CryoSat Characterization for FBR users

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CHANGE RECORDS

Issue	Date	Description	Author
1.0	23/05/2016	First Issue. Version C001 of the *CryoSat Users Characterization file.*	M. Scagliola, M. Fornari
2.0	13/06/2016	Second Issue. Version C002 of the *CryoSat Users Characterization file:* CAL1 P2P and CAL2 LPF corrections for SARin mode have been added. Typo corrected in formula (2).	M. Scagliola, M. Fornari
3.0	31/03/2022	Third Issue Document updated to be aligned with BaselineE FBR in NetCDF format	L.Recchia, L. Fioretti, M.Fornari

SUMMARY

LIST OF TABLES

1 Document overview

1.1 Purpose

This document contains the CryoSat system and SIRAL instrument characterization parameters that can be useful to CryoSat users to process BaselineD and BaselineE FBR data. The information provided in this document does not apply to FBR products from the previous Baselines.

With the release of Baseline C, CryoSat users have access to FBR products, in addition to L1B and L2. While L1B waveforms are fully calibrated and require no further actions from the users, FBR data are not calibrated and require the expert user to apply calibrations himself. Starting from the BaselineD, the FBR product format has been converted to NetCDF format. Current issue of this document has been updated to be aligned to the NetCDF format and is applicable to BaselineD and BaselineE FBR data.

Most of CryoSat calibrations can be retrieved within the FBR itself while, for the remaining, a dedicated file has been generated: the *CryoSat User Characterization file*. This document describes where each CryoSat calibration correction can be retrieved and provides a full description (NetCDF dump) of the *CryoSat User Characterization file*. The purpose of this document is not to describe the way the calibration corrections have to be applied to the FBR data, because expert users using FBR are assumed to be aware of the calibration application steps.

In addition, other satellite and instrument characterization parameters may be needed by the users. These are provided in Section 4 of this document, currently containing SIRAL and antenna characterization parameters.

1.2 Document organization

This document is organized as it follows:

- Section 2 describes the calibration corrections to be applied by the users to BaselineD and BaselineE FBR data, together with the indication of where they can be found.
- Section 3 details the *CryoSat User Characterization file* content and format, by means of its CDL dump.
- Section 4 describes the system characterization parameter that can be useful to the users.

1.3 Acronyms

ADC	Analog to Digital Converter
AGC	Automatic Gain Control
ARESYS	Advanced Remote Sensing and Systems
CCAL1	Complex CAL1
CDL	Common Data form Language
FBR	Full Bit Rate
IPF1	Instrument Processing Facility Level1
L1b	Level1B
LPF	Low Pass Filter
NetCDF	Network Common Data Format
P2P	Pulse-to-Pulse
RF	Radio Frequency
SIRAL	Synthetic Aperture Interferometric Radar Altimeter
SAR	Synthetic Aperture Radar
SARin	Synthetic Aperture Radar Interferometry

1.4 Reference documents

[RD1] C2-RS-ACS-ESL-5364, Instrument Processing Facility L1B, CryoSat Ice netCDF L1B Product Format Specification, issue 2.0, 23th December 2020

[RD2] M. Fornari, M. Scagliola, N. Tagliani, T. Parrinello and A. G. Mondejar, "CryoSat: Siral calibration and performance," *2014 IEEE Geoscience and Remote Sensing Symposium*, Quebec City, QC, 2014, pp. 702-705.

[RD3] C2-TR-SAA-SR-0065, SIRAL 2 Antenna Subsystem, RF Test Report, Issue 4, 20/11/2008

[RD4] D.J. Wingham, C.R. Francis, S. Baker, C. Bouzinac, D. Brockley, R. Cullen, P. de Chateau-Thierry, S.W. Laxon, U. Mallow, C. Mavrocordatos, L. Phalippou, G. Ratier, L. Rey, F. Rostan, P. Viau, D.W. Wallis, CryoSat: A mission to determine the fluctuations in Earth’s land and marine ice fields, Advances in Space Research, Volume 37, Issue 4, 2006, Pages 841-871

2 CryoSat Calibration corrections for FBR users

As defined in [RD1], SAR/SARin FBR products, which are intermediate outputs of the IPF1, contain the received echo data as complex numbers. In the FBR, complex echo waveforms are neither calibrated nor compressed: such processing operations are performed as following steps by the IPF1 in order to obtain the L1B products (which contain compressed, fully calibrated and multilooked waveforms).

