WP9300: Report on Validation of Covariance Software

Ref:
GOCE User Toolbox (GUT) Implementation and Supporting Scientific Studies,
Rider 1 to ESRIN Contract No 19568/06/I-OL

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1 INTRODUCTION

A full error variance-covariance matrix containing the coefficients of the spherical harmonic series of the GOCE gravity field model (HPF product ID EGM_GVC_2) will not be, at the time of the first GUT official release, ingested and handled inside the toolbox. The main reason for this is the huge size of the matrix. No effort has been put into the possibility of using the error matrix information from GUT due to the matrix size limitations.

In WP9000 a study was made in order to upgrade GUT such that the HPF error matrix products are ingested by the toolbox, and the information inside the matrix is correctly handled and used for propagation of GOCE products and further applications. During WP9000 two prototypes were developed and validated.

<u>WP9100 Rigorous computation</u>
In this sub-workpackage a computer programme was developed that can handle the GOCE error covariance matrix (EGM_GVC_2: Variance-covariance matrix file of the spherical harmonics coefficients) and compute errors and error covariances of the gravity field related quantities rigorously. The computer programme was written in FORTRAN.

<u>WP9200 Approximate computation</u>
In this sub-workpackage a computer programme was developed that can compute errors and error covariances of the gravity field related quantities using approximate expressions.

Both computer programme may form the basic tool for the implementation of the following GUT functionalities (**AD2** “User Requirement Consolidation Document”, Deliverable of GUTS WP2000):

- GUT_016: Geoid height errors in grid format.
- GUT_017: Geoid height errors in points.
- GUT_024: Geoid height error covariances on a grid.
- GUT_025: Geoid height error covariances in points.

The computer programmes were written in FORTRAN.

<u>WP9300 Validation</u>
In this sub-workpackage the functionality was tested using the prototypes. This validation was carried out by Georges Balmino and DTU. The results of the validation are summarised in this report. Furthermore this report contain recommendations on the use of software and for the development of a possible plug-in in WP9400.

2 VALIDATION

2.1 RIGOROUS METHOD

A computer programme was developed that can handle the GOCE error covariance matrix (EGM_GVC_2: Variance-covariance matrix file of the spherical harmonics coefficients) and compute errors and error covariances of the gravity field related quantities rigorously. The computer programme was written in FORTRAN.

According to Balmino the rigorous computation is carried out using the following equation:

$$\mathrm{cov}(q, q') = \sum_{i,j} f_i(q)\, \overline{P}_{ij}(\sin\phi) \sum_{trig=\cos}^{\sin} trig\,(j\lambda) \sum_{i',j'} \Gamma_{[i'j']}^{[ij]}\, f_{i'}(q')\, \overline{P}_{i'j'}(\sin\phi') \sum_{trig=\cos}^{\sin} trig\,(j'\lambda')$$

This expression only considers the full covariance matrix.

2.2 APPROXIMATE METHOD

A computer programme was developed that can compute errors and error covariances of the gravity field related quantities using approximate expressions.

According to Knudsen and approximate computation may be carried out using the following equation:

$$C_{TT} = \sum_{i=2}^{\infty} \left(\frac{R^2}{rr'}\right)^{i+1} \sum_{j=0}^{i} \overline{\sigma}_{ij}^{TT} \cos j(\lambda - \lambda')\overline{P}_{ij}(\sin\phi)\overline{P}_{ij}(\sin\phi')$$

This expression only considers the diagonal part of the covariance matrix.

2.3 COMPARISON

Using both expressions and the synthetic GOCE covariance matrix two dimensional error covariance functions were derived centred at three different latitudes. This was done using the EIGEN GL04S error covariance matrix. The results are shown below.

