Detecting and monitoring the time-variable gravity field of Greenland

- using reprocessed GOCE gradients -

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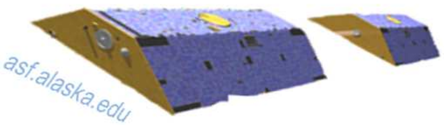
0. Motivation

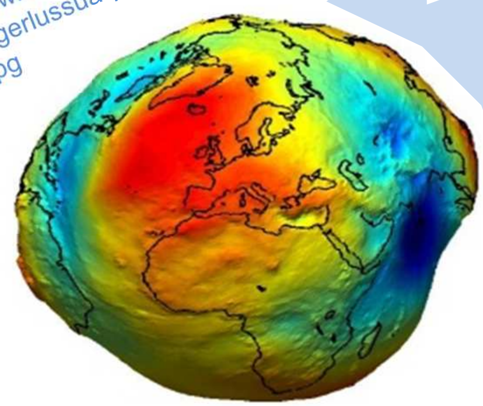
mass variations


http://en.wikipedia.org/wiki/File:Russells-gletscher-kangerlussuaq-greenland.jpg

different measurement systems


asf.alaska.edu


http://www.planetarium-laupheim.de/menu_ver_aktuelles_programm-Veranstaltungen-8.htm


http://geograficamente.word-press.com/2009/03/19/1621/

ice melting in Greenland

GOCE:
- sufficient time span
- reprocessing
  -> improved accuracy

time-variable gravity field of Greenland

1. Spectral sensitivity

- **resolution levels j** split spectrum into frequency bands
- upper boundary corresponds to the **maximum degree l** in a series expansion
- relation to the **spatial resolution r** on the Earth's surface
- GOCE covers higher frequencies than GRACE

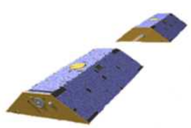
MBW: 5 … 30 mHz

$j = 5 \ldots 8$

$l = 2^j - 1$

$$r = \frac{20{,}000}{l} \; [km]$$

GRACE



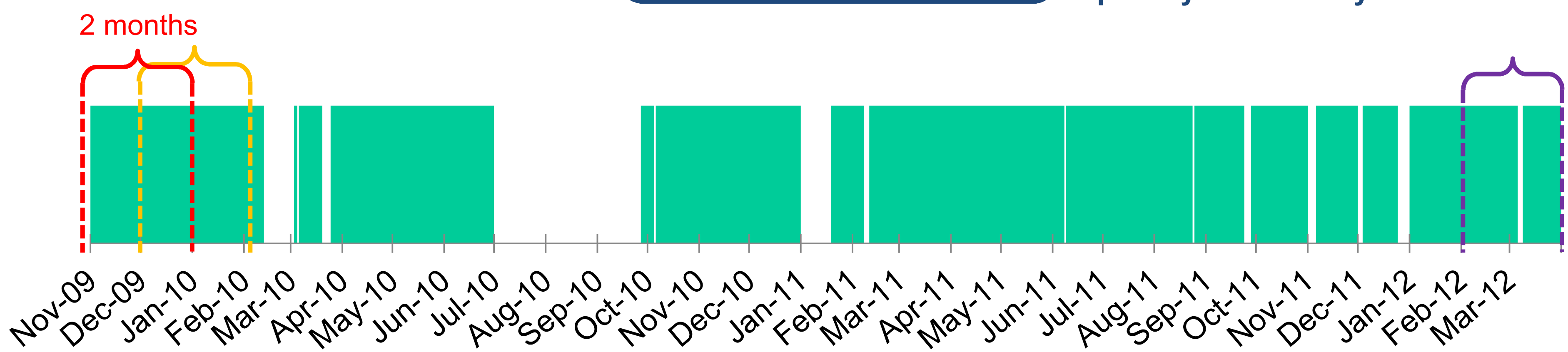
GOCE

j [level]	1	2	3	4	5	6	7	8	9	10	11	…
l [deg]	1	3	7	15	31	63	127	255	511	1023	2047	…
r [km]	20000	6667	2857	1333	645	317	157	78	39	20	10	…

frequency [deg] →

2. GOCE data set

Measuring:	Vzz	**GRF; Vzz ≈ Vrr**
Modifying:	Vzz_mod = (Vzz-Vxx-Vyy)/2	**40% noise reduction**
Filtering:	MBW: 5 … 30 mHz	**low frequencies: GOCO03S**
Creating 2-month data sets:	time span: 11/2009 - 03/2012 1 month overlap	**repeat cycle: ~ 61 days**



3. Regional gravity field modeling

GOCE

j [level]	1	2	3	4	5	6	7	8	9	10	11	12
l [deg]	1	3	7	15	31	63	127	255	511	1023	2047	4095
r [km]	20000	6667	2857	1333	645	317	157	78	39	20	10	5

frequency [deg] →

level j =7

- 2 month
- highest level within MBW (5 … 30 mHz ~ d/o 27 … 220)
- level j = 8 might be influenced by artefacts of non-equidistributed satellite tracks

1) Subtracting background model: **ΔVzz = Vzz - Vback**

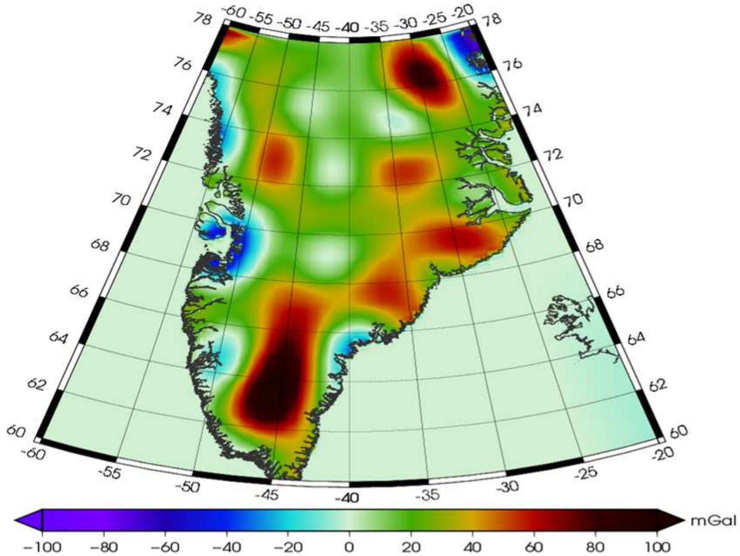
Vback: GOCO03S d/o 127

… the same as used for filling up low frequencies

… according to modeling resolution (**j = 7**)

(reduce static part completely)

3. Regional gravity field modeling

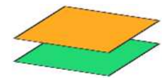


background model:

GOCO03S
Dg [mGal], d/o 127 (j=7)

3. Regional gravity field modeling

1) Subtracting background model: $\Delta Vzz = Vzz - Vback$



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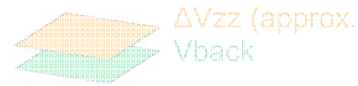

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Vback: GOCO03S
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… the same as used for filling up low frequencies

… according to modeling resolution (J = 7)

(reduce static part completely)

2) Analysis: series expansion in terms of **reproducing kernel**

krepro
d 140

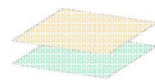
… estimating unknown scaling coefficients d_6

… avoiding omission errors

$$\Delta V_{zz}(P) + e_{Vzz}(P) = \sum_{q=1}^{N_6} d_{6,q}\, \widetilde{k}_{repro}(P,Q)$$

$$\widetilde{k}_{repro}(P,Q) = \sum_{l=0}^{140} \frac{2l+1}{4\pi R^2} \frac{(l+1)\,(l+2)}{r_p^2} \left(\frac{R}{r_p}\right)^{l+1} P_l(P,Q)$$

3. Regional gravity field modeling



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(reduce static part completely)

