




2nd Advanced Course on Radar Polarimetry -- 24 Jan 2013 -- irena.hajsek@dlr.de

Surface Parameter Estimation: Basics and Advanced Concepts

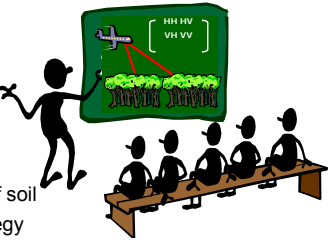
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Institute of Environmental Engineering, ETH Zürich
Microwaves and Radar Institute, DLR Oberpfaffenhofen



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Overview





Part I: Theory and basics

- Introduction into SAR polarimetric observables
- Decomposition techniques
- Introduction into material properties of soil
- Model description and inversion strategy

Part II: Exercises with L-band airborne data

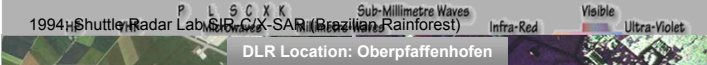
- Read the data
- Speckle filtering
- Oh, Dubois and X-Bragg inversion



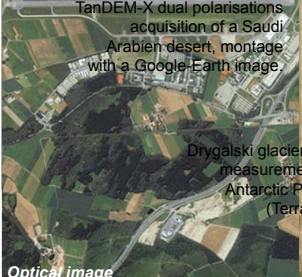
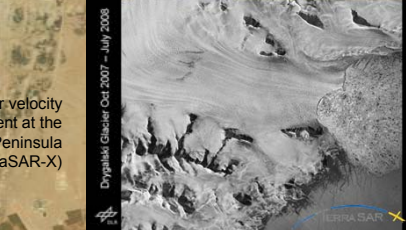
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Motivation of Radar Remote Sensing

- **Independent from Weather**
 - Data acquisition during cloud cover, rain, ...
- **Radar is complementary to optics**



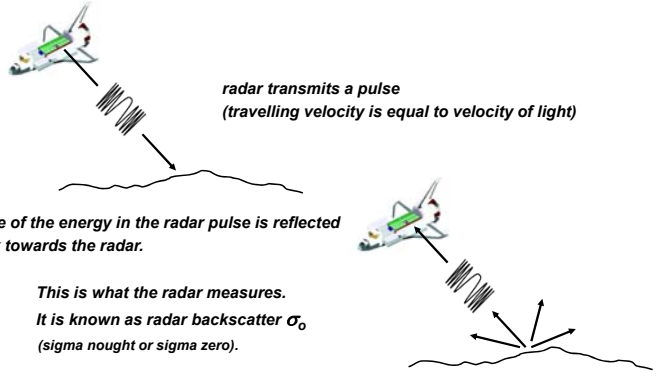
DLR Location: Oberpfaffenhofen

Optical image 

What does the Radar measure ?


Radar reflectivity (backscattered signal) of the target as a function of position.




radar transmits a pulse
(travelling velocity is equal to velocity of light)

some of the energy in the radar pulse is reflected back towards the radar.

This is what the radar measures.
It is known as radar backscatter σ_0
(sigma nought or sigma zero).



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What does the Radar measure ?

Normalized radar cross-section (backscattering coefficient) is given by:

$$\sigma_o (dB) = 10 \cdot \text{Log}_{10} (\text{energy ratio})$$

whereby

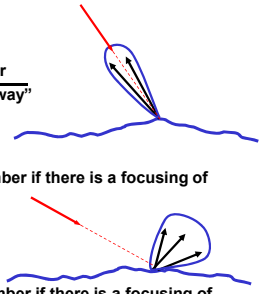
$$\text{energy ratio} = \frac{\text{received energy by the sensor}}{\text{“energy reflected in an isotropic way”}}$$

i.e.

The backscattered coefficient can be a positive number if there is a focusing of backscattered energy towards the radar

or

The backscattered coefficient can be a negative number if there is a focusing of backscattered energy away from the radar (e.g. smooth surface)



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Backscattering Coefficient σ_o

Levels of Radar backscatter	Typical scenario
Very high backscatter (above -5 dB)	<ul style="list-style-type: none"> ➤ man-made objects (urban) ➤ terrain slopes towards radar ➤ very rough surface ➤ radar looking very steep
High backscatter (-10 dB to 0 dB)	<ul style="list-style-type: none"> ➤ rough surface ➤ dense vegetation (forest)
Moderate backscatter (-20 to -10 dB)	<ul style="list-style-type: none"> ➤ medium level of vegetation ➤ agricultural crops ➤ moderately rough surfaces
Low backscatter (below -20 dB)	<ul style="list-style-type: none"> ➤ smooth surface ➤ calm water ➤ road ➤ very dry terrain (sand)



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Commonly Used Frequency Bands

Frequency band	Frequency range	Application examples
VHF	300 KHz - 300 MHz	foliage/ground penetration, biomass
P-Band	300 MHz - 1 GHz	soil moisture, biomass, penetration
L-Band	1 GHz - 2 GHz	agriculture, forestry, soil moisture
C-Band	4 GHz - 8 GHz	ocean, agriculture
X-Band	8 GHz - 12 GHz	agriculture, ocean, high resolution radar
Ku-Band	14 GHz - 18 GHz	glaciology (snow cover mapping)
Ka-Band	27 GHz - 47 GHz	high resolution radar



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Environmental Sensing from Air and Space

Airborne measurements

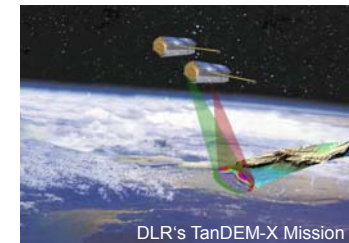
- Highly flexible operation
- Coverage of dedicated areas
- Experimental configuration
- Sensor specific data formats
- Short re-visit times

Spaceborne measurements

- Highly regular observation
- Wide area coverage
- Highly operational & reliable
- Standard product delivery
- Long term observations



DLR's E-SAR

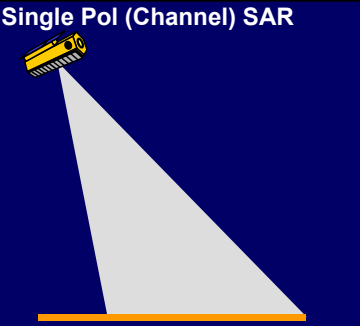


DLR's TanDEM-X Mission



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Single Pol (Channel) SAR



X-band
PI-SAR / Test Site: Gifu-Japan

$[S] = \begin{bmatrix} S_{HH} & S_{HV} \\ S_{VH} & S_{VV} \end{bmatrix}$

Observation Space

Distributed Scatterers:

- Norm. Backscattering Cross Section σ^0 ;
- Texture (σ^0 statistics).

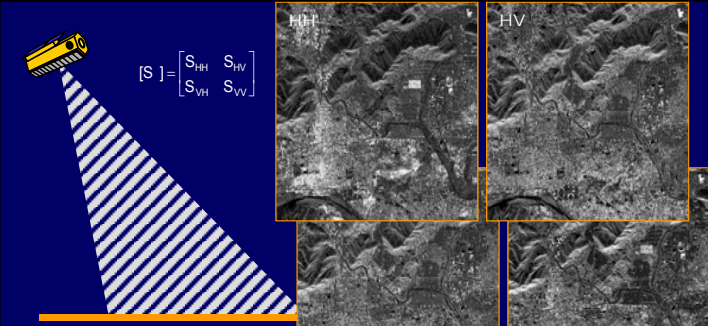
Deterministic Scatterers:

- Radar Cross Section (RCS) ;
- Phase of the SAR signal.

Applications

- Classification/Segmentation (Texture);
- Change detection (Multi temp. analysis);
- Glacier velocities (Feature tracking);
- Mapping (Feature extraction/Bordering) ;
- Ocean wave/Wind mapping;
- Coherent scatterers (CSs).

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$[S] = \begin{bmatrix} S_{HH} & S_{HV} \\ S_{VH} & S_{VV} \end{bmatrix}$

Observation Space

First Order Parameters:

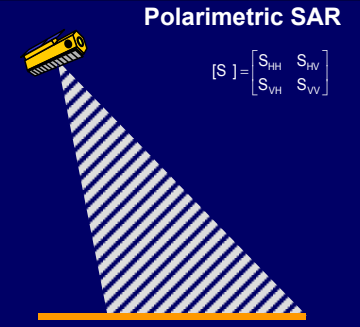
- Norm. Backscattering Cross Section σ^0 ;
- Intensity (Cross Section σ^0) ratios;
- Polarimetric phase differences.

Second Order Parameters:

- Polarimetric (inter channel) coherences

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Polarimetric SAR



X-band
PI-SAR / Test Site: Gifu-Japan

$[S] = \begin{bmatrix} S_{HH} & S_{HV} \\ S_{VH} & S_{VV} \end{bmatrix}$

R:HH-VV G:HV+VH B:HH+VV

Observation Space

First Order Parameters:

- Norm. Backscattering Cross Section σ^0
- Intensity (Cross Section σ^0) ratios;
- Polarimetric phase differences.

Second Order Parameters:

- Polarimetric (inter channel) coherences

Applications

- Soil moisture/roughness estimation;
- Wetland/Veg. characterisation/mapping;
- Snow & Ice mapping (type classification);
- Ship & Oil spill detection;
- Classification/Segmentation (Pol based).

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The Polarimetric Scattering Problem

Incident (Plane) Wave

$$\vec{E}_h^i(\vec{r}) = \begin{bmatrix} E_h^i(\vec{r}) \\ E_v^i(\vec{r}) \end{bmatrix}$$

(Jones Vector Representation)

Scattered Field


In the far zone region ($|\vec{r}| \gg \lambda$ $|\vec{r}| \gg |\vec{r}'|$)

$$\vec{E}_h^s(\vec{r}) = \begin{bmatrix} E_h^s(\vec{r}) \\ E_v^s(\vec{r}) \end{bmatrix}$$

(Jones Vector Representation)

Scatterer

Transforms the incident into the scattered wave



2x2 Complex Scattering Matrix

$$\begin{bmatrix} E_h^s(\vec{r}) \\ E_v^s(\vec{r}) \end{bmatrix} = \begin{bmatrix} \exp(i\kappa r) & S_{hh}(\vec{r}) - S_{vv}(\vec{r}) \\ r & S_{hv}(\vec{r}) - S_{vh}(\vec{r}) \end{bmatrix} \begin{bmatrix} E_h^i(\vec{r}) \\ E_v^i(\vec{r}) \end{bmatrix}$$

Mapping of the 2-dim incident vector $\vec{E}_h^i(\vec{r})$ into the 2-dim scattered vector $\vec{E}_h^s(\vec{r})$

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The Polarimetric Scattering Problem

Incident (Plane) Wave

$$\vec{E}_h^i(\vec{r}) = \begin{bmatrix} E_h^i(\vec{r}) \\ E_v^i(\vec{r}) \end{bmatrix}$$

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Scattered Field

In the far zone region
($|\vec{r}| \gg \lambda$, $|\vec{r}| \gg |\vec{r}'|$)

$$\vec{E}_h^s(\vec{r}) = \begin{bmatrix} E_h^s(\vec{r}) \\ E_v^s(\vec{r}) \end{bmatrix}$$

(Jones Vector Representation)

Scatterer

Transforms the Incident into the scattered wave

- 1: Changes the polarisation state of the incident wave
- 2: Changes the degree of polarisation of the incident wave

2x2 Complex Scattering Matrix

$$\begin{bmatrix} E_h^s(\vec{r}) \\ E_v^s(\vec{r}) \end{bmatrix} = \frac{\exp(i\kappa r)}{r} \begin{bmatrix} S_{HH}(\vec{r}) & S_{HV}(\vec{r}) \\ S_{VH}(\vec{r}) & S_{VV}(\vec{r}) \end{bmatrix} \begin{bmatrix} E_h^i(\vec{r}) \\ E_v^i(\vec{r}) \end{bmatrix}$$

Mapping of the 2-dim incident vector $\vec{E}_h^i(\vec{r})$ into the 2-dim scattered vector $\vec{E}_h^s(\vec{r})$

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Coherent Scattering Matrix

... also known as the Jones Matrix in the bistatic and Sinclair Matrix in the monostatic case

$$\begin{bmatrix} E_h^s \\ E_v^s \end{bmatrix} = \frac{\exp(i\kappa r)}{r} \begin{bmatrix} S_{HH} & S_{HV} \\ S_{VH} & S_{VV} \end{bmatrix} \begin{bmatrix} E_h^i \\ E_v^i \end{bmatrix}$$

Complex Scattering Amplitudes:
 $S_{ij} = f(\text{Frequency, Scattering Geometry})$

[S] is independent on the polarisation of the incident wave !!!
and depends only on the physical and geometrical properties of the scatterer

Total Scattered Power: $TP = \text{Span}([S]) = \text{Trace}([S][S]^+) = |S_{HH}|^2 + |S_{HV}|^2 + |S_{VH}|^2 + |S_{VV}|^2$

$$[S] = \frac{\exp(i\kappa r)}{r} \begin{bmatrix} |S_{HH}| \exp(i\phi_{HH}) & |S_{HV}| \exp(i\phi_{HV}) \\ |S_{VH}| \exp(i\phi_{VH}) & |S_{VV}| \exp(i\phi_{VV}) \end{bmatrix}$$

$$[S] = \frac{\exp(i\kappa r) \exp(i\phi_{VV})}{r} \begin{bmatrix} |S_{HH}| \exp(i(\phi_{HH} - \phi_{VV})) & |S_{HV}| \exp(i(\phi_{HV} - \phi_{VV})) \\ |S_{VH}| \exp(i(\phi_{VH} - \phi_{VV})) & |S_{VV}| \end{bmatrix}$$

Absolute Phase Factor Seven Parameters: 4 Amplitudes & 3 Phases

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Bi- & Mono-Static Measurement of the Scattering Matrix

