GOCE covariances

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Definition of covariances

Observation equation:

$$V(r,\theta,\lambda) = \frac{G\,M}{R}\sum_{n=0}^{n_{max}}\left(\frac{R}{r}\right)^{n+1}\sum_{m=0}^{n}\overline{P}_{nm}(\cos\,\theta)\cdot\left[\overline{C}_{nm}\cos(\,m\lambda) + \overline{S}_{nm}\sin(\,m\lambda)\right]$$

Satellite **observations**

Gravity field **parameters**

$$V(r,\theta,\lambda) = \sum_{n=0}^{n_{max}}\sum_{m=0}^{n} A_{nm}(r,\theta,\lambda)\cdot x_{nm}$$

ℓ A x

$$\ell + v = A\cdot x$$

Solution:

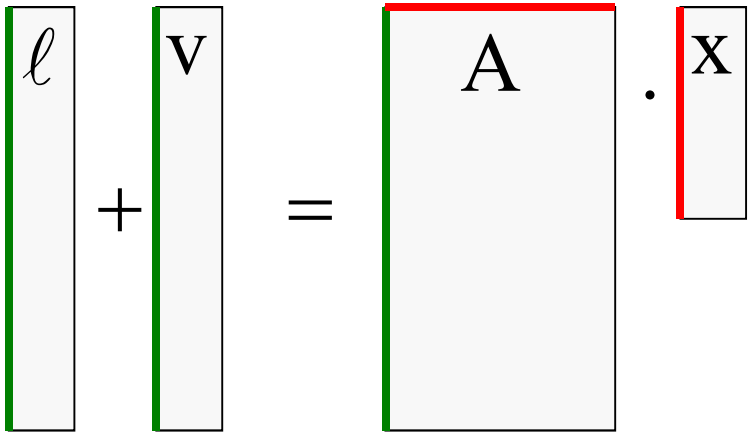
$$\hat{x} = \left(A^T\Sigma_\ell^{-1}A\right)^{-1}A^T\Sigma_\ell^{-1}\ell$$ … parameter estimates

$$\Sigma(\hat{x}) = \left(A^T\Sigma_\ell^{-1}A\right)^{-1}$$ … estimated accuracies of parameters

Definition of covariances

Dimensions of the system:

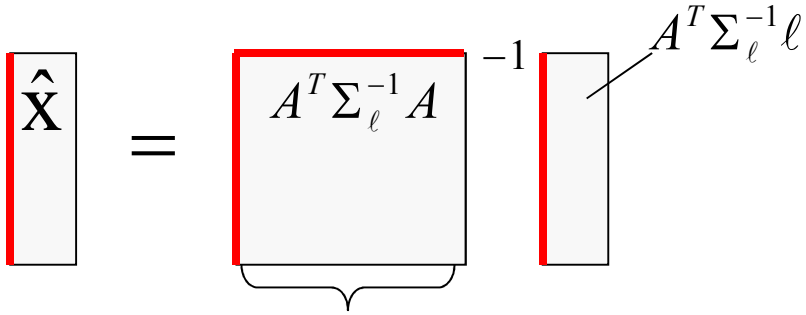
$$\ell + v = A \cdot x$$



number of observations
(~ several 100 000 000)

number of parameters
(~63 000)

$$\hat{x} = \left(A^T \Sigma_\ell^{-1} A\right)^{-1} A^T \Sigma_\ell^{-1} \ell$$

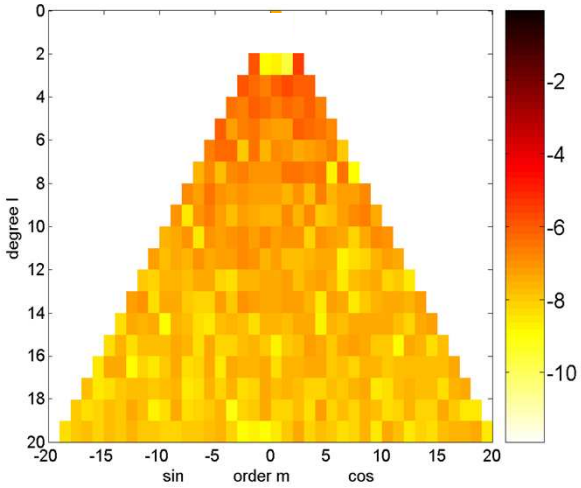


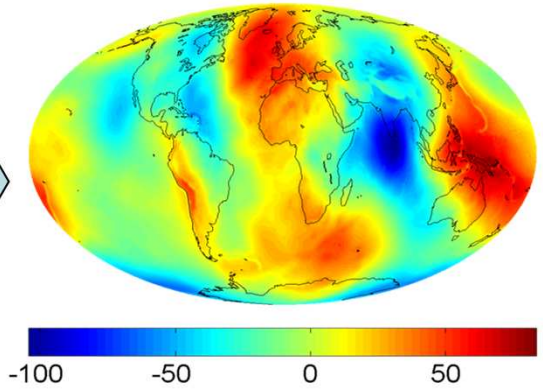
variance-covariance matrix (VCM)

Coefficients

$$\hat{x} = \begin{bmatrix} C_{lm} \\ \bullet \\ \bullet \\ S_{lm} \\ \bullet \\ \bullet \end{bmatrix}$$

Number of parameters (~63 000)



