→ 2nd ADVANCED COURSE ON RADAR POLARIMETRY

Speckle Noise Characterization and Filtering in Polarimetric SAR Data

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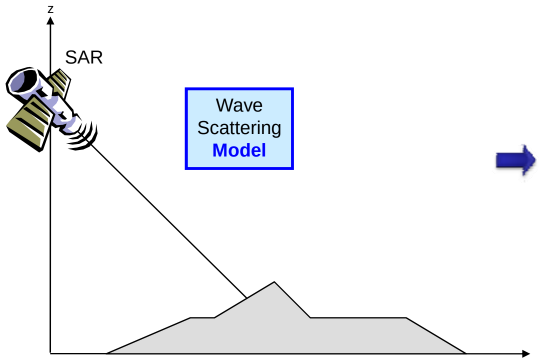
Outline

- Speckle Noise in SAR Data
- PolSAR Data Statistical Characterization
- PolSAR Information Estimation
- PolSAR Data Speckle Noise Characterization
- PolSAR Data Speckle Noise Filtering

Synthetic Aperture Radar Imaging

The analysis and understanding of data acquired by a SAR system needs from the following considerations

Model for the SAR imaging process/system



Wave Scattering **Model**

➡ **SAR Data Model**

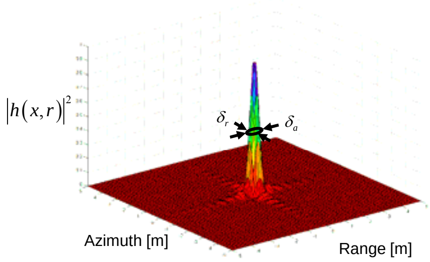
Model for the scatterer being imaged

SAR Imaging Process/System Model

Te impulse response of the SAR system embracing the acquisition and the focusing processes is

$$h(x,r) = \exp\left(j\frac{4\pi}{\lambda}r\right)\text{sinc}\left(\frac{\pi r}{\delta R}\right)\text{sinc}\left(\frac{\pi r}{\delta X}\right)$$

- Range resolution: $\delta R = \dfrac{c}{2B}$
- Azimuth resolution: $\delta X = \dfrac{D_a}{2}$



$\delta_a = \delta_r = 1m$

Point scatter

How it appears in the SAR image $s(x,r)$

Distributed scatter

Idea of resolution cell $\delta_a \times \delta_r$

The resolution cell is not the pixel of the SAR image. The pixel properties depend on how the SAR impulse response is sampled. Over sampling induces image spatial correlation.

Point Scatters Model

Examples of point targets imaged by SAR systems

Power lines | Vehicles | Railways | Houses

Types of microwave scattering
- Point scattering
- Complex scattering

$\sigma_s(x_0, r_0) = \sqrt{\sigma} e^{j\theta} \delta(x - x_0, r - r_0)$ Object description (Deterministic description)

Man-made media present a strong point scattering behaviour

↓

Scattered field dominated by canonical scattering mechanisms

Distributed Scatters Model

Examples of natural targets imaged by SAR systems

Rocks | Rough surface | Snow | Sea ice | Vegetation cover

Types of microwave scattering
- Surface scattering
- Volume scattering

$u(\vec{r})$ Object scattering function. (Random function - *microscopic structure*) NOT ACCESIBLE

Distributed scatterers have complex geometries and are randomly distributed

$\langle u(\vec{r}) \cdot u(\vec{r})^* \rangle = \sigma^0 \cdot \delta(\vec{r} - \vec{r}\,')$ Object description. (2^{nd} order descriptor - *macroscopic structure*)

Geophysical media present complicate structures and/or compositions

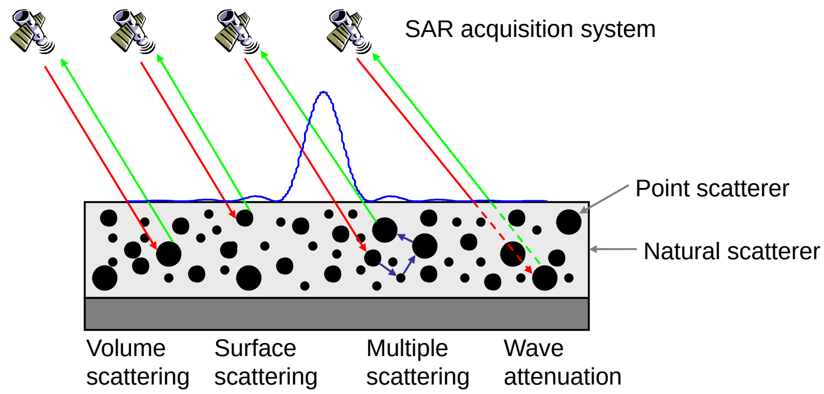
↓

Exact knowledge of the scattered field very difficult

Wave Scattering Model

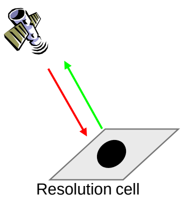
Scattering based on the Born approximation or single scattering approximation

- The scattering is supposed to be the linear coherent addition of the individual scattered waves from a set of discrete or point scatters



- The model does not consider attenuation nor multiple scattering

SAR Imagery Analysis



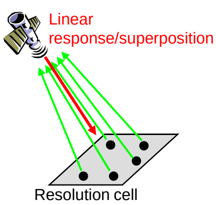
Deterministic Scatterer → Image formation process → DATA

Described in a **deterministic** way

Deterministic process

Can be studied using **deterministic** tools

Man Made Media



Distributed Scatterer → Image formation process → DATA

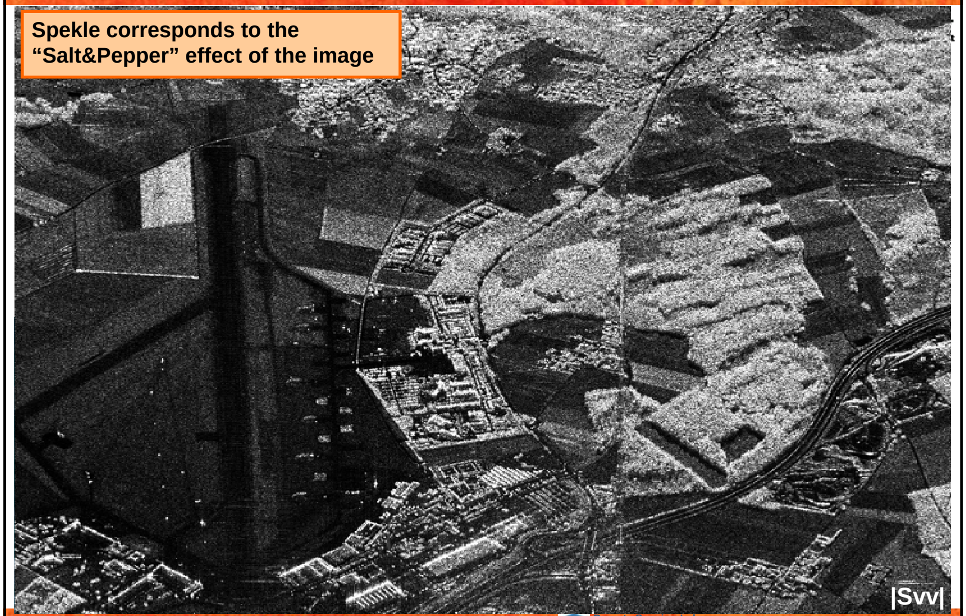
Described in a **stochastic** way

Deterministic electromagnetic process but can only be described in a **stochastic** way

Can be studied using **stochastic** tools

Natural Media

Speckle Noise



Speckle Noise

On the basis of a discrete scatter description

$$S(x,r)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}u(x',r')h(x-x',r-r')\mathrm{d}x'\mathrm{d}r' \Rightarrow S(x,r)=\frac{1}{\sqrt{L}}\sum_{k=1}^{L}\sqrt{\sigma_k}e^{j\theta_k}h(x-x_k,r-r_k)$$

Normalizing factor

L: Number of point scatters embraced by the resolution cell

- L as a deterministic quantity
  - L = 1: or a dominating point scatter: Deterministic scattering
    - Rice/Rician model
  - L >1: Partially developed speckle
    - Not solved model. Even numerical solution difficult
  - L >>1: Fully developed speckle
    - Gaussian model
- L as a stochastic quantity
  - L characterized by a pdf: Image texture
    - K-distribution model