2) Analysis: series expansion in terms of **reproducing kernel**

krepro d 140

… estimating unknown scaling coefficients **d_6**

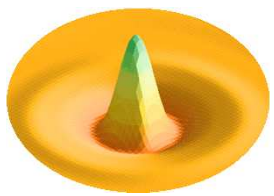
… avoiding omission errors

3) Synthesis: series expansion in **scaling functions**

Blackman d 127

… modeling approximation signal **F_7**

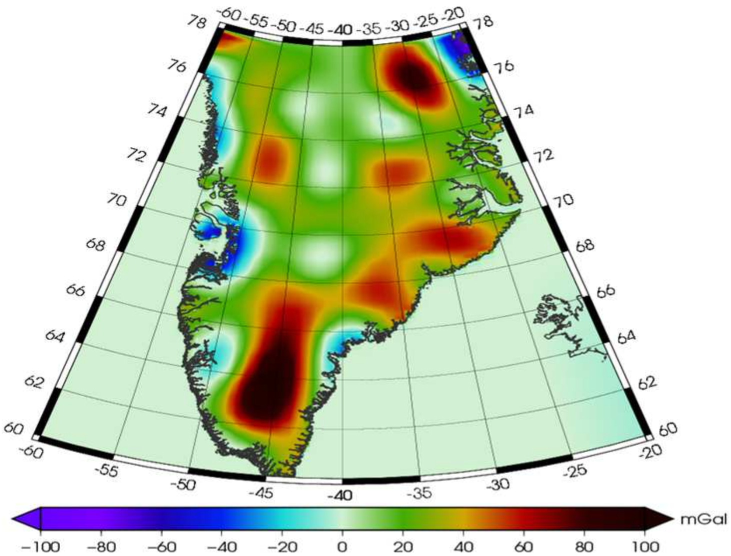
… low-pass filter



$$F_7(P) = \sum_{q=1}^{N_6} \widehat{d}_{6,q}\, \widetilde{\phi}_7(P,Q)$$

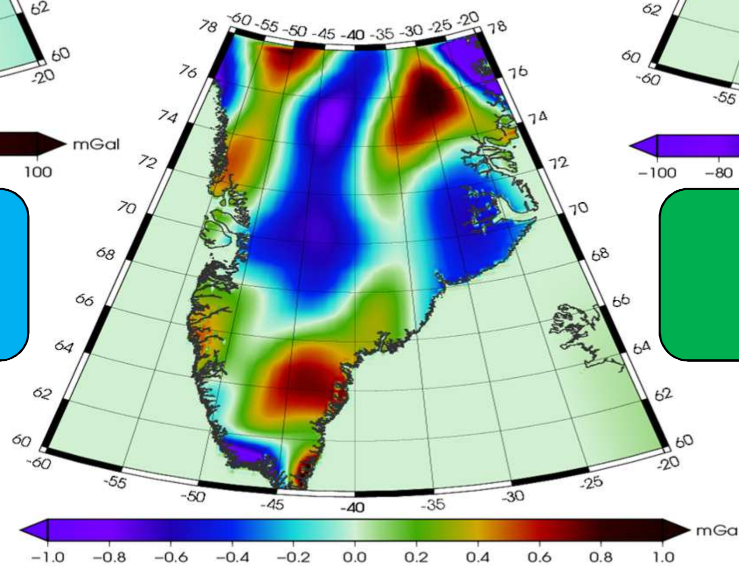
$$\widetilde{\phi}_7(P,Q) = \sum_{l=0}^{127} \frac{2l+1}{4\pi R^2}\,\frac{(l-1)}{r_p}\left(\frac{R}{r_p}\right)^{l+1} \Phi_{7,l}\, P_l(P,Q)$$

3. Regional gravity field modeling



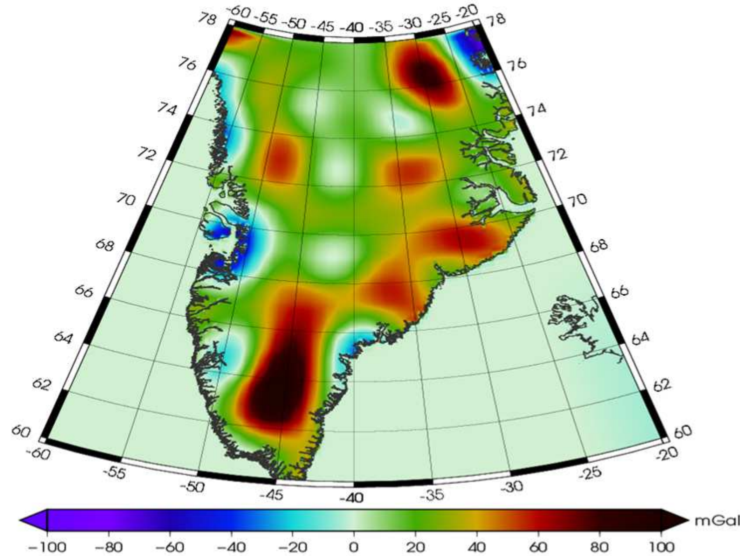
full modeled signal:

In: Goce Vzz
Out: Dg [mGal], d 127 (j=7)



approximation signal $F_{J=7}$:

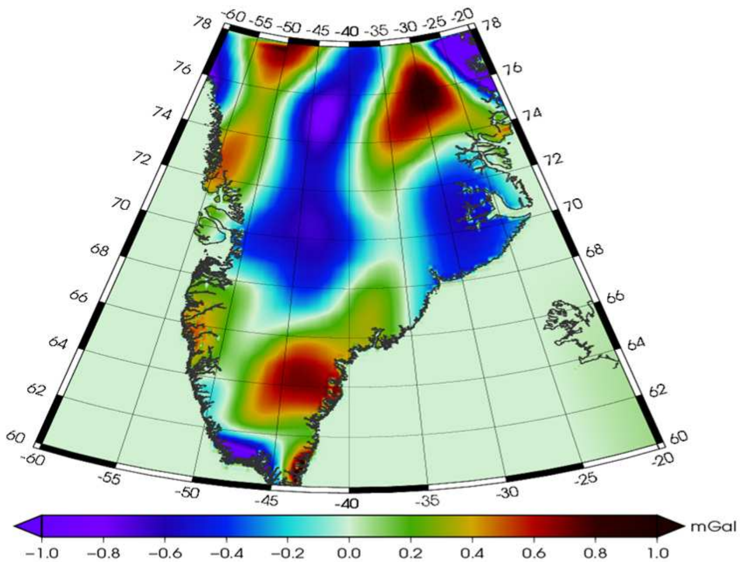
Dg [mGal], d 127 (j=7)



background model:

GOCO03S
Dg [mGal], d/o 127 (j=7)

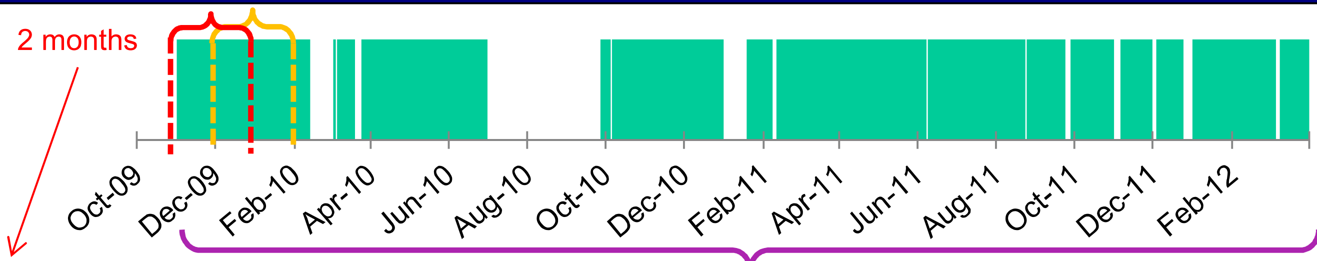
3. Regional gravity field modeling

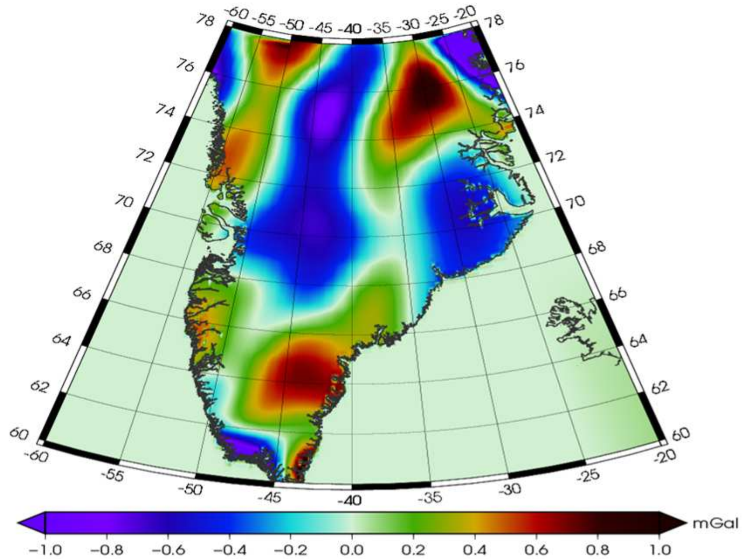


approximation signal $F_{J=7}$:
Dg [mGal], d 127 (j=7)

