l\zeta}$$

$$C_{ll} = \frac{1}{R^2} \left(\cos \psi C'_{\zeta\zeta} - \sin^2 \psi C''_{\zeta\zeta} \right)$$

$$C_{qq} = \frac{1}{R^2} C'_{\zeta\zeta}$$

$$C_{l\zeta} = \frac{-1}{R} \sin \psi C'_{\zeta\zeta}$$

Where the α_{PQ} is the azimuth between the two points and ψ is the spherical distance between them. The error covariances are given between two specified points.

Input:

- Specification of the two points for which the covariance is to be calculated.

Output:

- The covariance of the two input points.

GUT_100: Grid addition.

Form sums of the values in two grids:

$$\text{Grid3} = \text{grid1} + \text{grid2}$$

Input:

- Name of input file of grid1 in gravsoft format
- Name of input file of grid2 in gravsoft format
- Name of output file

Output:

- Values of grid3 in points in the output file in gravsoft grid format.

GUT_101: Grid subtraction.

Form differences between values in two grids:

$$\text{Grid3} = \text{grid1} - \text{grid2}$$

Input:

- Name of input file of grid1 in gravsoft format
- Name of input file of grid2 in gravsoft format
- Name of output file

Output:

- Values of grid3 in points in the output file in gravsoft grid format.

GUT_102: Grid filtering.

The toolbox will include two filters, a standard box-car filter and a Gaussian filter.

The box-car filter takes an average of the values of the field inside a user defined square box. This means that the user specifies two variables, $\Delta\theta$ and $\Delta\lambda$, which defines the box size. If the point of calculation is called (θ, λ) this leads to the new value at this point given by

$$f(\theta, \lambda) = \frac{1}{n} \sum_{\theta_i, \lambda_i} f(\theta_i, \lambda_j) \quad \text{with}$$

$$\theta - \Delta\theta/2 < \theta_i < \theta + \Delta\theta/2, \quad \lambda - \Delta\lambda/2 < \lambda_i < \lambda + \Delta\lambda/2$$

where n is the number of points inside the box.

The Gaussian filter provides a weighted average where the user defines a radius, R , for the filter. Again the user defines a box size with $\Delta\theta$ and $\Delta\lambda$ and the point of calculation is (θ, λ) . This gives the equation

$$f(\theta, \lambda) = \frac{\sum_{\theta_i, \lambda_i} e^{r^2/2R^2} f(\theta_i, \lambda_j)}{\sum_{\theta_i, \lambda_i} e^{r^2/2R^2}} \quad \text{with}$$

$$\theta - \Delta\theta/2 < \theta_i < \theta + \Delta\theta/2, \quad \lambda - \Delta\lambda/2 < \lambda_i < \lambda + \Delta\lambda/2$$

$$r^2 = (\lambda - \lambda_j)^2 + (\theta - \theta_i)^2$$

Input:

- Name of input file of grid1 in gravsoft format
- Specification of boxcar or Gaussian filter.
- Filter parameters, $\Delta\theta$, $\Delta\lambda$ and R for the gaussian
- Name of output file

Output:

- Values of grid2 in points in the output file in gravsoft grid format.

GUT_103: Going from geographical space to spectral.

This functionality will allow the user to get the coefficients of a spherical harmonic expansion from a grid in geographical space. The formulas for this are built on the concept that the spherical harmonics form a complete set. What this means is that if the gridded field is given by $f(\theta, \lambda)$ then the formula to get the coefficient of degree n and order m is

$$\bar{a}_{nm} = \frac{1}{4\pi} \iint_{\sigma} f(\theta, \lambda) \bar{P}_{nm}(\cos \theta) \cos(m\lambda) d\sigma$$

$$\bar{b}_{nm} = \frac{1}{4\pi} \iint_{\sigma} f(\theta, \lambda) \bar{P}_{nm}(\cos \theta) \sin(m\lambda) d\sigma$$

This is the general solution that will not give any factors like those that are usually associated with, e.g. the expansion of the geoid.