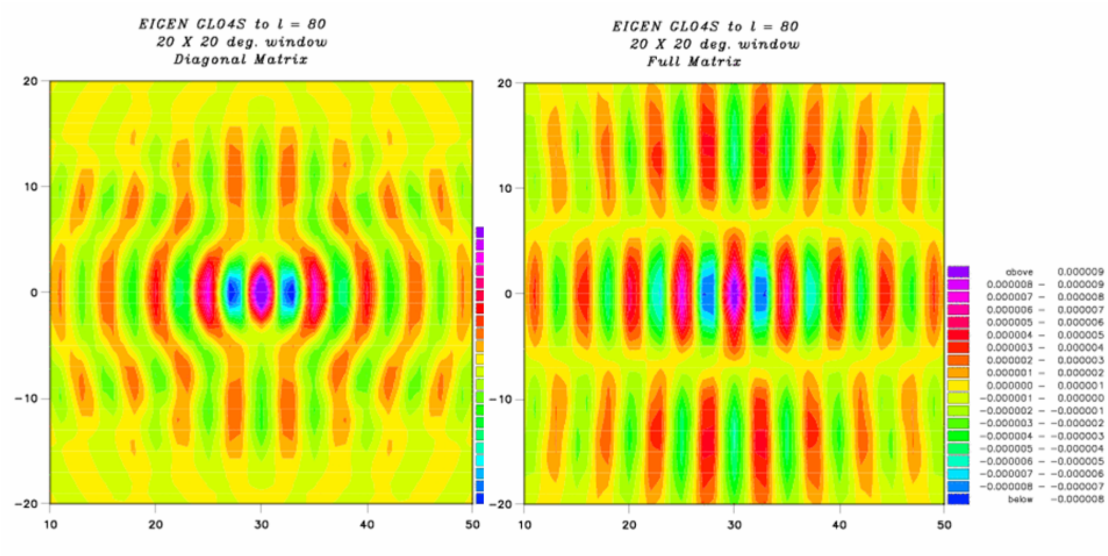

Figure 1. 2D geoid error covariances from EIGEN GL04S to harmonic degree 80 using diagonal (left) and full matrix (right) centred at latitude 0.

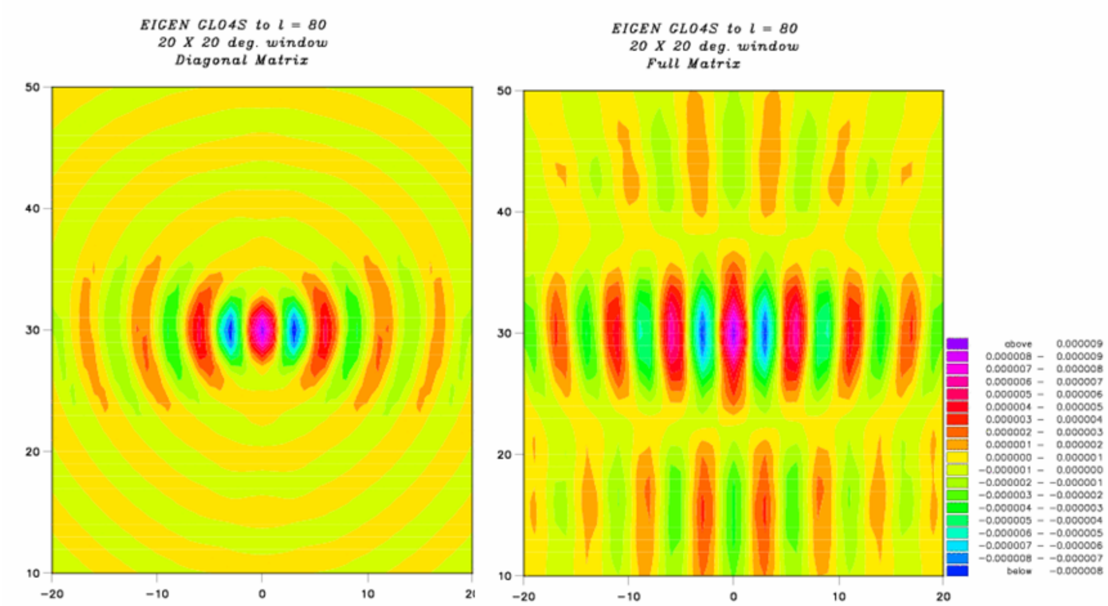

Figure 2. 2D geoid error covariances from EIGEN GL04S to harmonic degree 80 using diagonal (left) and full matrix (right) centred at latitude 30.