$T = 1/\text{PRF}$

H	V	H	V
V	V	V	V
H	H	H	H

Tx
Rx

$$\begin{bmatrix} S_{HH} & S_{HV} \\ S_{VH} & S_{VV} \end{bmatrix}$$

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Scattering Amplitude Images

Azimuth

Range

HH
HV

VH
VV

Backscattering (FSA & BSA)

In the case of monostatic backscattering from reciprocal scatterers:

Reciprocity Theorem $S_{HV}^{BSA} = S_{VH}^{BSA} = S_{XX}^{BSA}$ ($S_{HV}^{FSA} = -S_{VH}^{FSA} = S_{XX}^{FSA}$)

$[S] = \begin{bmatrix} S_{HH} & S_{XX} \\ S_{XX} & S_{VV} \end{bmatrix} \rightarrow [S] = \frac{\exp(i\alpha r) \exp(i\phi_{VV})}{r} \begin{bmatrix} |S_{HH}| \exp(i(\phi_{HH} - \phi_{VV})} & |S_{XX}| \exp(i(\phi_{XX} - \phi_{VV})} \\ |S_{XX}| \exp(i(\phi_{XX} - \phi_{VV})} & |S_{VV}| \end{bmatrix}$



Absolute Phase Factor **Five Parameters: 3 Amplitudes & 2 Phases**

The scattering problem can be addressed in the 3-dim complex space:

3-dim Lexicographic (L) and Pauli (P) Scattering Vectors:

$\vec{k}_{4L} = \begin{bmatrix} S_{HH} \\ S_{XX} \\ S_{XX} \\ S_{VV} \end{bmatrix} \rightarrow \vec{k}_{3L} = \begin{bmatrix} S_{HH} \\ \sqrt{2} S_{XX} \\ S_{VV} \end{bmatrix}$ $\vec{k}_{3P} = \frac{1}{\sqrt{2}} \begin{bmatrix} S_{HH} + S_{VV} \\ S_{HH} - S_{VV} \\ 2S_{XX} \\ 0 \end{bmatrix} \rightarrow \vec{k}_{3P} = \frac{1}{\sqrt{2}} \begin{bmatrix} S_{HH} + S_{VV} \\ S_{HH} - S_{VV} \\ 2S_{XX} \end{bmatrix}$

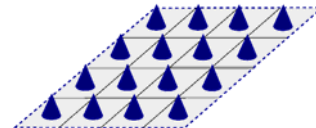
Note: The factor $\sqrt{2}$ is required to keep the vector norm of \vec{k}_{3L} invariant

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Partial Scatterers

Deterministic Scatterers



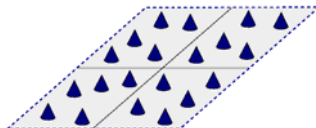
Point Scatterers

- Change the polarisation state of the wave
- Do not change the degree of polarisation

Monochromatic Incident Wave \rightarrow Monochromatic Scattered Wave

Completely described by [S]

Partial Scatterers





Scatterers with Space or Time Variability

- Change the polarisation state of the wave and also change the degree of polarisation

Depolarisation described by second order statistics

Cannot be described by a single [S]

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Covariance & Coherency Matrices in Backscattering



Lexicographic Scattering Vector: $\vec{k}_{4L} = [S_{HH}, \sqrt{2}S_{XX}, S_{VV}]^T$ \rightarrow **Covariance Matrix [C₃]:** $[C_3] := \langle \vec{k}_{3L} \cdot \vec{k}_{3L}^+ \rangle$

$$[C_3] := \begin{bmatrix} \langle |S_{HH}|^2 \rangle & \sqrt{2} \langle S_{HH} S_{VV}^* \rangle & \langle S_{HH} S_{XX}^* \rangle \\ \sqrt{2} \langle S_{HH} S_{VV}^* \rangle & 2 \langle |S_{VV}|^2 \rangle & \sqrt{2} \langle S_{VV} S_{XX}^* \rangle \\ \langle S_{HH} S_{XX}^* \rangle & \sqrt{2} \langle S_{VV} S_{XX}^* \rangle & \langle |S_{XX}|^2 \rangle \end{bmatrix}$$

Pauli Scattering Vector: $\vec{k}_{3P} = \frac{1}{\sqrt{2}} [S_{HH} + S_{VV}, S_{HH} - S_{VV}, 2S_{XX}]^T$ \rightarrow **Coherency Matrix [T₃]:** $[T_3] := \langle \vec{k}_{3P} \cdot \vec{k}_{3P}^+ \rangle$

$$[T_3] := \begin{bmatrix} \langle (S_{HH} + S_{VV})^2 \rangle & \langle (S_{HH} + S_{VV})(S_{HH} - S_{VV})^* \rangle & 2 \langle (S_{HH} + S_{VV}) S_{XX}^* \rangle \\ \langle (S_{HH} - S_{VV})(S_{HH} + S_{VV})^* \rangle & \langle (S_{HH} - S_{VV})^2 \rangle & 2 \langle (S_{HH} - S_{VV}) S_{XX}^* \rangle \\ 2 \langle S_{XX} (S_{HH} + S_{VV})^* \rangle & 2 \langle S_{XX} (S_{HH} - S_{VV})^* \rangle & 4 \langle |S_{XX}|^2 \rangle \end{bmatrix}$$

$[C_3]$ and $[T_3]$ are by definition 3x3 hermitian positive semi-definite matrices & contain in general 9 independent parameters

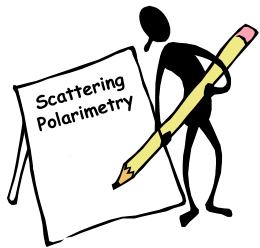





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Scattering Polarimetry

INTERPRETATION OF SCATTERING MECHANISMS

- Directly from the Scattering Matrix
- Model based

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Interpretation of Scattering Mechanisms

Scattering Matrix: $[S] = \begin{bmatrix} S_{HH} & S_{HV} \\ S_{HV} & S_{VV} \end{bmatrix}$

Scattering Vector: $\bar{k}_p = \frac{1}{\sqrt{2}} [S_{HH} + S_{VV} \quad S_{HH} - S_{VV} \quad 2S_{XX}]^T$

Unitary Representation:

$$\bar{e} = \frac{1}{|\bar{k}_p|} \bar{k}_p = \frac{1}{\sqrt{2}} \begin{bmatrix} \cos\alpha \exp(i\phi_1) \\ \sin\alpha \cos\beta \exp(i\phi_2) \\ \sin\alpha \sin\beta \exp(i\phi_3) \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} S_{HH} + S_{VV} \\ S_{HH} - S_{VV} \\ 2S_{XX} \end{bmatrix}$$

Parameterisation of \bar{e} in terms of five angles: $\alpha, \beta, \phi_1, \phi_2, \phi_3$

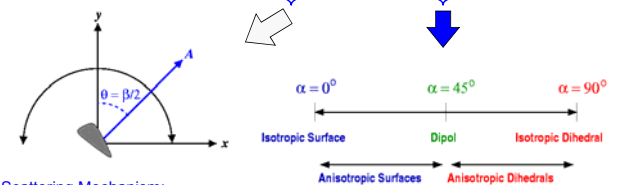


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Interpretation of Scattering Mechanisms

Point Reduction Theorem:

$$\bar{e} = \begin{bmatrix} \exp(i\phi_1) & 0 & 0 \\ 0 & \exp(i\phi_2) & 0 \\ 0 & 0 & \exp(i\phi_3) \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\beta & -\sin\beta \\ 0 & \sin\beta & \cos\beta \end{bmatrix} \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$



Change of Scattering Mechanism:

$$\bar{e}' = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\Delta\beta & -\sin\Delta\beta \\ 0 & \sin\Delta\beta & \cos\Delta\beta \end{bmatrix} \bar{e} \quad \bar{e}' = \begin{bmatrix} \cos\Delta\alpha & -\sin\Delta\alpha & 0 \\ \sin\Delta\alpha & \cos\Delta\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \bar{e}$$



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Interpretation of Scattering Mechanisms

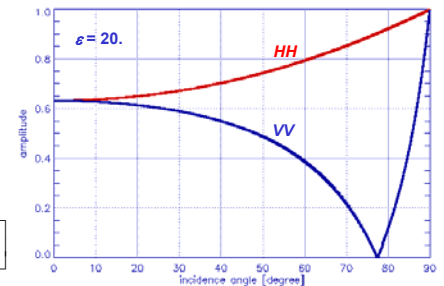
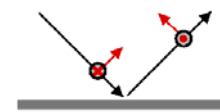
$$\bar{e}' = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\beta & -\sin\beta \\ 0 & \sin\beta & \cos\beta \end{bmatrix} \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad \text{where} \quad \bar{e} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} S_{HH} + S_{VV} \\ S_{HH} - S_{VV} \\ 2S_{HH} \end{bmatrix}$$

- $\alpha=90^\circ$ and $\beta=0^\circ$ \rightarrow $\bar{e}' = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ **Dihedral Scatterer**
- $\alpha=45^\circ$ and $\beta=0^\circ$ \rightarrow $\bar{e}' = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{bmatrix}$ **H-Dipole Scatterer**
- $\alpha=45^\circ$ and $\beta=180^\circ$ \rightarrow $\bar{e}' = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \\ 0 \end{bmatrix}$ **V-Dipole Scatterer**



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Scattering Processes: Fresnel Scattering



Scattering Matrix: $[S] = \begin{bmatrix} R_H & 0 \\ 0 & R_V \end{bmatrix}$

Fresnel Reflection Coefficients:

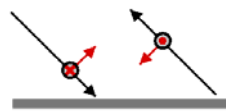
$$R_H = \frac{\cos\theta - \sqrt{\epsilon - \sin^2\theta}}{\cos\theta + \sqrt{\epsilon - \sin^2\theta}} \quad \text{and} \quad R_V = \frac{\epsilon \cos\theta - \sqrt{\epsilon - \sin^2\theta}}{\epsilon \cos\theta + \sqrt{\epsilon - \sin^2\theta}}$$

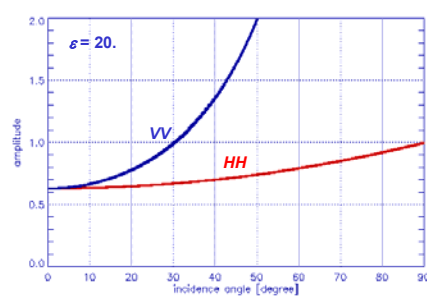
... where ϵ is the dielectric constant of the surface



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Scattering Processes: Bragg Scattering





Scattering Matrix: $[S] = \begin{bmatrix} R_H & 0 \\ 0 & R_V \end{bmatrix}$

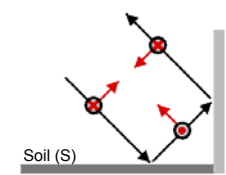
Bragg Scattering Coefficients:

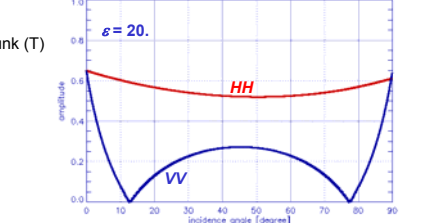
$$R_H = \frac{\cos\theta - \sqrt{\epsilon - \sin^2\theta}}{\cos\theta + \sqrt{\epsilon - \sin^2\theta}} \quad \text{and} \quad R_V = \frac{(\epsilon - 1)[\sin^2\theta - \epsilon(1 + \sin^2\theta)]}{\epsilon \cos\theta + \sqrt{\epsilon - \sin^2\theta}}$$

... where ϵ is the dielectric constant of the surface

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Scattering Processes: Dihedral Scattering





Scattering Matrix:

$$[S] = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{bmatrix} \begin{bmatrix} R_{HS} & 0 \\ 0 & R_{VS} \end{bmatrix} \begin{bmatrix} R_{HT} & 0 \\ 0 & R_{VT} \end{bmatrix} = \begin{bmatrix} R_{HS}R_{HT} & 0 \\ 0 & -R_{VS}R_{VT}e^{i\theta} \end{bmatrix}$$

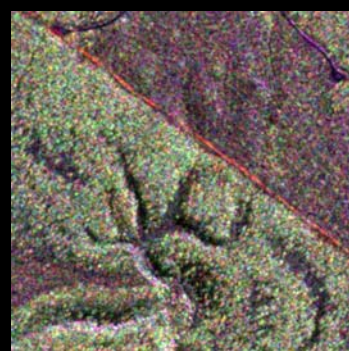
Fresnel Coefficients:

$$R_{HS} = \frac{\cos\theta - \sqrt{\epsilon_S - \sin^2\theta}}{\cos\theta + \sqrt{\epsilon_S - \sin^2\theta}} \quad \text{and} \quad R_{VS} = \frac{\epsilon_S \cos\theta - \sqrt{\epsilon_S - \sin^2\theta}}{\epsilon_S \cos\theta + \sqrt{\epsilon_S - \sin^2\theta}}$$

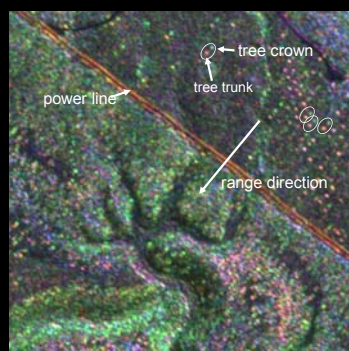
$$R_{HT} = \frac{\cos(90-\theta) - \sqrt{\epsilon_T - \sin^2(90-\theta)}}{\cos(90-\theta) + \sqrt{\epsilon_T - \sin^2(90-\theta)}} \quad \text{and} \quad R_{VT} = \frac{\epsilon_S \cos(90-\theta) - \sqrt{\epsilon_S - \sin^2(90-\theta)}}{\epsilon_S \cos(90-\theta) + \sqrt{\epsilon_S - \sin^2(90-\theta)}}$$

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Dihedral vs Volume Scattering (L-/P-Band E-SAR – Kryklan/Sweden)