FBR users, in order to compensate the complex echoes for the instrument's imperfections, have thus to apply the calibration corrections by themselves. A description of the SIRAL internal calibrations and of the calibration corrections available of ground is given in [RD1].

Amongst the needed calibration corrections, only a subset is annotated in the FBR products. The calibration corrections that cannot be extracted from the FBR product have been included in the *CryoSat Users Characterization file*.

2.1 CryoSat Calibration corrections for SAR FBR

A complete list of the calibration corrections to be applied to SAR complex echoes contained in BaselineD and Baseline E FBR products and their source is given in Tab.1.

SAR mode			
Calibration	**Correction**	**FBR**	**CryoSat Users Characterization file**
CAL1	Internal path delay	instr_cor_range_tx_rx_85_ku *1-way instrument range correction (tx-rx chain)*	Not provided
	Power gain variation	instr_cor_gain_tx_rx_85_ku *instrument gain correction (tx-rx chain)*	Not provided
	Pulse-to-pulse amplitude	Not provided	cal1_p2p_amplitude_sar
	Pulse-to-pulse phase	Not provided	cal1_p2p_phase_sar
CCAL1	AGC power gain	instr_cor_gain_tx_rx_85_ku *instrument gain correction (tx-rx chain)*	Not provided
CAL2	Low Pass Filter amplitude gain	Not provided	cal2_lpf_sar

Tab.1 Calibration correction to be applied to SAR complex echoes from FBR.

It is worth noting that instr_cor_gain_tx_rx_85_ku variable of FBR product contains the sum of the following corrections:

- Power gain variation correction from CAL1;
- AGC power gain correction from CCAL1.

The total power gain of the RF section of the instrument in SAR mode can be thus computed as

$$\begin{aligned} P_{tot,RF} &= FixGain_{Rx1} + AGC_1 + AGC_2 + GainCorr_{Rx1} = \\ &= tot\_gain\_ch1\_85\_ku + agc\_1\_85\_ku + agc\_2\_85\_ku \\ &\quad + instr\_cor\_gain\_rx\_85\_ku \qquad [dB] \end{aligned} \tag{1}$$

where the variables above refer to the FBR product [RD1].

For the calibration on FBR data of power gain for the digital section of the instrument, the ADC multiplier factor needs to be taken into account

$$P_{tot,Dig} = 10\,log_{10}(\,ADC\_MULT^2) = 60 \qquad [dB] \tag{2}$$

with *ADC_MULT*=1000.

The total power gain results in the summation of the power gain of the RF section and the power gain of the digital section

$$P_{tot} = P_{tot,RF} + P_{tot,Dig} \qquad [dB] \tag{3}$$

It is worth noticing that, while the calibration corrections extracted from the FBR are forwarded from the Calibration L1B products that are continuously generated in operations, the calibration corrections from the *CryoSat Users Characterization file* are static. In fact, taking advantage from the fact that the calibration corrections for CAL1 P2P and CAL2 LPF in SAR mode are stable from the beginning of the operations, in the *CryoSat Users Characterization file* the reference values valid for the whole mission lifetime have been provided. For more details on corrections contained in the *CryoSat Users Characterization file*, refer to Section 3.

2.2 CryoSat Calibration corrections for SARin FBR

In SARin mode, SIRAL transmits the signals using one antenna, while it acquires data from two antennas and two independent receiving chains, which have to be calibrated independently. Throughout this document, the following convention is used:

- the signal acquired by the transmitting and receiving antenna is identified as Rx1
- the signal acquired by the receiving only antenna is identified as Rx2

A complete list of the calibration corrections to be applied to SARin complex echoes contained in BaselineD and Baseline E FBR products and their source is given in Tab.2 for Rx1 and in Tab.3 for Rx2.

SARin mode Rx1			
Calibration	**Correction**	**FBR**	**CryoSat Users Characterization file**
CAL1	Internal path delay	instr_cor_range_tx_rx_85_ku *1-way instrument range correction (tx-rx chain)*	Not provided
	Power gain variation	instr_cor_gain_tx_rx_85_ku *instrument gain correction (tx-rx chain)*	Not provided
	Pulse-to-pulse amplitude	Not provided	cal1_p2p_amplitude_sarin_rx1
	Pulse-to-pulse phase	Not provided	cal1_p2p_phase_sarin_rx1
CCAL1	AGC power gain	instr_cor_gain_tx_rx_85_ku *instrument gain correction (tx-rx chain)*	Not provided
CAL2	Low Pass Filter amplitude gain	Not provided	cal2_lpf_sarin_rx1

Tab.2 Calibration correction to be applied to SARin Rx1 complex echoes from FBR.