Fully Developed Speckle Noise

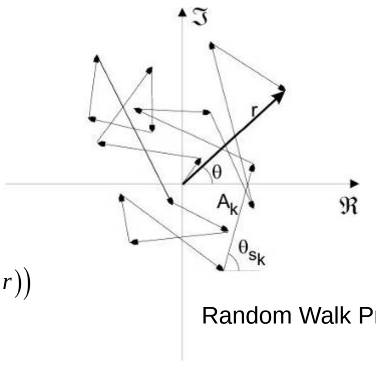
esa

- SAR image formation process

$$S(x,r)=\frac{1}{\sqrt{L}}\sum_{k=1}^{L}\sqrt{\sigma_k}e^{j\theta_k}h(x-x_k,r-r_k)$$

- Complex SAR data for L>>1

$$S(r(x,r),\theta(x,r))=\Re\{S\}+j\Im(S)$$
$$=r(x,r)\exp(j\theta(x,r))$$



Random Walk Process

- Real part

$$\Re\{S\}=\frac{1}{\sqrt{L}}\sum_{k=1}^{L}A_k\cos(\theta_{S_k})$$

- Imaginary part

$$\Im\{S\}=\frac{1}{\sqrt{L}}\sum_{k=1}^{L}A_k\sin(\theta_{S_k})$$

$$r(x,r)\exp(j\theta(x,r))=\frac{1}{\sqrt{L}}\sum_{k=1}^{L}A_k\exp(j\theta_{S_k})$$

Fully Developed Speckle Noise

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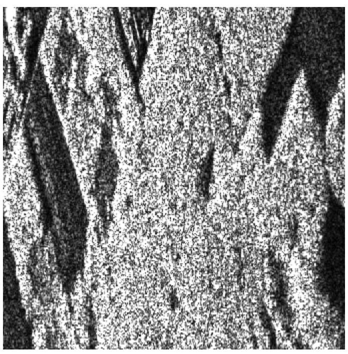

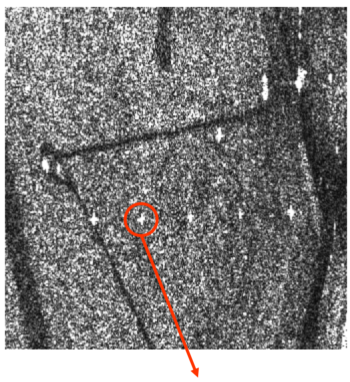

Fully Developed speckle

Bright points: Points where the interference is constructive
Dark points: Points where the interference is destructive

Corner reflector
Dominant scatter
No speckle

Speckle is the interference or fading pattern

S_{hh} amplitude
E-SAR L-band system

Fully Developed Speckle Noise

- Completely developed Speckle (large L and no dominant scatter)
  - Hypotheses
    - The amplitude A_k and the phase θ_{s_k} of the kth scattered wave are statistically independent of each other and from the amplitudes and phases of all other elementary waves (Uncorrelated point scatters)
    - The phases of the elementary contributions θ_{s_k} are equally likely to lie anywhere in the primary interval *[-π, π)*

$$S = \mathcal{N}_{C^2}\left(0, \sigma^2/2\right)$$

- Central Limit Theorem
  - Real Part

$$p_{\Re\{S\}}\left(\Re\{S\}\right) = \frac{1}{\sqrt{\pi\sigma^2}} \exp\left(-\left(\frac{\Re\{S\}}{\sigma}\right)^2\right) \quad \Re\{S\} \in (-\infty, \infty) \qquad \text{Gaussian pdf}$$

  - Imaginary Part

$$p_{\Im\{S\}}\left(\Im\{S\}\right) = \frac{1}{\sqrt{\pi\sigma^2}} \exp\left(-\left(\frac{\Im\{S\}}{\sigma}\right)^2\right) \quad \Im\{S\} \in (-\infty, \infty) \qquad \text{Gaussian pdf}$$

  - Real and imaginary parts are uncorrelated $E\left\{\Re\{S\}\Im\{S\}\right\} = 0$

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Fully Developed Speckle Noise

- Amplitude: Rayleigh pdf

$$p_r(r) = \frac{2r}{\sigma^2} \exp\left(-\left(\frac{r}{\sigma}\right)^2\right) \quad r \in [0, \infty)$$

$$E\{r\} = \frac{\sqrt{\pi}}{2}\sigma$$

$$E\{r^2\} = \sigma^2$$

$$\sigma_r^2 = E\{r^2\} - E^2\{r\} = \left(1 - \frac{\pi}{4}\right)\sigma^2$$

- Intensity ($I=r^2$): Exponential pdf

$$p_I(I) = \frac{1}{\sigma^2} \exp\left(-\frac{I}{\sigma^2}\right) \quad I \in [0, \infty)$$

$$E\{I\} = \sigma^2 \equiv \sigma$$

$$E\{I^2\} = 2\left(\sigma^2\right)^2$$

$$\sigma_I^2 = E\{I^2\} - E^2\{I\} = \sigma^2$$

- Phase: Uniform pdf. Contains NO information

$$p_\theta(\theta) = \frac{1}{2\pi} \quad \theta \in [-\pi, \pi)$$

- Amplitude and phase are uncorrelated

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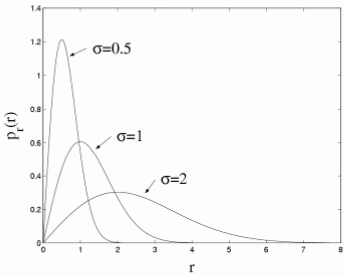
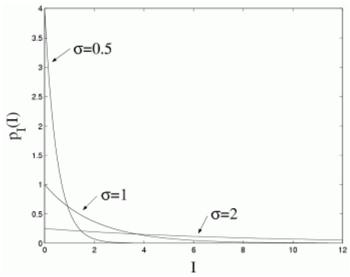
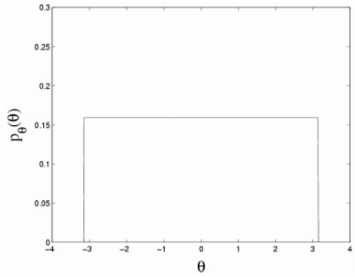
Fully Developed Speckle Noise



Amplitude: Rayleigh pdf



Intensity ($I=r^2$): Exponential pdf



Phase: Uniform pdf

Fully Developed Speckle Noise

Important considerations

- Speckle is a deterministic electromagnetic effect, but due to the complexity of the image formation process, it must be analysed statistically
- Considering completely developed speckle, a SAR image pixel does not give information about the target. Only statistical moments can describe the target or the process

Information

What does it mean information in the presence of Speckle?

- Phase contains no information
- Intensity exponentially distributed

$$p_I(I)=\frac{1}{2\sigma^2}\exp\left(-\frac{I}{2\sigma^2}\right) \quad I\in[0,\infty) \quad \Rightarrow \quad E\{I\}=2\sigma^2 \quad \sigma_I=2\sigma^2$$

Exponential pdf — First and second order moments

- Intensity, under the previous hypotheses, is completely determined by the exponential pdf
  - Pdf completely determined by the pdf shape
  - Pdf shape parameterized by σ ⇒ INFORMATION ⇒ RCS σ^0
- Not useful information is considered as NOISE

Fully Developed Speckle Noise Model

Objectives of a Noise Model

- To embed the data distribution into a noise model, that is, a function that allows identifying of the useful information to be retrieved, the noise sources, and how these terms interact
- Optimize the information extraction process, i.e., the noise filtering process

SAR image intensity noise model

SAR image intensity ($I=r^2$)

$$p_I(I)=\frac{1}{2\sigma^2}\exp\left(-\frac{I}{2\sigma^2}\right) \quad I\in[0,\infty) \quad E\{I\}=2\sigma^2 \quad \sigma_I=2\sigma^2$$

$$I=2\sigma^2 n \qquad p_n(n)=\exp(-n) \quad n\in[0,\infty) \quad E\{I\}=1 \quad \sigma_I=1$$

One dimensional speckle noise model (Model over the SAR image intensity - 2nd moment) ⇒ $I(x,r)=\sigma(x,r)n(x,r)$