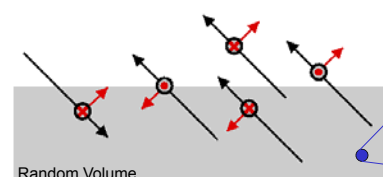
08biosar0201x1_t11 L-band, hh hv vv

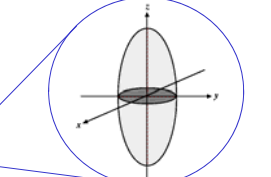


08biosar0302x1_t12 P-band, hh hv vv

DLR Slide 27

Scattering Processes: Volume Scattering





Random Volume

$$\bar{k}(\alpha, \beta, \gamma) = [R_x(\alpha)] [R_y(\beta)] [R_z(\gamma)] \bar{k} \quad \leftarrow \quad [S] = \begin{bmatrix} a & b \\ b & d \end{bmatrix} \rightarrow \bar{k} = \frac{1}{\sqrt{2}} \begin{bmatrix} a+b \\ a-b \\ 2c \end{bmatrix}$$

Coherency Matrix: $[T] = \langle \bar{k}(\alpha, \beta, \gamma) \cdot \bar{k}^+(\alpha, \beta, \gamma) \rangle$

- $0 \leq \alpha \leq 2\pi$ Rotation about x-axis
- $0 \leq \beta \leq \pi$ Rotation about y-axis
- $0 \leq \gamma \leq 2\pi$ Rotation about z-axis

$$[R_x(\alpha)] = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad [R_y(\beta)] = \begin{bmatrix} \cos\beta & 0 & -\sin\beta \\ 1 & 0 & 0 \\ \sin\beta & 0 & \cos\beta \end{bmatrix} \quad [R_z(\gamma)] = \begin{bmatrix} \cos\gamma & \sin\gamma & 0 \\ -\sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

DLR Slide 28

Scattering Processes: Volume Scattering

Principal Polarisability Matrix:

$$[P] = \begin{bmatrix} \rho_x & 0 & 0 \\ 0 & \rho_y & 0 \\ 0 & 0 & \rho_z \end{bmatrix} \quad \text{with} \quad \rho_i = \frac{V}{4\pi(L_i + 1/(\epsilon_r - 1))}$$

where $V = \frac{3}{4}\pi \frac{L_x}{2} \frac{L_y}{2} \frac{L_z}{2}$ is the particle volume and

$$L_i = \int_0^\infty \frac{(l_x l_y l_z) / 8}{2(s + l_x/2)^{3/2} (s + l_y/2)^{3/2} (s + l_z/2)^{3/2}} ds \quad L_x : L_y : L_z = \frac{l_x}{l_x} : \frac{l_y}{l_y} : \frac{l_z}{l_z} \quad L_x + L_y + L_z = 1$$

Particle Anisotropy: $A_p = \frac{L_x(\epsilon_r - 1) + 1}{L_y(\epsilon_r - 1) + 1} = \frac{m\epsilon_r + 2}{m + \epsilon_r + 1}$

Particle Shape Ratio: $m = \frac{L_x}{L_y} = \frac{l_y}{l_x} \quad 0 \leq m \leq \infty$

- $m > 1$ oblate spheroids
- $m < 1$ prolate spheroids

DLR Slide 29

Scattering Processes: Volume Scattering

$$[P](\alpha, \beta, \gamma) = [R_s(\alpha)] [R_s(\beta)] [R_s(\gamma)] [P] [R_s(\gamma)]^T [R_s(\beta)]^T [R_s(\alpha)]^T$$

Coherency Matrix: $[T] = \iiint_{\alpha, \beta, \gamma} [P](\alpha, \beta, \gamma) p(\alpha) p(\beta) p(\gamma) d\alpha d\beta d\gamma$

where $p(\alpha), p(\beta), p(\gamma)$ are the pdfs for the corresponding angle distributions

Special Case: Random Volume

$$[T] = a \begin{bmatrix} 1 & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & b \end{bmatrix} \quad \text{where} \quad b = \frac{(A_p - 1)^2}{2(2 + 6A_p + 7A_p^2)} \quad 0 \leq b \leq 0.5$$

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Scattering Processes: Volume Scattering

Prolate Particles

H/a Loci for varying particle shape and width of distribution

DLR

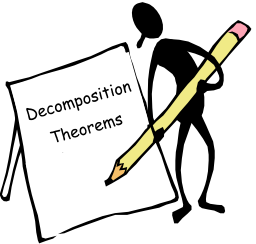
Decomposition Theorems



<p>[S]</p> <p>COHERENT DECOMPOSITION</p> <p>E. KROGAGER (1990)</p> <p>W.L. CAMERON (1990)</p>	<p>[T]</p> <p>EIGENVECTORS BASED DECOMPOSITION</p> <p>S.R. CLOUDE (1985)</p> <p>W.A. HOLM (1988)</p> <p>EIGENVECTORS / EIGENVALUES ANALYSIS & MODEL BASED DECOMPOSITION</p> <p>EIGENVECTORS / EIGENVALUES ANALYSIS ENTROPY / ANISOTROPY</p> <p>S.R. CLOUDE - E. POTTIER (1996-1997)</p>	<p>[C]</p> <p>AZIMUTHAL SYMMETRY</p> <p>MODEL BASED DECOMPOSITION</p> <p>A.J. FREEMAN (1992)</p> <p>J.J. VAN ZYL (1992)</p>
<p>[K]</p> <p>TARGET DICHOTOMY</p> <p>J.R. HUYNEN (1970)</p> <p>R.M. BARNES (1988)</p>		

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Decomposition Theorems

- Pauli Matrix Decomp.
- Model Based Decomp.
- Eigenvector Decomp.





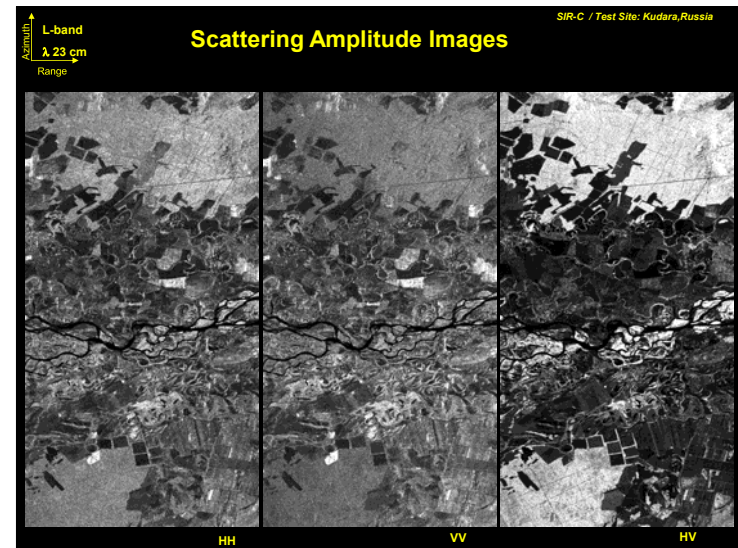
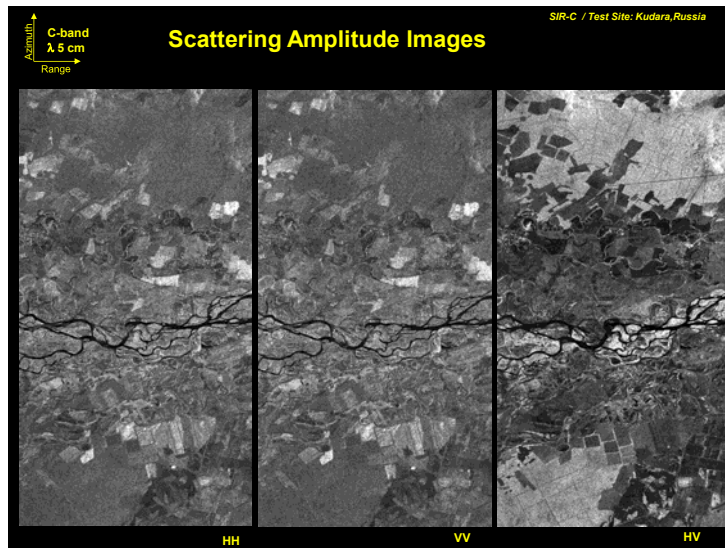


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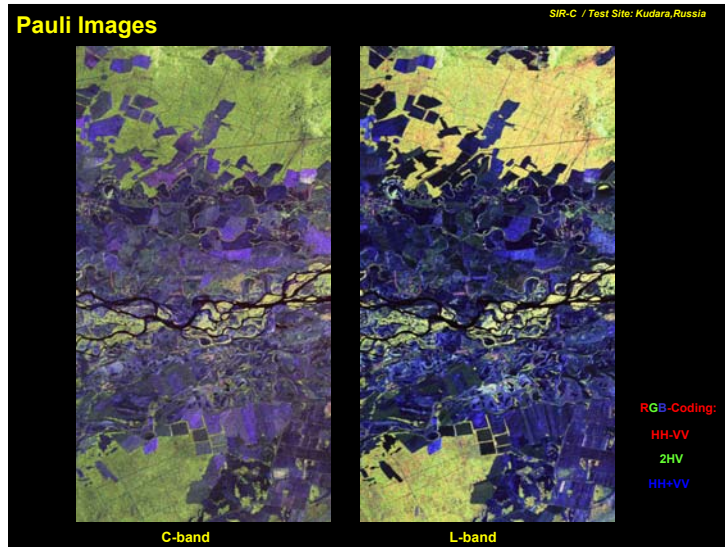
Pauli Matrices Decomposition

$$[S] = \begin{bmatrix} S_{HH} & S_{HV} \\ S_{VH} & S_{VV} \end{bmatrix} = \begin{bmatrix} a+b & c-id \\ c+id & a-b \end{bmatrix} = a \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + b \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} + c \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} + d \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

- $[S_s] = a \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ Single Scattering: $S_{HH} = S_{VV}$
- $[S_d] = b \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ Dihedral Scattering: $S_{HH} = -S_{VV}$
- $[S_r] = c \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ Dihedral Scattering (... rotated by $\pi/2$ about the LOS)
- $[S_j] = d \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$ Transforms all polarisation states into their orthogonal states (disappears in backscattering)



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Decomposition Theorems

- Pauli Matrix Decomp.
- **Eigenvector Decomp.**
- Model Based Decomp.

DLR

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Eigenvector Decomposition

Coherence Matrix: $[T_3] := \langle \vec{k}_{3p}, \vec{k}_{3p}^* \rangle$

Diagonalisation: $[T_3] = [U_3][\Lambda][U_3]^t$

$$[T_3] = \sum_{i=1}^3 \lambda_i (\vec{e}_i \cdot \vec{e}_i^*) = \lambda_1 (\vec{e}_1 \cdot \vec{e}_1^*) + \lambda_2 (\vec{e}_2 \cdot \vec{e}_2^*) + \lambda_3 (\vec{e}_3 \cdot \vec{e}_3^*) = [T_3^1] + [T_3^2] + [T_3^3]$$

↓ ↓ ↓
 $[S_1]$ $[S_2]$ $[S_3]$

- 3 real positive eigenvalues
- 3 orthonormal eigenvectors

$$\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq 0$$

$$\vec{e}_1 = \begin{bmatrix} e_{11} \\ e_{21} \\ e_{31} \end{bmatrix} \quad \vec{e}_2 = \begin{bmatrix} e_{12} \\ e_{22} \\ e_{32} \end{bmatrix} \quad \vec{e}_3 = \begin{bmatrix} e_{13} \\ e_{23} \\ e_{33} \end{bmatrix}$$

where: $[\Lambda] = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}$ $[U_3] = \begin{bmatrix} e_{11} & e_{12} & e_{13} \\ e_{21} & e_{22} & e_{23} \\ e_{31} & e_{32} & e_{33} \end{bmatrix}$ $[U_3]^{-1} = [U_3]^* = \begin{bmatrix} e_{11}^* & e_{21}^* & e_{31}^* \\ e_{12}^* & e_{22}^* & e_{32}^* \\ e_{13}^* & e_{23}^* & e_{33}^* \end{bmatrix}$

DLR

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Scattering Entropy / Anisotropy

Coherency Matrix Diagonalisation:

$$[T_3] = \sum_{i=1}^3 \lambda_i (\vec{e}_i \cdot \vec{e}_i^*) = \lambda_1 (\vec{e}_1 \cdot \vec{e}_1^*) + \lambda_2 (\vec{e}_2 \cdot \vec{e}_2^*) + \lambda_3 (\vec{e}_3 \cdot \vec{e}_3^*) = [T_3^1] + [T_3^2] + [T_3^3]$$

↓

Scattering Entropy: $H := \sum_{i=1}^3 P_i \log_3 P_i$ with: $P_i := \frac{\lambda_i}{\lambda_1 + \lambda_2 + \lambda_3}$

$0 \leq H \leq 1$ where:

- $H = 0$ → Totally Polarised Scatterer
- $H = 1$ → Totally Unpolarised Scatterer

