THE FREQUENCY ANALYSIS OF GRAVITY GRADIENTS AND THE METHODS OF FILTERING PROCESSING

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Abstract: A new and simple filtering method is investigated in this paper, since the low frequency error in GOCE gradients data must be removed for efficient gravity field recovery. Considering the observation data is given as a time series, the frequency of gradients data in time domain will be analyzed firstly. Then two main filtering strategies which are remove-restore method and forward-backward filtering technique will be introduced. The remove-restore method is used to prevent low frequency signal loss and the forward-backward filtering effectively avoids phase drift. The final result shows our approach is effective.

Keywords: GOCE; gravity gradients; frequency analysis; filtering

Introduction

GOCE satellite has provided much gravity gradients data, but they only achieve mE-accuracy in a given measurement bandwidth (MBW). Beyond MWB, the accuracy is a little poor, especially the low frequency error which must be removed for high accuracy of gravity field recovery from GOCE data. Some scientists have done much research on this, e.g., Schuh W. D et al 2003[1] whose method is very effective to least square method. However, in this paper we do harmonic analysis to recover gravity field model. Generally speaking, our approach is based on the following boundary value problems derived by Yu et al.2010[2],

$$\begin{cases} \mathrm{Lap}T = 0 \\ \left.\dfrac{\partial^2 T}{\partial r^2}\right|_S = -\dfrac{r^3}{3}\Delta B \\ T = O(r^{-1}), \qquad \text{at infinity} \end{cases} \tag{1}$$

and

$$\begin{cases} \mathrm{Lap}T = 0 \\ \left.\dfrac{\partial^2 T}{\partial r^2}\right|_S = -\dfrac{r^6}{3}\Delta C \\ T = O(r^{-1}), \qquad \text{at infinity} \end{cases} \tag{2}$$

where

$$\begin{cases} T = V - U \\ B = V_{XX}V_{YY} + V_{YY}V_{ZZ} + V_{XX}V_{ZZ} - ({V_{XY}}^2 + {V_{YZ}}^2 + {V_{XZ}}^2) \\ C = V_{XY}^2 V_{ZZ} + V_{YZ}^2 V_{XX} + V_{XZ}^2 V_{YY} - V_{XX}V_{YY}V_{ZZ} - 2V_{XY}V_{YZ}V_{XZ} \\ \Delta B = B - B_0 \\ \Delta C = C - C_0 \end{cases}$$

V is actual potential and U is normal potential, correspondingly T is disturbing potential. B and C are invariants of gravity gradients which are introduced by Baur et al.2008[3] and Yu et al.2010[2], B_0 and C_0 can be calculated using reference model which is the former 300 degrees of EGM08 in our work. We solve these boundary value problems using harmonic analysis introduced by Yu et al. 2010[2]. Since the approach is new, we must find new filtering strategies. Some work will be done in this paper.

Firstly, the frequency of gradients data in time domain will be analyzed. Which orders of the gravity field model will be influenced by the low frequency error? Does the high orders part contain low frequency information in time domain? All of these are what we care in this part. Secondly, the methods of filtering processing will be introduced. In short, what we adopt

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is the method of remove-restore. It’ s that at first we simulate gravity gradients data using a reference gravity field model such as EGM08, and then let observation data subtract the simulated values and get residual values which are then filtered with method of forward-backward filtering in order to solve phase drift problem. Correspondingly, the final values used in gravity field recovery are the sums of simulated values and residual values. Finally some results will be given to show effectiveness of our approach.

Frequency analysis

In order to show influence on gravity field recovery from the low frequency error, we do a gravity field recovery using raw data (***EGG NOM 2 and SST PSO 2.Time period is 1.11.2009~31.12.2009***) which has never been preprocessed. The result is showed in Fig. 1.where

$$\xi_n = \sqrt{\frac{\sum_{m=0}^{n}\left(C_{nm}^{New} - C_{nm}^{08}\right)^2 + \sum_{m=1}^{n}\left(S_{nm}^{New} - S_{nm}^{08}\right)^2}{2n+1}}.$$

The accuracy is poor, not only the low degrees but also the high degrees.