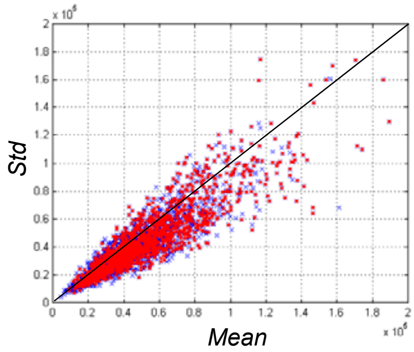
Multiplicative Speckle Noise Model

Fully Developed Speckle Noise Model

Moments calculated over local 7x7 local windows

Statistics area

Grass area



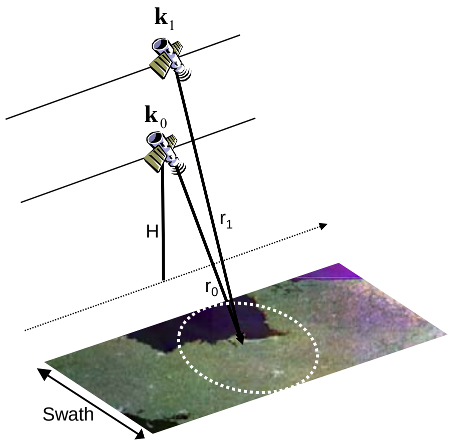
Blue: $|S_{hh}|^2$
Red: $|S_{vv}|^2$

S_{hh} amplitude
E-SAR L-band system

Polarimetric SAR Systems

A Polarimetric SAR system acquires 3 complex SAR images

Target vector $\mathbf{k} = [S_{hh}, 2S_{hv}, S_{vv}]^T$



The properties of the target vector follow from the properties of a single SAR image:

- **k** is deterministic for point scatters. It contains all the necessary information to characterize the scatter
- **k** is a multidimensional random variable for distributed scatters due to speckle. A single sample does not characterize the scatter

SAR images characterized through second order moments:

- Second order moments in multidimensional SAR data are matrix quantities

Mathematical Representation

PDF for non-correlated SAR images

- Zero-mean multidimensional complex (also circular) Gaussian pdf

$$p_{\mathbf{k}}(\mathbf{k})=\prod_{k=1}^{3}\frac{1}{\pi\sigma^2}\exp\left(-\frac{S_k S_k^H}{\sigma^2}\right)=\frac{1}{\pi^m\sigma^{2m}}\exp\left(-\sum_{k=1}^{m}\frac{S_k S_k^H}{\sigma^2}\right)=\frac{1}{\pi^m\sigma^{2m}}\exp\left(-\frac{1}{\sigma^2}\operatorname{tr}\left(\mathbf{k}\mathbf{k}^H\right)\right)$$

Independent SAR images with the same power $S_k=\mathcal{N}_{C^2}\left(0,\sigma^2/2\right)$

- First order moment

$$E\{\mathbf{k}\}=\mathbf{0}$$

- Second order moment: Covariance matrix

$$\mathbf{C}=E\left\{\mathbf{k}\mathbf{k}^H\right\}=\sigma^2\mathbf{I}_{m\times m}$$

Multidimensional Gaussian pdf Properties

Characterization of random variables

- Probability Density Function (pdf)
- Moment-generating function
- Statistical moments (mean, power, kurtosis, skewness…)

Zero-mean multidimensional complex Gaussian pdf

$$p_{\mathbf{k}}(\mathbf{k})=\frac{1}{\pi^3|\mathbf{C}|}\exp\left(-\mathbf{k}^H\mathbf{C}^{-1}\mathbf{k}\right)$$

- First order moment $E\{\mathbf{k}\}=\mathbf{0}$
- Second order moment: Covariance matrix

$$\mathbf{C}=E\left\{\mathbf{k}\mathbf{k}^H\right\}=\begin{bmatrix} E\left\{|S_{hh}|^2\right\} & E\left\{S_{hh}S_{hv}^*\right\} & E\left\{S_{hh}S_{vv}^*\right\} \\ E\left\{S_{hv}S_{hh}^*\right\} & E\left\{|S_{hv}|^2\right\} & E\left\{S_{hv}S_{vv}^*\right\} \\ E\left\{S_{vv}S_{hh}^*\right\} & E\left\{S_{vv}S_{hv}^*\right\} & E\left\{|S_{vv}|^2\right\} \end{bmatrix}$$

$E\left\{S_k S_l^*\right\}\neq 0 \quad k,l\in\{1,\ldots,m\}, k\neq l$

Correlated SAR images

Multidimensional Gaussian pdf Properties

A zero-mean multidimensional complex Gaussian pdf is completely characterized by the second order moments, i.e., the covariance matrix

- Moment theorem for complex Gaussian processes, given Q correlated SAR images
  - For $k \neq l$, where m_k and n_l are integers from $\{1, 2, \ldots, Q\}$

$$E\left\{S_{m_1} S_{m_2} \cdots S_{m_k} S_{n_1}^* S_{n_2}^* \cdots S_{n_l}^*\right\} = 0$$

  - For $k = l$, where π is a permutation of the set of integers $\{1, 2, \ldots, Q\}$

$$E\left\{S_{m_1} S_{m_2} \cdots S_{m_k} S_{n_1}^* S_{n_2}^* \cdots S_{n_l}^*\right\} = \sum_{\pi} E\left\{S_{m_{\pi(1)}} S_{n_1}^*\right\} E\left\{S_{m_{\pi(2)}} S_{n_2}^*\right\} \cdots E\left\{S_{m_{\pi(l)}} S_{n_l}^*\right\}$$

- Considering the covariance matrix
  - Higher order moments are function of the covariance matrix

Mathematical Representation

The covariance matrix contains the correlation structure of the set of m SAR images

$$\mathbf{C} = E\left\{\mathbf{k}\mathbf{k}^H\right\} = \begin{bmatrix} E\left\{|S_{hh}|^2\right\} & E\left\{S_{hh}S_{hv}^*\right\} & E\left\{S_{hh}S_{vv}^*\right\} \\ E\left\{S_{hv}S_{hh}^*\right\} & E\left\{|S_{hv}|^2\right\} & E\left\{S_{hv}S_{vv}^*\right\} \\ E\left\{S_{vv}S_{hh}^*\right\} & E\left\{S_{vv}S_{hv}^*\right\} & E\left\{|S_{vv}|^2\right\} \end{bmatrix}$$

All the information characterizing the set of 3 SAR images is contained in the covariance matrix

Information

- Diagonal elements: Power information

$$E\left\{S_k S_k^H\right\} = E\left\{|S_k|^2\right\} \quad k \in \{1, 2, \ldots, m\}$$

- Off-diagonal elements: Correlation information

$$E\left\{S_k S_l^H\right\} \quad k, l \in \{1, 2, \ldots, m\}, k \neq l$$

Complex Correlation Coefficient

How to consider the correlation information

- Off-diagonal covariance matrix elements

$$E\left\{S_k S_l^H\right\} \quad k,l \in \{1,2,\ldots,m\}, k \neq l$$

  - Absolute correlation information

- Complex correlation coefficient

$$\rho_{k,l} = \frac{E\left\{S_k S_l^*\right\}}{\sqrt{E\left\{|S_k|^2\right\} \cdot E\left\{|S_l|^2\right\}}} = |\rho_{k,l}| e^{j\theta_{k,l}}$$

$0 \leq |\rho_{k,l}| \leq 1$ Coherence

$-\pi \leq \theta_{k,l} \leq \pi$

  - Normalized correlation information

- The complex correlation information represents the most important observable for multidimensional SAR data. Its physical interpretation depends on the multidimensional SAR system configuration

Information Content

SAR Interferometry

- Phase $\theta_{k,l}$ contains topographic information
- Coherence $|\rho_{k,l}|$ is sensitive to different properties of the imaged area
  - Study and retrieval of stem volume over forested areas
  - Study of dry and wet snow covered areas
  - Characterization of glaciers, valleys, and fjord ice