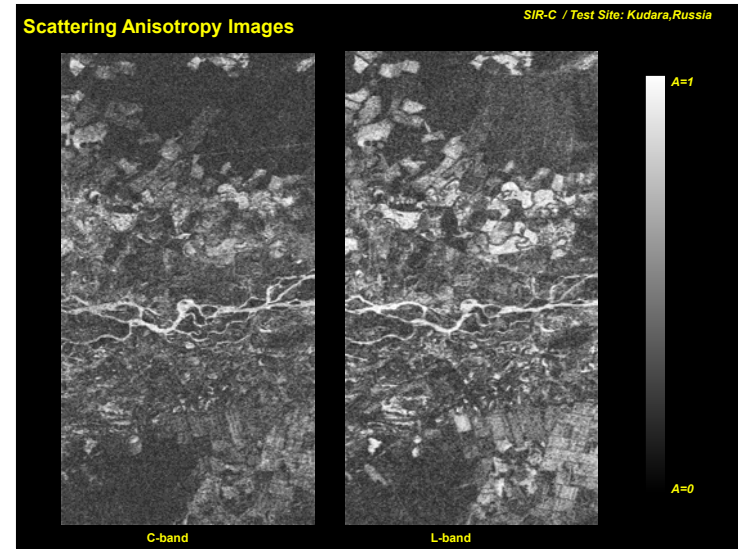
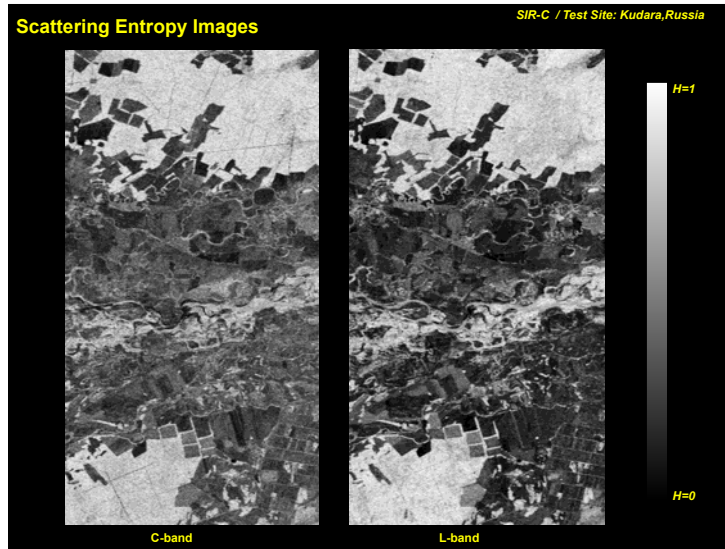
Scattering Anisotropy: $A := \frac{\lambda_2 - \lambda_3}{\lambda_2 + \lambda_3} = \frac{P_2 - P_3}{P_2 + P_3}$

$0 \leq A \leq 1$ where:

- $A = 0$ → 2 Equal Secondary Scattering Processes
- $A = 1$ → Only 1 Secondary Scattering Process

DLR

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Mean Scattering Parameters: Eigenvector

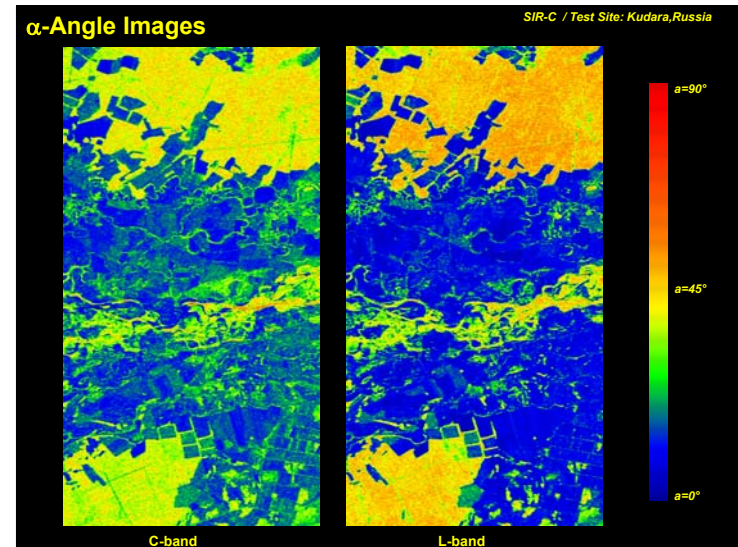
Coherency Matrix Diagonalisation: $[T_3] = \lambda_1(\bar{e}_1 \cdot \bar{e}_1^*) + \lambda_2(\bar{e}_2 \cdot \bar{e}_2^*) + \lambda_3(\bar{e}_3 \cdot \bar{e}_3^*)$

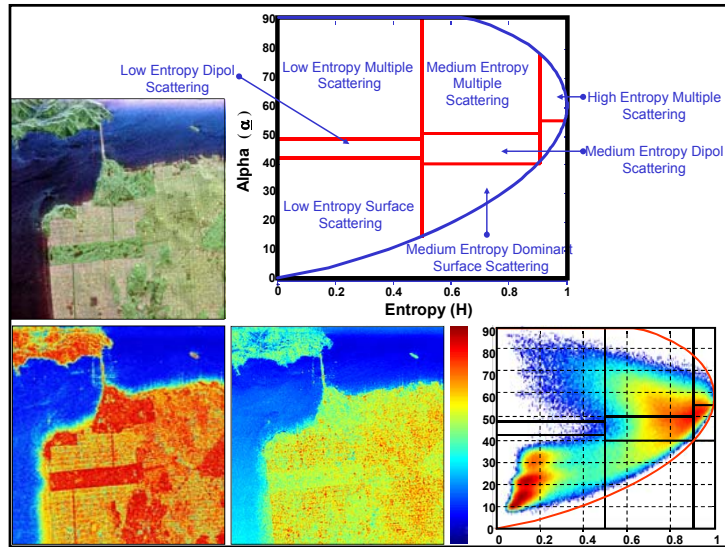
Eigenvectors $\bar{e}_i = \begin{bmatrix} \cos(\alpha_i) \exp(i\gamma_i) \\ \sin(\alpha_i) \cos(\beta_i) \exp(i\delta_i) \\ \sin(\alpha_i) \sin(\beta_i) \exp(i\varepsilon_i) \end{bmatrix}$ Appearance Probabilities: $P_i := \frac{\lambda_i}{\lambda_1 + \lambda_2 + \lambda_3}$

Mean α -Angle: $\alpha = P_1 \alpha_1 + P_2 \alpha_2 + P_3 \alpha_3$
Mean β -Angle: $\beta = P_1 \beta_1 + P_2 \beta_2 + P_3 \beta_3$
Mean γ -Angle: $\gamma = P_1 \gamma_1 + P_2 \gamma_2 + P_3 \gamma_3$
Mean δ -Angle: $\delta = P_1 \delta_1 + P_2 \delta_2 + P_3 \delta_3$
Mean ε -Angle: $\varepsilon = P_1 \varepsilon_1 + P_2 \varepsilon_2 + P_3 \varepsilon_3$

} Mean Scattering Mechanism $\bar{e} = \begin{bmatrix} \cos(\alpha) \exp(i\gamma) \\ \sin(\alpha) \cos(\beta) \exp(i\delta) \\ \sin(\alpha) \sin(\beta) \exp(i\varepsilon) \end{bmatrix}$

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Decomposition Theorems

- Pauli Matrix Decomp.
- Eigenvector Decomp.
- Model Based Decomp.

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Freeman 3 Component Decomposition

Vegetation Scattering : Bragg-Scattering + Dihedral Scattering + Random Volume of Dipoles

$$[T] = [T_s] + [T_D] + [T_v]$$

where $[T_s] = f_s \begin{bmatrix} \beta^2 & \beta & 0 \\ \beta & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ $[T_D] = f_D \begin{bmatrix} \alpha^2 & -\alpha & 0 \\ -\alpha & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ and $[T_v] = f_v \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$[T] = \begin{bmatrix} f_s \beta^2 + f_D \alpha^2 + 2f_v & f_s \beta - f_D \alpha & 0 \\ f_s \beta - f_D \alpha & f_s + f_D + f_v & 0 \\ 0 & 0 & f_v \end{bmatrix}$$

4 Equations for 5 Unknowns

- $\text{Re}(\text{HHVV}) - f_v/3 > 0 \rightarrow$ Single bounce is dominant \rightarrow alpha = 1
- $\text{Re}(\text{HHVV}) - f_v/3 < 0 \rightarrow$ Double bounce is dominant \rightarrow beta = 1

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3 Component Freeman Decomposition (San Francisco)

AirSAR JPL

Pauli $|\text{HH-VV}|, |\text{HV}|, |\text{HH+VV}|$ Freeman/Durden, P_D, P_V, P_S

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Freeman 2 Component Decomposition

$$[T] = [T_{S/D}] + [T_V] \quad \begin{bmatrix} T_{11} & T_{12} & 0 \\ T_{12}^* & T_{22} & 0 \\ 0 & 0 & T_{33} \end{bmatrix} = f_G \begin{bmatrix} |\alpha|^2 & \alpha & 0 \\ \alpha^* & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} + f_V \begin{bmatrix} 1+\rho & 0 & 0 \\ 0 & 1-\rho & 0 \\ 0 & 0 & 1-\rho \end{bmatrix}$$

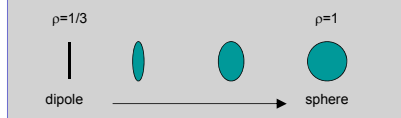
Scattering Power Components:

$$P_G = f_G(1 + |\alpha|^2) \quad P_V = f_V(3 - \rho)$$

Distinction between dihedral and surface of the ground component:

$|\alpha| < 1$ dihedral
 $|\alpha| \geq 1$ surface

Volume shape parameter ρ of a randomly oriented volume:



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Summary: Polarimetric Parameters / Scattering Matrix

$$[S] = \begin{bmatrix} S_{HH} & S_{XX} \\ S_{XX} & S_{VV} \end{bmatrix} \rightarrow [S] = \frac{\exp(i\kappa r) \exp(i\varphi_{VV})}{r} \begin{bmatrix} |S_{HH}| \exp(i(\varphi_{HH} - \varphi_{VV})} & |S_{XX}| \exp(i(\varphi_{XX} - \varphi_{VV})} \\ |S_{XX}| \exp(i(\varphi_{XX} - \varphi_{VV})} & |S_{VV}| \end{bmatrix}$$

Absolute Phase Factor **Five Parameters: 3 Amplitudes & 2 Phases**

• Scattring Amplitudes $\sigma_{HH}^0 := |S_{HH} S_{HH}^*|_N$ $\sigma_{XX}^0 := |S_{XX} S_{XX}^*|_N$ $\sigma_{VV}^0 := |S_{VV} S_{VV}^*|_N$

• Total Power $TP := |S_{VV}|^2 + 4|S_{XX}|^2 + |S_{HH}|^2$

• Amplitude Ratios $\sigma_{HH}^0 / \sigma_{VV}^0$ $\sigma_{XX}^0 / \sigma_{VV}^0$ $\sigma_{XX}^0 / (\sigma_{HH}^0 + \sigma_{VV}^0)$

• Pol Phase Differences $\varphi_{HHVV} := \varphi_{HH} - \varphi_{VV}$

• Helicity $Hel := |S_{LL}| - |S_{RR}|$



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Summary: Second Order Polarimetric Parameters

$$\vec{k}_3 = \begin{bmatrix} A \\ B \\ C \end{bmatrix} \rightarrow [X_3] := \langle \vec{k}_3 \cdot \vec{k}_3^* \rangle = \begin{bmatrix} \langle AA^* \rangle & \langle AB^* \rangle & \langle AC^* \rangle \\ \langle BA^* \rangle & \langle BB^* \rangle & \langle BC^* \rangle \\ \langle CA^* \rangle & \langle CB^* \rangle & \langle CC^* \rangle \end{bmatrix}$$

Pauli Scat. Vector	Lexico. Scat. Vector
$A := \frac{1}{\sqrt{2}}(S_{HH} + S_{VV})$	$A := S_{HH}$
$B := \frac{1}{\sqrt{2}}(S_{HH} - S_{VV})$	$B := \sqrt{2}S_{XX}$
$C := \sqrt{2}S_{XX}$	$C := S_{VV}$

Nine Parameters: 3 Real & 3 Complex Elements

• Correlation Coefficients $\gamma_{HHVV} := \frac{|\langle S_{HH} S_{VV}^* \rangle|}{\sqrt{|\langle S_{HH} S_{HH}^* \rangle| |\langle S_{VV} S_{VV}^* \rangle|}}$ $\gamma_{LLRR} := \frac{|\langle S_{LL} S_{RR}^* \rangle|}{\sqrt{|\langle S_{LL} S_{LL}^* \rangle| |\langle S_{RR} S_{RR}^* \rangle|}}$

• Scattering Entropy $H := \sum_{i=1}^3 P_i \log_3 P_i$ $P_i := \frac{\lambda_i}{\lambda_1 + \lambda_2 + \lambda_3}$

• Scattering Anisotropy $A := \frac{\lambda_2 - \lambda_3}{\lambda_2 + \lambda_3} = \frac{P_2 - P_3}{P_2 + P_3}$

• Polarimetric Alpha Angle $\alpha := P_1 \alpha_1 + P_2 \alpha_2 + P_3 \alpha_3$

• Polarimetric Beta Angle $\beta := P_1 \beta_1 + P_2 \beta_2 + P_3 \beta_3$



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Applications of Radar Polarimetry

Agriculture/Land-Use

- Crop Classification/Moisture Content Estimation
- Urban Area mapping
- Urban Topography for Mobilecomms

Forestry

- Biomass Estimation: (Saturates For High Biomass)
 - C-band saturation at 50 tons/hectare
 - L-band saturation at 100 tons/hectare
 - P band saturation at 200 tons/hectare
- Deforestation
- Forest Canopy Height Estimation
- Tree Species Discrimination
- Forest Re-growth Monitoring

Geology

- Plays: Smooth Natural Surfaces (rms = 1cm)
- Alluvial fans, Sand Dunes, Moraines
- Sedimentary Rock Formations
- Lava Flows (extreme in surface roughness)
- Weathering Erosion Studies
- Surface Roughness Estimates

Hydrology

- Flood mapping/Forest Inundation
- Snow Hydrology
- Soil Moisture

Sea Ice/Oceanography

- Ice Roughness/Thickness Studies
- Polar Ice Cap Studies
- Extra-Terrestrial Ice/Water Studies

Meteorology

- Rain rate estimation
- Water/Ice particle studies
- Severe Storm/Flood warning

Topography/Cartography

- Direct Surface Slope Estimation
- Accurate DEM Generation
- Difference of DEMs for Vegetation mapping

Humanitarian Demining


- Surface Penetrating Radar (SPR)
- SAR for Mine Field Detection



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Soil Moisture Estimation

- Surface characterisation
- mv estimation over bare surface
- mv estimation under the vegetation

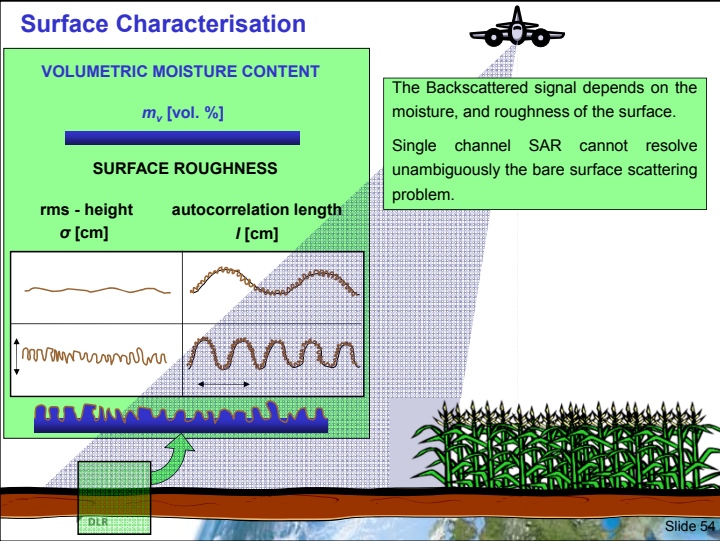


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Surface Characterisation

VOLUMETRIC MOISTURE CONTENT
 m_v [vol. %]

SURFACE ROUGHNESS
 rms - height σ [cm] autocorrelation length l [cm]



The Backscattered signal depends on the moisture, and roughness of the surface.
 Single channel SAR cannot resolve unambiguously the bare surface scattering problem.