4. Time series

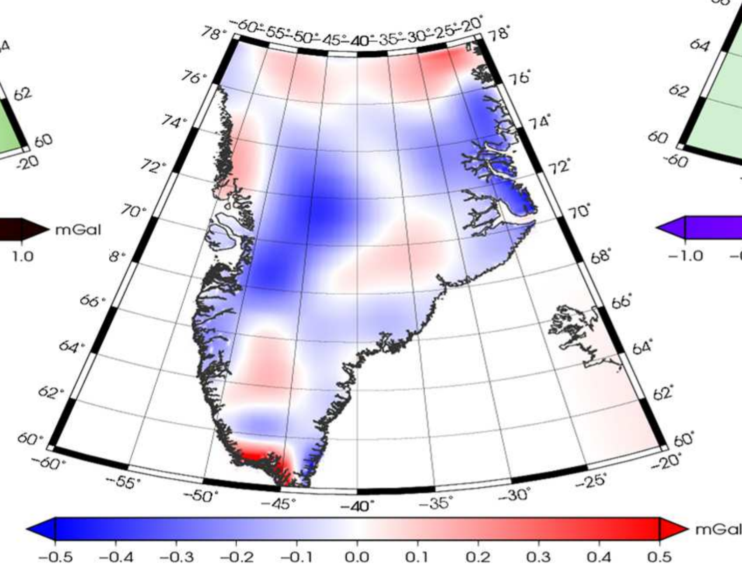
i. **Regional approach – GOCE**
ii. Regional approach – GRACE

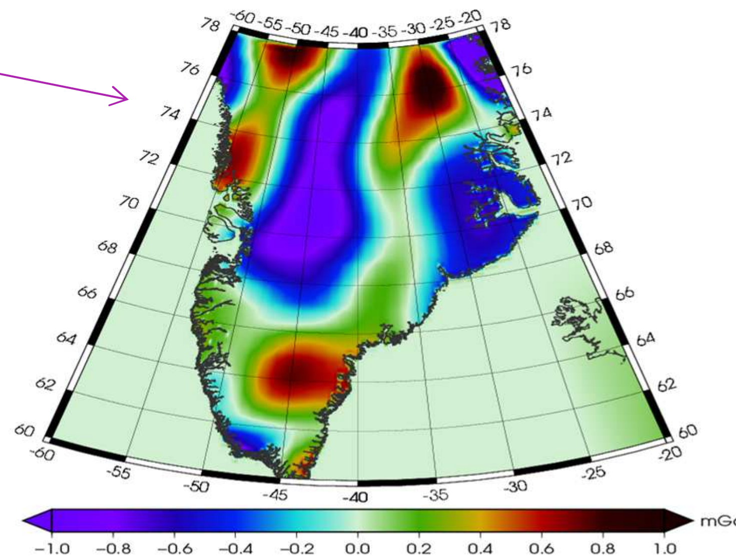




2 months (11/09, 12/09)
ΔDg [mGal], $j = 7, l = 127$

2 months – static
ΔDg [mGal], $j = 7, l = 127$





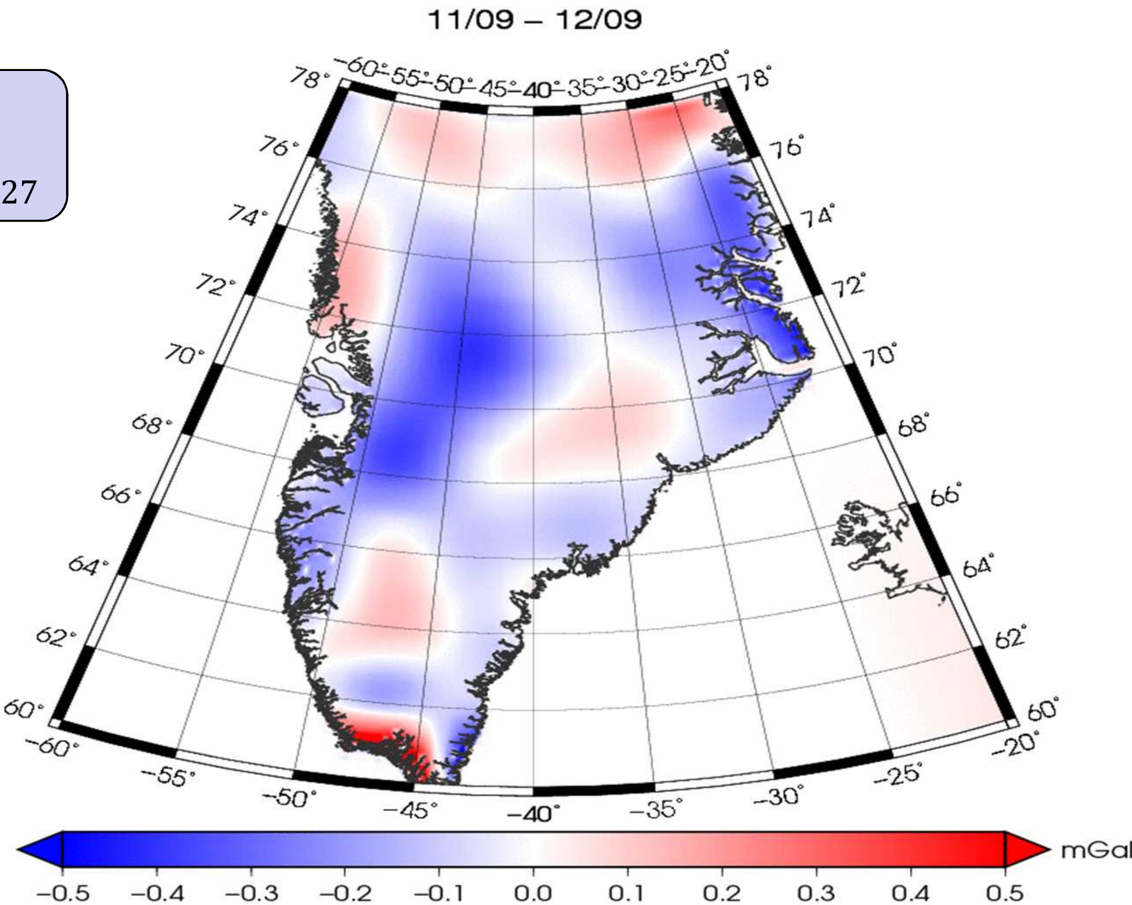
static (11/09 … 03/12)
ΔDg [mGal], $j = 7, l = 127$