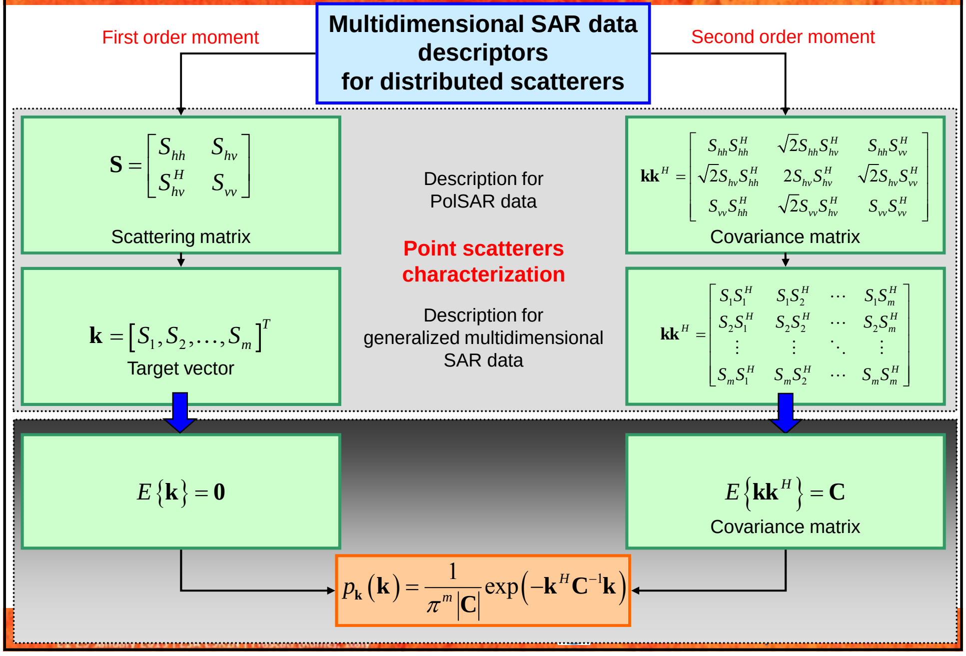
SAR Polarimetry

- Off-diagonal information related with the geometry and the electrical properties of the target being imaged

Polarimetric SAR Interferometry

- Complex correlation coefficient related with the vegetation height and the vegetation structural properties

Mathematical Representation



Information Estimation/Filtering

For multidimensional SAR data, under the hypothesis of Gaussian scattering, all the information is contained in the covariance matrix

$$\mathbf{C} = E\{\mathbf{kk}^H\} = \begin{bmatrix} E\{|S_{hh}|^2\} & E\{S_{hh}S_{hv}^*\} & E\{S_{hh}S_{vv}^*\} \\ E\{S_{hv}S_{hh}^*\} & E\{|S_{hv}|^2\} & E\{S_{hv}S_{vv}^*\} \\ E\{S_{vv}S_{hh}^*\} & E\{S_{vv}S_{hv}^*\} & E\{|S_{vv}|^2\} \end{bmatrix}$$

This matrix must be estimated from the available information

- The scattering vector for each pixel/sample of the SAR data

$$\mathbf{k} = [S_{hh}, 2S_{hv}, S_{vv}]^T$$

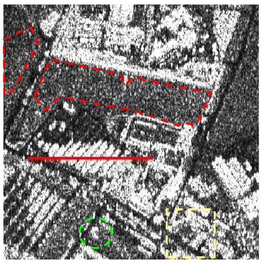
- The estimation process reduces to estimate the ensemble average (expectation operator) $E\{\cdot\}$
- The estimation process also receives the name of data filtering process

Information Estimation/Filtering

Considerations about speckle noise reduction

Optical image DLR OP

SAR images reflex the Nature's complexity



SAR image DLR OP

Homogeneous areas	Image details	Heterogeneous areas
Maintain useful information (σ)	Maintain spatial details (Shape and value)	Maintain both
RADIOMETRIC RESOLUTION	SPATIAL RESOLUTION	LOCAL ANALYSIS

Image data: S_{hh} amplitude. E-SAR L-band system

Information Estimation/Filtering

Multidimensional SAR data information estimation, i.e., data filtering, based on two main hypotheses

- Ergodicity in mean: The different time/space averages of each process converge to the same limit, i.e., the ensemble average $E\{\}$
  - The statistics in the realizations domain can be calculated in the time/spatial domain
  - Necessary to assume ergodicity since there are not multiple data realizations over the same area
  - Applied to the processes $E\left\{|S_k|^2\right\}$, $E\left\{|S_l|^2\right\}$ and $E\left\{S_k S_l^H\right\}$ $\quad k,l \in \{1,2,\ldots,m\}$
- Wide-sense stationary: Given a spatial domain statistical moments do not depend on the sample location
  - SAR images can not be considered as wide-sense stationary processes since they are a reflex of the data heterogeneity
  - SAR images can be considered locally wide-sense stationary
  - Applied to the processes $E\left\{|S_k|^2\right\}$, $E\left\{|S_l|^2\right\}$ and $E\left\{S_k S_l^H\right\}$ $\quad k,l \in \{1,2,\ldots,m\}$
- Homogeneity: Refers to non-textured data
  - Gaussian distributed data

Sample Covariance Matrix

Covariance matrix estimation by means of a MultiLook (BoxCar)

- Maximum likelihood estimator: Sample covariance matrix

$$\mathbf{Z}_n = \frac{1}{n}\sum_{k=1}^{n}\mathbf{k}\mathbf{k}^H = \begin{bmatrix} \frac{1}{n}\sum_{k=1}^{n}S_1(k)S_1^*(k) & \frac{1}{n}\sum_{k=1}^{n}S_1(k)S_2^*(k) & \cdots & \frac{1}{n}\sum_{k=1}^{n}S_1(k)S_m^*(k) \\ \frac{1}{n}\sum_{k=1}^{n}S_2(k)S_1^*(k) & \frac{1}{n}\sum_{k=1}^{n}S_2(k)S_2^*(k) & \cdots & \frac{1}{n}\sum_{k=1}^{n}S_2(k)S_m^*(k) \\ \vdots & \vdots & \ddots & \vdots \\ \frac{1}{n}\sum_{k=1}^{n}S_m(k)S_1^*(k) & \frac{1}{n}\sum_{k=1}^{n}S_m(k)S_2^*(k) & \cdots & \frac{1}{n}\sum_{k=1}^{n}S_m(k)S_m^*(k) \end{bmatrix}$$

  - n represents the total number of samples employed to estimate the covariance matrix, taken a region (square, rectangular, adapted…)
  - $\mathbf{Z}_n$ as estimator of $\mathbf{C}$
    - Does not consider signal morphology/heterogeneity
    - Loss of spatial resolution

The sample covariance matrix $\mathbf{Z}_n$ is itself a multidimensional random variable

Sample Covariance Matrix Distribution

The sample covariance matrix $\mathbf{Z}_n$ is characterized by the complex Wishart distribution $\mathbf{Z}_n \sim \mathcal{W}(n,\mathbf{C})$

$$p_{\mathbf{Z}_n}(\mathbf{Z}_n) = \frac{n^{mn}|\mathbf{Z}_n|^{n-m}}{|\mathbf{C}|^n \tilde{\Gamma}_m(n)}\mathrm{etr}\left(-n\mathbf{C}^{-1}\mathbf{Z}_n\right) \qquad \tilde{\Gamma}_m(n) = \pi^{m(m-1)/2}\Pi_{i=1}^{m}\Gamma(n-i+1)$$

- Multidimensional data distribution
- Valid for $n \geq m$, otherwise $|\mathbf{Z}_n|^{n-m}$ is equal to zero and the Wishart pdf is undetermined
  - Equivalent to *Rank*($\mathbf{Z}_n$)=*m*, i.e., the sample covariance matrix is a full rank matrix
  - The higher the data dimensionality m the higher the number of looks n for the Wishart pdf to be defined

Multidimensional SAR Data Description

$$S_k = \mathcal{N}_{C^2}\left(0, \sigma^2/2\right)$$

Single SAR image

$$\mathbf{k} = \left[S_1, S_2, \ldots, S_m\right]^T$$

Multidimensional SAR dataset

$$p_{\mathbf{k}}(\mathbf{k}) = \frac{1}{\pi^m |\mathbf{C}|} \exp\left(-\mathbf{k}^H \mathbf{C}^{-1} \mathbf{k}\right)$$