SARin mode Rx2			
Calibration	**Correction**	**FBR**	**CryoSat Users Characterization file**
CAL1	Internal path delay	instr_cor_range_rx_85_ku *1-way instrument range correction (rx only chain)*	Not provided
	Power gain variation	instr_cor_gain_rx_85_ku *instrument gain correction (rx only chain)*	Not provided
	Pulse-to-pulse amplitude	Not provided	cal1_p2p_amplitude_sarin_rx2
	Pulse-to-pulse phase	Not provided	cal1_p2p_phase_sarin_rx2
CCAL1	AGC power gain	instr_cor_gain_rx_85_ku *instrument gain correction (rx only chain)*	Not provided
CAL2	Low Pass Filter amplitude gain	Not provided	cal2_lpf_sarin_rx2

Tab.3 Calibration correction to be applied to SARin Rx2 complex echoes from FBR.

It is worth noting that instr_cor_gain_tx_rx_85_ku andinstr_cor_gain_rx_85_ku of FBR product respectively contain the sum of the following corrections for Rx1 and Rx2:

- Power gain variation correction from CAL1;
- AGC power gain correction from CCAL1.

The total power gain of the RF section of the instrument for Rx1 in SARin mode can be thus computed as

$$P_{tot,RF,Rx1} = FixGain_{Rx1} + AGC_1 + AGC_2 + GainCorr_{Rx1} = \\ = (tot\_gain\_ch1\_85\_ku + agc\_1\_85\_ku + agc\_2\_85\_ku \\ + instr\_cor\_gain\_rx\_85\_ku) \qquad [dB] \tag{4}$$

The total power gain of the RF section of the instrument for Rx2 in SARin mode can be thus computed as

$$P_{tot,RF,Rx2} = FixGain_{Rx2} + AGC_1 + AGC_2 + GainCorr_{Rx2} = \\ = (tot\_gain\_ch2\_85\_ku + agc\_1\_85\_ku + agc\_2\_85\_ku \\ + \mathrm{instr\_cor\_gain\_rx\_85\_ku}) \qquad [dB] \tag{5}$$

where the variables above refer to the FBR product [RD1]. It is worth noting that the AGC1 and AGC2 commands, that are stored in FBR agc_1_85_ku and agc_2_85_ku variables respectively, are the same for both Rx1 and Rx2; on the other side, the calibration correction differs for each channel as belonging to different physical components.

For the calibration on FBR data of power gain for the digital section of the instrument, the ADC multiplier factor needs to be taken into account

$$P_{tot,Dig} = 10\,log_{10}(\,ADC\_MULT^2) = 60 \qquad [dB] \tag{6}$$

with *ADC_MULT*=1000.

The total power gain results in the summation of the power gain of the RF section and the power gain of the digital section

$$\begin{aligned} P_{tot,Rx1} &= P_{tot,RF,Rx1} + P_{tot,Dig} \quad [dB] \\ P_{tot,Rx2} &= P_{tot,RF,Rx2} + P_{tot,Dig} \quad [dB] \end{aligned} \tag{7}$$

It is worth noticing that, while the calibration corrections extracted from the FBR are forwarded from the Calibration L1B products that are continuously generated in operations, the calibration corrections from the *CryoSat Users Characterization file* are static. In fact, taking advantage from the fact that the calibration corrections for CAL1 P2P and CAL2 LPF for both the receiving channels in SARin mode are stable from the beginning of the operations, in the *CryoSat Users Characterization file* the reference values valid for the whole mission lifetime have been provided. For more details on corrections contained in the *CryoSat Users Characterization file*, refer to Section 3.

2.2.1 Calibration of the interferometric phase difference

The echoes acquired in SARin mode from the two receiving antennas can be combined in order to extract the interferometric phase difference of the echo return so that information of the angle of arrival can be retrieved [RD4]. An accurate interferometric phase difference requires the two receiving channels to be cross-calibrated in phase. As discussed in [RD1], SIRAL provides measurements for the transfer functions in phase as well as for the phase difference between the two receiving antennas. These calibration measurements are processed on-ground prior application to the echoes.