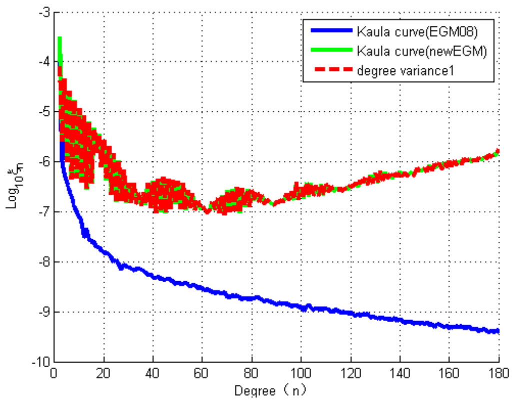

Fig 1. Degree Variance

In order to show the relationship between the frequency in time domain with degrees, we express the radial gradients in function of time t, as follows

$$V_{rr} = \frac{GM}{r^3}\left[2 + \sum_{n=2}^{\infty}(n+1)(n+2)\left(\frac{R}{r}\right)^n \sum_{m=0}^{n}\left(C_{nm}\cos(m\omega_\lambda t) + S_{nm}\sin(m\omega_\lambda t)\right)\overline{P}_{nm}\left(\cos(\omega_\theta t)\right)\right]$$

$$= \frac{GM}{r^3}\left[2 + \sum_{n=2}^{\infty}(n+1)(n+2)\left(\frac{R}{r}\right)^n \sum_{m=0}^{n}\left(C_{nm}\cos(m\omega_\lambda t) + S_{nm}\sin(m\omega_\lambda t)\right)\right.$$

$$\left.\times\sqrt{(2-\delta_{0m})(2n+1)\frac{(n-m)!}{(n+m)!}}\sin^m(\omega_\theta t)\sum_{s=0}^{\left[\frac{n-m}{2}\right]}(-1)^s\frac{(2n-2s)!}{2^n s!(n-s)!(n-2s-k)!}\cos^{n-2s-m}(\omega_\theta t)\right]$$

Where ω_λ can be seen as velocity of earth rotation and ω_θ can be seen as velocity of satellite movement since GOCE satellite can be seen as nearly polar orbit. All of ω_λ and ω_θ are almost constant, so the temporal frequency of V_{rr} will be mainly decided by n and m. For a special degree n, when m is changed, the time frequency will be much different. From other perspective, the high degrees still contain low frequency signal. Concerning this, a simulation has been done and the result is as follows,

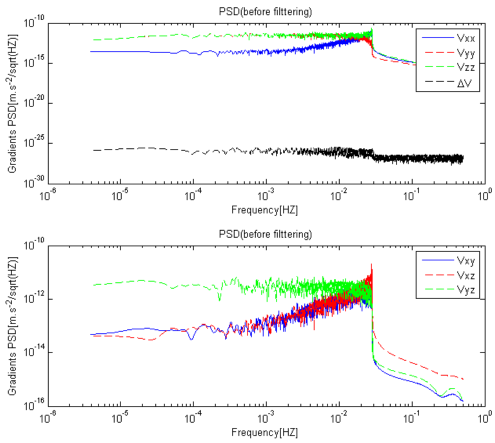

Fig 2.Simulated GGs in GRF on GOCE’s orbit using 150th degree of EGM08

Definitely, 150^{th} degree contains low frequency signal and low frequency error will not only influence the accuracy of low degrees but also high degrees.

Filtering method

We adopt 1000 orders’ band pass FIR filter to remove error beyond MBW. However we also remove the signal beyond MBW at the same time, so we adopt the remove-store method to solve loss of useful information during filtering processing.