Multidimensional complex Gaussian pdf

$$\mathbf{Z}_n = \frac{1}{n} \sum_{i=1}^{n} \mathbf{k}_i \mathbf{k}_i^H$$

$$\mathbf{Z}_n \sim \mathcal{W}(n, \mathbf{C})$$

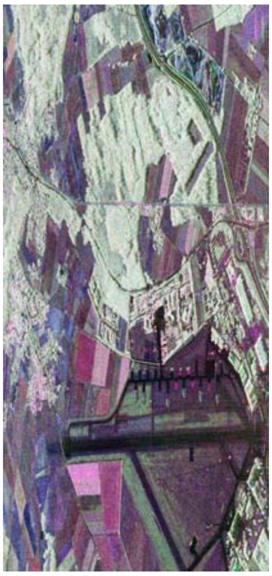
Sample covariance matrix

$$p_{\mathbf{Z}_n}(\mathbf{Z}_n) = \frac{n^{mn} |\mathbf{Z}_n|^{n-m}}{|\mathbf{C}|^n \tilde{\Gamma}_m(n)} \operatorname{etr}\left(-n \mathbf{C}^{-1} \mathbf{Z}_n\right)$$

Complex Wishart pdf

Multilook/Boxcar Example

|Shh-Svv| 2|Shv| | Shh +Svv|



Original data

7x7 MLT data

L-band (1.3 GHz) fully PolSAR data
E-SAR system. Oberpfaffenhofen test area (D)

Multilook/Boxcar Example

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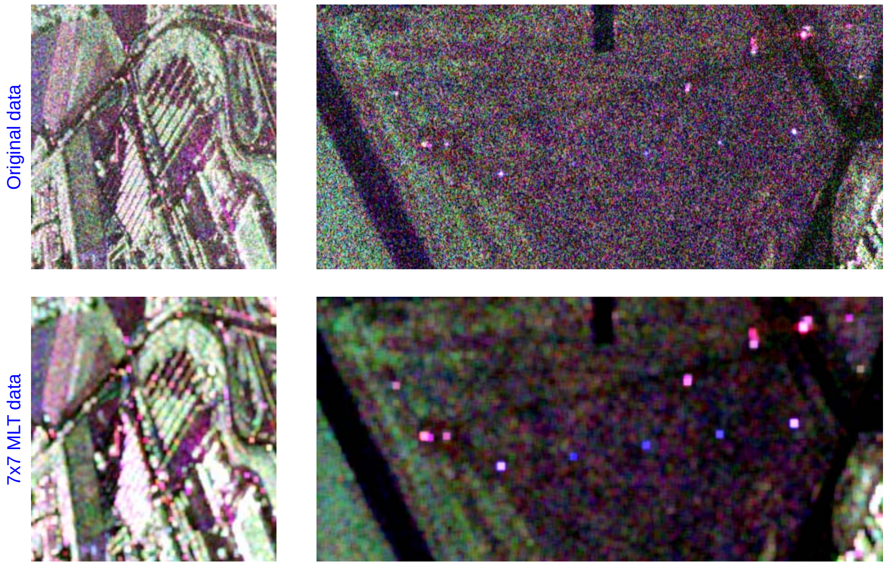

|Shh-Svv| 2|Shv| | Shh +Svv|

Multidimensional Speckle Noise Model

esa

Objective of a multidimensional speckle noise model

Useful information ← → Noise sources

$$\mathbf{Z}_n = f\left(\mathbf{C}, n_1, n_2, \ldots, n_p\right)$$

- Overcome the limitations of the fully multiplicative speckle noise model. Noise model independent of the data dimensionality and valid for any correlation structure for the data
  - Observation: Any matrix entry consists of the Hermitian product of two complex SAR images

One-look sample covariance matrix $\mathbf{Z}_1 = \mathbf{k}\mathbf{k}^H = \begin{bmatrix} S_1S_1^H & S_1S_2^H & \cdots & S_1S_m^H \\ S_2S_1^H & S_2S_2^H & \cdots & S_2S_m^H \\ \vdots & \vdots & \ddots & \vdots \\ S_mS_1^H & S_mS_2^H & \cdots & S_mS_m^H \end{bmatrix}$

- Speckle noise model for the Hermitian product of a pair of SAR images

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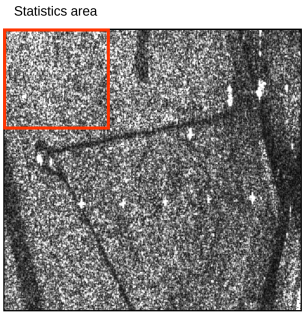
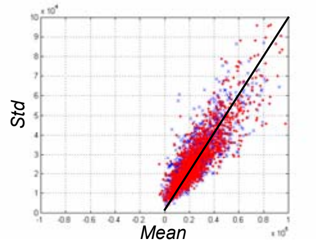
Extension to model the sample covariance matrix independently of its dimensions

Multidimensional Speckle Noise Model



Grass area

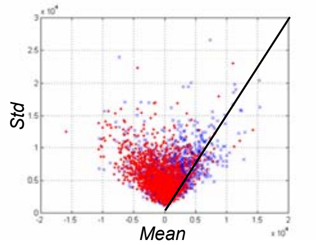
Statistics calculated over 7x7 pixel windows



Copolar

Blue: Real($S_{hh}S_{vv}$*)
Red: Imag($S_{hh}S_{vv}$*)

$|\rho_{hhvv}| = 0.77 e^{j0.807}$



Crosspolar

Blue: Real($S_{hh}S_{hv}$*)
Red: Imag($S_{hh}S_{hv}$*)

$|\rho_{hhvv}| = 0.118 e^{j0.638}$

S_{hh} amplitude
E-SAR L-band system

Hermitian Product Speckle Noise Model

Hermitian product speckle noise model: $S_i S_j^* = \underbrace{\psi \bar{z}_n n_m N_c e^{j\phi_x}}_{\text{Multiplicative term}} + \underbrace{\psi\left(|\rho| - N_c \bar{z}_n\right) e^{j\phi_x} + \psi\left(n_{ar} + j n_{ai}\right)}_{\text{Additive term}}$

Multiplicative speckle component: n_m ➡ High coherence areas $E\{n_m\} = 1$, $\sigma_{n_m}^2 = 1$

Stationary

$$\Re\{ze^{j\phi}\}_1 = z_c \cos(\phi_x) = \psi N_c \bar{z}_n n_m \cos(\phi_x)$$

$$\Im\{ze^{j\phi}\}_1 = z_c \sin(\phi_x) = \psi N_c \bar{z}_n n_m \sin(\phi_x)$$

Additive speckle components: n_{ar}, n_{ai} ➡ Low coherence areas $E\{n_{ar}\} = E\{n_{ai}\} = 0$, $\sigma_{n_{ar}}^2 = \sigma_{n_{ai}}^2 \simeq \frac{1}{2}\left(1 - |\rho|^2\right)^{1.32}$

Non stationary

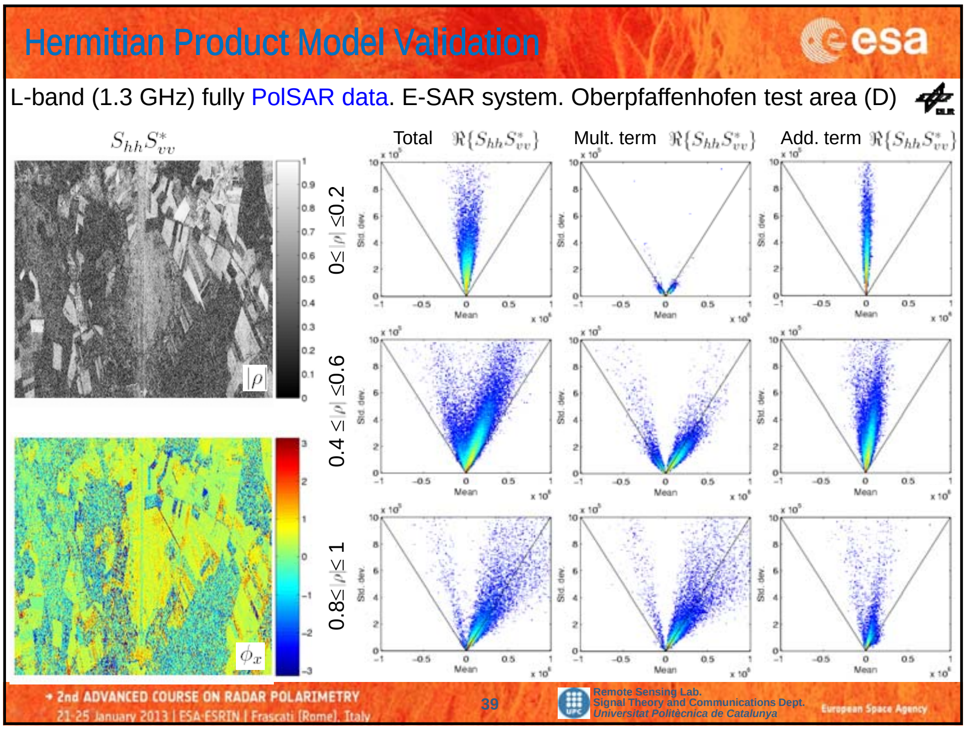
Final speckle noise behaviour { Combination of multiplicative and additive noise components, determined by ρ

