

GOCE EXPLOITATION FOR MOHO MODELING AND APPLICATIONS

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ABSTRACT

New ESA missions dedicated to the observation of the Earth from space, like the gravity-gradiometry mission GOCE and the radar altimetry mission CRYOSAT 2, foster research, among other subjects, also on inverse gravimetric problems and on the description of the nature and the geographical location of gravimetric signals.

In this framework the GEMMA project (GOCE Exploitation for Moho Modeling and Applications), funded by the European Space Agency and Politecnico di Milano, aims at estimating the boundary between Earth's crust and mantle (the so called Mohorovičić discontinuity or Moho) from GOCE data in key regions of the world.

In the project a solution based on a simple two layer model in spherical approximation is proposed. This inversion problem based on the linearization of the Newton's gravitational law around an approximate mean Moho surface will be solved by exploiting Wiener-Kolmogorov theory in the frequency domain where the depth of the Moho discontinuity will be treated as a random signal with a zero mean and its own covariance function. The algorithm can be applied in a numerically efficient way by using the Fast Fourier Transform.

As for the gravity observations, we will consider grids of the anomalous gravitational potential and its second radial derivative at satellite altitude. In particular this will require first of all to elaborate GOCE data to obtain a local grid of the gravitational potential field and its second radial derivative and after that to separate the gravimetric signal due to the considered discontinuity from the gravitational effects of other geological structures present into the observations. The first problem can be solved by applying the so called space-wise approach to GOCE observations, while the second one can be achieved by considering a priori models and geophysical information by means of an appropriate Bayesian technique. Moreover other data such as ground gravity anomalies or seismic profiles can be combined, in an efficient way, to gridded satellite data in order to obtain better results. The research has to be firstly performed on case studies where existing data allow the calibration of the approach.

Among other things this project represents a careful study of the prior weighting of different information

sources, including the rather qualitative geological information.

1. INTRODUCTION

The Mohorovičić discontinuity (from the name of the Croatian seismologist who discovered it in 1909) is the boundary between the crust and mantle. It separates rocks having V_p (P-wave) velocities of 6 km/s from those having velocities of about 7-8 km/s.

Since the upper part of the mantle is denser than the crust the Moho depth can be inferred also from gravity data. The main aim of the GOCE Exploitation for Moho Modeling and Applications (GEMMA) project, funded by ESA and Politecnico di Milano, is to map the crust-mantle discontinuity in key regions of the world by means of observation coming from the innovative satellite mission GOCE. This will permit, for the first time, to estimate a Moho model without any need for geophysical interpretation (avoiding the uncertainties connected with this operation). Moreover since the main input are gravimetric data collected by the satellite mission GOCE it will be possible to estimate the Moho almost worldwide (also where gravity or seismic observations are not available for political or economical reasons).

Usually, in literature, global and local models describing the Earth crust and the Moho are based on seismic refraction data, for example CRUST 2.0 (see Bassin et. al. [1]) a global model of USGS (United States Geological Survey) is based on 360 one-dimensional seismic refraction profiles. This model consists of $2^\circ \times 2^\circ$ tiles in which the crust and uppermost mantle are described. Topography and bathymetry are adopted from a standard database (ETOPO5). Compressional wave velocity in each layer is based on field measurements, and shear wave velocity and density are estimated using empirical V_P - V_S and V_P -density relationships. Statistics are used to predict crustal structures in areas without field measurements. In these unsurveyed areas, the thickness of ice, water and sediments is taken from published compilations, and the velocity structure of the crystalline crust and uppermost mantle is estimated from the statistical average of regions with a similar crustal age and tectonic setting.

Another example of Moho estimation from seismic data is the new Moho map of the European plate (see Grad et al. [2]). This model is based on several one-dimensional

seismic profiles taken from 1970 to present years. Even if a huge number of observations have been considered, the dishomogeneity (in time and space) of the different data sets implies large errors in the final result.

A further problem of this kind of models is that refraction and reflection of seismic waves, from which these models are inferred, need to be interpreted. This operation is usually done in a subjective way and implies great uncertainties that lead to large errors in the final Moho model (i.e. errors in the European Moho map go up to 10 km). A possible alternative solution for the Moho estimation avoiding the interpretation of seismic profiles is the one based on gravimetric information.