For the above formulas to be useful, numerical integration needs to be used. If the field is defined on a grid (θ_i, λ_j) with grid spacings $\Delta\theta$ and $\Delta\lambda$ then the above formulas become

$$\bar{a}_{nm} = \frac{1}{4\pi} \sum_{i,j} f(\theta_i, \lambda_j) \bar{P}_{nm}(\cos \theta_i) \cos(m\lambda_j) \Delta\theta \Delta\lambda$$

$$\bar{b}_{nm} = \frac{1}{4\pi} \sum_{i,j} f(\theta_i, \lambda_j) \bar{P}_{nm}(\cos \theta_i) \sin(m\lambda_j) \Delta\theta \Delta\lambda$$

Input:

- Name of input file containing grid in gravsoft format
- The maximum degree and order for the coefficients

Output:

- A file containing the calculated coefficients in ICGEM format.

GUT_104: Change of reference ellipsoid in grid

To change the reference ellipsoid the following formula will be used.

$$\begin{pmatrix} a\delta\varphi \\ a\cos(\varphi)\delta\lambda \\ \delta h \end{pmatrix} = -\bar{\mathbf{A}}^{-1}\delta\mathbf{r}_0 + \mathbf{C} \begin{pmatrix} \delta\varepsilon_{\bar{x}} \\ \delta\varepsilon_{\bar{y}} \\ \delta\varepsilon_{\bar{z}} \end{pmatrix} + \mathbf{F} \begin{pmatrix} \delta a \\ a\delta f \end{pmatrix}$$

$$\bar{\mathbf{A}}^{-1} = \begin{pmatrix} -\sin\varphi\cos\lambda & -\sin\varphi\sin\lambda & \cos\varphi \\ -\sin\lambda & \cos\lambda & 0 \\ \cos\varphi\cos\lambda & \cos\varphi\sin\lambda & \sin\varphi \end{pmatrix}$$

$$\mathbf{C} = a \begin{pmatrix} \sin\lambda & -\cos\lambda & 0 \\ -\sin\varphi\cos\lambda & -\sin\varphi\sin\lambda & \cos\varphi \\ 0 & 0 & 0 \end{pmatrix}$$

$$\mathbf{F} = \begin{pmatrix} 0 & \sin 2\varphi \\ 0 & 0 \\ -1 & \sin^2\varphi \end{pmatrix}$$

Here $\delta\mathbf{r}_0$ is the translation vector to the new reference frame and the $\delta\varepsilon$ are the three rotation angles. This gives a correction needed to get from the coordinates of the original reference frame to the new one. The first two parts here are only necessary for a datum shift, that is, moving the Earth center and rotating the axis. Thus, usually only the third term is needed.

This will be done on every point of a grid.

Input:

- Grid in gravsoft format
- Transformation parameters, $\delta\mathbf{r}_0$, $\delta\varepsilon$, a and f

Output:

- Transformed grid in gravsoft format

GUT_105: Change of reference ellipsoid in points

To change the reference ellipsoid the following formula will be used.

$$\begin{pmatrix} a\delta\varphi \\ a\cos(\varphi)\delta\lambda \\ \delta h \end{pmatrix} = -\bar{\mathbf{A}}^{-1}\delta\mathbf{r}_0 + \mathbf{C} \begin{pmatrix} \delta\varepsilon_{\bar{x}} \\ \delta\varepsilon_{\bar{y}} \\ \delta\varepsilon_{\bar{z}} \end{pmatrix} + \mathbf{F} \begin{pmatrix} \delta a \\ a\delta f \end{pmatrix}$$

$$\bar{\mathbf{A}}^{-1} = \begin{pmatrix} -\sin\varphi\cos\lambda & -\sin\varphi\sin\lambda & \cos\varphi \\ -\sin\lambda & \cos\lambda & 0 \\ \cos\varphi\cos\lambda & \cos\varphi\sin\lambda & \sin\varphi \end{pmatrix}$$