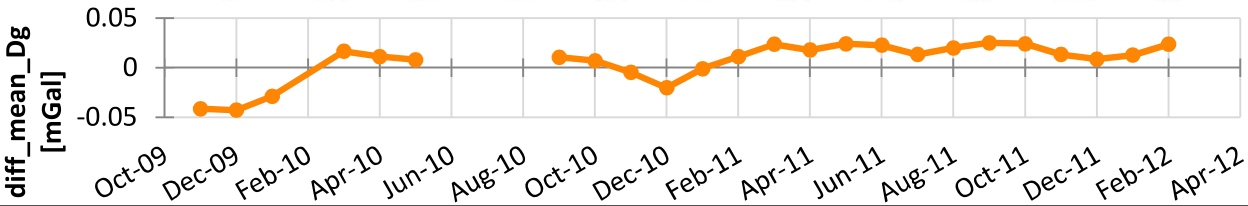
4. Time series

i. **Regional approach – GOCE**
ii. Regional approach – GRACE

Goce ΔVzz

2 months – static
ΔDg [mGal], $j = 7, l = 127$



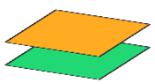


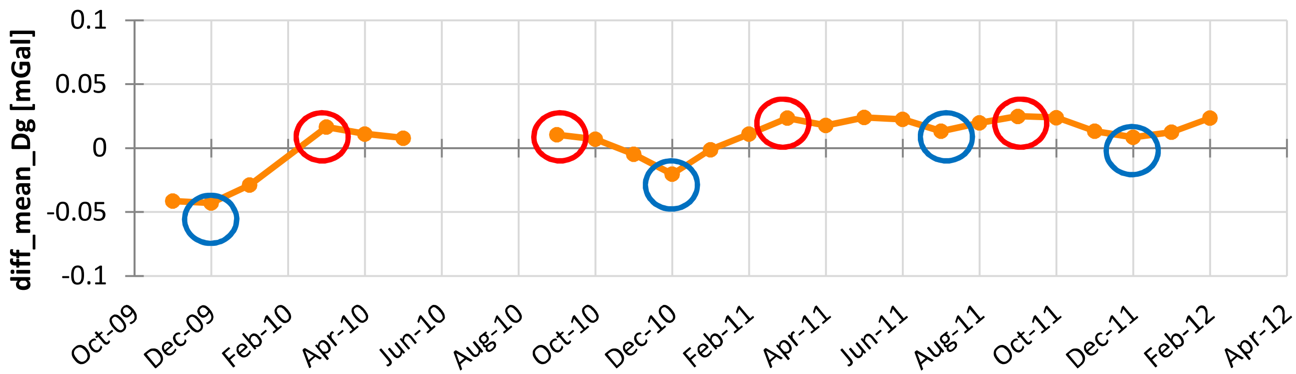
4. Time series

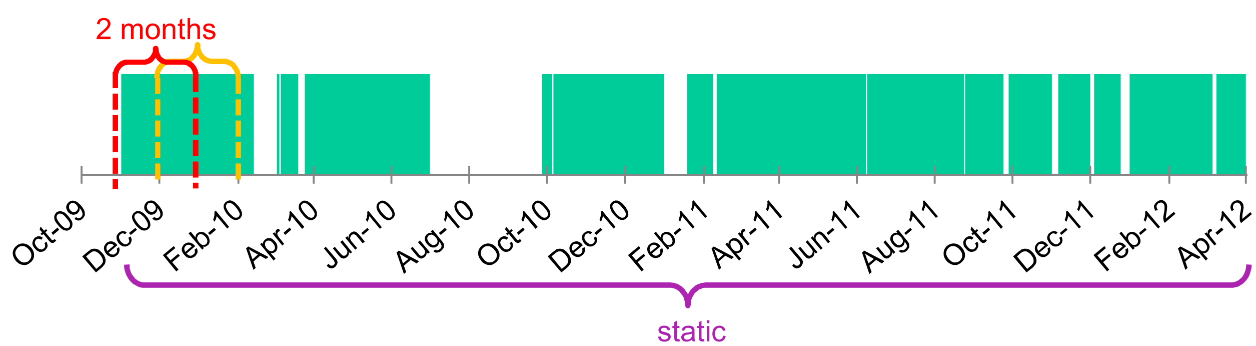
Regional approach - GOCE ($j = 7$)

Goce ΔVzz

2 months – static
ΔDg [mGal], $j = 7, l = 127$



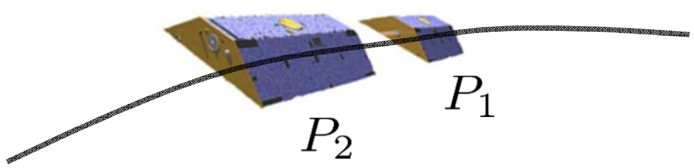




- variation: ~ 30 µGal
- seasonal variations
  max: Mar/Apr, Sep/Oct
  min: Dec/Jan, Jul/Aug
- no trend

Regional approach – GRACE ($j = 6$)

1) Computing the potential differences $dV(P_1, P_2) = V(P_1) - V(P_2)$ from GSM potential fields



2) Subtracting a background model: $\Delta dV(P_1, P_2) = dV(P_1, P_2) - dV_{back}(P_1, P_2)$

GRACE

j [level]	1	2	3	4	5	6	7	8	9	10	11	...
l [deg]	1	3	7	15	31	63	127	255	511	1023	2047	...
r [km]	20000	6667	2857	1333	645	317	157	78	39	20	10	...

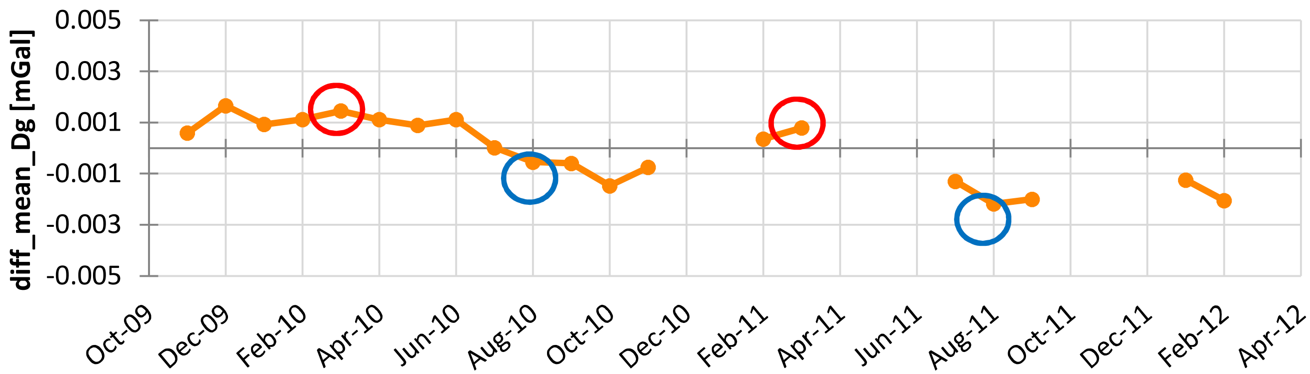
frequency [deg] →

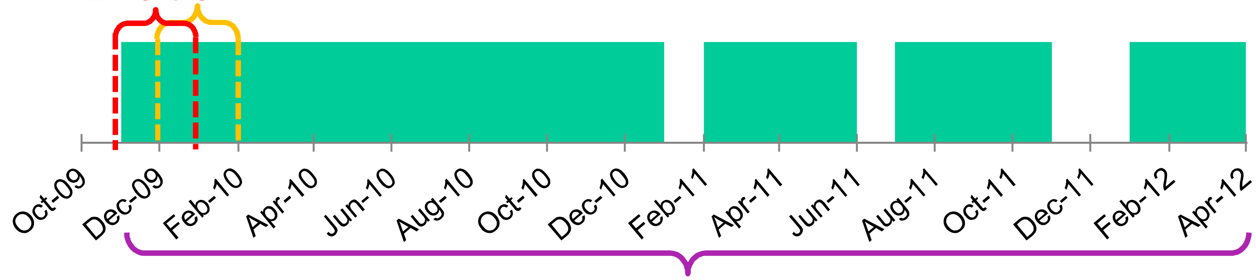
Regional approach – GRACE ($j = 6$)

Grace ΔdV

2 months – static
ΔDg [mGal], $j = 6, l = 63$

mean ΔdV (approx.)
Vback subtracted



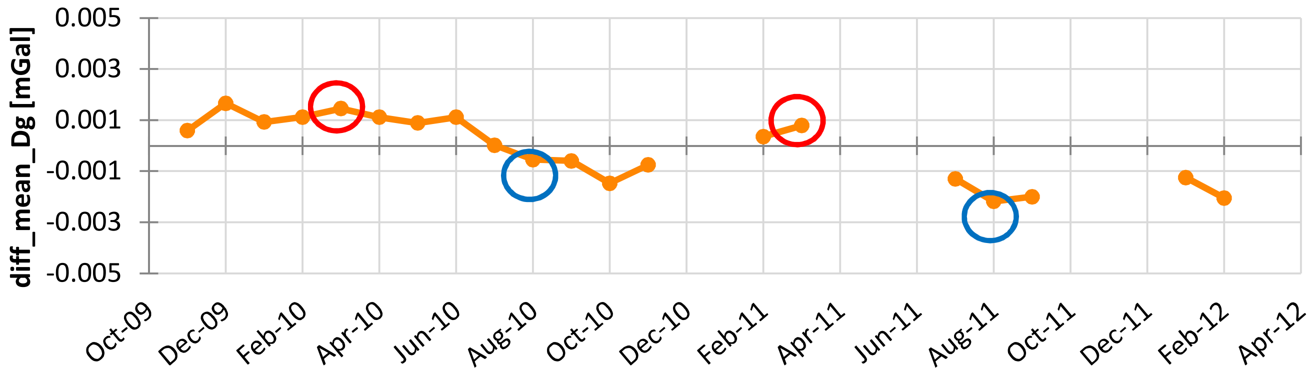


- variation: ~ 5 µGal
- seasonal variations
  max: Mar/Apr
  min: Aug/Sep
- descending trend ?