This problem, based on the inversion of Newton's laws of gravitation, is called in general inverse gravimetric problem and consists of the determination of the internal density distribution of a body ρ from its exterior gravity field V . Different approaches to solve this problem can be found in literature, see among the others Parker [3], Oldenburg [4], or the more recent works of Shin et al. [5] and Shin et al. [6] and the reference therein.

Another thorny issue to face in order to solve the inverse gravimetric problem is how to separate the various signals, coming from different geological structures, mixed up into the observed gravimetric data. As a matter of fact this can be achieved only with the help of additional geological information by modeling crustal dishomogeneities, as well as unwrapping the contributions of large deep features from those closer to the surface. In general this can be done by simple stripping from the gravity data a reference (global) model and the effects of known geological structures (Ioane et al. [7] or Simeoni and Brückl [8]).

2. THE PROPOSED APPROACH

If we consider the inversion of Newton's gravitational potential it turns out that each of Hadamard's criteria for a well-posed problem (existence, uniqueness and stability of the solution) is violated, in particular if the problem is solvable, then the space of all solutions $\rho(Q)$ corresponding to a fixed potential $V(P)$ is infinite-dimensional. This non-uniqueness of the solution can, for instance, be treated by considering very simplifying hypotheses. Among other the uniqueness of the solution has been proved (Sampietro and Sansò [10]) for three simple cases:

1. the recovery of the interface between two layers of known density;
2. the recovery of the density distribution, in a two layers model, given the geometry of the problem (topography and depth of compensation);
3. the recovery of the distribution of the vertical gradient of density, in a two layers model, given the geometry of the problem (topography and depth of compensation) and the density distribution at sea level.

Even if this three cases are very rough hypothesis, in principle one can think that it is better to use one of these geophysical approximation and find a unique solution rather than accepting a solution that can be very far from reality because it corresponds to a purely mathematical criterion (Sansò et al. [10]). Once the uniqueness is guaranteed, we are entitled to apply to the corresponding inverse problem a regularization method and we know from literature (e.g. Schock [11]) that in this way we can approximate the true solution, dominating the inherent instabilities.

In the proposed approach the first hypothesis has been considered: we neglect the effect due to the atmosphere, we consider a mean reference Moho (computed for example from an isostatic model or from the CRUST 2.0 model) and we suppose to know (and subtract from the observations) the gravitational effect of the layers from the center of the Earth to the bottom of the lithosphere (e.g. using a preliminar reference Earth model). In this way the reduced observations contain the effect of only two layers: the first one from the bottom of the lithosphere to the reference Moho and the second one from the reference Moho to the top of the topography.

The main problem related to this two-layer approximation is connected to the regularity of the Moho. In fact in the areas of collision between continental plates subduction zones are often present: this involve doubling or fragmentation of the Moho (and as a consequence the two layer hypothesis does not hold anymore). However it can be notice that in these areas the concept of Moho itself becomes meaningless since there is no more a net division between mantle and crust.

A second problem is that if real data are considered the effect of anomalous masses can be present in the observations as well. These local anomalies such as intrusion of granites, or the presence of sedimentary basins, etc., can cause significant distortions in the prediction of the depth of the Moho. To solve this problem a simple solution can be implemented. First of all the effect, in terms of potential and second radial derivatives, due to the known anomalies should be removed from the observations. After that a spatial-frequency analysis should be carried out. In fact, since in general these anomalies are due to well localized phenomena, it can be expected that the field generated by such anomalies is different (from the spatial-frequency spectral point of view) from the one due to the Moho.

The last problem is related to the heterogeneities present in the mantle. In this case however we know, from high-resolution seismic tomography and mantle convection modeling, that the convective flow involve the whole mantle. This means that large-scale mantle heterogeneities especially those formed early in Earth history are unlikely to have been preserved (van Keken

et al. [12]). Moreover, since these heterogeneities are deeper than the Moho (and therefore the distance between the heterogeneity and the observation point is greater than the one from the Moho and the observation point), their effect on the gravitational potential and its second radial derivative are small and can probably be disregarded.

Once the uniqueness of the solution is guaranteed the inverse gravimetric problem can be solved using a proper regularization method.