$$\mathbf{C} = a \begin{pmatrix} \sin\lambda & -\cos\lambda & 0 \\ -\sin\varphi\cos\lambda & -\sin\varphi\sin\lambda & \cos\varphi \\ 0 & 0 & 0 \end{pmatrix}$$

$$\mathbf{F} = \begin{pmatrix} 0 & \sin 2\varphi \\ 0 & 0 \\ -1 & \sin^2\varphi \end{pmatrix}$$

Here $\delta\mathbf{r}_0$ is the translation vector to the new reference frame and the $\delta\varepsilon$ are the three rotation angles. This gives a correction needed to get from the coordinates of the original reference frame to the new one. The first two parts here are only necessary for a datum shift, that is, moving the Earth center and rotating the axis. Thus, usually only the third term is needed.

This will be done on each point in the list.

Input:

- Point list in gravsoft format
- Transformation parameters, $\delta\mathbf{r}_0$, $\delta\varepsilon$, a and f

Output:

- Transformed points in gravsoft format

GUT_106: Change of tide system in a grid

The three tidal models used are mean tide, zero tide, and non-tide. Conversion of geoid height between these three models use the following formulas.

The first for conversion from zero tide geoid to mean tide geoid:

$$N_m - N_z = \frac{GMr^2}{4gd} \left(\frac{3}{2} \sin^2 \varepsilon - 1 \right) (3 \sin^2 \varphi - 1)$$

Next for non-tidal geoid to zero geoid:

$$N_z - N_n = k \left(\frac{GMr^2}{4gd} \left(\frac{3}{2} \sin^2 \varepsilon - 1 \right) (3 \sin^2 \varphi - 1) \right)$$

The third is for non-tidal geoid to mean geoid:

$$N_m - N_n = (1 - k) \left(\frac{GMr^2}{4gd} \left(\frac{3}{2} \sin^2 \varepsilon - 1 \right) (3 \sin^2 \varphi - 1) \right)$$

Here d is the geocentric distance to the moon, ε is the inclination of the ecliptic to the equator, g is the gravity, and k is a Love's number.

This will be calculated at every point of the grid

Input:

- Grid in gravsoft format
- The parameter k
- The current tidal model and the wanted tidal model

Output:

- Transformed grid in gravsoft format

GUT_107: Change of tide system at points

The three tidal models used are mean tide, zero tide, and non-tide. Conversion of geoid height between these three models use the following formulas.

The first for conversion from zero tide geoid to mean tide geoid:

$$N_m - N_z = \frac{GMr^2}{4gd} \left(\frac{3}{2} \sin^2 \varepsilon - 1 \right) (3 \sin^2 \varphi - 1)$$

Next for non-tidal geoid to zero geoid:

$$N_z - N_n = k \left(\frac{GMr^2}{4gd} \left(\frac{3}{2} \sin^2 \varepsilon - 1 \right) (3 \sin^2 \varphi - 1) \right)$$

The third is for non-tidal geoid to mean geoid:

$$N_m - N_n = (1 - k) \left(\frac{GMr^2}{4gd} \left(\frac{3}{2} \sin^2 \varepsilon - 1 \right) (3 \sin^2 \varphi - 1) \right)$$

Here d is the geocentric distance to the moon, ε is the inclination of the ecliptic to the equator, g is the gravity, and k is a Love's number.