4. Time series

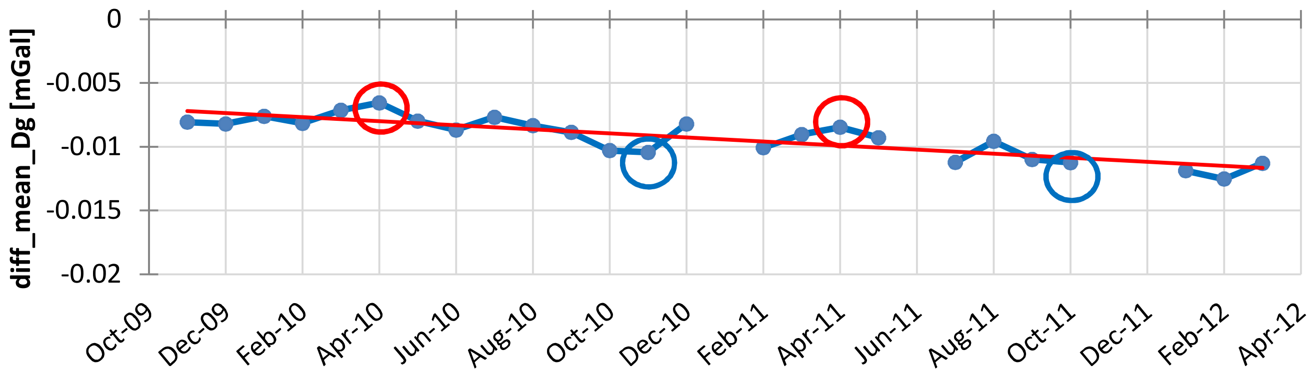
Grace ΔdV
2 months – static
ΔDg [mGal], $j = 6, l = 63$

mean ΔdV (approx.)
Vback subtracted



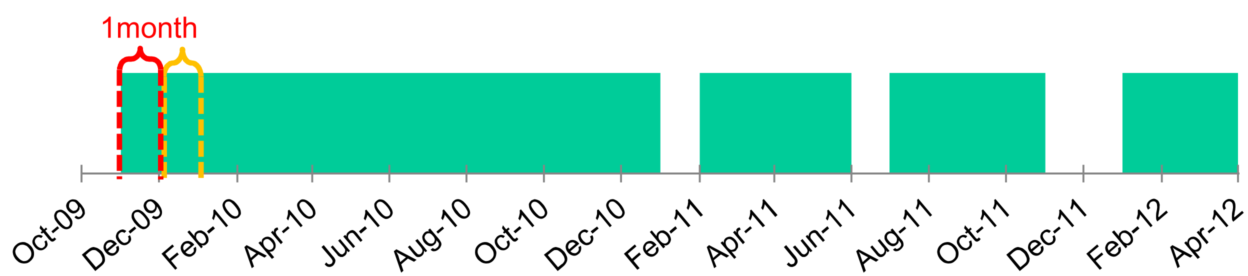
Grace GSM
1 month – GOCO03S
ΔDg [mGal], $j = 6, l = 63$

spherical harmonics



- variation: 5 µGal
- seasonal variations
  (not so clear)
  max: Apr
  min: Oct
- descending trend: 5 µGal

-> parallels detectable



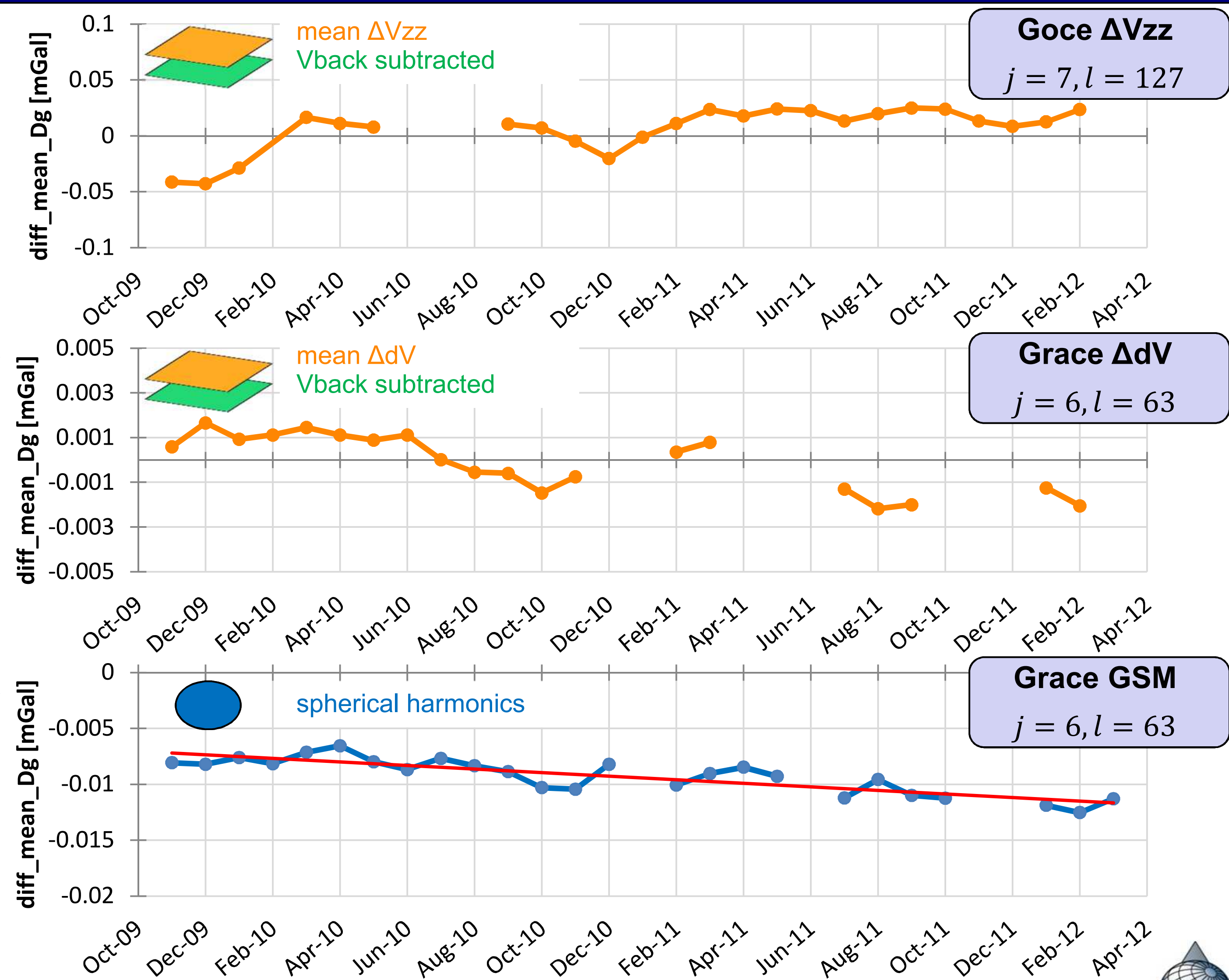
5. Aim: combination

regional: j=7 vs. j=6

consistent maxima and minima

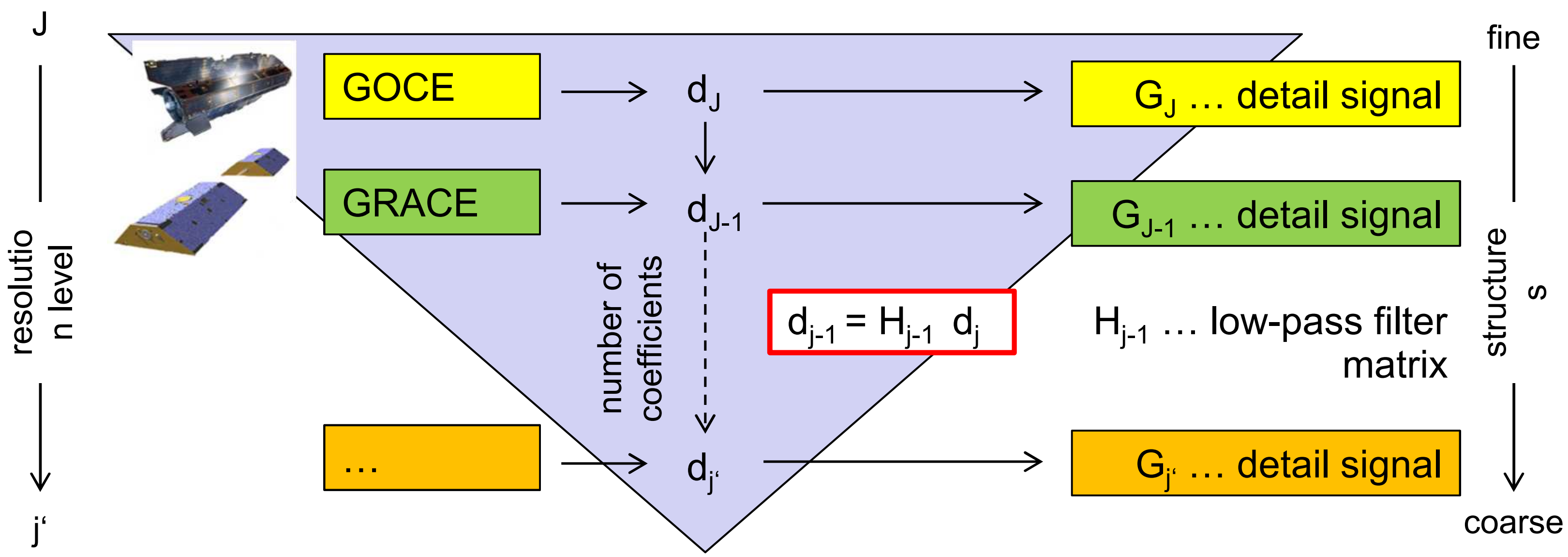
j=6: regional vs. global

consistent trend



5. Aim: combination

Pyramidal algorithm



Summary

GOCE: originally planned to observe the Earth's static gravity field, but…

… time variations visible??