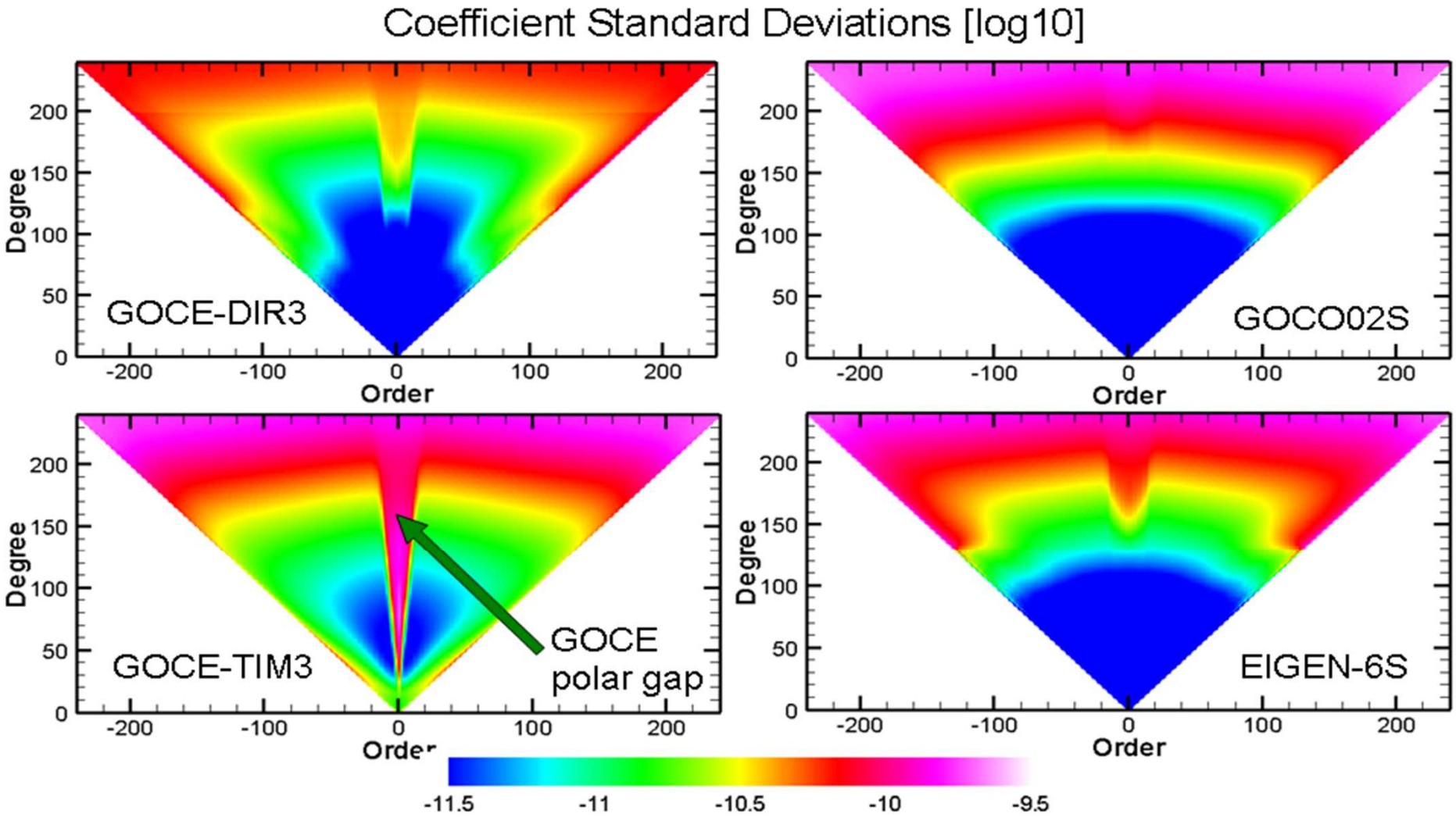


VCM

$$\Sigma(\hat{x}) = \begin{bmatrix} \sigma^2_{C_{lm}} & & & & \\ & \bullet & & & \\ & & \bullet & & \\ & & & \sigma^2_{S_{lm}} & \\ & & & & \bullet \\ & & & & & \bullet \end{bmatrix}$$

- GOCE provides the full VCM
- The main diagonal elements contain the error estimates (variances)

Estimated coefficient errors



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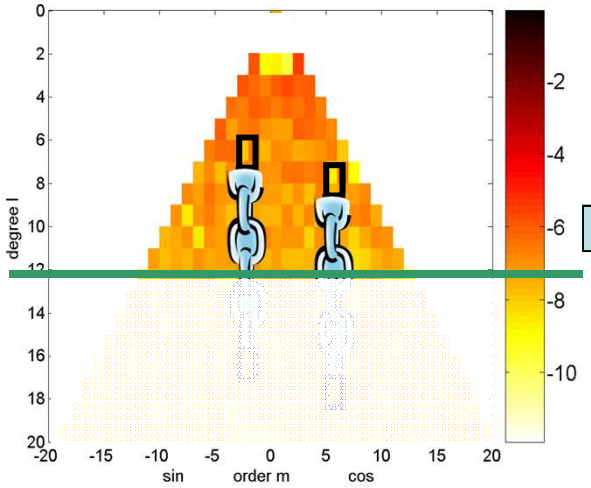
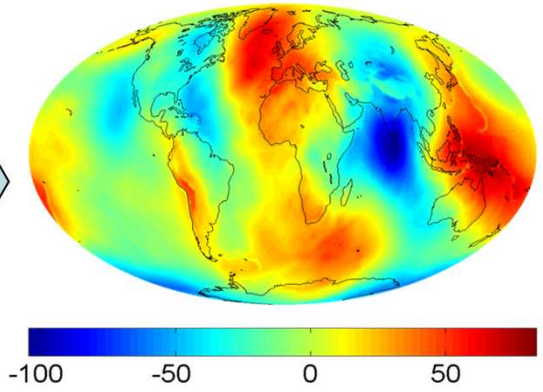