This will be calculated at every point of the grid

Input:

- Point list in gravsoft format
- The parameter k
- The current tidal model and the wanted tidal model

Output:

- Transformed point list in gravsoft format

GUT_108: Calculation of reference potential, grid

A necessary part of the toolbox is being able to calculate a reference potential from a set of variables with values set by the user. There are different sets of variables that can be used namely

- GM, a, J_2 , ω
- GM, a, f, ω
- γ_a , a, f, ω

The potential is then calculated through use of

$$U = \frac{GM}{r} \left(1 - \sum_{n=1}^{\infty} J_{2n} \left(\frac{a}{r} \right)^{2n} P_{2n}(\cos \theta) \right)$$
$$J_{2n} = (-1)^{2n} \frac{3e^{2n}}{(2n+1)(2n+3)} \left(1 - n + 5n \frac{C-A}{ME^2} \right), \quad e = \frac{E}{a}$$

where C and A are moments of inertia, C with respect to the rotation axis and A with respect to any axis in the equatorial plane, E is the eccentricity, and e is the first eccentricity. Dependent on the variables given different ways to find the J_{2n} 's must be used. However they all use the formulas

$$G(C-A) = \frac{1}{3} GME^2 \left(1 - \frac{2}{15} \frac{me'}{q_0} \right), \quad m = \frac{\omega^2 a^2 b}{GM}, \quad e' = \frac{E}{b}$$
$$q_0 = \frac{1}{2} \left[\left(1 + 3 \frac{b^2}{E^2} \right) \tan^{-1} \frac{E}{b} - 3 \frac{b}{E} \right]$$

Input:

- The specifications for the reference ellipsoid, this input can be in several forms. The choices are
 - GM, a, J_2 , ω
 - GM, a, f, ω
 - γ_a , a, f, ω
- Maximum harmonic degree, N_{\max}
- Altitude of the grid points. This altitude should be above the terrain.
- Wanted grid specifications:
 - Starting latitude, φ_1
 - Ending latitude, φ_2

- Starting longitude, λ_1
- Ending longitude, λ_2
- Grid spacing in latitude direction, $\Delta\varphi$
- Grid spacing in longitudinal direction, $\Delta\lambda$

Output:

- The specified grid in gravsoft grid format. This format contains a line at the start of the file containing in order and in decimal degree notation
 - Starting latitude, φ_1
 - Ending latitude, φ_2
 - Starting longitude, λ_1
 - Ending longitude, λ_2
 - Grid spacing in latitude direction, $\Delta\varphi$
 - Grid spacing in longitudinal direction, $\Delta\lambda$

GUT_109: Calculation of reference potential, point

A necessary part of the toolbox is being able to calculate a reference potential from a set of variables with values set by the user. There are different sets of variables that can be used namely

- GM, a, J_2 , ω
- GM, a, f, ω
- γ_a , a, f, ω

The potential is then calculated through use of

$$U = \frac{GM}{r} \left(1 - \sum_{n=1}^{\infty} J_{2n} \left(\frac{a}{r} \right)^{2n} P_{2n}(\cos \theta) \right)$$
$$J_{2n} = (-1)^{2n} \frac{3e^{2n}}{(2n+1)(2n+3)} \left(1 - n + 5n \frac{C-A}{ME^2} \right), \quad e = \frac{E}{a}$$

where C and A are moments of inertia, C with respect to the rotation axis and A with respect to any axis in the equatorial plane, E is the eccentricity, and e is the first eccentricity. Dependent on the variables given different ways to find the J_{2n} 's must be used. However they all use the formulas

$$G(C-A) = \frac{1}{3} GME^2 \left(1 - \frac{2}{15} \frac{me'}{q_0} \right), \quad m = \frac{\omega^2 a^2 b}{GM}, \quad e' = \frac{E}{b}$$
$$q_0 = \frac{1}{2} \left[\left(1 + 3 \frac{b^2}{E^2} \right) \tan^{-1} \frac{E}{b} - 3 \frac{b}{E} \right]$$

Input:

- EGM_GCF_2.
- The specifications for the reference ellipsoid, this input can be in several forms. The choices are
 - GM, a, J_2 , ω
 - GM, a, f, ω
 - γ_a , a, f, ω
- Maximum harmonic degree and order, N_{\max}
- Name of input point file, containing heights of the points
- Name of output file

Output:

- Values in points in the output file in gravsoft point list format.