Special cases

- Covariance matrix diagonal element

$$\rho = 1 \exp(j0) \Rightarrow S_k S_k^* = \psi n_m$$

- By construction, the complex Hermitian product phase difference is characterized by an additive noise model


Hermitian Product Model Validation
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L-band (1.3 GHz) fully PolSAR data. E-SAR system. Oberpfaffenhofen test area (D)
$S_{hh}S_{vv}^*$
Total $\Re\{S_{hh}S_{vv}^*\}$
Mult. term $\Re\{S_{hh}S_{vv}^*\}$
Add. term $\Re\{S_{hh}S_{vv}^*\}$
$0 \leq |\rho| \leq 0.2$
$0.4 \leq |\rho| \leq 0.6$
$0.8 \leq |\rho| \leq 1$
$|\rho|$
ϕ_x
Std. dev.
Mean
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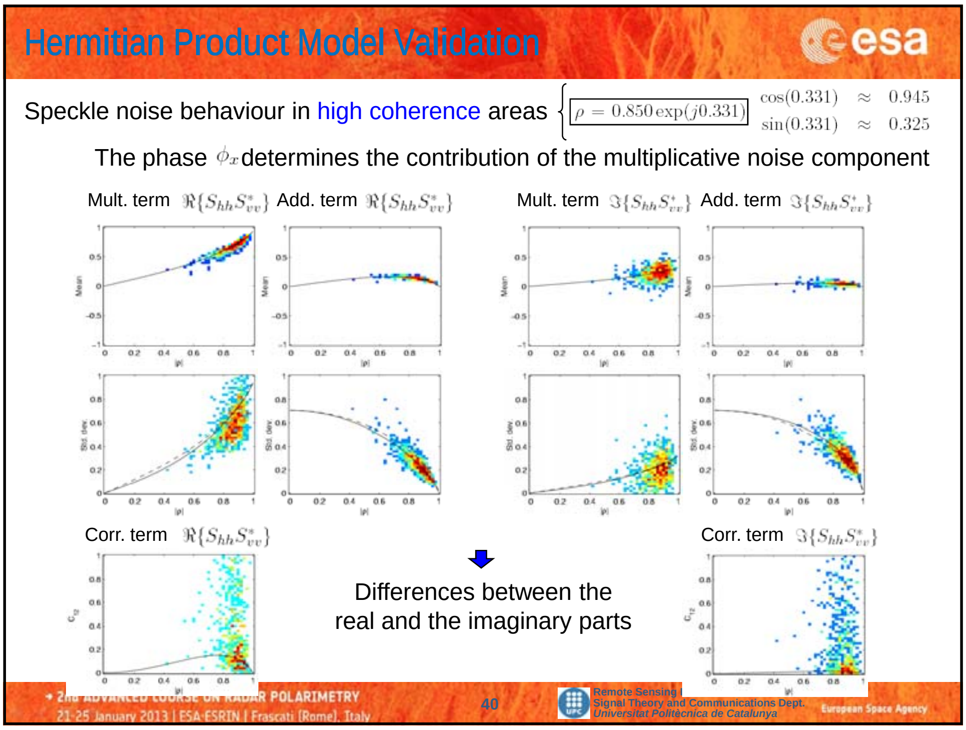
Signal Theory and Communications Dept.
Universitat Politècnica de Catalunya
European Space Agency


Hermitian Product Model Validation
esa
Speckle noise behaviour in high coherence areas
$\rho = 0.850 \exp(j0.331)$
$\cos(0.331) \approx 0.945$
$\sin(0.331) \approx 0.325$
The phase ϕ_x determines the contribution of the multiplicative noise component
Mult. term $\Re\{S_{hh}S_{vv}^*\}$
Add. term $\Re\{S_{hh}S_{vv}^*\}$
Mult. term $\Im\{S_{hh}S_{vv}^*\}$
Add. term $\Im\{S_{hh}S_{vv}^*\}$
Corr. term $\Re\{S_{hh}S_{vv}^*\}$
Corr. term $\Im\{S_{hh}S_{vv}^*\}$
Differences between the
real and the imaginary parts
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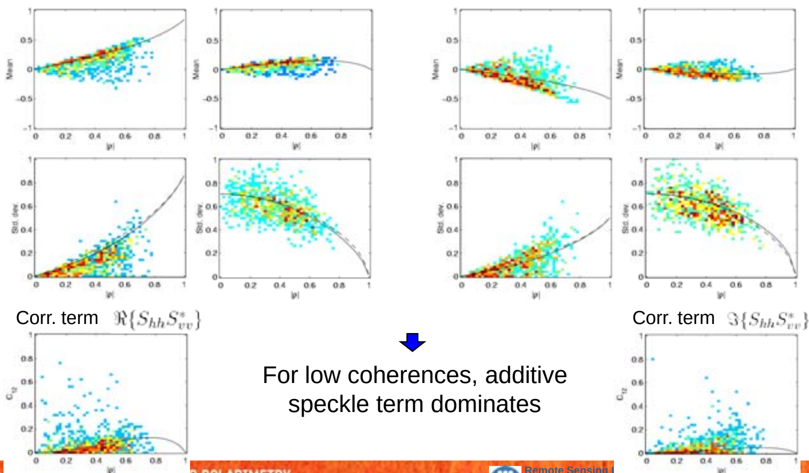
Hermitian Product Model Validation

Speckle noise behaviour in low coherence areas $\left\{ \boxed{\rho = 0.389\exp(-j0.528)} \right.$ $\begin{array}{lll} \cos(-0.528) & \approx & 0.863 \\ \sin(-0.528) & \approx & -0.503 \end{array}$

Low influence of the average phase ϕ_x in low coherence areas



For low coherences, additive speckle term dominates

Multilook Multidimensional Speckle Noise Model

Hermitian product speckle noise model: $\boxed{\left\langle S_i S_j^* \right\rangle_n = \underbrace{\psi n_m \exp(j\phi_x)}_{\text{Multiplicative term}} + \underbrace{\psi\left(|\rho| - N_c \bar{z}_n\right)\exp(j\phi_x) + \psi\left(n_{ar} + jn_{ai}\right)}_{\text{Additive term}}}$

Multiplicative speckle noise component

- Dominant for high coherences
- Modulated by phase information

$E\{n_m\} = N_c \bar{z}_n \qquad \sigma_{n_m}^2 = N_c^2 \frac{\left(1+|\rho|^2\right)}{2n}$

Additive speckle noise component

- Dominant for low coherences
- Not affected by phase information

$E\{n_{ar}\} = E\{n_{ai}\} = 0 \qquad \sigma_{n_{ar}}^2 = \sigma_{n_{ai}}^2 \simeq \frac{1}{2n}\left(1-|\rho|^2\right)^{1.32\sqrt{n}}$

Effect of the approximations

- Mean value IS NOT approximated ➡ No loss of information

$$\lim_{n\to\infty}\left\{\psi n_m \exp(j\phi_x) + \psi\left(|\rho| - N_c \bar{z}_n\right)\exp(j\phi_x) + \psi\left(n_{ar} + jn_{ai}\right)\right\} = \psi|\rho|\exp(j\phi_x)$$

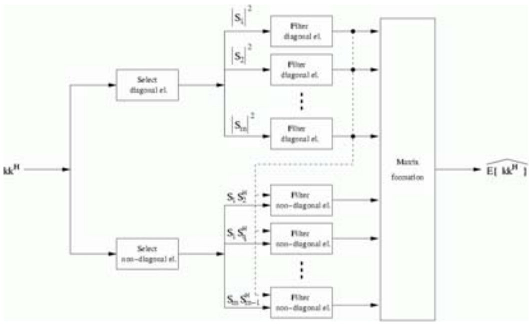
- Std. Dev. ARE approximated

Multidimensional Speckle Noise Filtering

Define a multidimensional SAR data filtering strategy based on the multidimensional speckle noise model