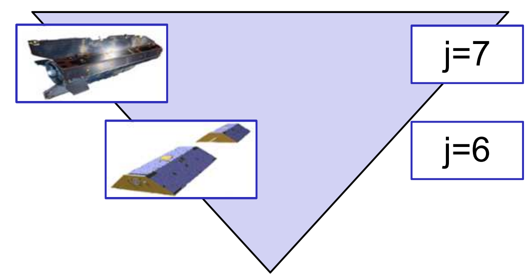
- 2-months solutions compared with static solution

- semi-seasonal variations detected with high sensitivity by GOCE
- good consistency to GRACE

- combination of GOCE + GRACE
- exploit highest degree of information from each measurement technique

Open questions:

- consistent data sets (2-months GOCE vs. 1-month GRACE-GSM)?
- Vzz_mod: systematic errors caused by Vyy?
- using full GOCE gradient tensor information?
- combination by pyramidal algorithm
  (step-by-step introducing new observation techniques)?



Appendix

3. Regional gravity field modeling

- With all these assumptions we obtain the observation equation for the **modified GOCE gravity gradient** $V_{zz} = V_{rr}$ with $\Delta V_{zz}(\boldsymbol{x}(t)) = V_{zz}(\boldsymbol{x}(t)) - V_{zz,\mathrm{GOCO03S}}(\boldsymbol{x}(t))$

$$\Delta V_{zz}(\boldsymbol{x}(t)) + e_{zz}\big(\boldsymbol{x}(t)\big) = \Delta V_{zz,7}(\boldsymbol{x}(t)) = \sum_{q=1}^{N_6} d_{6,q}\ \tilde{\phi}_7\big(\boldsymbol{x}(t), \boldsymbol{x}_q\big)$$

(globally the condition $N_6 \geq 16{,}384 = 128^2$ $(19{,}881 = 141^2)$ has to be fulfilled).

The **modified scaling functions** $\tilde{\phi}_7\big(\boldsymbol{x}(t), \boldsymbol{x}_q\big)$ are defined as

$$\tilde{\phi}_7\big(\boldsymbol{x}(t), \boldsymbol{x}_q\big) = \sum_{l=0}^{l'_{127}} \frac{2l+1}{4\pi\, R^2} \frac{(l+1)(l+2)}{r(t)^2} \left(\frac{R}{r(t)}\right)^{l+1} \Phi_{7,l}\, P_l\big(\boldsymbol{r}(t)^T \boldsymbol{r}_q\big) \quad .$$

- With the $N_6 \times 1$ vectors $\boldsymbol{a}^T(\boldsymbol{x}) = \left[\tilde{\phi}_7(\boldsymbol{x}, \boldsymbol{x}_1), \tilde{\phi}_7(\boldsymbol{x}, \boldsymbol{x}_2), \ldots, \tilde{\phi}_7(\boldsymbol{x}, \boldsymbol{x}_{N_6})\right]$ and $\boldsymbol{d}_6{}^T = \left[d_{6,1}, d_{6,2}, \ldots, d_{6,N_6}\right]$ the **general observation equation** reads

$$\Delta V_{zz}(\boldsymbol{x}(t)) + e_{zz}\big(\boldsymbol{x}(t)\big) = \boldsymbol{a}^T(\boldsymbol{x}(t))\, \boldsymbol{d}_6\, .$$

3. Regional gravity field modeling

- Considering the prior information $E(\boldsymbol{d}_J) = \boldsymbol{\mu}_d$ and $D(\boldsymbol{d}_J) = \boldsymbol{\Sigma}_d$ for the expectation vector and the covariance matrix of the vector $\boldsymbol{d}_J$ the linear model

$$\begin{bmatrix} \boldsymbol{y} \\ \boldsymbol{\mu}_d \end{bmatrix} + \begin{bmatrix} \boldsymbol{e} \\ \boldsymbol{e}_d \end{bmatrix} = \begin{bmatrix} \boldsymbol{A} \\ \boldsymbol{I} \end{bmatrix} \boldsymbol{d}_J \qquad D\left(\begin{bmatrix} \boldsymbol{y} \\ \boldsymbol{\mu}_d \end{bmatrix} \right) = \sigma_y^2 \begin{bmatrix} \boldsymbol{P}_y^{-1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} + \sigma_d^2 \begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{P}_d^{-1} \end{bmatrix}$$

results, wherein σ_y^2 and σ_d^2 are unknown variance components, $\boldsymbol{P}_y$ is the given positive weight matrix of the observations.

- **Variance component estimation** yields the estimation

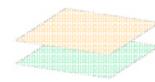
$$\widehat{\boldsymbol{d}}_J = \left(\frac{1}{\widehat{\sigma}_y^2} \boldsymbol{A}^T \boldsymbol{P}_y \boldsymbol{A} + \frac{1}{\widehat{\sigma}_d^2} \boldsymbol{P}_d \right)^{-1} \left(\frac{1}{\widehat{\sigma}_y^2} \boldsymbol{A}^T \boldsymbol{P}_y \boldsymbol{y} + \frac{1}{\widehat{\sigma}_d^2} \boldsymbol{P}_d \boldsymbol{\mu}_d \right)$$

of the coefficient vector and its covariance matrix $D(\widehat{\boldsymbol{d}}_J)$.

Introducing the parameter $\lambda = \hat{\sigma}_y^2 / \hat{\sigma}_d^2$ the solution can be rewritten as

3. Regional gravity field modeling

i. General approach
ii. MRR



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d/o 127

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… according to modeling resolution (J = 7)

(reduce static part completely)

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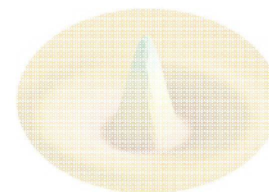