VCM

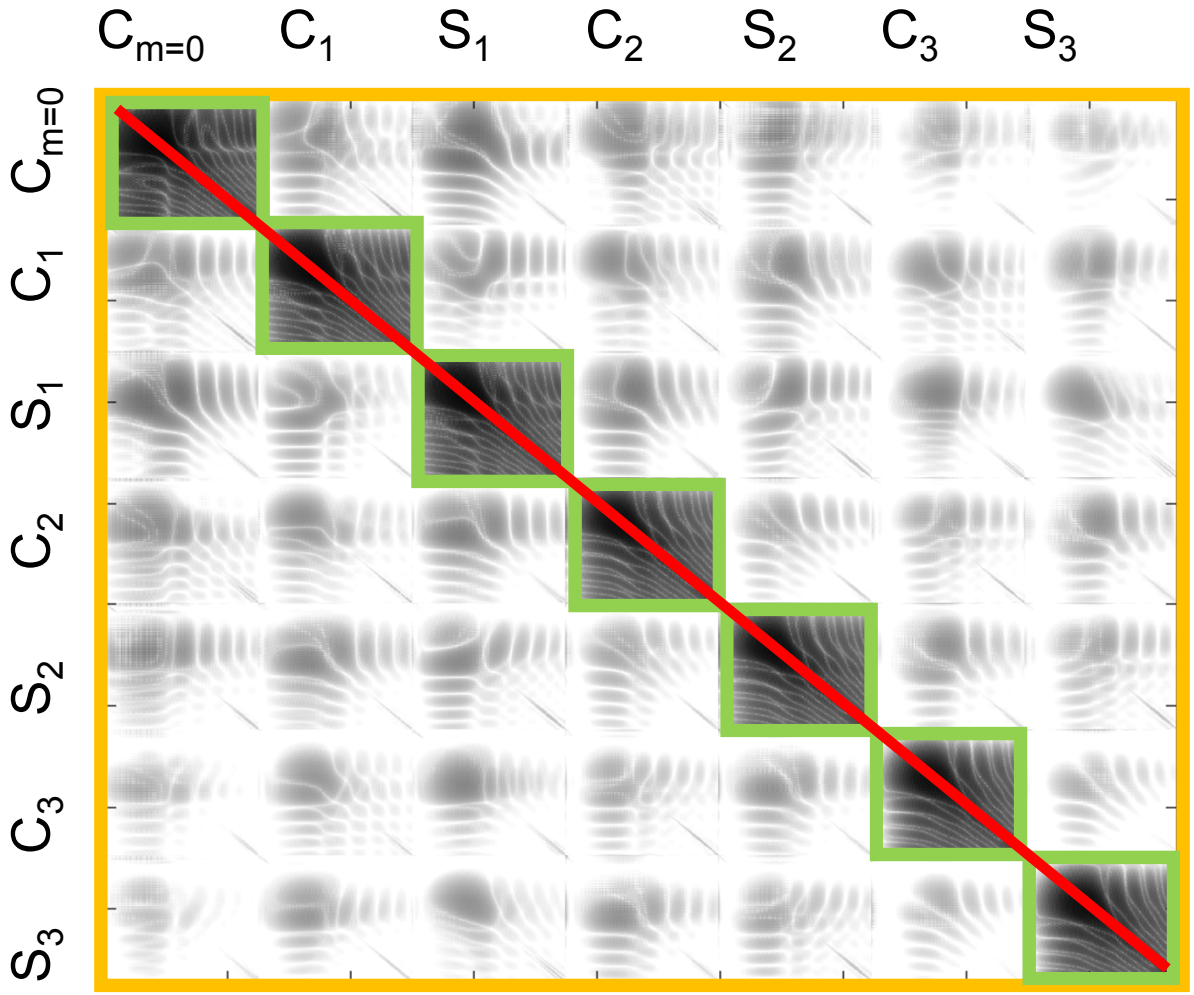
$$\Sigma(\hat{x}) = \begin{bmatrix} \sigma^2_{C_{lm}} & \sigma_{ij} & \\ \bullet & \bullet & \\ & \sigma^2_{S_{lm}} & \sigma_{ij} \\ & \bullet & \bullet \end{bmatrix}$$

- GOCE provides the full VCM
- The main diagonal elements contain the error estimates (variances)
- The coefficient estimates are **correlated** !!! (covariances)

If possible, do not cut coefficient models to use only one part!

Estimated coefficient errors

Example: GOCE VCM for orders m={0,1,2,3}



Why should you care about the VCM?

Let us compute a **derived quantity** from the coefficients, e.g.:

- geoid heights: $N = A_N \cdot \hat{x}$
- gravity anomalies: $\Delta g = A_{\Delta g} \cdot \hat{x}$