In the proposed approach we consider the gravitational potential observed at point P (e. g. at satellite altitude). According to the notation presented in Fig. 1 we can write:

$$T(P) = \iint d\varphi d\lambda \int_{H_Q}^{M_Q} \frac{\rho G r_P^2}{\sqrt{r_P^2 + r_Q^2 + 2r_P r_Q \cos\psi}} dr \quad (1)$$

Linearizing Eq. 1 with respect to r around a mean Moho depth \bar{R} we obtain:

$$T(P) = \iint d\varphi d\lambda \int_{H_Q}^{\bar{R}} \frac{\rho G r_P^2}{\sqrt{r_P^2 + r_Q^2 + 2r_P r_Q \cos\psi}} dr + \iint d\varphi d\lambda \frac{\Delta\rho G r_P^2}{\sqrt{r_P^2 + \bar{R}^2 + 2r_P \bar{R} \cos\psi}} \delta D(Q) \quad (2)$$

where $\delta D(Q)$ is the unknown depth of the Moho with respect to \bar{R} . Note that since every term in the first integral of Eq. 2 is known it can be numerically computed and subtracted from the original potential obtaining:

$$\delta T(P) = \iint d\varphi d\lambda \frac{\Delta\rho G r_P^2}{\sqrt{r_P^2 + \bar{R}^2 + 2r_P \bar{R} \cos\psi}} \delta D(Q). \quad (3)$$

An analogous reasoning can be applied to the second radial derivative of the gravitational potential.

$$\delta T_{rr}(P) = \frac{\partial^2}{\partial r_P^2} \iint d\varphi d\lambda \frac{\Delta\rho G r_P^2}{\sqrt{r_P^2 + \bar{R}^2 + 2r_P \bar{R} \cos\psi}} \delta D(Q) \quad (4)$$

To estimate the Moho we have to solve the system obtained by inverting Eqs. 3-4 degraded with the corresponding observation noise. This can be achieved by means of collocation, treating $\delta D(Q)$ as a random signal:

$$\hat{\delta D}(Q) = C_{y,\delta D}^T C_{y,y}^{-1} y \quad (5)$$

where:

$$C_{y,\delta D} = \begin{bmatrix} C_{T,\delta D} \\ C_{T_{rr},\delta D} \end{bmatrix}, \quad C_{y,y} = \begin{bmatrix} \bar{C}_{T,T} & C_{T,T_{rr}} \\ C_{T_{rr},T} & \bar{C}_{T_{rr},T_{rr}} \end{bmatrix}, \quad y = \begin{bmatrix} \delta T \\ \delta T_{rr} \end{bmatrix}$$

$\delta D(Q)$ is the unknown Moho depth, y are the observations, $C_{a,b}$ is the covariance matrix between a and b and $\bar{C}_{a,b}$ the covariance matrix between a and b plus the error covariance matrix C_{v_a,v_b} . Since we

consider the noise of the two observations uncorrelated, the error covariances are present only on the diagonal blocks of the matrix $C_{y,y}$. It can be noticed that all the needed covariance matrices, with the exception of the error covariance matrices, can be computed by propagating $C_{\delta D,\delta D}$ through Eq. 3 and Eq. 4.

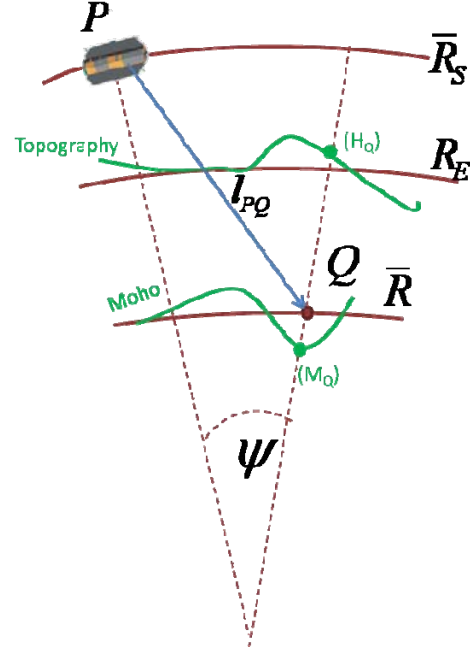


Figure 1 Geometry and notation used.

Note that if some simplifications are introduced into the problem i.e. we neglect the Earth curvature, we introduce a Cartesian coordinates system, we suppose observations on a regular sampled grid and we estimate the Moho on the same gridded points, Eq. 5 can be efficiently computed in terms of Multiple Input Single Output (MISO) Wiener filter in the frequency domain (Papoulis [13]; Sideris [14], Reguzzoni and Sampietro [15]). In addition a more complex approximation to reduce the observation equation to convolution integral taking into account the sphericity of the Earth has been studied and implemented.