GUT_110: Interpolation

Interpolation of some sort will be needed in the GUT to change grids and make subgrids of already existing ones.

The algorithm given here is just an example and is only of use on regular grids. The idea is to be able to find the value of a function $F(x,y)$ at some point (x,y) that is not located on the grid. It will however be contained in a rectangle where the four corners are grid points. These will be called – in the order lower left corner, lower right corner, upper right corner, and upper left corner – (x_1,y_1) , (x_2,y_1) , (x_2,y_2) , and (x_1,y_2) with the values $F(x_1,y_1)$, $F(x_2,y_1)$, $F(x_2,y_2)$, and $F(x_1,y_2)$. Define

$$t = \frac{(x - x_1)}{(x_2 - x_1)}$$
$$u = \frac{(y - y_1)}{(y_2 - y_1)}$$

The value of the function F at the point (x,y) will then be given by

$$F(x, y) = (1-t)(1-u)F(x_1, y_1) + t(1-u)F(x_2, y_1) + tuF(x_2, y_2) + (1-t)uF(x_1, y_2)$$

Input:

- Grid to be interpolated from
- Points or grid wanted in output
 - If grid then the grid specifications need to be given, that is
 - Starting latitude, ϕ_1
 - Ending latitude, ϕ_2
 - Starting longitude, λ_1
 - Ending longitude, λ_2
 - Grid spacing in latitude direction, $\Delta\phi$
 - Grid spacing in longitudinal direction, $\Delta\lambda$
 - If points then specification of latitude and longitude for the points

Output:

- Grid or points specified as input.

Prioritizing the algorithms

The above algorithms, and the functionalities they represent, are in essence what the ideal toolbox would contain. The toolbox will of course be useful even if it does not contain all of the functionalities described. Which of these is included in the toolbox is partly determined by the amount of money allocated to the building of the toolbox, and partly determined by ongoing research. Some of the functionalities above, particularly the ones involving the variance-covariance matrix, are not completely agreed upon in the scientific community and for some programs with easy access do not exist.

Therefore these algorithms will need to be prioritized, to determine which are absolutely vital, which are less vital, and which are still subject to research. This will be done in the WP3000.

Additionally the algorithms will be grouped into larger functions that closely relate to already existing programs.

Appendix A

Calculating sums of spherical harmonic functions

The starting points for finding the sums of the spherical harmonic expansions are the recurrence relations for the associated Legendre functions

$$\begin{aligned}\bar{P}_{nm}(\theta) &= a_{nm}t\bar{P}_{n-1,m}(\theta) - b_{nm}\bar{P}_{n-2,m}(\theta), \quad n > m \\ \bar{P}_{mm}(\theta) &= u\sqrt{\frac{2m+1}{2m}}\bar{P}_{m-1,m-1}(\theta) = u^m\sqrt{3}\prod_{i=2}^m\sqrt{\frac{2i+1}{2i}}, \quad m > 1 \\ a_{nm} &= \sqrt{\frac{(2n-1)(2n+1)}{(n-m)(n+m)}}, \quad b_{nm} = \sqrt{\frac{(2n+1)(n+m-1)(n-m-1)}{(n-m)(n+m)(2n-3)}} \\ t &= \cos\theta, \quad u = \sin\theta\end{aligned}$$

These relations are stable up to about degree 2700. The full sum on a grid is then found through

$$\begin{aligned}S &= c \sum_{m=0}^{N_{\max}} \Omega_m \\ \Omega_m &= \sum_{\alpha=1}^2 \begin{cases} X_{m\alpha} \cos m\lambda & \text{for } \alpha = 1 \\ X_{m\alpha} \sin m\lambda & \text{for } \alpha = 2 \end{cases} \\ X_{m\alpha} &= \sum_{n=\mu}^{N_{\max}} \bar{E}_{nm\alpha} \bar{P}_{nm}(\theta)\end{aligned}$$