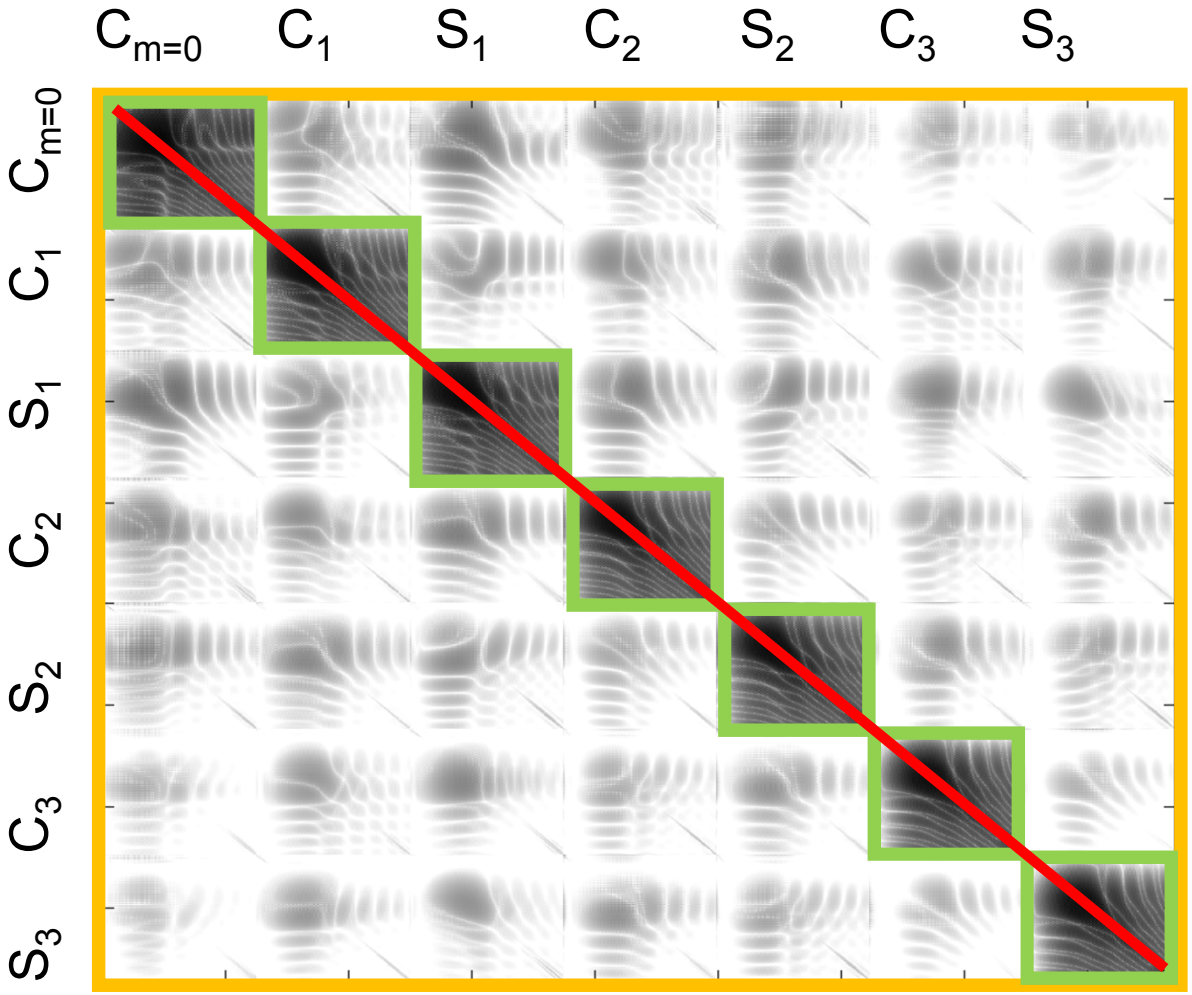
with $\hat{x} = \left\{ \overline{C}_{lm} ; \overline{S}_{lm} \right\}$

We can also derive the **(estimated) accuracy** of the resulting derived quantity by covariance propagation:

- geoid heights: $\rightarrow \Sigma(N) = A_N \cdot \Sigma(\hat{x}) \cdot A_N^T$
- gravity anomalies: $\rightarrow \Sigma(\Delta g) = A_{\Delta g} \cdot \Sigma(\hat{x}) \cdot A_{\Delta g}^T$