3. INPUT DATA AND THE SPACE-WISE MODEL

As for the gravity data the two different quantities considered are the potential and its second derivatives collected from GOCE mission. It is important to underline that these observations have complementary spectral characteristics: the potential carries the low frequencies of the signal while its second derivatives contains the medium-high frequencies. Note that none of the two quantities is a direct observation of the GOCE mission: in fact the potential T is derived from GPS tracking data, for example by applying the so called energy integral approach (Visser et al. [16]),

while the second radial derivatives T_{zz} are obtained by preprocessing the gradiometer observations taken in the instrumental reference frame (Pail [17]).

In this work the two grid of potential and its second radial derivative are computed as by-product of the so called space-wise approach: a multi-step collocation procedure, developed in the framework of the GOCE HPF data processing for the estimation of the spherical harmonic coefficients of the Earth gravitational field and their error covariance matrix (Migliaccio et al. [18]).

As for the noise in Eqs. 3-4 the resulting potential is known to have an almost white error, while the second radial derivatives have a time-correlated error with spectral characteristics almost identical to the original observations (Migliaccio et al. [19]).

4. SOME NUMERICAL EXPERIMENTS

The method has been tested using the first two months of GOCE observations for a numerical example in a region of $45^\circ \times 65^\circ$ in the centre of Europe (latitude between 30° and 75° North and longitude between 20° West and 45° East). This area presents a complex geology characterized among the other by the presence of the Alps and of the Fennoscandian Shield.

In this experiment, the noise is derived by Montecarlo algorithm with 400 samples (Reguzzoni and Tselfes [20]).

The inverse problem has been solved using three different approximations: in particular a spherical solution (obtained with a collocation procedure) and two solutions that allow to solve the problem exploiting the properties of the FFT in planar and almost spherical approximations have been tried.

Considering the FFT solutions, in order to avoid edge effects, a proper border area for the two grids (the potential and its second radial derivative) has to be considered. In order to determine the dimension of this border area, a numerical experiment has been performed: first a small area of $3^\circ \times 3^\circ$ has been considered and the inverse gravimetric problem has been solved at the center of that region. Then larger regions are considered until the convergence of the solution (always in the center of the region) is reached. The experiment has been repeated in different areas of the world (central Europe, Tibetan region, Australian plate). Results for the different areas are very similar and are shown in Fig. 2 where it can be seen that the stability of the solution is reached only when a border area of about 15° is considered. Note that the edge effect (and consequently the size of the border area) depends on the correlation of the signal: in regions where an appropriate geological a priori knowledge exists the original signal can be properly reduced (e.g removing an isostatic Moho model and removing the effects of all the other known geological structures) and

as a consequence also the border area can be considerably reduced (Fig. 3).

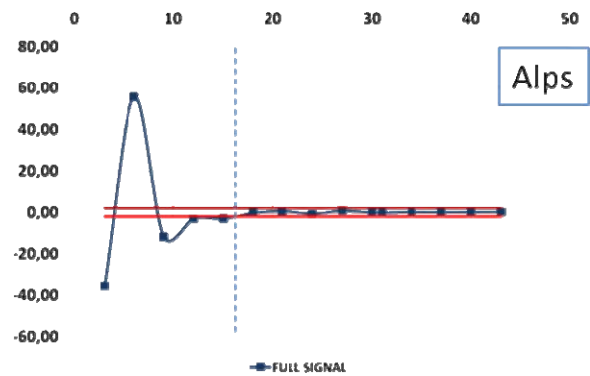


Figure 2 Convergence of the solution of the inverse problem for the Alpine region.

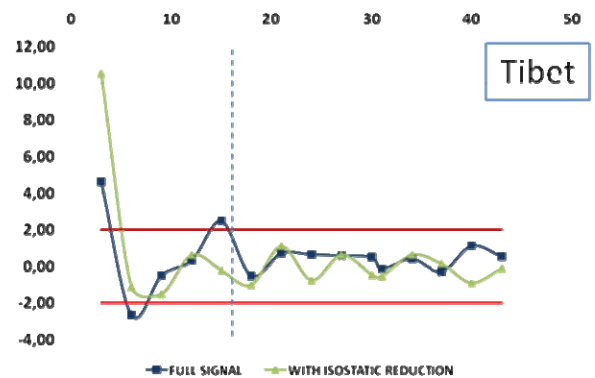


Figure 3 Convergence of the solution of the inverse problem for the Tibetan region removing an isostatic Moho model from the observations.