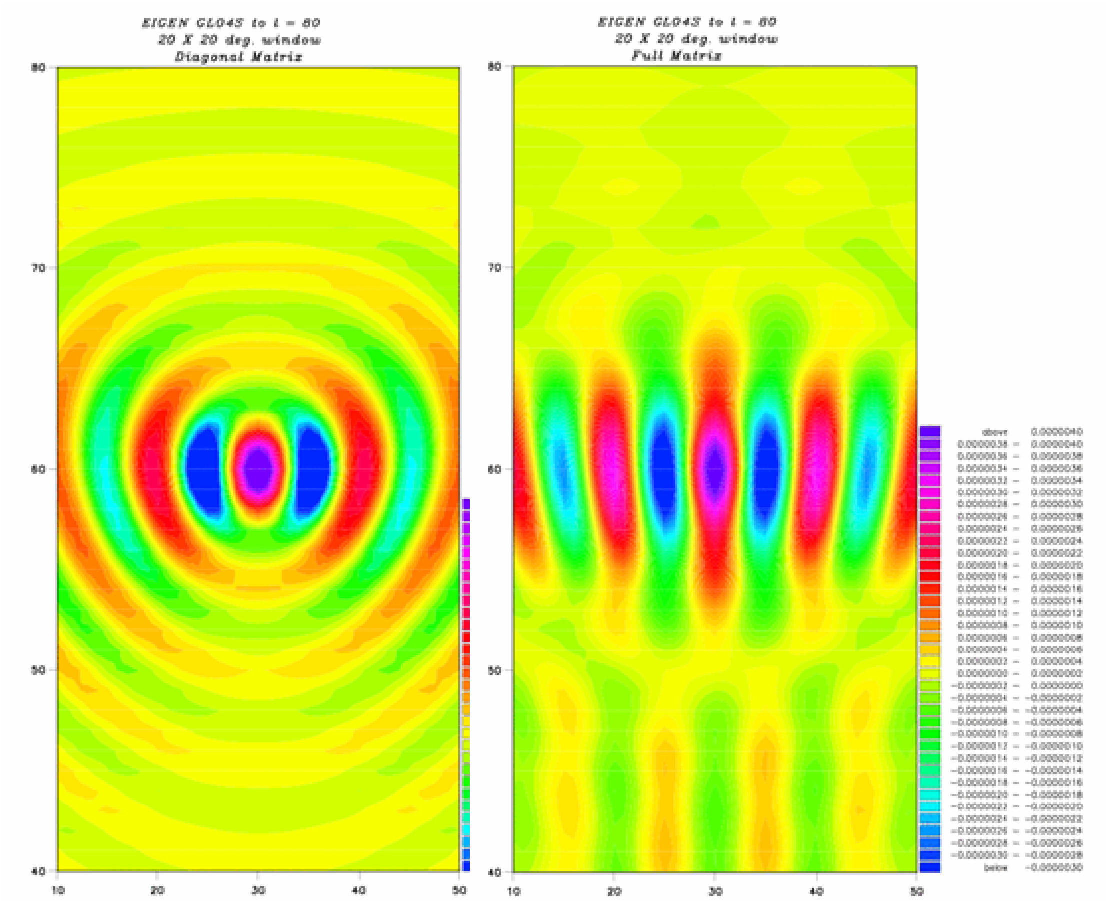

Figure 3. 2D geoid error covariances from EIGEN GL04S to harmonic degree 80 using diagonal (left) and full matrix (right) centred at latitude 60.

The 2D geoid error covariance functions shown in Figure 1-3 show that both expressions (rigorous and approximate) may be used to compute anisotropic covariance function where the characteristic pattern is that the North-south error correlations are larger that the East-west error correlations. Also the error variances are reproduced by the approximate expression compared to the rigorous expression.

However, there are also differences that are quite visible. The approximate expression does not reproduce the North-south correlations fully and it appear to produce more circular patterns that the rigorous expression. The curves shown in Figure 4 verify that.

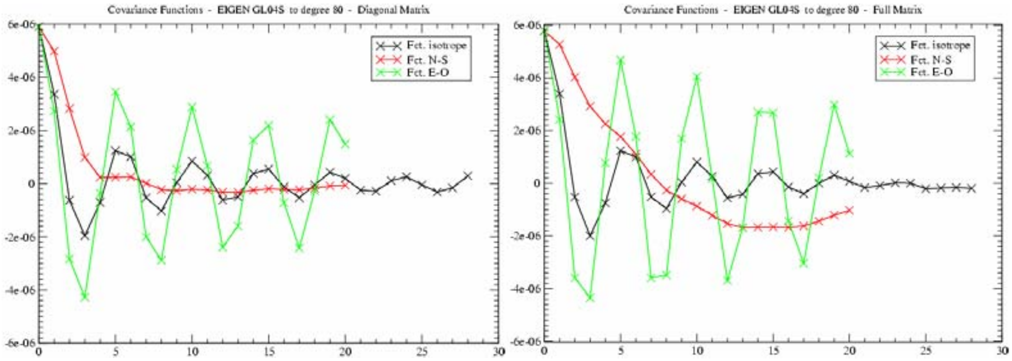

Figure 4. North-south, East-west, and isotropic profiles of the 2D geoid error covariances from EIGEN GL04S to harmonic degree 80 using diagonal (left) and full matrix (right) centred at latitude 0.

2.4 CONCLUSIONS AND RECOMMENDATIONS

The results show the approximate expression may be used to compute anisotropic covariance function having the characteristic pattern that the North-south error correlations are larger that the East-west error correlations. However, the comparisons above show that there are some differences that are quite visible. The approximate expression does not reproduce the North-south correlations fully and it appear to produce more circular patterns that the rigorous expression.

Since the implementation of the rigorous expression in the software by G. Balmino is not much more computer resource demanding it is recommended to apply this software and compute the error covariances using this method. The software is not suited for implementation in GUT as a plug-in though. It is recommended to supply the software separately to GUT.