Estimated coefficient errors

Example: GOCE VCM for orders m={0,1,2,3}



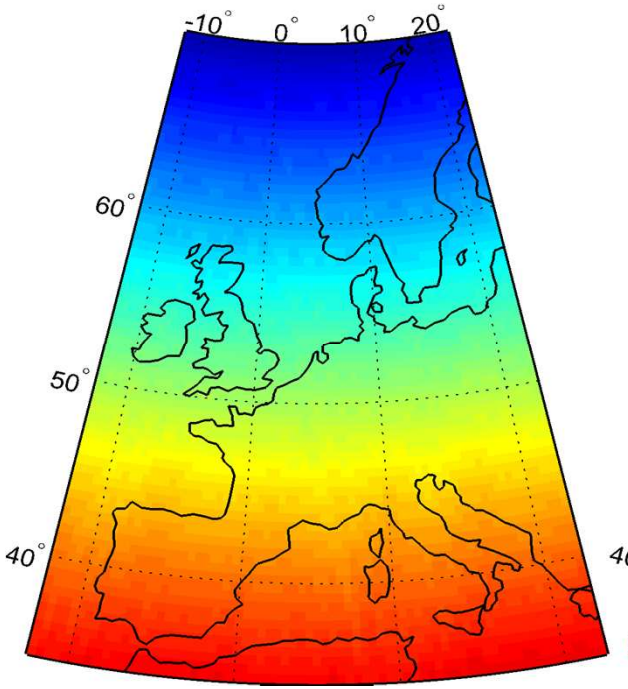
Error propagation usir

full VCM

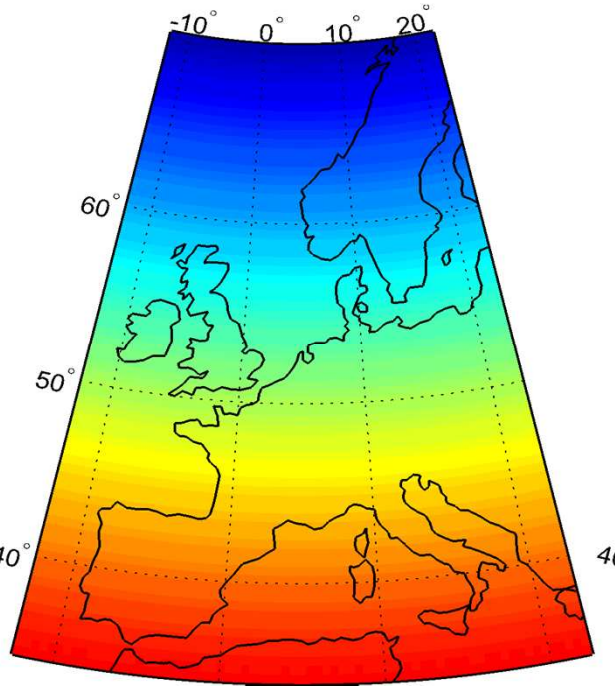
- m-blocks
- diagonal

Full matrix vs. only variances

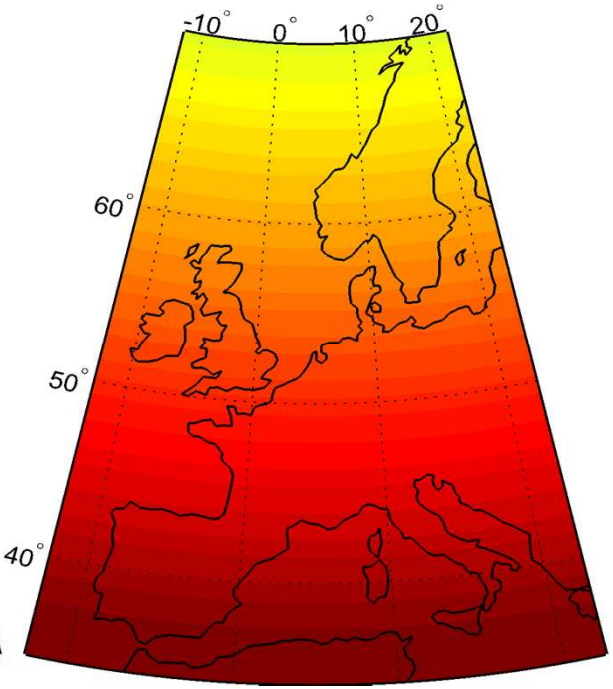
full

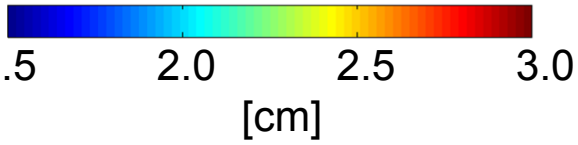


m-blocks



diagonal

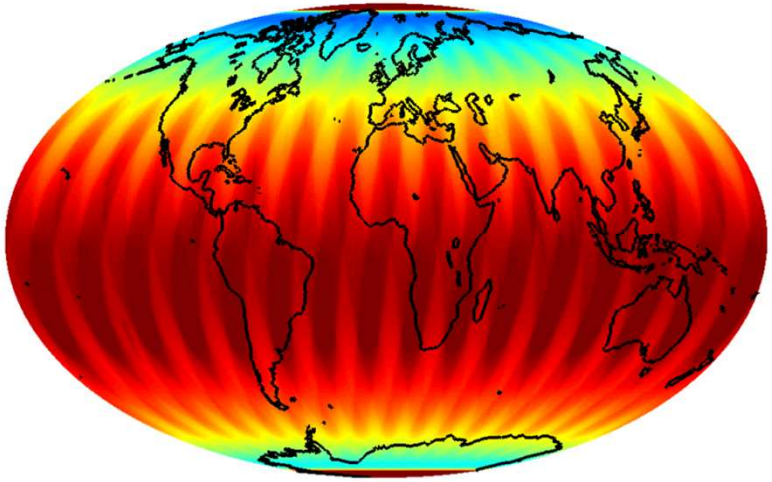




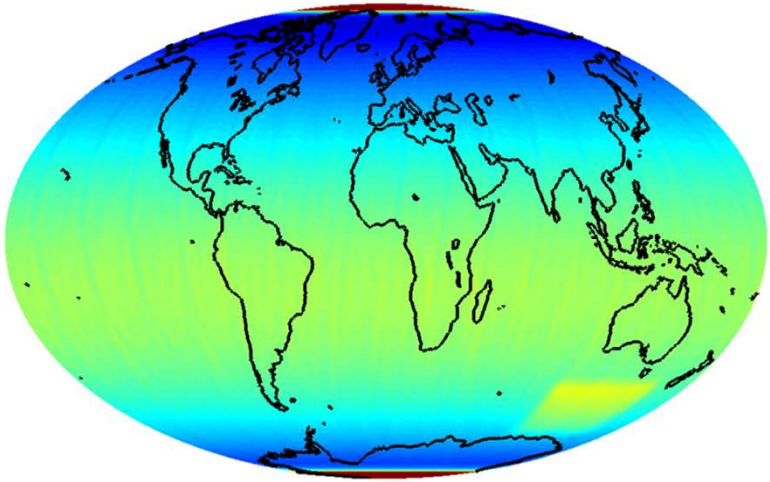
→ Disregarding the full matrix leads to incorrect amplitudes of estimated errors.

Estimated geoid height errors

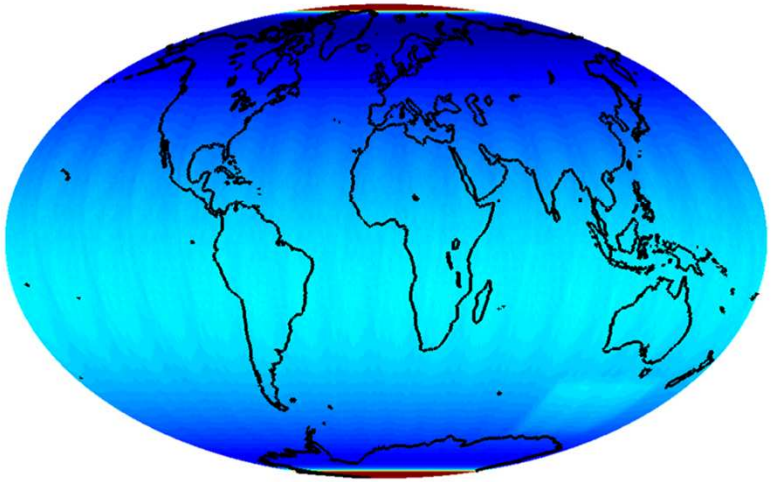
Release 1 (D/O 200)



Release 2 (D/O 200)



Release 3 (D/O 200)