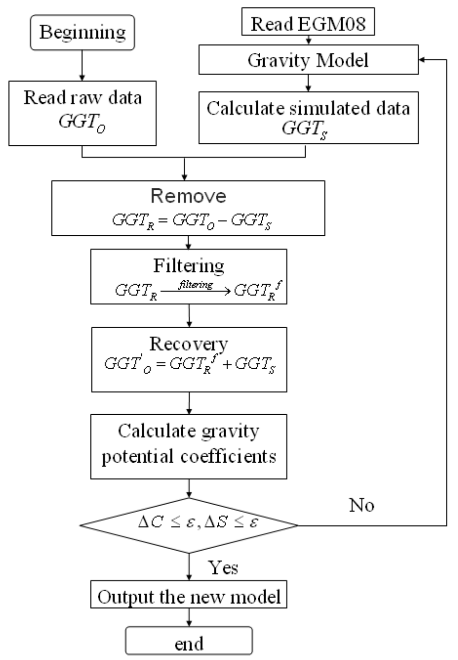

Fig 3. Flow chart

In order to avoid phase drift problem, we adopt the forward-backward filtering. It's that we not only filter in forward direction but also in backward direction, the final output is the time reverse of the output of the second filtering operation. By this strategy, there is precisely zero-phase distortion. The PSD comparison between raw data and the data after filtering is shown in Fig 4 ,

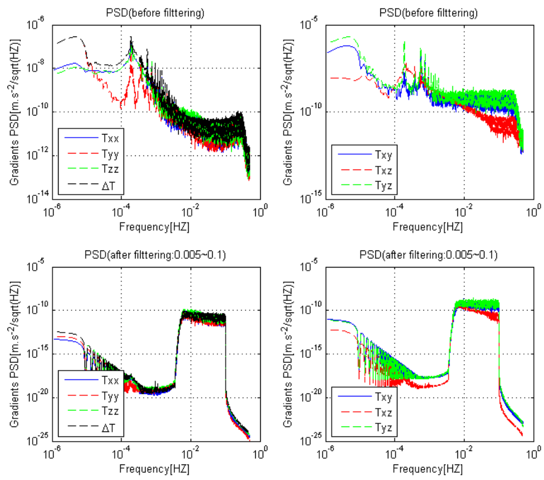

Fig 4. PSD comparison. The filter is 1000 orders band pass FIR filter and time period of raw data is1.11.2009~10.11.2009

Furthermore, in order to show the relationship between the frequency and degree, we changed the pass band and the result is as follows,

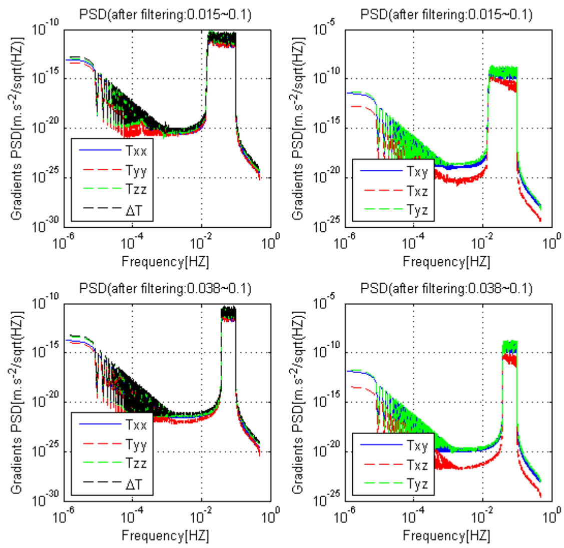

Fig 5. PSD of residual data filtered by1000 orders band pass FIR filter. Time period of raw data is1.11.2009~10.11.2009

Results

In this part, some results will be given.

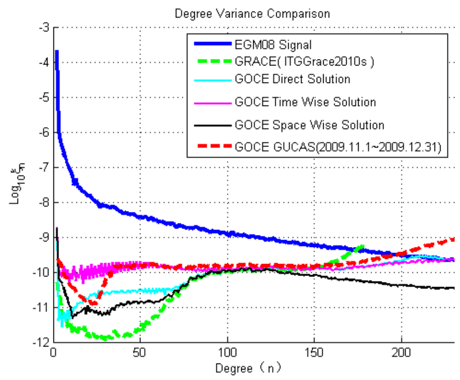

Fig 6. Comparison with Models from ESA(pass band:0.005~0.1HZ)