Here μ is a number that varies with m , and $E_{nm\alpha}$ incorporates the coefficients of the expansion. As an example look at the Earth's gravity potential as given in the beginning of this document. Here one would get

$$\begin{aligned}\bar{E}_{nm\alpha} &= \begin{cases} \left(\frac{a}{r}\right)^n \bar{C}_{nm} & \text{for } \alpha = 1 \\ \left(\frac{a}{r}\right)^n \bar{S}_{nm} & \text{for } \alpha = 2 \end{cases} \\ X_{m\alpha} &= \sum_{n=\mu}^{N_{\max}} \left(\frac{a}{r}\right)^n \bar{E}_{nm\alpha} \bar{P}_{nm}(\theta)\end{aligned}$$

The reason the sums are done this way is because of computing time. In this way only $X_{m\alpha}$ depend on the latitude and much computation time can be saved. The sum can be found in several ways. One way is to just calculate the Legendre functions to the needed

degree and order and then manually doing the sum. Another way is to use Clenshaw summation. Clenshaw summation is useful because it greatly reduces the amount of calculations necessary. Clenshaw summation is tailored to easily do sums in the form of

$$S = \sum_{n=m}^N y_n P_n(x) = y^T p$$

where the y_n 's are coefficients of some series and P_n is a series of polynomials for which a recurrence relation of the following form exists

$$P_n(x) + a_n(x)P_{n-1}(x) + b_n(x)P_{n-2}(x) = 0$$

The sum is then given by the equations

$$s_n = -a_{n+1}s_{n+1} - b_{n+2}s_{n+2} + y_n$$

$$S = s_m P_m(x)$$

$$s_{N+1} = s_{N+2} = 0$$

This is a recursive relation which when the termination condition is reached will only depend on the coefficients and the polynomial P_m .

For finding the sum of the derivative polynomial another set of formulas are needed together with the assumption that $a_n(x)$ is linear in x .

$$a_n(x) = a'_n x + a''_n$$

$$s'_n = -a_{n+1}s'_{n+1} - b_{n+2}s'_{n+2} - a'_{n+1}s_{n+1}$$

$$dS = s'_m P_m + s_m dP_m$$

$$s'_{N+1} = s'_{N+2} = 0$$

This will only depend on the coefficients and the polynomial and its first derivative.

It can be seen from the above that Clenshaw summation can be used to find expressions for the $X_{m\alpha}$ only from knowledge of the sectorial associated Legendre functions.

Appendix B

The GRAVSOF T formats

From Document: GOCE_s1.doc Date: 2001.05.08, Author: C.C.Tscherning,

URL: http://www.gfy.ku.dk/~cct/GOCE_s1.htm.

With slight modifications

GRAVSOF T Standard.

The basis for some of the specifications proposed below are the so-called GRAVSOF T standards, (Tscherning et al., 1994). In brief they are:

All data are in ASCII code with blanks as delimiters.

Spherical harmonic coefficients:

Degree n , order m , C_{nm} , S_{nm} , $\sigma (C_{nm}, S_{nm})$. Coefficients are unitless. All coefficients are included (also (0,0)).

Geo-located data:

Geographical coordinates:

<Unique id, integer> <latitude, decimal degrees (number of significant digits to be specified)>, <longitude, decimal degrees from 0 to 360 degrees, positive East > < altitude in m, significant digits to be specified , ellipsoidal height or orthometric height must be specified> <data 1> <data 2> <data n >

UTM coordinates: like geographical coordinates, but Northing instead of latitude and Easting instead of longitude.

Cartesian Coordinates:

Like geographical coordinates, but <unique id. >, <X>, <Y>, <Z>, and then data.