This document version does not provide indications on the phase difference calibration between the two receiving chains.

In fact the calibration corrections for SARin mode that are detailed in Section 2.2 allow to calibrate the FBR data with respect to the internal path delay and the instrument power gain, so that the interferometric phase difference cannot be correctly retrieved by simply combining the echoes from the two receiving chains according to Section 2.2 of this document, as the phase cross-calibration is missing.

3 CryoSat User Characterization file: CDL dump

This section contains the CDL dump of CryoSat User Characterization file, version E001:

- cs_users_characterization_E001.nc

The reference corrections provided in this version of the CryoSat Users Characterization file have been computed by averaging all the corresponding corrections read from BaselineD and BaselineE Calibration L1B products between 01/03/2011 and 31/08/2021.

3.1 Global Attributes

```
   :ncdf_filename = "cs_users_characterization_E001.nc";
   :creation_date = "14-Apr-2022 10:00:15";
   :satellite_name = "CryoSat-2";
   :altimeter_name = "SIRAL";
   :altimeter_mode = "SAR/SARin";
   :baseline = "BaselineE";
   :version = "v1.0";
   :general_description = "This file contains the reference values for CAL1
Pulse-to-Pulse corrections (amplitude and phase)and CAL2 Low Pass Filter
corrections for CryoSat-2 SAR and SARin acquisition modes.These reference
corrections are valid for the whole mission lifetime due to the stability of the
instrument.These reference corrections have been computed by averaging all the
corresponding corrections read from BaselineC to BaselineE CAL L1B models
(extracted from products) between 01/03/2011 and 31/08/2021.";
```

3.2 Dimensions

```
n_pulses = 64;
n_samples_SAR = 128;
n_samples_SARin = 512;
```

3.3 Variables

SAR charachterization variables

```
double cal1_p2p_amplitude_sar(n_pulses=64);
      :long_name = "CAL1 SAR pulse-to-pulse amplitude correction";
      :units = "unitless";
      :scale_factor = 1.0E-6; // double
      :source = " Computed by averaging all CAL1 SAR pulse-to-pulse amplitude
corrections read from BaselineC to BaselineE CAL L1B models (extracted from
products) between 01/03/2011 and 31/08/2021";
      :comment = "CAL1 Amplitude Pulse-to-Pulse correction to be applied to
bursts acquired in SAR mode: the k-th echo in the received burst has to be
multiplied by the k-th values of the correction.";
```

```
double cal1_p2p_phase_sar(n_pulses=64);
      :long_name = "CAL1 SAR pulse-to-pulse phase correction";
      :units = "rad";
      :scale_factor = 1.0E-6; // double
      :source = " Computed by averaging all CAL1 SAR pulse-to-pulse phase
corrections read from BaselineC to BaselineE CAL L1B models (extracted from
products) between 01/03/2011 and 31/08/2021";
      :comment = "CAL1 Phase Pulse-to-Pulse correction to be applied to bursts
acquired in SAR mode: the k-th echo in the received burst has to be multiplied
by complex exponential of the k-th values of the correction.";
```

```
double cal2_lpf_sar(n_samples_SAR=128);
      :long_name = "CAL2 Low Pass Filter amplitude correction for SAR mode.";
      :units = "unitless";
      :scale_factor = 1.0E-6; // double
      :source = " Computed by averaging all CAL2 SAR LPF corrections read from
BaselineC to BaselineE CAL L1B models (extracted from products) between
01/03/2011 and 31/08/2021";
      :comment = "CAL2 LPF amplitude correction to be applied to each echo in
the bursts acquired in SAR mode.This correction has to be applied to the Fourier
transform of each echo in the acquired bursts.The k-th sample of the Fourier
Transform of the echo has to be multiplied by the k-th values of the
correction.The echo can go then through Inverse Fourier Transform.";
```