Another numerical experiment has been performed in order to study the differences between the planar and almost spherical solutions (that make use of the FFT) and the solution of the inverse problem obtained with the collocation procedure (that consider the right geometry of the problem).

In Fig. 4 the differences between the two approximation and the collocation solution has been computed for areas with different dimension.

It can be notice that the standard deviations of these differences are practically the same for the planar and almost spherical approximation. Moreover results show that for relative small region (area smaller than 10°) the mean of the differences between the two FFT solutions and the collocation solutions are practically the same (and smaller than 0.5 km). As expected for bigger regions the planar approximation introduce an error in the low frequencies of the signal that cannot be neglected.

Finally a numerical example to estimate the Moho in the central Europe has been performed. The inversion has been carried out in almost spherical approximation, exploiting the properties of the FFT transform.

The normal potential, the topography (i.e. the effects of ocean/sea, ice, sediments and bedrocks) as well as an isostatic Moho model have been removed from the observations. Lateral density variations according to the Crust 2.0 model have been considered.

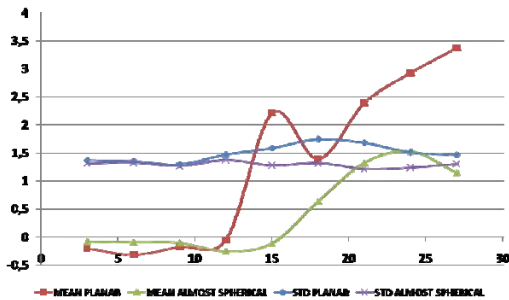


Figure 4 Differences between collocation and different FFT solutions

All in all, the estimated Moho (see Fig. 5) presents differences with respect to the Moho depth of the European plate (here considered as reference model) of less than 5 km (with a mean value of only -0.4 km and a rms of 1.64 km) on a grid of $0.5^\circ \times 0.5^\circ$ resolution under the Alps.

Larger differences (up to more than 10 km) are obtained under the Fennoscandian Shield where the isostatic assumption and the two-layer approximation used to solve the inverse problem are known to be too rough to describe the complexity of the actual crust.

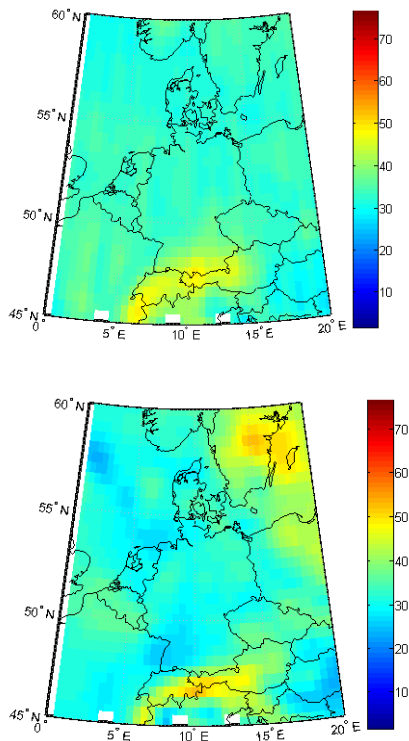


Figure 5 Estimated Moho model (up) and reference model (down) [km].

5. CONCLUSIONS

A method based on collocation and FFT has been implemented to evaluate the contribution of GOCE data in the Moho estimation. The method has been implemented both in planar and in spherical approximation. It has been shown by means of numerical experiments that if no reduction of the signal is applied a border area of about 15° is required in order to avoid edge effects in the solution of the inverse problem. If the signal can be reduced (i.e. by means of a proper model of the crust) the border area can be reduced to 5° .

Regarding the different solution it has been shown that if the resulting area is larger than 10° , spherical approximation has to be used.

Finally the Moho in central Europe has been computed using the GOCE data grids. The differences with respect to the present Moho model are of less than 5 km in the Alpine region and are principally due to the different resolution between the estimated Moho and the reference model. Larger discrepancies can be found under Fennoscandian Shield where it is known that the isostatic assumption is not satisfied.

In order to improve the solution also in this area, the fundamental problem of the isolation of the gravimetric signal due to the Moho discontinuity has to be treated in detail.

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