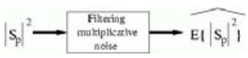
Element to consider: Covariance matrix

- Diagonal element: Multiplicative noise source
- Non-diagonal element: Multiplicative and additive noise sources combined according to the complex correlation coefficient



Multidimensional Speckle Noise Filtering

Diagonal element processing

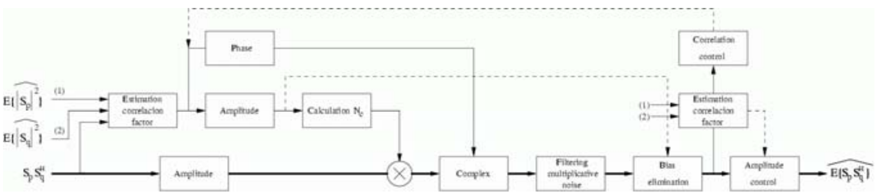


Any alternative to filter multiplicative noise can be considered

Non-iterative scheme

Off-diagonal element processing

The filter uses the Hermitian product speckle model: $S_i S_j^* = \underbrace{\psi \bar{z}_n n_m N_c e^{j\phi_x}}_{Multiplicative\ term} + \underbrace{\psi\left(|\rho| - N_c \bar{z}_n\right) e^{j\phi_x} + \psi\left(n_{ar} + j n_{ai}\right)}_{Additive\ term}$



Iterative scheme to take benefit of the improved coherence estimation

This strategy filters differently the covariance matrix elements

Results: Simulated Multidimensional SAR Data

Quantitative evaluation of the filter difficult with experimental SAR data due to speckle

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Necessity to consider an evaluation with simulated multidimensional SAR data

PolSAR data

Nevertheless results and conclusions may be extended to any multidimensional SAR

PolSAR data simulated according to the covariance matrix

$$\mathbf{C} = E\left\{\mathbf{k}\mathbf{k}^H\right\} = \begin{bmatrix} 1 & 0 & |\rho| e^{j\phi_x} \\ 0 & 0.75 & 0 \\ |\rho| e^{-j\phi_x} & 0 & 1 \end{bmatrix}$$

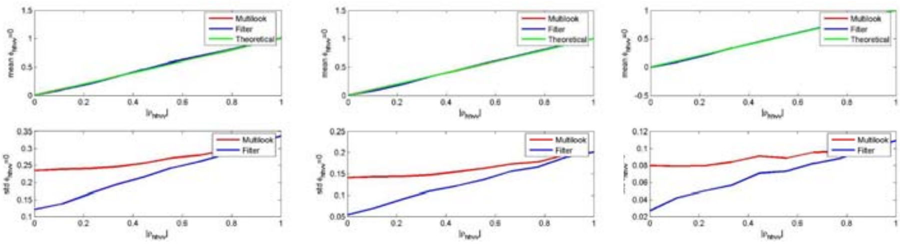
Matrix parameterized by the co-polar complex correlation coefficient

Performed tests:

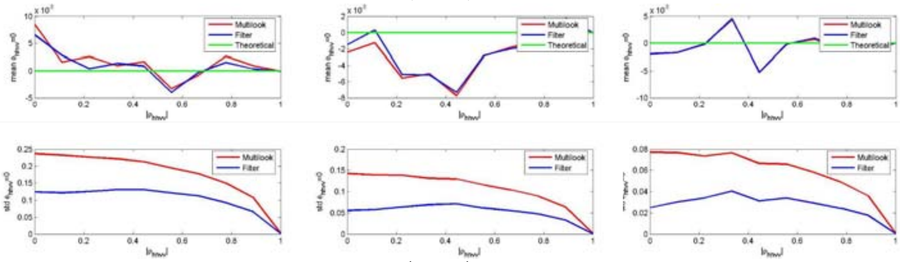
- Covariance matrix elements
  - Analysis of: Real and imaginary parts, amplitude, phase, correlation
- Covariance matrix
  - Analysis of: Eigendecomposition, polarimetric signatures

Results: Simulated Multidimensional SAR Data

3x3 looks 5x5 looks 9x9 looks



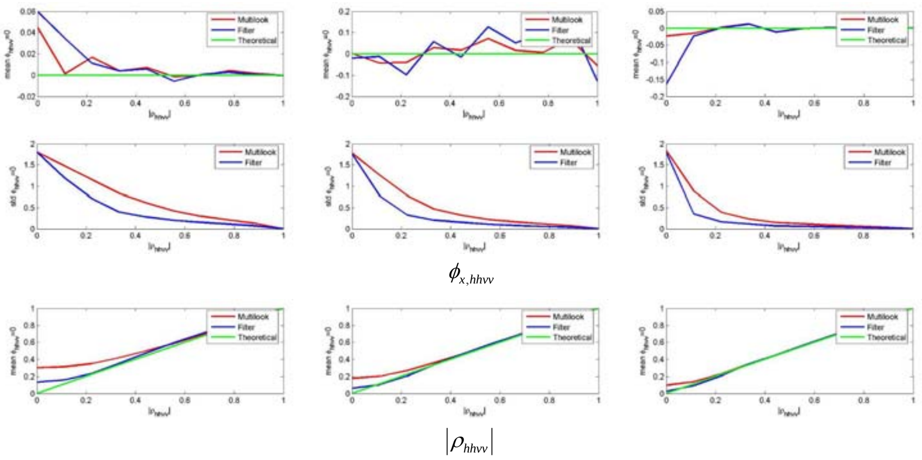
$\Re\left\{S_{hh}S_{vv}^*\right\} \quad \phi_x = 0$



$\Im\left\{S_{hh}S_{vv}^*\right\} \quad \phi_x = 0$

Results: Simulated Multidimensional SAR Data

3x3 looks | 5x5 looks | 9x9 looks

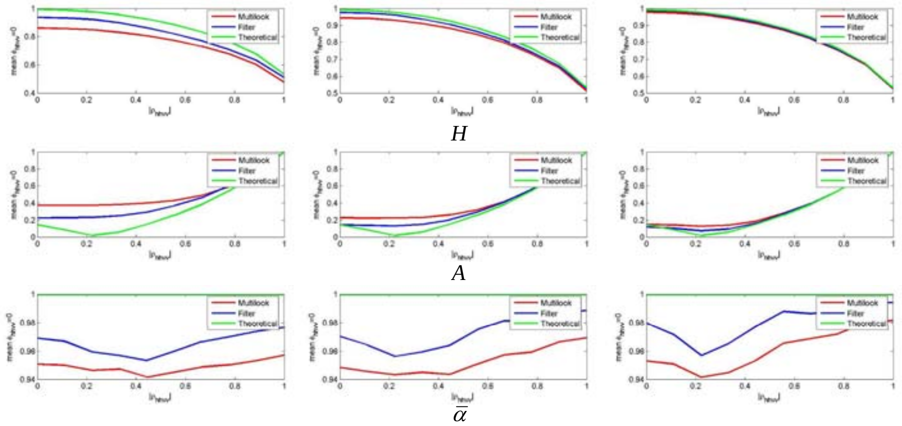


The multidimensional speckle noise model improves the estimation of covariance matrix components, but what happens with the whole covariance matrix?