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Models for the Estimation of Bare Surface

Modeling and Inversion

<p>THEORETICAL MODELS</p> <p>Geometrical-Optic 1963 Physical-Optic 1963 <u>Small Perturbation 1988</u> Integral Equation 1992</p> <ul style="list-style-type: none"> • very complex • inversion is restricted possible 	<p>EMPIRICAL EXTENSIONS</p> <p>Shi Model 1997</p> <ul style="list-style-type: none"> • inversion with dual pol observables based on regressions coefficients of a specific test site 	<p>MODEL BASED EXTENSIONS</p> <p>Oh Model 1992 Dubois Model 1995 X-Bragg 1999</p> <ul style="list-style-type: none"> • simple model extension • based on SPM (1st order) with an roughness extension
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Time →

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Models for the Estimation of Vegetated Surface


Modeling and Inversion

<p>THEORETICAL MODELS</p> <p>Vegetation Models (Stiles 2000)</p> <ul style="list-style-type: none"> • very complex scat. models • inversion is not possible 	<p>SCATTERING DECOMPOSITION APPROACHES</p> <p>Eigen-Decomposition Model based-Decomposition</p> <p>Hajnsek 2009 Jagdhuber 2012</p> <ul style="list-style-type: none"> • simple canonical scat. models • inversion possible 	<p>SEMI-MODEL BASED EXTENSIONS</p> <p>Optimisation (physical and numerical)</p> <p>Ari 2010 Neumann 2010</p> <ul style="list-style-type: none"> • extended scat. models • inversion with higher computation time
--	---	--

Time →

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Small Perturbation Model




$$[S] = \begin{bmatrix} S_{HH} & S_{HV} \\ S_{VH} & S_{VV} \end{bmatrix} = j2k \cos(\theta) Z \begin{bmatrix} R_s(\theta, \epsilon_r) & 0 \\ 0 & R_p(\theta, \epsilon_r) \end{bmatrix}$$


Exact solution of Maxwell Equation for $s \rightarrow 0$

$$R_s = \frac{\cos\theta - \sqrt{\epsilon_r - \sin^2\theta}}{\cos\theta + \sqrt{\epsilon_r - \sin^2\theta}} \quad R_p = \frac{(\epsilon_r - 1)(\sin^2\theta - \epsilon_r(1 + \sin^2\theta))}{(\epsilon_r \cos\theta + \sqrt{\epsilon_r - \sin^2\theta})^2}$$

$Z = F.T.(E(x, y))$
 $\theta =$ Incidence angle and $k = 2\pi/\lambda$

Roughness:  Depolarises the incident wave introducing (depolarised) HV and decreasing polarimetric coherence.

Ambiguity: Terrain Slopes introduce also a HV component but this is co-related to HH and VV.

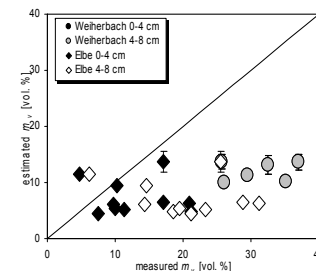
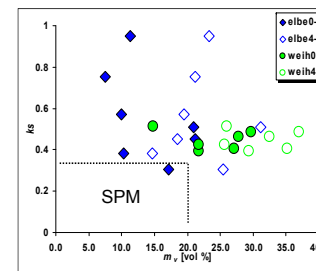
Moisture content:  Effects the scattering amplitude of all polarisations. For "smooth" (with respect to wavelength) surfaces the co-polarimetric ratio (HH/VV) becomes independent of roughness

$\frac{S_{HH}}{S_{VV}} = \frac{R_s}{R_p}$ is independent of roughness

Polarimetry provides an observation space that allows to separate roughness from moisture effects.

DLR Slide 58

Small Perturbation Model (Bragg-Model)

Measured versus estimated volumetric moisture content m_v [vol. %] using SPM

Validity range of the SPM for the ground measurements - surface roughness against volumetric soil moisture

DLR Slide 59

Semi-Empirical Models

Oh-Model

Y. Oh, K. Sarabandi and F.T. Ulaby, "An Empirical Model and an Inversion Technique for Radar Scattering from Bare Soil Surfaces", IEEE Transactions on Geoscience and Remote Sensing, vol. 30, no. 2, pp. 370-381, 1992.

Cross-Pol Amplitude Relation

$$p = \frac{\sigma_{hh}^0}{\sigma_{vv}^0} = 1 - \left(\frac{20}{\pi} \frac{1}{\sin^2\theta} e^{-ks} \right)^2$$

$$q = \frac{\sigma_{hv}^0}{\sigma_{vh}^0} = 0.23 \sqrt{1 - e^{-ks}}$$

s_{vv}^0 : backscattering coefficient for different polarisations
 q : incidence angle in radians
 ϵ_r : real part of the dielectric constant
 s : RMS height
 k : wavenumber ($2\pi/\lambda$)
 G_0 : Fresnel reflectivity of the surface at nadir

Dubois-Model

P. C. Dubois, J. J. van Zyl and J. Engman, "Measuring Soil Moisture with Imaging Radar", IEEE Transactions on Geoscience and Remote Sensing, vol. 33, no. 4, pp. 915-926, 1995.

Co-Pol Amplitude Relation

$$\sigma_{hh}^0 = 10^{-2.75} \frac{\cos^2\theta}{\sin^2\theta} 10^{0.028s \tan\theta} (k \sin\theta)^{1.4} \lambda^{-0.7}$$

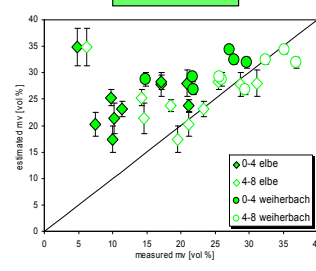
$$\sigma_{vv}^0 = 10^{-2.35} \frac{\cos^2\theta}{\sin^2\theta} 10^{0.046s \tan\theta} (k \sin\theta)^{1.1} \lambda^{-0.7}$$

q : incidence angle in radians
 ϵ_r : real part of the dielectric constant
 s : RMS height
 k : wave number ($2\pi/\lambda$)
 λ : wavelength [cm]

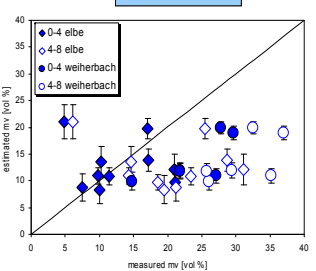
DLR Slide 60

Quantitative Estimation of Volumetric Moisture m_v [vol %]

Dubois



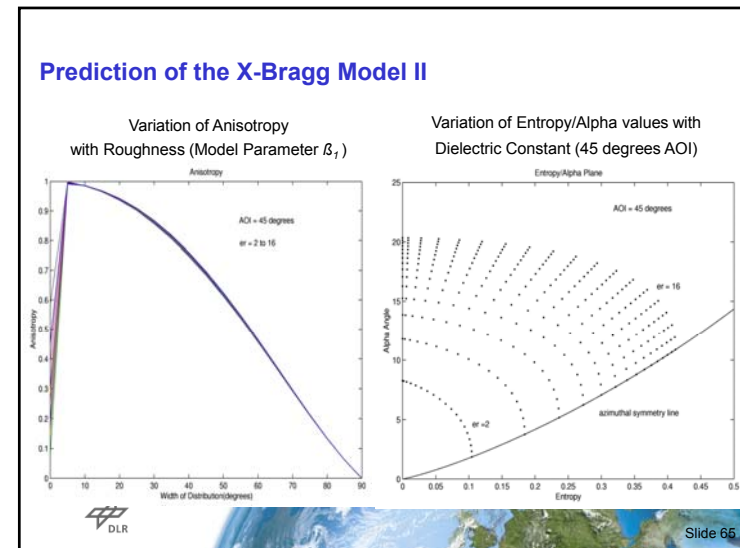
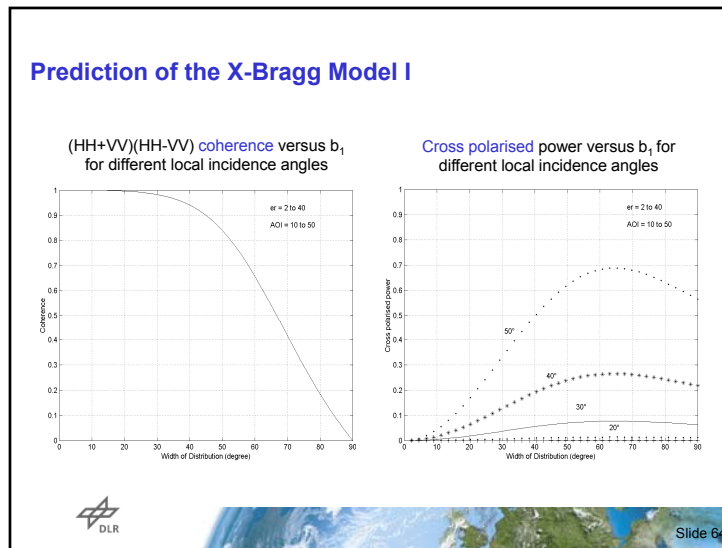
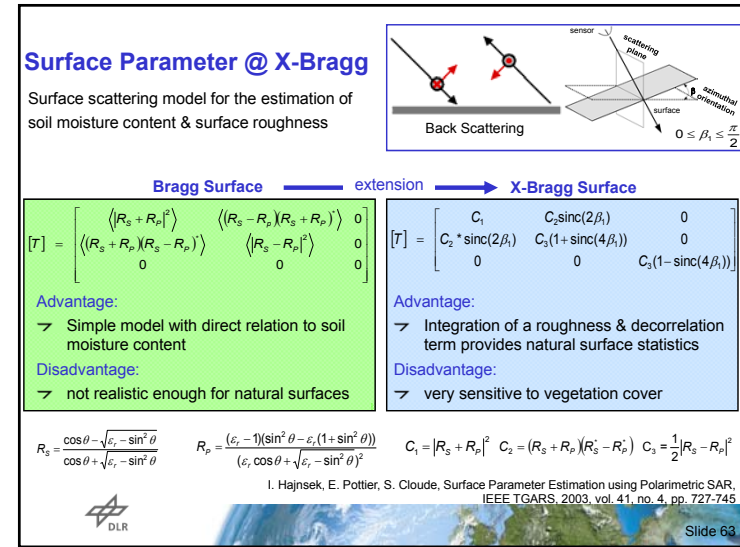
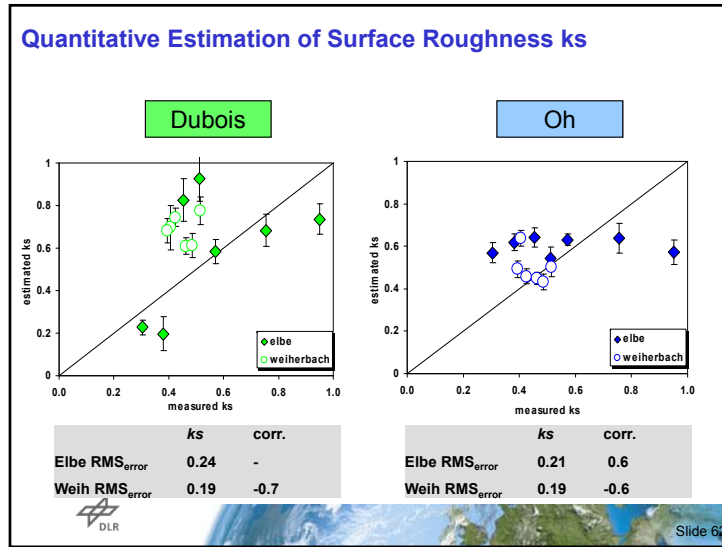
Oh



	0-4 cm	4-8 cm	corr.
Elbe RMS_{error}	14	10	-/-
Weih RMS_{error}	11	17	0.2/0.4

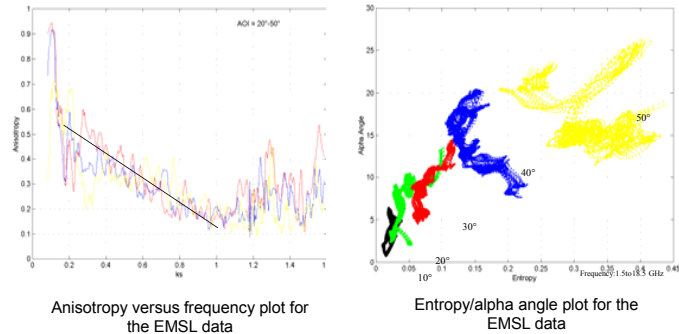
	0-4 cm	4-8 cm	corr.
Elbe RMS_{error}	7	12	-/-
Weih RMS_{error}	18	25	0.5/0.6