SARin charachterization variables

```
double cal1_p2p_amplitude_sarin_rx1(n_pulses=64);
       :long_name = "CAL1 SARin Rx1 pulse-to-pulse amplitude correction";
       :units = "unitless";
       :scale_factor = 1.0E-6; // double
       :source = " Computed by averaging all CAL1 SARin Rx1 pulse-to-pulse
amplitude corrections read from BaselineC to BaselineE CAL L1B models (extracted
from products) between 01/03/2011 and 31/08/2021";
       :comment = "CAL1 Amplitude Pulse-to-Pulse correction to be applied to
bursts acquired by Rx channel 1 in SARin mode: the k-th echo in the received
burst has to be multiplied by the k-th values of the correction.";
```

```
double cal1_p2p_phase_sarin_rx1(n_pulses=64);
       :long_name = "CAL1 SARin Rx1 pulse-to-pulse phase correction";
       :units = "rad";
       :scale_factor = 1.0E-6; // double
       :source = " Computed by averaging all CAL1 SARin Rx1 pulse-to-pulse phase
corrections read from BaselineC to BaselineE CAL L1B models (extracted from
products) between 01/03/2011 and 31/08/2021";
       :comment = "CAL1 Phase Pulse-to-Pulse correction to be applied to bursts
acquired by Rx channel 1 in SARin mode: the k-th echo in the received burst has
to be multiplied by complex exponential of the k-th values of the correction.";
```

```
double cal1_p2p_amplitude_sarin_rx2(n_pulses=64);
        :long_name = "CAL1 SARin Rx2 pulse-to-pulse amplitude correction";
        :units = "unitless";
        :scale_factor = 1.0E-6; // double
        :source = " Computed by averaging all CAL1 SARin Rx1 pulse-to-pulse
amplitude corrections read from BaselineC to BaselineE CAL L1B models (extracted
from products) between 01/03/2011 and 31/08/2021";
        :comment = "CAL1 Amplitude Pulse-to-Pulse correction to be applied to
bursts acquired by Rx channel 2 in SARin mode: the k-th echo in the received
burst has to be multiplied by the k-th values of the correction.";


double cal1_p2p_phase_sarin_rx2(n_pulses=64);
        :long_name = "CAL1 SARin Rx2 pulse-to-pulse phase correction";
        :units = "rad";
        :scale_factor = 1.0E-6; // double
        :source = " Computed by averaging all CAL1 SARin Rx2 pulse-to-pulse phase
corrections read from BaselineC to BaselineE CAL L1B models (extracted from
products) between 01/03/2011 and 31/08/2021";
        :comment = "CAL1 Phase Pulse-to-Pulse correction to be applied to bursts
acquired by Rx channel 2 in SARin mode: the k-th echo in the received burst has
to be multiplied by complex exponential of the k-th values of the correction.";


double cal2_lpf_sarin_rx1(n_samples_SARin=512);
        :long_name = "CAL2 Low Pass Filter amplitude correction on Rx1 for SARin
mode.";
        :units = "unitless";
        :scale_factor = 1.0E-6; // double
        :source = " Computed by averaging all CAL2 SARin Rx1 LPF corrections read
from BaselineC to BaselineE CAL L1B models (extracted from products) between
01/03/2011 and 31/08/2021";
        :comment = "CAL2 LPF amplitude correction to be applied to each echo in
the bursts acquired by Rx channel 1 in SARin mode.This correction has to be
applied to the Fourier transform of each echo in the acquired bursts.The k-th
sample of the Fourier Transform of the echo has to be multiplied by the k-th
values of the correction.The echo can go then through Inverse Fourier
Transform.";


double cal2_lpf_sarin_rx2(n_samples_SARin=512);
        :long_name = "CAL2 Low Pass Filter amplitude correction on Rx2 for SARin
mode.";
        :units = "unitless";
        :scale_factor = 1.0E-6; // double
        :source = " Computed by averaging all CAL2 SARin Rx2 LPF corrections read
from BaselineC to BaselineE CAL L1B models (extracted from products) between
01/03/2011 and 31/08/2021";
        :comment = "CAL2 LPF amplitude correction to be applied to each echo in
the bursts acquired by Rx channel 2 in SARin mode.This correction has to be
applied to the Fourier transform of each echo in the acquired bursts.The k-th
sample of the Fourier Transform of the echo has to be multiplied by the k-th
values of the correction.The echo can go then through Inverse Fourier
Transform.";
```

4 System characterization

Throughout this section, the system characterization parameter that can be useful to FBR users are described:

- the main instrument parameters for SIRAL are described in Section 4.1
- the antenna pattern for the SIRAL antennas is described in Section 4.2

4.1 SIRAL instrument parameters

Tab.4 gives the main SIRAL instruments parameters for SAR and SARin modes, which have been constant along the mission.