Full covariance propagation (**D/O 200**):

$$\Sigma(N) = A_N \cdot \Sigma(\hat{x}) \cdot A_N^T$$

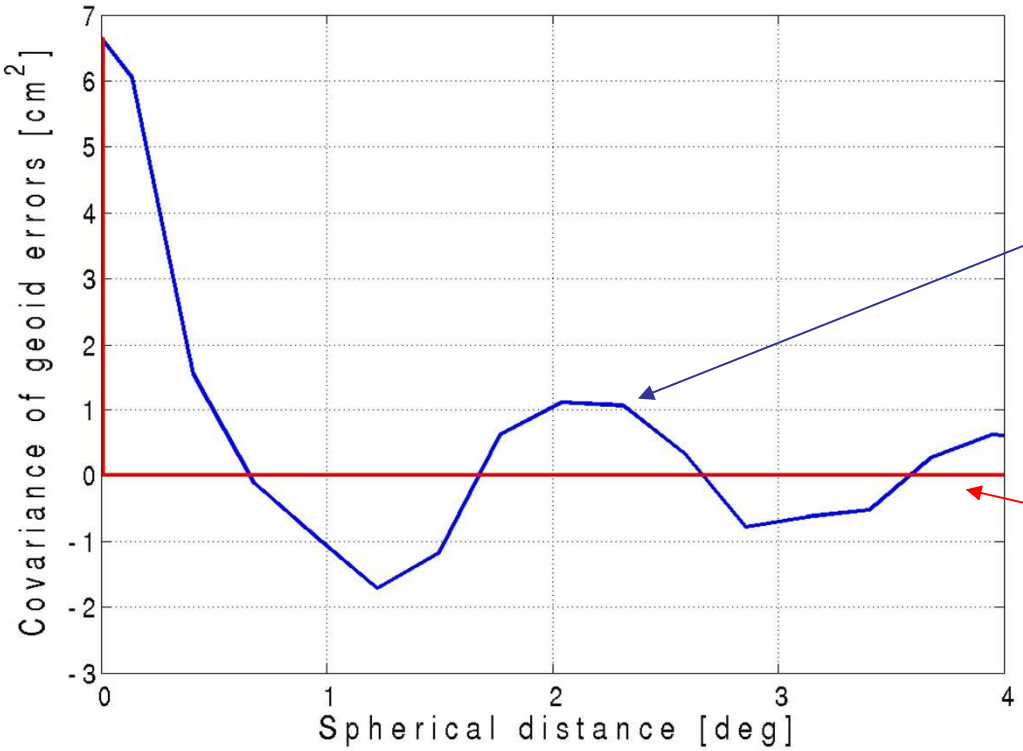
→ Only using the full matrix can express irregular data distribution.

What is a covariance ?

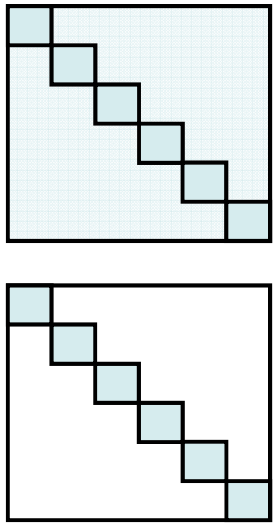
$$N = A_N \cdot \hat{x} \quad \text{with} \quad \hat{x} = \left\{ \overline{C}_{lm} ; \overline{S}_{lm} \right\}$$

$$\rightarrow \Sigma(N) = A_N \cdot \Sigma(\hat{x}) \cdot A_N^T$$

Geoid height error covariances (off-diagonal elements of $\Sigma(N)$)



based on:



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$$N = A_N \cdot \hat{x} \quad \text{with} \quad \hat{x} = \left\{ \overline{C}_{lm} ; \overline{S}_{lm} \right\}$$

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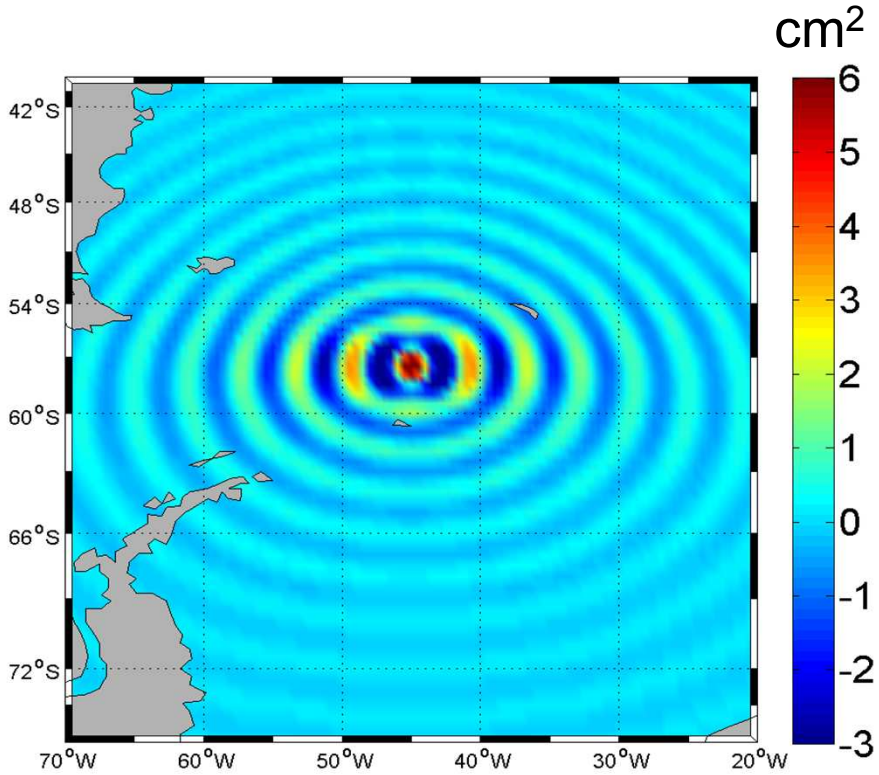
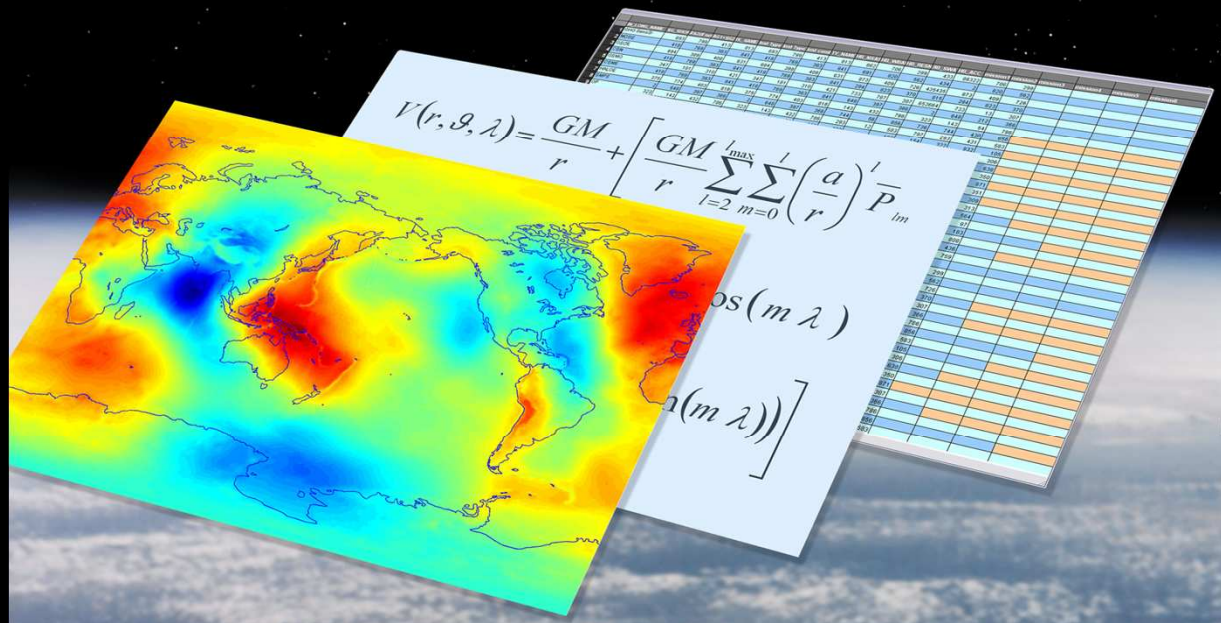
Geoid height error covariances (off-diagonal elements of $\Sigma(N)$)