DLR Slide 61



Experimental Data from Anechoic Chamber (JRC - Ispra)

Collected in European Microwave Signature Laboratory (EMSL)
quad-pol scattering matrix(HH,VV,HV,VH); $s = 0.4$ cm; surface correlation length $l = 6$;
dielectric constant $\epsilon' = 8$; frequency range = 1.5-18.5



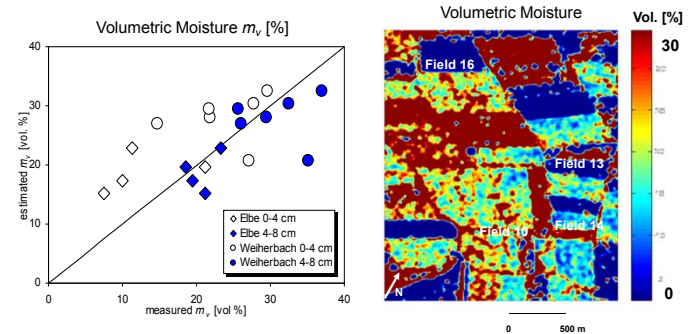
Anisotropy versus frequency plot for the EMSL data

Entropy/alpha angle plot for the EMSL data



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Inversion Results @ X-Bragg – Early Example on ELBE 2000



Volumetric Moisture m_v [%]

Volumetric Moisture

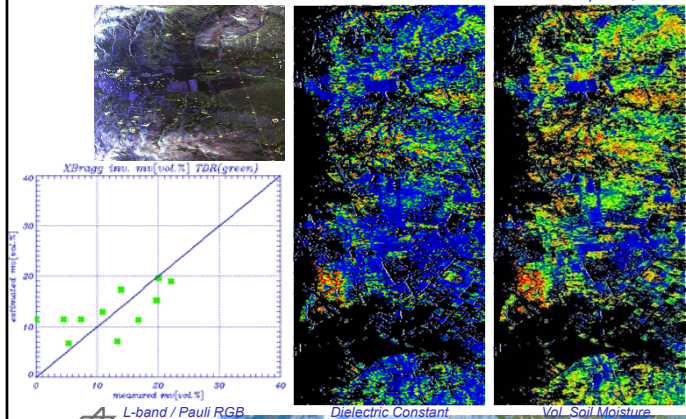
Vol. [%]



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Bare Surfaces: Soil Moisture Estimation @ AQUIFEREX 05

E-SAR / Test Site: Aquiferex, Tunisia



L-band / Pauli RGB

Dielectric Constant

Vol. Soil Moisture



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Soil Moisture Estimation

- Surface Characterisation
- m_v estimation over bare surface
- m_v estimation under the vegetation
 - 3 component model decomp.
 - improvement of the 3 component model decomp.
 - multi-angle approach combined with 3 comp. model decomp.
 - hybrid decomposition



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3 Component Freeman Decomposition

$$[T_{Bragg}] = f_s \begin{bmatrix} 1 & \beta^* & 0 \\ \beta & |\beta|^2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \beta = \frac{R_H - R_V}{R_H + R_V}$$

$$R_H = \frac{\cos\theta - \sqrt{\epsilon_r - \sin^2\theta}}{\cos\theta + \sqrt{\epsilon_r - \sin^2\theta}}$$

$$R_V = \frac{(\epsilon_r - 1)(\sin^2\theta - \epsilon_r(1 + \sin^2\theta))}{(\epsilon_r \cos\theta + \sqrt{\epsilon_r - \sin^2\theta})^2}$$

$$f_s \begin{bmatrix} 1 & \beta^* & 0 \\ \beta & |\beta|^2 & 0 \\ 0 & 0 & 0 \end{bmatrix} + f_D \begin{bmatrix} |\alpha|^2 & \alpha & 0 \\ \alpha^* & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \frac{f_V}{4} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} & 0 \\ T_{12}^* & T_{22} & 0 \\ 0 & 0 & T_{33} \end{bmatrix}$$

Bragg
Surface

Fresnel
Dihedral

Random Volume of Dipoles
Volume

$$[S] = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} R_{SS} & 0 \\ 0 & R_{PS} \end{bmatrix} \begin{bmatrix} R_{ST} & 0 \\ 0 & R_{PT} \end{bmatrix} = \begin{bmatrix} R_{SS}R_{ST} & 0 \\ 0 & -R_{PS}R_{PT}e \end{bmatrix}$$

$$R_{S(S/T)} = \frac{\cos\theta - \sqrt{\epsilon_{S/T} - \sin^2\theta}}{\cos\theta + \sqrt{\epsilon_{S/T} - \sin^2\theta}}$$

$$R_{P(S/T)} = \frac{\epsilon_{S/T} \cos\theta - \sqrt{\epsilon_{S/T} - \sin^2\theta}}{\epsilon_{S/T} \cos\theta + \sqrt{\epsilon_{S/T} - \sin^2\theta}}$$

$$f_s \begin{bmatrix} 1 & \beta^* & 0 \\ \beta & |\beta|^2 & 0 \\ 0 & 0 & 0 \end{bmatrix} + f_D \begin{bmatrix} |\alpha|^2 & \alpha & 0 \\ \alpha^* & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \frac{f_V}{4} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} & 0 \\ T_{12}^* & T_{22} & 0 \\ 0 & 0 & T_{33} \end{bmatrix}$$

Bragg
Surface

Fresnel
Dihedral

Random Volume of Dipoles
Volume

AGRISAR Campaign in Northern Germany 2006

1. Agricultural data base over a whole vegetation growth period - April - August 2006
2. 16 data acquisitions (DLR's E-SAR) & ground measurements
3. Support by ESA for the space segment - Sentinel Program

DEMMIN

Thermal Infrared
Soil Moisture Station
Bowen-Ratio Station
LAS
ASD
Laser-Profiler

DLR

Freeman-Durden 3-Component Decomposition (AGRISAR 2006)

C-band

03/5/2006 14/6/2006 02/8/2006

L-band

03/5/2006 14/6/2006 02/8/2006

Legend: dihedral (red), volume (green), surface (blue)

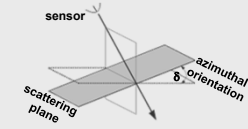
DLR

3 Component Decomposition and Modifications

Surface Dihedral Volume

Decision rule = $\text{Re}\langle S_{HH}S_{VV}^* \rangle > 0$ or $\text{Re}\langle S_{HH}S_{VV}^* \rangle < 0$
 $\alpha = 0$ or $\beta = 0$


Surface modification (X-Bragg) ($\epsilon_s, \theta, \delta$):

$$[T_{SB}] = f_s \begin{bmatrix} 1 & \beta \sin(2\delta) & 0 \\ \beta \sin(2\delta) & \frac{1}{2}|\beta|^2(1 + \sin(4\delta)) & 0 \\ 0 & 0 & \frac{1}{2}|\beta|^2(1 - \sin(4\delta)) \end{bmatrix}$$


→ Incorporation of a roughness and HV-decorrelation term (δ) [Hajsek et al., 2001]

Dihedral modification (modified Fresnel coefficients) ($\epsilon_s, \epsilon_r, \theta, \phi, L_S$):


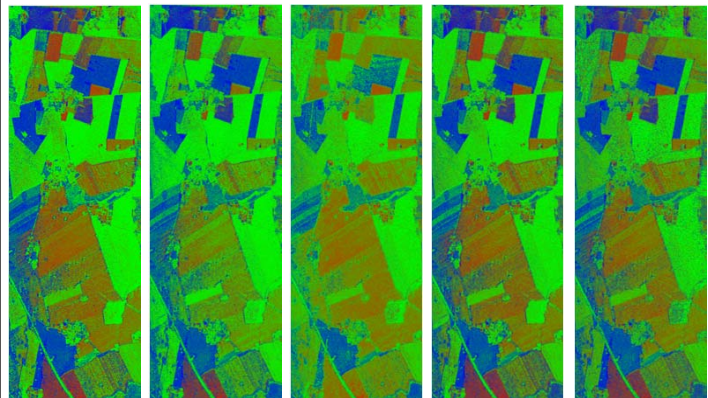
$$[T_{mod}] = f_D \cdot [L_S]^2 \begin{bmatrix} |\alpha|^2 & \alpha & 0 \\ \alpha^* & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad 0 < L_S \leq 1$$

$$L_S = \exp(-2 \cdot (ks)^2 \cdot \cos(\theta)^2) \quad ks = 1 - A$$


→ Incorporation of a surface scattering loss L_S [Lee et al., 2005] [Freeman & Durden, 1998; Yamaguchi et al., 2006]

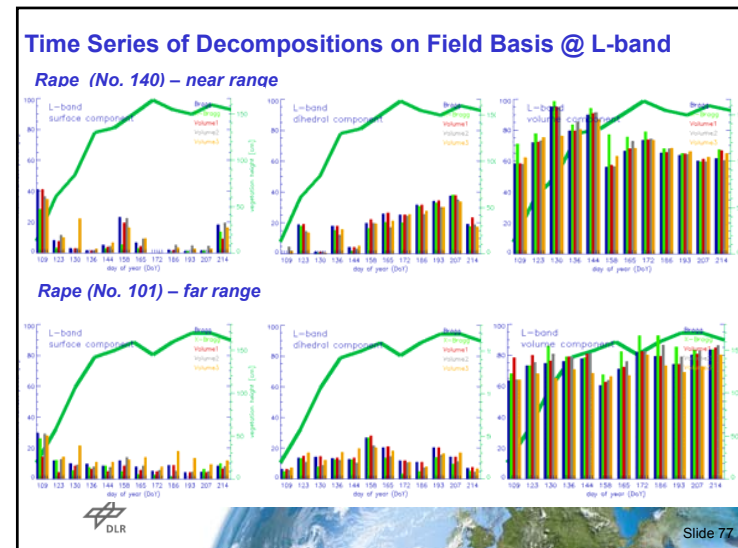
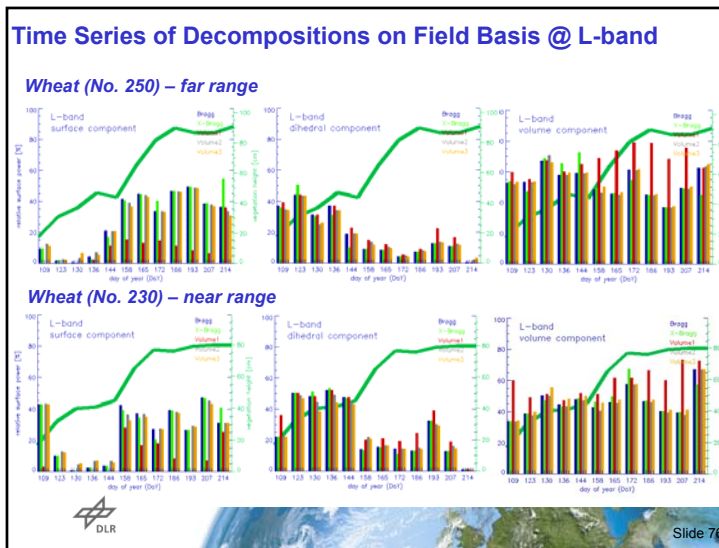
DLR Slide 74

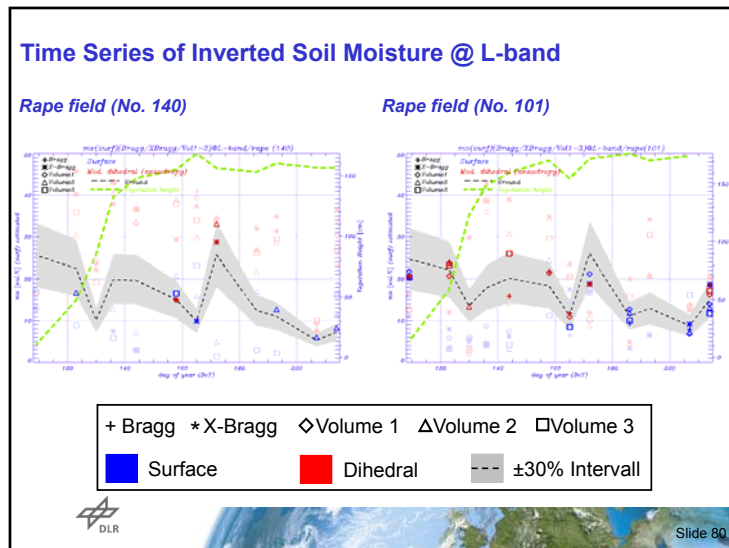
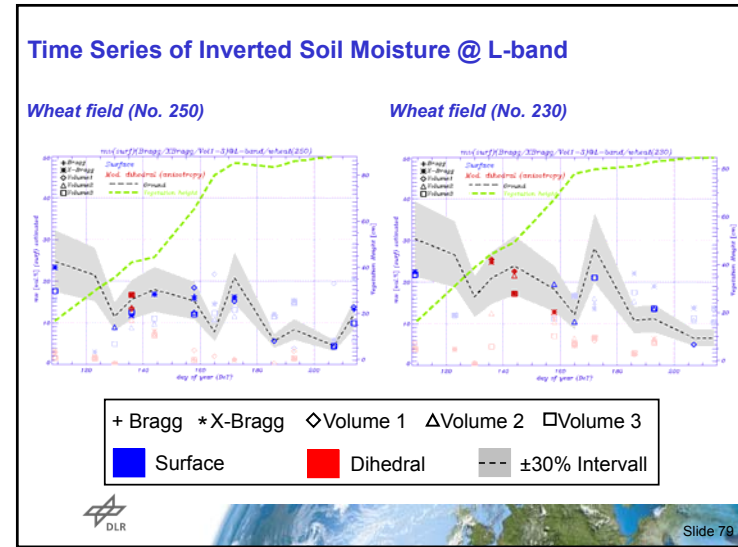
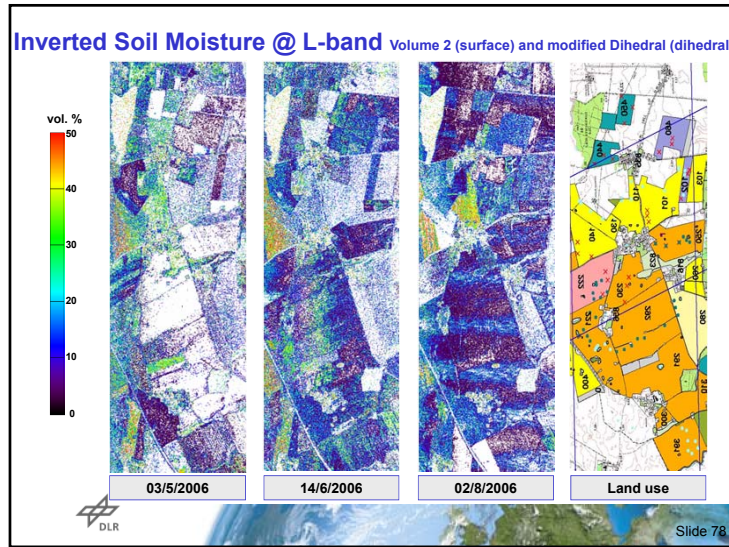
Modified Decompositions @ L-band for 03/5/2006