Parameter	Value	Unit
PRF	18.1818	Hz
PRI	55	microsec
Tracking cycle duration	47.17185	millisec
Nb of bursts in 1 radar cycle	4 (SAR), 1 (SARin)	#
Nb of emitted Ku pulses in 1 burst	64	#
Nb of samples in 1 echo	128 (SAR), 512 (SARin)	#
Wavelength	0.022	m
Carrier Frequency	13.575	GHz
Transmission Chirp Bandwidth	350	MHz
Reception Chirp Bandwidth	320	MHz
Reception Chirp Duration	44.8	microsec
Range Resolution	0.4684	m
Sampling clock	320	MHz
Range Sampling	0.4684	m

Tab.4 SIRAL instrument main parameters.

4.2 Antenna Pattern characterization

SIRAL antenna sub-systems have been characterised on-ground by the antenna manufacturer and the test results have been provided in [RD3].

Tab.5 and Tab.6 respectively provide the parameters needed for computing the antenna pattern for the transmitting and receiving antenna (Antenna 1), which is used in SAR and SARin mode, and the receiving only antenna (Antenna 2), which is used in SARin mode only.

Antenna 1 (Tx and Rx)			
Symbol	**Description**	**Value** ± σ	**Unit**
θ_{3dB}	Along Track Antenna beam width at -3dB	19.133 ± 0.484	milliradian
α_{3dB}	Across Track Antenna beam width at -3dB	21.466 ± 0.420	milliradian
G_{dB}	Antenna Maximum gain in dB	42.814 ± 0.076	dB

Tab.5 SIRAL antenna 1 sub-system parameters.

Antenna 2 (Rx only)			
Symbol	**Description**	**Value** ± σ	**Unit**
θ_{3dB}	Along Track Antenna beam width at -3dB	19.090 ± 0.427	milliradian
α_{3dB}	Across Track Antenna beam width at -3dB	21.488 ± 0.413	milliradian
G_{dB}	Antenna Maximum gain in dB	42.924 ± 0.077	dB

Tab.6 SIRAL antenna 2 sub-system parameters.

It is worth noticing that during the on-ground characterization of the antenna subsystems, the antenna pattern was tested across the entire transmission chirp bandwidth. The parameters provided in Tab.5 and Tab.6 have been obtained averaging the values for the different frequencies from [RD3] and, for sake of completeness, the standard deviation on those values has been also reported.

Starting from the parameters in Tab.5 and Tab.6, the antenna pattern can be defined according to different formulas. Sections 5.1.1 and 5.1.2 provide two different equations used in literature.

4.2.1 Antenna Pattern – Equation 1

In [RD3], the antenna pattern is assumed elliptical and separable in the along-track and across-track directions, so that it can described by the following equation:

$$G(\theta, \alpha) = G_0 \cdot EXP\left[-K \cdot \left(\frac{2 \cdot \theta}{\theta_{3dB}}\right)^2\right] \cdot EXP\left[-K \cdot \left(\frac{2 \cdot \alpha}{\alpha_{3dB}}\right)^2\right] \quad (8)$$

where:

- θ is the along-track angle in radians
- α is the across-track angle in radians
- $G_0 = 10^{\frac{G_{dB}}{10}}$ is the maximum gain
- $K = \ln(2) = 0.693147$

4.2.2 Antenna Pattern – Equation 2

In [RD4], the antenna pattern is assumed elliptical and separable in the along-track and across-track directions, so that it can described by the following equation:

$$G(\psi, \omega) = G_0 \cdot EXP\left[-\psi^2 \cdot \left(\frac{\cos^2(\omega)}{\gamma_1^2} + \frac{\sin^2(\omega)}{\gamma_2^2}\right)\right] \quad (9)$$

where:

- ψ is the polar angle in radians, measured from the antennas' boresight
- ω is the azimuthal angle in radians, measured from the along-track direction
- $\gamma_1 = \theta_{3dB} / \left(2 \cdot \sqrt{K}\right)$ determines the along-track width of the illumination
- $\gamma_2 = \alpha_{3dB} / \left(2 \cdot \sqrt{K}\right)$ determines the across-track width of the illumination
- $G_0 = 10^{\frac{G_{dB}}{10}}$ is the maximum gain