3) Synthesis: series expansion in **scaling functions**

Blackman
d 127

… modeling approximation signal F_7

… low-pass filter

4) MRR: series expansion in **wavelet functions**

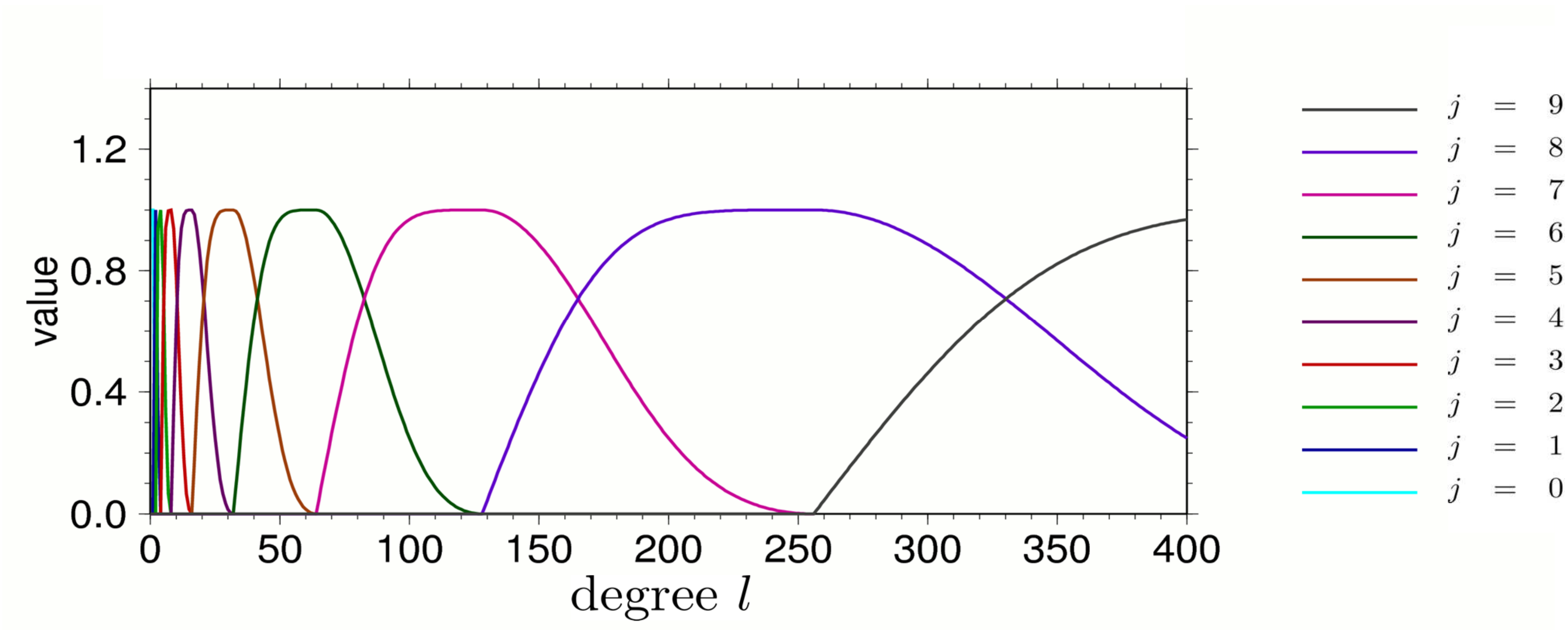
Vback: GOCO03S
d/o 127

… splitting approximation signal into detail signals $G_{j=0,\ldots,6}$

… using Blackman wavelet functions

… band-pass filter

3. Regional gravity field modeling



4) MRR: series expansion in **wavelet functions**

Vback: GOCO03S
d/o 127

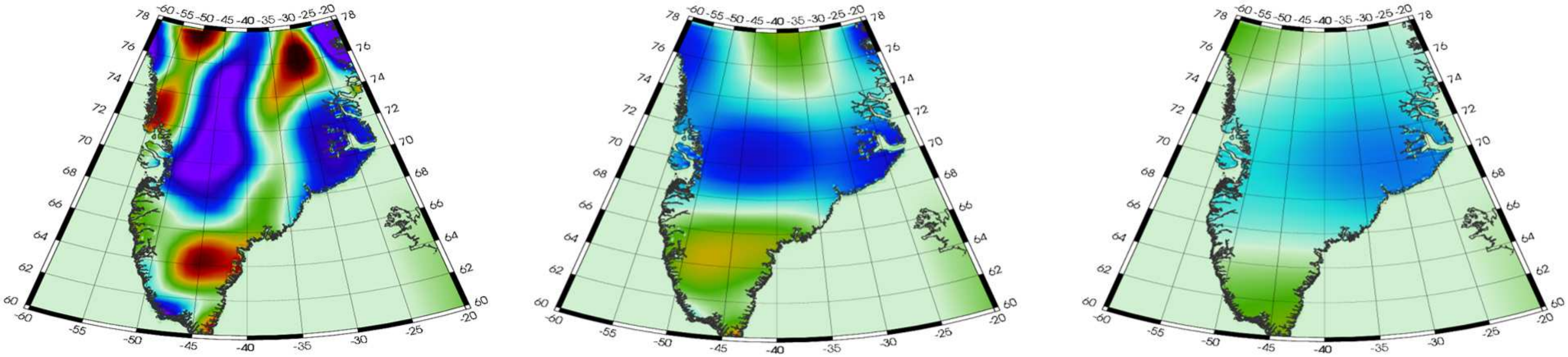
… splitting approximation signal into detail signals $G_{j=0,...,6}$

… using Blackman wavelet functions

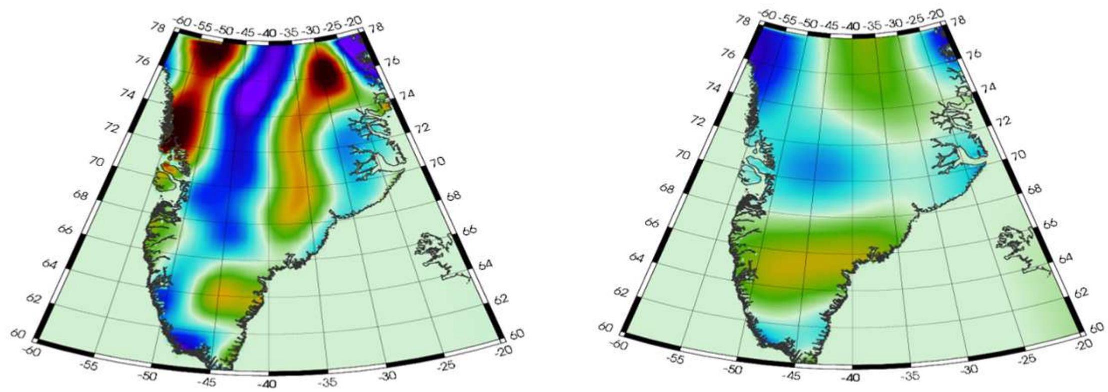
… band-pass filter

3. Regional gravity field modeling

F_7 up to $l = 127$ | F_6 up to $l = 63$ | F_5 up to $l = 31$



G_6 $l = 32 \dots 127$ | G_5 $l = 16 \dots 63$

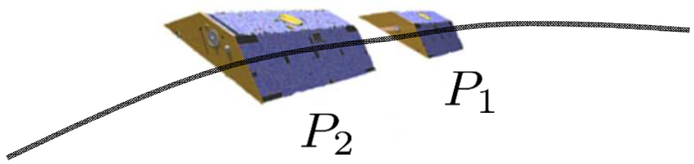


Multi-resolution representation (MRR)

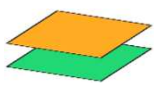
$$F_7 = F_5 + \sum_{j=5}^{6} G_j$$

Regional approach – GRACE ($j = 6$)

1) Computing the potential differences $dV(P_1, P_2) = V(P_1) - V(P_2)$ from GSM potential fields



2) Subtracting a background model: $\Delta dV(P_1, P_2) = dV(P_1, P_2) - dV_{back}(P_1, P_2)$



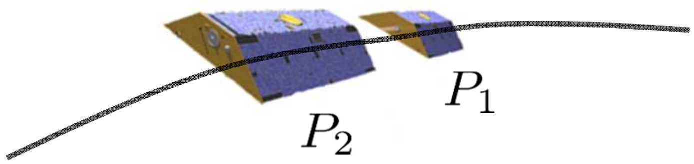
GRACE

j [level]	1	2	3	4	5	6	7	8	9	10	11	…
l [deg]	1	3	7	15	31	63	127	255	511	1023	2047	…
r [km]	20000	6667	2857	1333	645	317	157	78	39	20	10	…

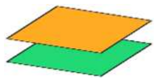
frequency [deg] →

Regional approach – GRACE ($j = 6$)

1) Computing the potential differences $dV(P_1, P_2) = V(P_1) - V(P_2)$ from GSM potential fields



2) Subtracting a background model: $\Delta dV(P_1, P_2) = dV(P_1, P_2) - dV_{back}(P_1, P_2)$



3) Analysis: series expansion in terms of **reproducing kernel** (up to degree $l = 75$)

$$\Delta dV(P_1, P_2) + e_{dV}(P_1, P_2) = \sum_{q=1}^{N_5} d_{5,q} \, \tilde{k}_{repro}(P_1, P_2, Q)$$

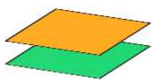
$$\tilde{k}_{repro}(P_1, P_2, Q) = k_{repro}(P_1, Q) - k_{repro}(P_2, Q)$$