X-Bragg Modified Dihedral Volume 1 Volume 2 Volume 3

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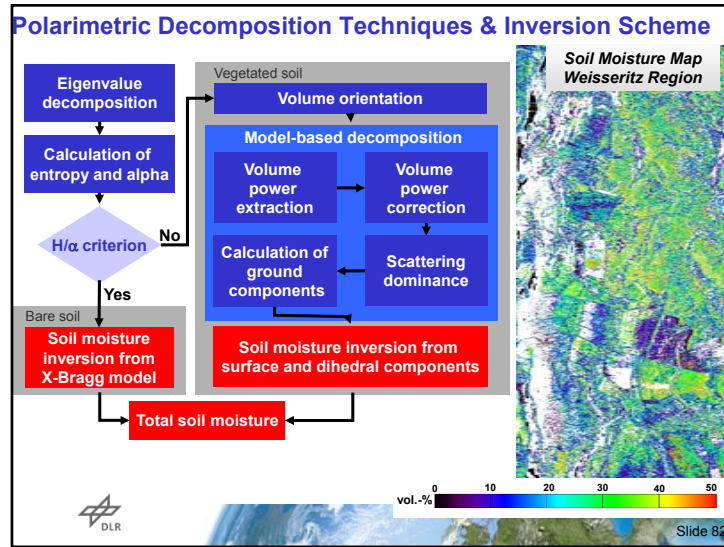




Soil Moisture Estimation

- Surface Characterisation
- mv estimation over bare surface
- **mv estimation under the vegetation**
 - 3 component model decomp.
 - **improvement of the 3 component model decomp.**
 - multi-angle approach combined with 3 comp. model decomp.
 - hybrid decomposition

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Modeling of Volume Orientation

Volume modelling of a cloud of uniformly shaped (ρ) particles with an orientation (τ) in azimuthal direction

Selection of volume orientation by polarisation power ratio P_r : $P_r = 10 \cdot \log \frac{\langle |S_{VV}|^2 \rangle}{\langle |S_{HH}|^2 \rangle}$

General Case	Vertical dipoles [Pr < -2dB]	Random dipoles [-2dB < Pr < 2dB]	Horizontal dipoles [Pr > 2dB]
	$pdf(\tau) = \frac{1}{2} \cdot \sin(\tau)$ $0 < \tau \leq \pi$	$pdf(\tau) = \frac{1}{2\pi}$ $0 < \tau \leq 2\pi$	$pdf(\tau) = \frac{1}{2} \cdot \cos(\tau)$ $-\pi/2 < \tau \leq \pi/2$
$[T_v] = f_v \begin{bmatrix} C1 & C4 & 0 \\ C4 & C2 & 0 \\ 0 & 0 & C3 \end{bmatrix}$	$[T_v] = \frac{f_v}{30} \begin{bmatrix} 15 & 5 & 0 \\ 5 & 7 & 0 \\ 0 & 0 & 8 \end{bmatrix}$	$[T_v] = \frac{f_v}{4} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$[T_v] = \frac{f_v}{30} \begin{bmatrix} 15 & -5 & 0 \\ -5 & 7 & 0 \\ 0 & 0 & 8 \end{bmatrix}$

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Correction of Volume Intensity f_v with Eigen-based analysis

$$\begin{bmatrix} T_{11} & T_{12} & 0 \\ T_{12}^* & T_{22} & 0 \\ 0 & 0 & T_{33} \end{bmatrix} - f_v \cdot \begin{bmatrix} C_1 & C_4 & 0 \\ C_4 & C_2 & 0 \\ 0 & 0 & C_3 \end{bmatrix} = \begin{bmatrix} T_{Surface} \\ T_{Dihedral} \end{bmatrix}$$

Eigenanalysis of Ground Components

Set Eigenvalues to zero (lower calculus limit)
Solutions for f_v :

$$f_{v1} = 4T_{33}$$

$$f_{v2,3} = T_{11} + 2T_{22} \pm \sqrt{T_{11}^2 + 8T_{12} \cdot T_{12}^* - 4T_{11} \cdot T_{22} + 4T_{22}^2}$$

Negative Powers = Too Strong Volume Subtraction!

Find Minimum

Decomposition with Corrected Volume Intensity f_v^{corr}

[van Zyl et al., 2008]

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3 Component Decomposition and Modifications

Surface Dihedral Volume

Scattering dominance = $\text{Re}\langle S_{HH} S_{VV}^* \rangle > 0$ or $\text{Re}\langle S_{HH} S_{VV}^* \rangle < 0$
 $\alpha = 0$ or $\beta = 0$

Surface model (X-Bragg) ($\epsilon_s, \theta, \delta$):

$$[T_{13}] = f_s \begin{bmatrix} 1 & \beta^* \sin c(2\delta) & 0 \\ \beta \sin c(2\delta) & \frac{1}{2} |\beta|^2 (1 + \sin c(4\delta)) & 0 \\ 0 & 0 & \frac{1}{2} |\beta|^2 (1 - \sin c(4\delta)) \end{bmatrix}$$

Dihedral model (Fresnel with vegetation attenuation) ($\epsilon_s, \epsilon_v, \theta, \varphi, L_v$):

$$[T_{mod}] = f_D \cdot L_v \cdot \begin{bmatrix} |\alpha|^2 & \alpha & 0 \\ \alpha^* & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$L_v = \exp(-I(\mu_{max} - \mu_{min}))$

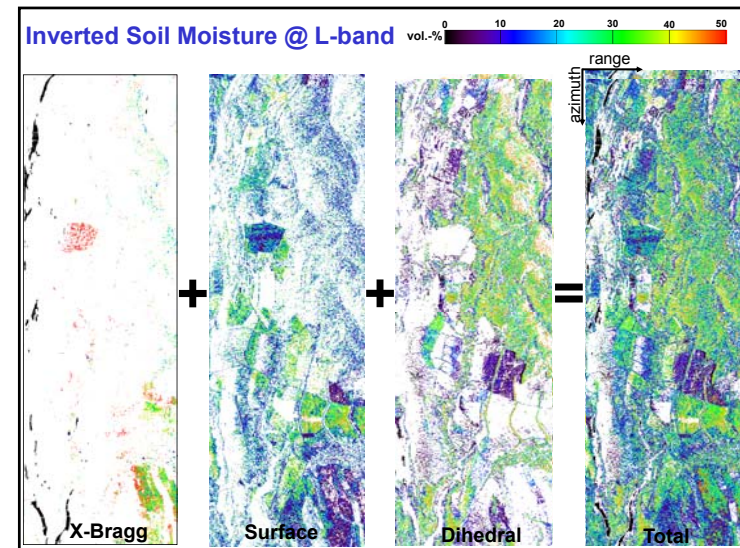
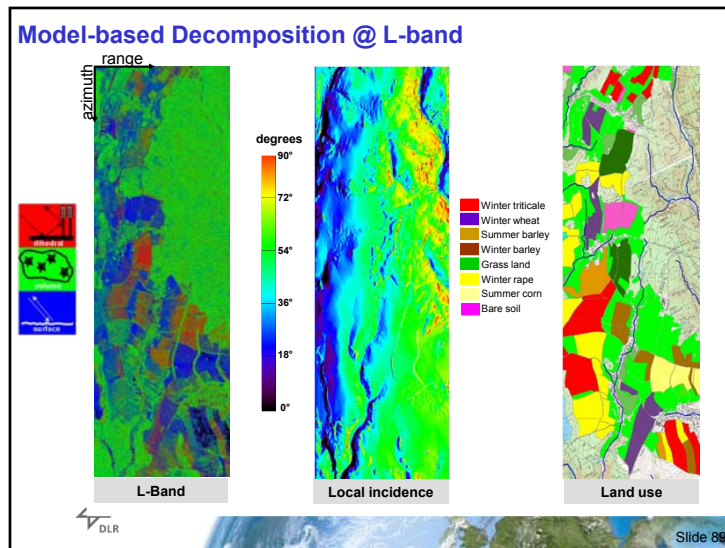
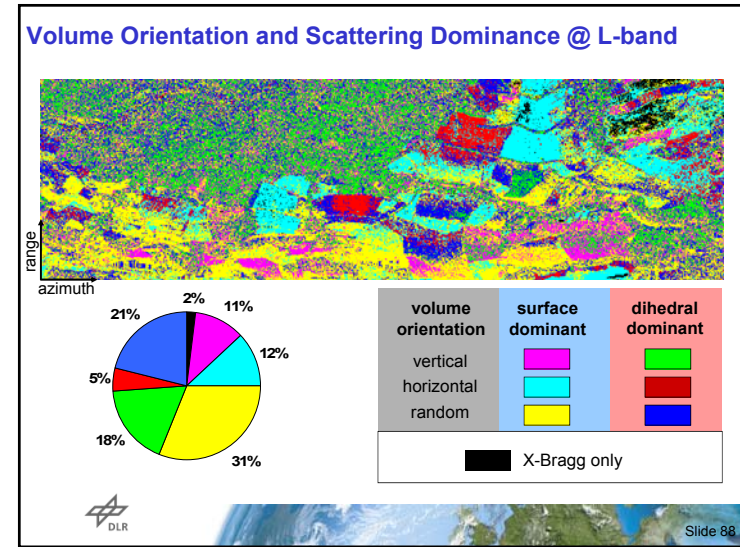
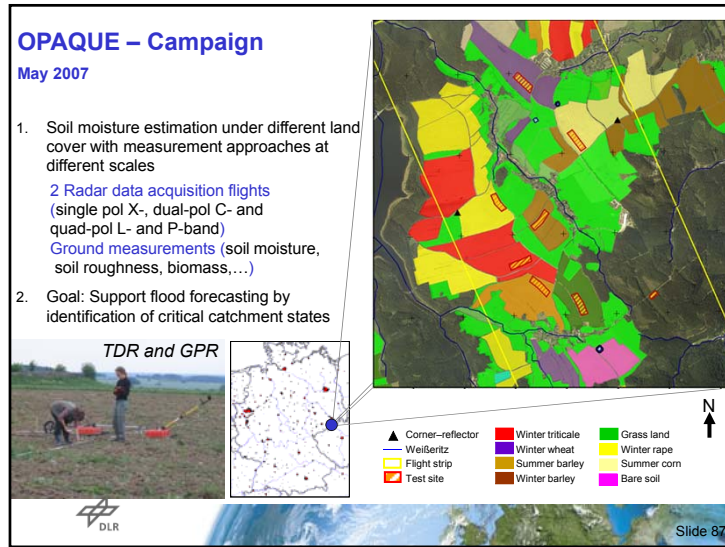
$\mu_{max} = \frac{\lambda_1}{P_r}$