Results: Simulated Multidimensional SAR Data

Eigendecomposition applied to the estimated covariance matrices

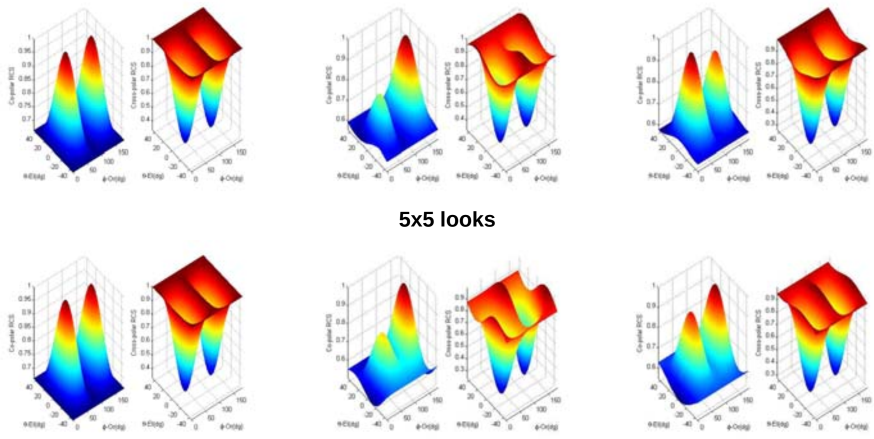
3x3 looks | **5x5 looks** | **9x9 looks**



To filter covariance elements differently does not damages information

Results: Simulated Multidimensional SAR Data

Theoretical PS | **Multilook PS** | **Filter PS**



5x5 looks

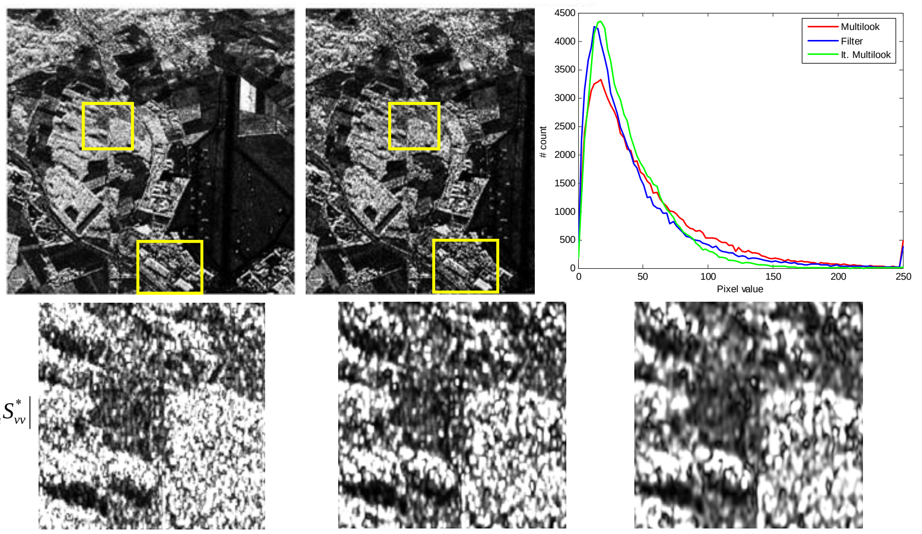
11x11 looks

What happens in this particular case?

Results: Experimental PolSAR Data

Full-polar ESAR L-Band SAR data in Oberpfaffenhofen (DE)

5x5 Multilook | **Filter** | **5x5 It. multilook**

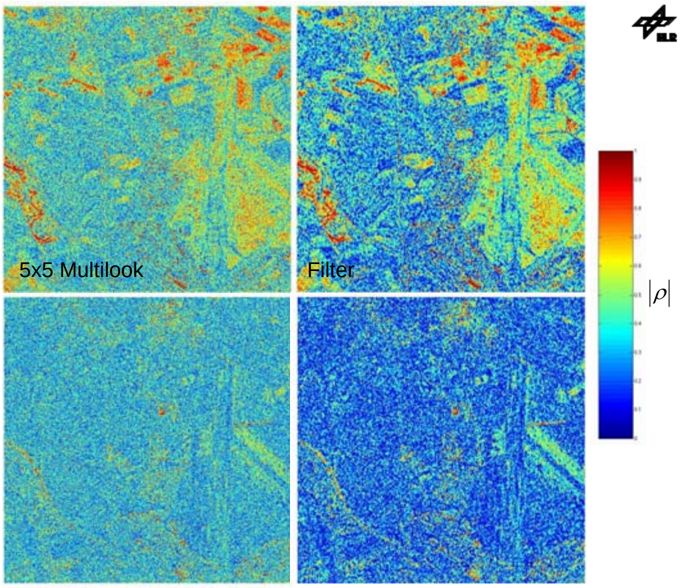


$|S_{hh}S_{vv}^*|$

Results: Experimental PolSAR Data

Co-polar correlation $|\rho_{hhvv}|$

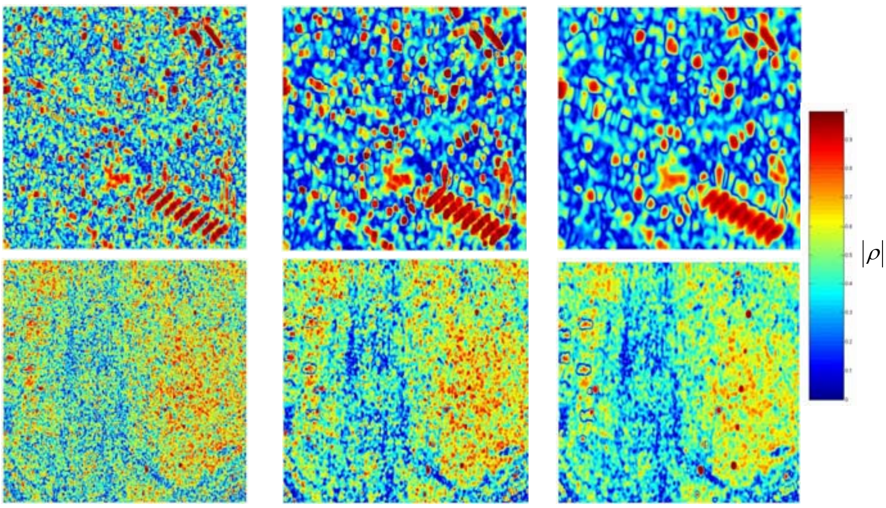
Cross-polar correlation $|\rho_{hhhv}|$



Results: Experimental PolSAR Data

Co-polar correlation $|\rho_{hhvv}|$: Details analysis

5x5 Multilook | Filter | 5x5 It. multilook

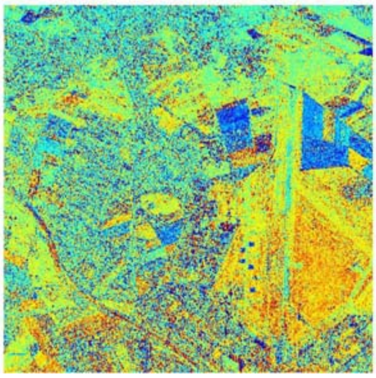


Results: Experimental PolSAR Data

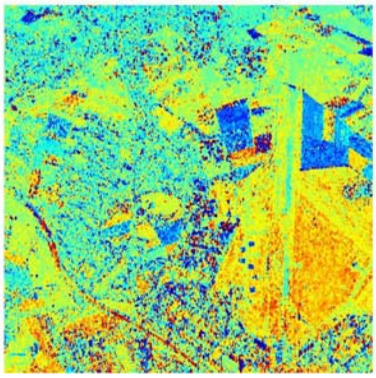
Co-polar correlation phase

$|\rho_{hhvv}|$

5x5 Multilook

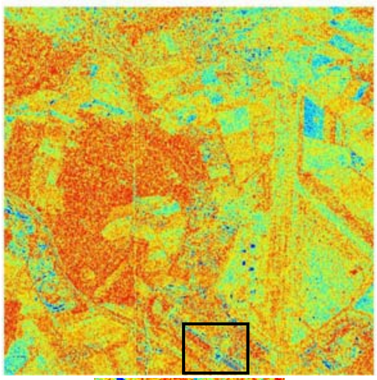


Filter

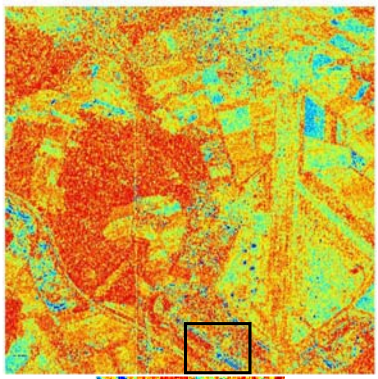


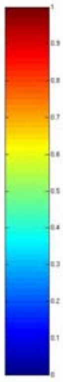
Results: Experimental PolSAR Data

5x5 Multilook

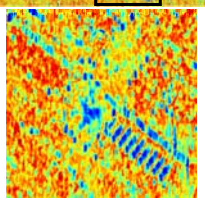


Filter



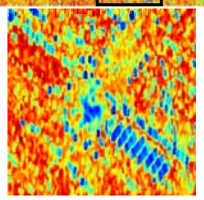


H



Entropy (H)

Details



Results: Experimental PolSAR Data