4. Time series

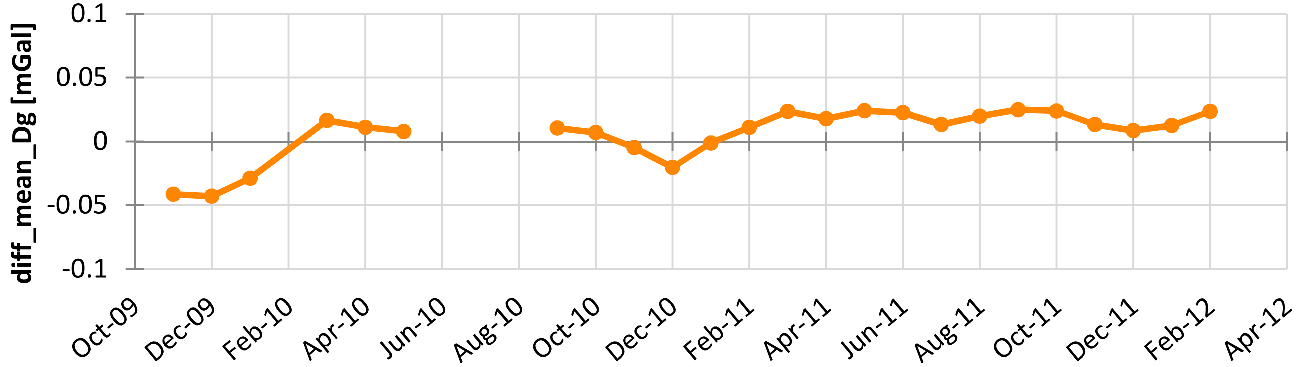
i. Regional approach – GOCE
ii. Regional approach – GRACE

Goce ΔVzz

2 months – static
ΔDg [mGal], j=7 (d/o127)

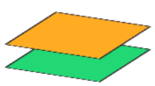


mean ΔVzz (approx.)
Vback subtracted

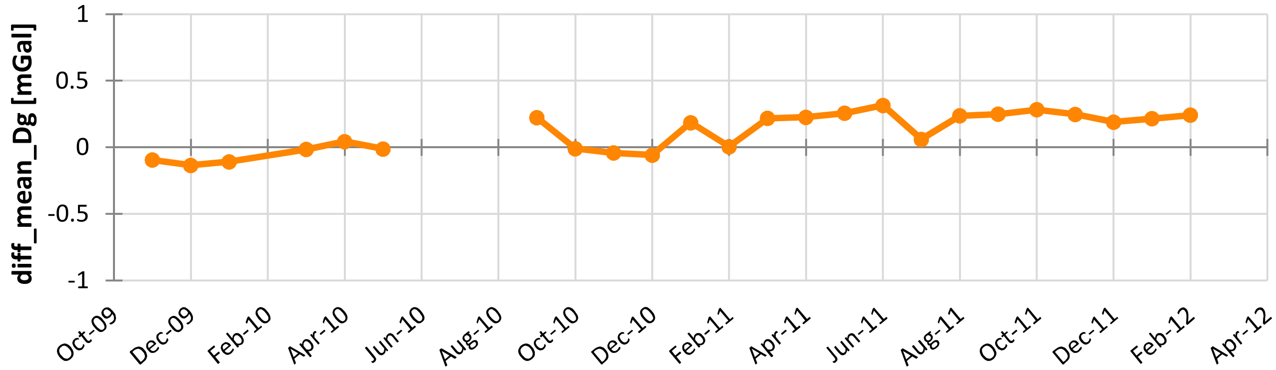


Goce ΔVzz

2 months – static
ΔDg [mGal], j=6 (d/o63)

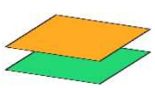


mean ΔVzz (approx.)
Vback subtracted

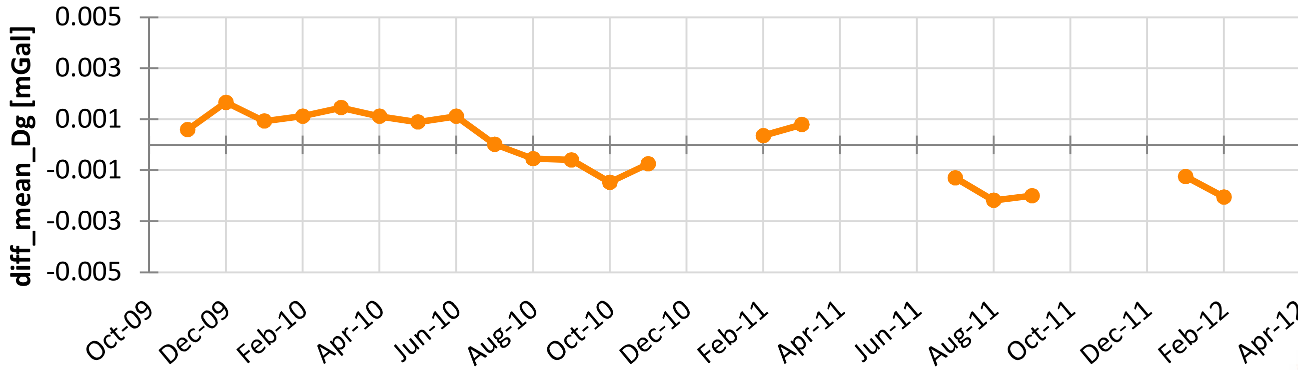


Grace ΔdV

2 months – static
ΔDg [mGal], j=6 (d/o63)



mean ΔdV (approx.)
Vback subtracted

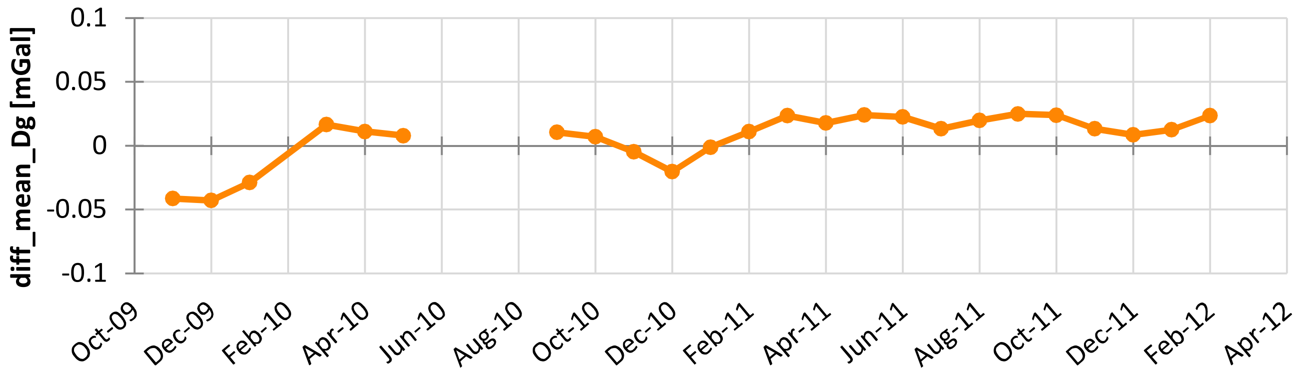


5. Aim: combination

Goce ΔVzz

2 months – static
ΔDg [mGal], j=7 (d/o127)

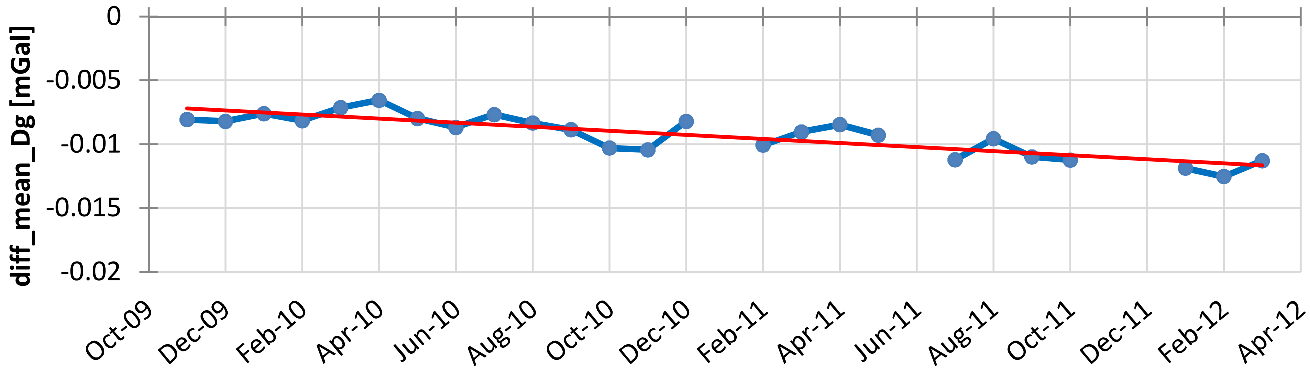
mean ΔVzz (approx.)
Vback subtracted



Grace GSM

1 month – GOCO03S
ΔDg [mGal], j=6 (d/o63)

spherical harmonics



GOCE j=7

- variation: 75 µGal
- sensitive at high frequencies
- seasonal variations

GRACE j=6

- variation smaller
- clear trend