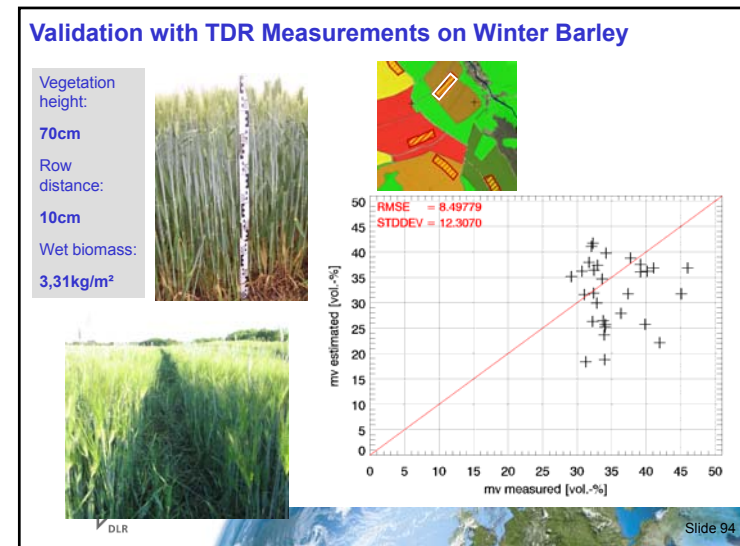
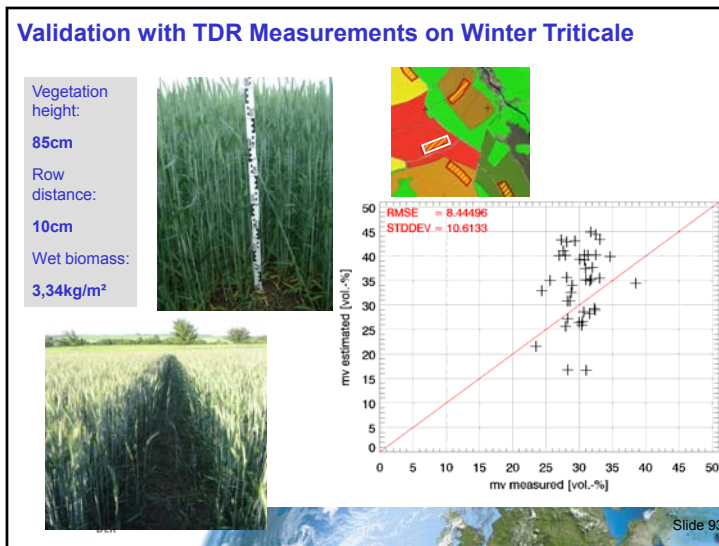
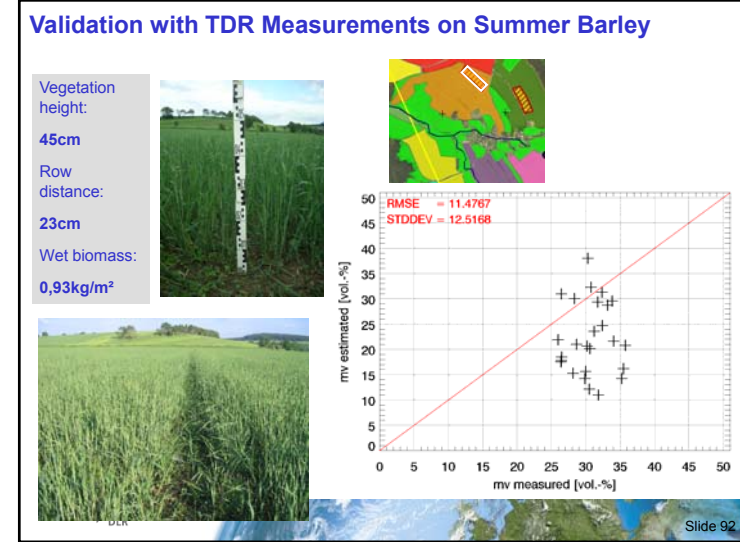
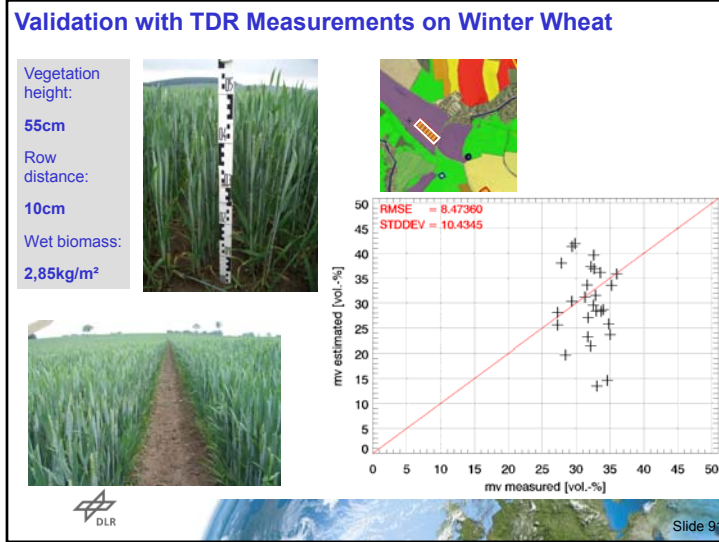
$\mu_{min} = \frac{\lambda_2}{P_r}$

→ Incorporation of a roughness and HV-decorrelation term [Hajnsek et al., 2001]

→ Incorporation of a vegetation attenuation L_v


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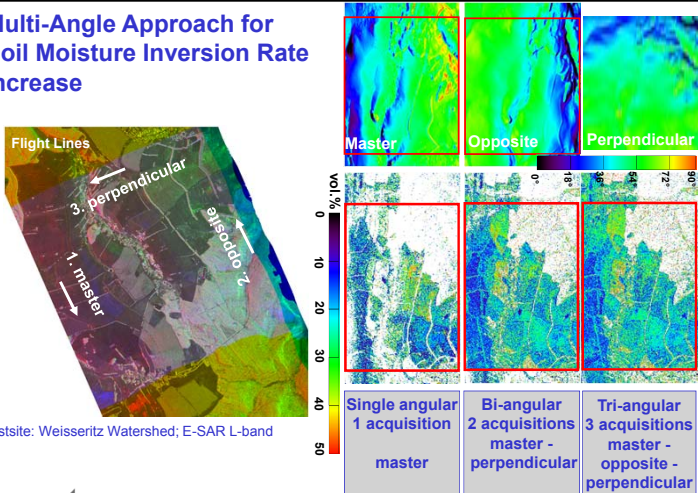
Soil Moisture Estimation

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Multi-Angle Approach for Soil Moisture Inversion Rate Increase

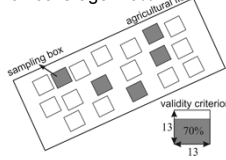


Testsite: Weissertz Watershed; E-SAR L-band

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Inversion Rate of Soil Moisture Estimation @ different AOI

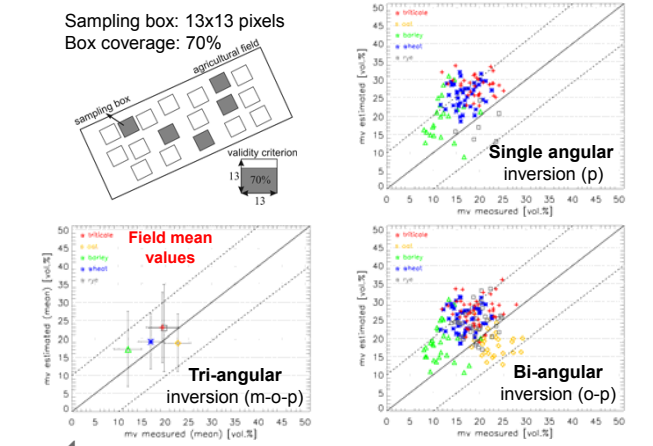
One acquisition	master	opposite	perpendicular
Single angular	40.32%	29.87%	48.53%
Two acquisitions	master-opposite	master-perpendicular	opposite-perpendicular
Bi-angular	55.87%	63.39%	60.71%
Three acquisitions	master-opposite-perpendicular		
Tri-angular	70.89%		



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Validation of Soil Moistures @ different Incidence Angles

Sampling box: 13x13 pixels
Box coverage: 70%




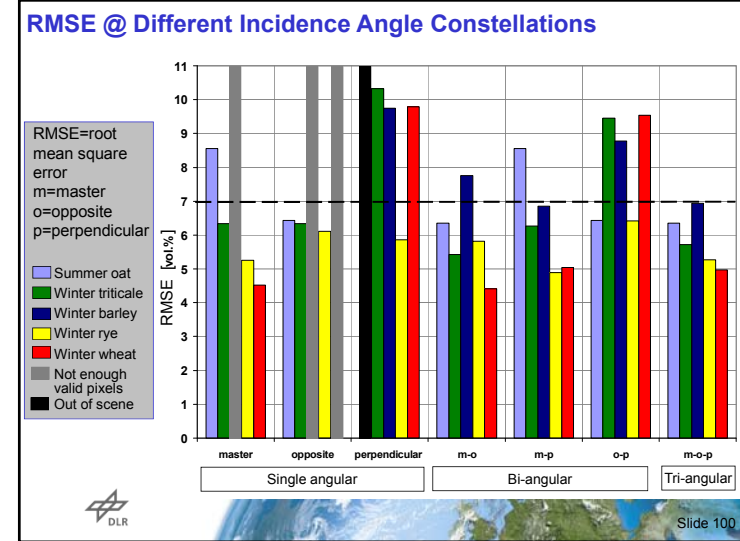
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RMSE [vol.%] between Estimated and Measured Soil Moistures for Different Incidence Angle Constellations

Fields	m	o	p	m-o	m-p	o-p	m-o-p
Winter Triticale	6.34	6.34	10.33	5.42	6.27	9.46	5.72
Winter Barley	<	<	9.75	7.76	6.85	8.78	6.94
Winter Rye	5.25	6.11	5.86	5.81	4.89	6.42	5.27
Winter Wheat	4.52	<	9.79	4.41	5.04	9.54	4.98
Summer Oat	8.55	6.43	-	6.35	8.55	6.43	6.35
Mean	6.17	6.29	8.93	5.95	6.32	8.13	5.85



- = out of scene
 < = too less values for a valid analysis

Single angular: m, o, p
 Bi-angular: m-o, m-p, o-p
 Tri-angular: m-o-p

Soil Moisture Estimation

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 - **hybrid decomposition**

Basic Principle of Hybrid Polarimetric Decomposition

Polarimetric Signature = Surface + Dihedral + Volume


signature = ground + vegetation

Removal of Vegetation Component

signature - f_v = ground + vegetation

Hybrid Polarimetric Decomposition

Physically consistent volume intensity (f_{vc}) with surface scattering (α_s)

$$f_{vc} = \frac{2}{3} \frac{f_v}{\sin^2 \alpha_s} \sqrt{1 - \cos^2(2\alpha_s)} \quad \text{Not physically consistent} \quad \cos(2\alpha_s) \sin(2\alpha_s) \cos(2\alpha_s)$$


Retrieval of the Ground Scattering Components

$$\begin{bmatrix} f_s \cos \alpha_s + f_d \sin \alpha_s & (f_d - f_s) \cos \alpha_s \sin \alpha_s & 0 \\ (f_d - f_s) \cos \alpha_s \sin \alpha_s & f_s \cos \alpha_s + f_d \sin \alpha_s & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Hybrid Polarimetric Decomposition

Eigen-based Decomposition of Ground Components

- ➡ From eigenvalues:
- ➡ From eigenvectors: **Intensity of ground** (f_d, f_s)

Scattering mechanisms of ground (α_d, α_s)

Physically Meaningful Separation of Scattering Mechanisms (α_d, α_s)

$\alpha_d + \alpha_s = \pi/2$ (Orthogonality condition)

$\alpha \in [0, \pi/4]$ Surface scattering ➡ α_s

$\alpha \in [\pi/4, \pi/2]$ Dihedral scattering ➡ α_d

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Soil Moisture Inversion from Surface Scattering Component

Polarimetric SAR data

Surface scattering component from hybrid polarimetric decomposition

$$\beta = -\tan(\alpha_s)$$

Surface scattering model

Bragg scatter modeling with θ_{loc} and a variety of soil dielectric constants ϵ_s

$$\beta_m = \frac{R_{HH} - R_{VV}}{R_{HH} + R_{VV}}$$

$R_{HH}, R_{VV} = f(\epsilon_s, \theta_{loc})$

data ➡ **Minimization** $\min_{\epsilon_s} \{|\beta - \beta_m|\}$ **model**

↓

Pedo-Transfer Function of ϵ_s

Topp et al. ($\epsilon_s < 40$)
Roth et al. ($\epsilon_s > 40$)

Soil moisture [vol.%]

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Soil Moisture Inversion along Vegetation Growth Cycle

Land use

July, 5th

AgriSAR 2006 L-band Quad-pol

Height: 172cm
Wet biomass: 6.28kg/m²

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Validation of Moisture Inversion with Field Measurements (TDR)

APRIL

RMSE=6.75

JUNE

RMSE=6.98

JULY

RMSE=4.43

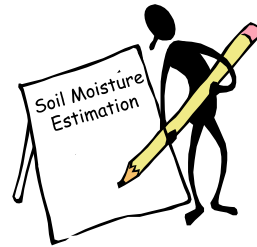
RMSE [vol.%]	APRIL	JUNE	JULY
sugar beet	7.55	11.90	4.68
winter wheat	8.06	6.15	3.40
winter rape	5.70	2.62	2.06
summer corn	5.34	3.20	5.25

validation box: 13x13 pixels

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Summary Soil Moisture Estimation

- **Soil moisture inversion is still object of scientific research:**
 - Main research focuses on the estimation of m_v under the vegetation
 - One attempt is to use SAR polarimetry for scattering mechanism decomposition in order to characterise and subtract volume from a ground component (surface/dihedral)
 - PolSARPro provides the possibility to invert soil moisture over bare fields and allows a decomposition of scattering mechanisms
 - Outlook: Inversion procedures for vegetated areas are still missing



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- Bronstert, A., C. Creutzfeldt, T. Gräff, I. Hajnsek, M. Heistermann, S. Iizerott, T. Jagdhuber, D. Kneis, E. Lück & D. Reusser, 'Potentials and constraints of different type of soil moisture observations for flood simulations in headwater catchments', *Natural Hazards*, Vol. 60, pp.879-914, 2012.



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Acknowledgement to the AgriSAR Team



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Acknowledgement to the OPAQUE Team



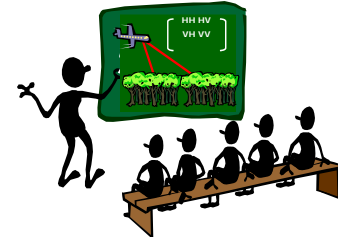

30/05/2007



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Part II: Exercises with L-band Airborne Data




- Read the airborne SAR data
- Speckle Filtering (refined Lee)
- Oh, Dubois and X-Bragg Inversion

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Test Date Used for the Exercise

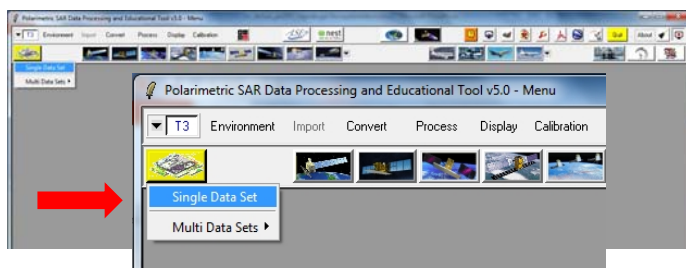

- **Testsite: Demmin**
 - Location: Northern Germany
 - Acquisition Date: May 2012
 - Frequency: L-band
 - Data size: az: 2.75km rg: 2.2 km
 - Polarisation: 4 SLC
 - Resolution: az: 60cm x rg: 3.8m
 - Rows and columns: 7981 x 1837

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First Steps in PoISARPro

- Please open PoISARPro
- Define your environment
- Open the DLR's acquired test Radar data

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