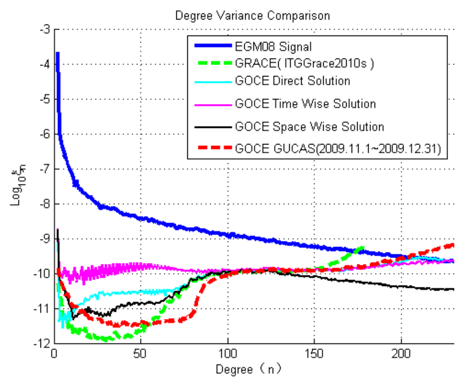

Fig 7. Comparison with Models from ESA(pass band:0.015~0.1HZ)

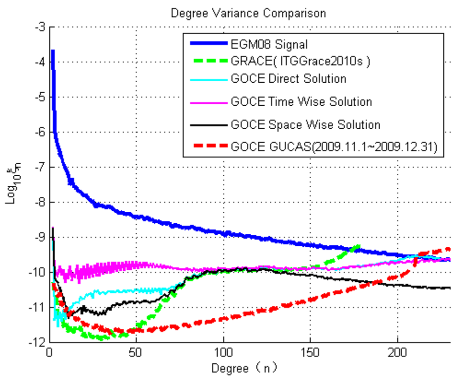

Fig 8. Comparison with Models from ESA(pass band:0.038~0.1HZ)

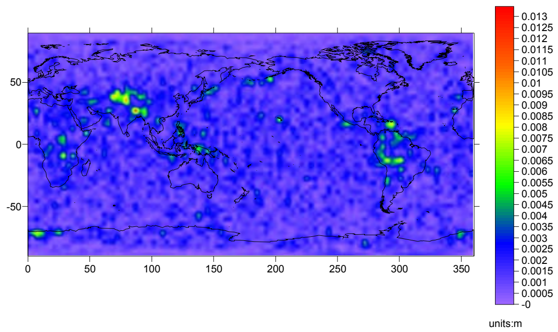

Fig 10. Geoid height differences compared with EGM08 (Pass band:0.015~0.1HZ)

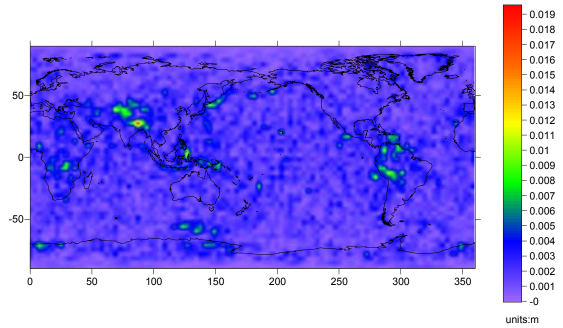

Fig 9. Geoid height differences compared with EGM08 (Pass band:0.005~0.1HZ)

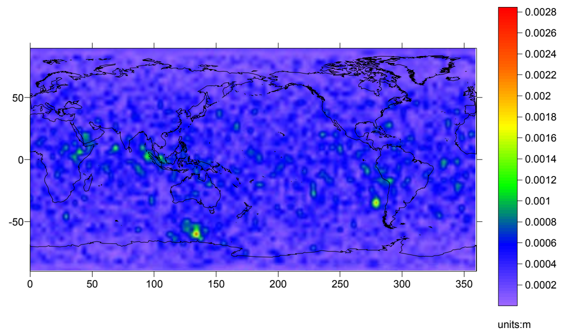

Fig 11. Geoid height differences compared with EGM08 (Pass band:0.038~0.1HZ)

From these results we can conclude that,

1, GOCE data does a better job than GRACE after degree 150;

2, Large differences of geoid height appear in Himalaya, South America and Africa, which can be seen as contribution from GOCE;

3, High degrees of our models are consistent with ESA's, but the low degrees will mainly depend on reference model;

4, From Fig. 9, it seems that the signal in 38mHz-100mHz plays little role in recovering the gravity field below 210 degree.

Conclusions

From these works, we can conclude the approach of gravity field recovery based on gradients' invariants as well as our filtering method is effective. But we should need to consider how to remove white noise in MBW. Refinement should be made on the filtering strategy to avoid loss of useful information.

Note: our model can be downloaded from:

http://people.gucas.ac.cn/~yjh?language=en

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