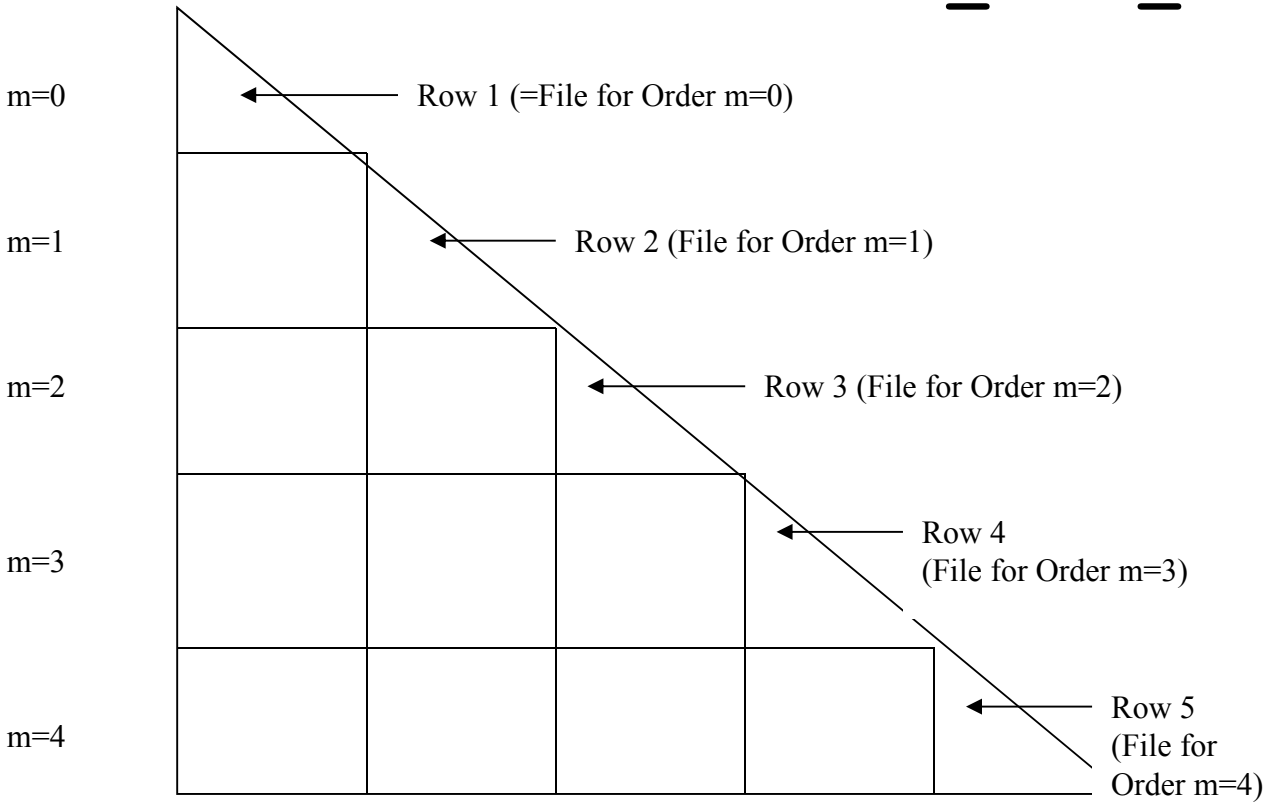


VCM formats


$V(r,\vartheta,\lambda)=\frac{GM}{r}+\left[\frac{GM}{r}\sum_{l=2}^{l_{max}}\sum_{m=0}^{l}\left(\frac{a}{r}\right)^{l}\overline{P}_{lm}\right.$
$\left.\left(m\lambda\right)\right]$

General product layout

- GO_CONS_EGM_DVC_2__*
- GO_CONS_EGM_TVC_2__*
- GO_CONS_EGM_SVC_2__*



→ Separate files for each order *m*.