Equal-angular grids of data: Data start with a grid label which describes the extend and spacing of the data grid, see the precise description below. Then the data are organized in "bands" of equal latitude (or UTM Northing) from North to South where the bands start in west and move east. The coordinates refer to the center of the cell.

Grid label: Geographical coordinates: minimum, maximum latitude, θ_1 and θ_2 , minimum, maximum longitude, λ_1 and λ_2 , spacing in latitude, $\Delta\theta$, spacing in longitude, $\Delta\lambda$. UTM: Northing instead of latitude, Easting instead of longitude, last value is the UTM zone.

This results in a grid that has

$$\frac{\lambda_2 - \lambda_1}{\Delta\lambda} + 1.5 \text{ entries per line and}$$

$$\frac{\theta_2 - \theta_1}{\Delta\theta} + 1.5 \text{ lines.}$$

Example: Grid with 4 elements.

54.0 56.0 10.0 12.0 2.0 2.0

4.1 5.0

4.5 5.6

Data-specifications which agree with GRAVSOF:

Auxiliary data needed are:

A-priori set of spherical harmonic coefficients and associated error-estimates.

Global digital mean terrain/depth data in a 5' x 5' grid, globally. Units m.

For all the following data, the horizontal coordinates must be given as geographical coordinates in WGS84/ITRF ???. All associated heights are heights above mean-sea level. (Datum to be specified if not mean value).

5' mean Free-air gravity anomalies with <data 1> = <gravity anomaly, mgal with 1 decimal>, <data 2> = < standard error>.

Height anomalies from GPS and levelling <data 1> = < height anomaly in units of m with 3 decimals> <data 2> = < error estimates > <data 3> = < datum name (ISO)>

Deflections of the vertical: Meridian and Prime-vertical components.

<data 1> = <Meridian component, arcseconds with 2 decimals >, <data 2> = < Prime-vertical component >, <data 3 >, <data 4> = <error-estimates of the components > <datum id.>

Deflection of the vertical, one component only. As above with only two data-items.

Satellite data:

Geo-located observational data on GRAVSOF T format.

SGG data: Geographical coordinates. Altitude is ellipsoidal height in mm. <data 1> ... <data n>, <standard deviations of data 1 ... data n> <3 attitude angles: tilt, roll, pitch in decimal degrees with 4 decimals>

SST data: Cartesian coordinates

Corrections to geo-located data on same format.

Products:

Spherical harmonic coefficients and their variance-covariance matrix.

Grids of free-air anomalies, deflections of the vertical and height anomalies referring to the mean terrain height.

Corresponding grids of error-estimates. The grid-labels will tell whether the grids are global or local.

Geo-located and corrected satellite data on the same format as above.

Non-GRAVSOF T data-types.

The gravity field statistics.

The physical correlation of gravity field quantities may be expressed through a global or regional covariance functions. Such functions may be specified using the following parameters:

<error-degree-variance modifiers> < depth to Bjerhammar-sphere (m) > < gravity anomaly variance (mgal²) >. The degree-variance modifiers are the following

< weight factor on error degree-variances > < number of error-degree-variances used > < indicator of whether the scale factor is to be applied on all degree-variances or so that it changes linearly from degree zero to the maximal degree >

A file should be established with the current values (they depend on the a-priori spherical harmonic model used). It should in one record hold the boundaries of the areas of validity and the parameters.

Variance-covariance matrices of a-priori given or estimated quantities.

These matrices must be on binary form, REAL*8. Since the matrices are symmetric, the matrix elements should be stored from column 1 to "N" , from element no. 1 to the diagonal element.

Example: The symmetric 2 x 2 matrix

1 3 becomes 1, 3, 4.

3 4

Reference:

Tscherning, C.C., P.Knudsen and R.Forsberg: Description of the GRAVSOFIT package. Geophysical Institute, University of Copenhagen, Technical Report, 1991, 2. Ed. 1992, 3. Ed. 1993, 4. ed, 1994.