Meta Data File (IIH)

Keyword [30 character (IIHG)s fixed length, filled with blanks]	**Meaning of parameters** [<70 characters]
product_type	Variance-covariance matrix
modelname	Name of the model
earth_gravity_constant	Earth gravity constant multiplied by mass, which has been used for the gravity field model (GM). This value might be required for error propagation.
radius	Radius of sphere which has been used for the gravity field model. This value might be required for error propagation.
max_degree	Maximum degree of the spherical harmonic series
errors	Must be set to: “formal”, “calibrated” or “calibrated_and_formal” depending on the type of the errors
covariance_matrix_type	Covariance file type: “full” or “block” Full means a full system, block means a block-diagonal system.
sequence_number_entries	Number of coefficients in the matrix. After this entry the sequence of the coefficients is provided. Each coefficient is described in one line. The sequence of the coefficients must be according to the order of the spherical harmonic series (see figure above).
[C,S]_nnn_mmm	Coefficient description with degree nnn and order mmm
sequence_number_files	Number of files that belong to the variance-covariance matrix file. Each file name is then provided in a subsequent record. Files are stored per harmonic order.
	List of file names (each in a new record without keyword)

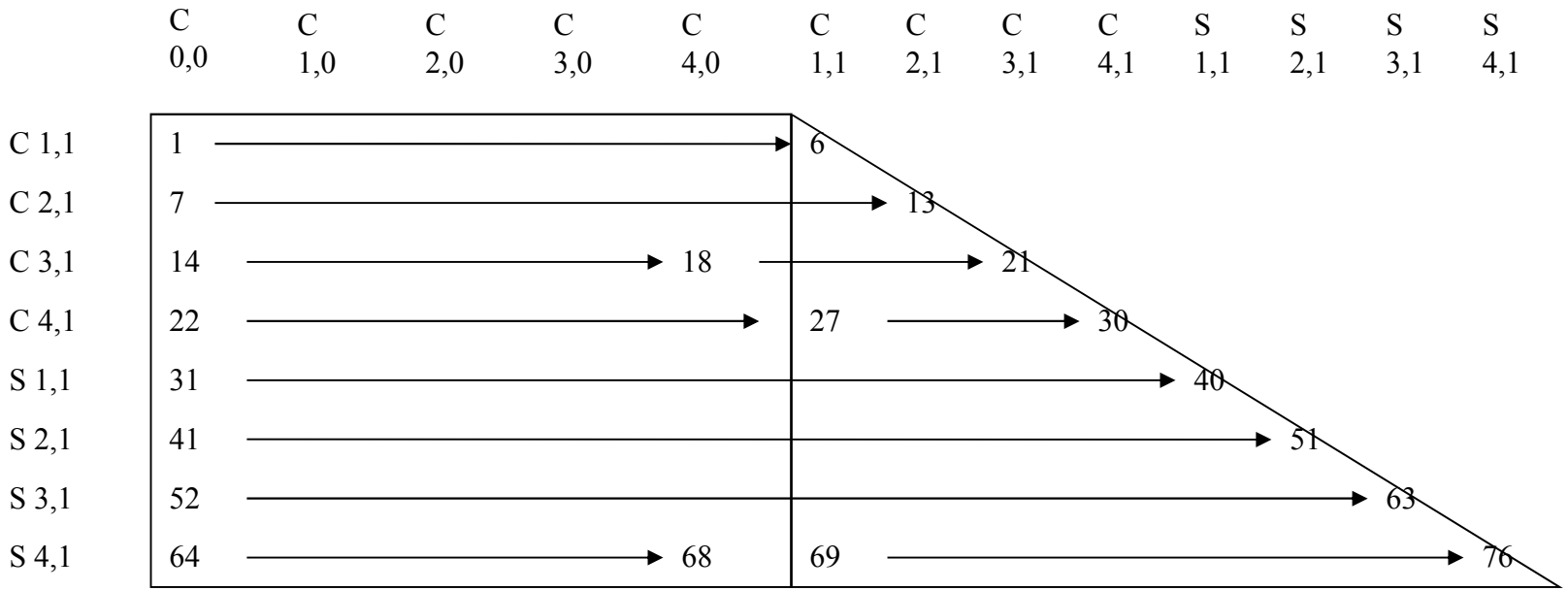
Meta Data File (IIH) – Example: d/o 4

```
product_type                    variance-covariance matrix
modelname                       EXAMPLE-MODEL-1
earth_gravity_constant          0.3986004415E+15
radius                          0.6378136460E+07
max_degree                      4
errors                          formal
covariance_matrix_type          full
sequence_number_entries         25
                  C_000_000
                  C_001_000
                  C_002_000
                  C_003_000
                  C_004_000
                  C_001_001
                  C_002_001
                  C_003_001
                  C_004_001
                  S_001_001
                  S_002_001
                  S_003_001
                  S_004_001
                  C_002_002
                  C_003_002
                  C_004_002
                  S_002_002
                  S_003_002
                  S_004_002
                  C_003_003
                  C_004_003
                  S_003_003
                  S_004_003
                  C_004_004
                  S_004_004
sequence_number_files           5
                  data_file_1_000
                  data_file_1_001
                  data_file_1_002
                  data_file_1_003
                  data_file_1_004
```

Data File (IDF)

Keyword [30 characters fixed length, filled with blanks]	**Meaning of parameters** [<70 characters]
meta_data_file_name	Name of the meta data file
order	Harmonic order of the file
number_entries	Total number of data values in the file (for control information)
begin_data	Keyword indicating where the data section starts
end_data	Keyword indicating where the data section ends.

Example d/o 4: Data File Storage Sequence



Example d/o 4: data file (IDF)

```
meta_data_file_name              meta_data_file_1.IIH
order                            1
number_entries                   76
begin_data
-0.765788765432OE-09
…
…
…
…              (all together 76 numbers in ASCII format)
…
…
+1.056775876689OE-14
end_data
```

GOCE covariances