Noise covariance model for time-series InSAR analysis

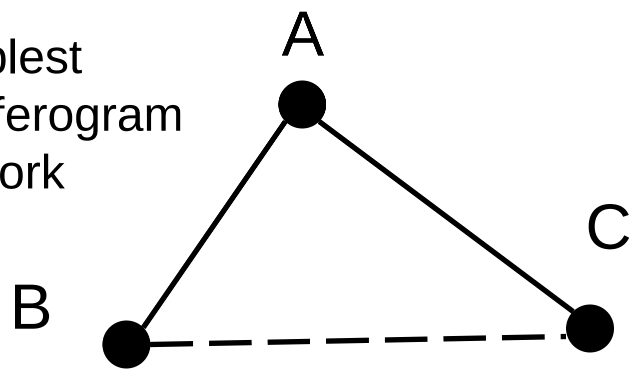
Piyush Agram, Mark Simons

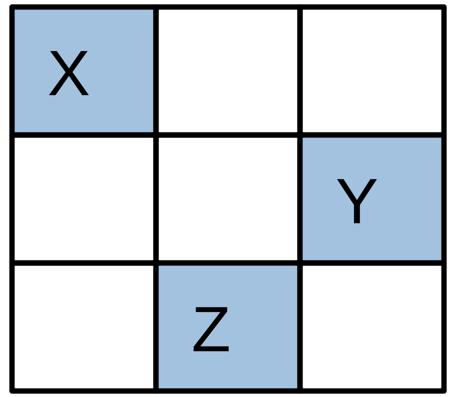
Motivation

- Noise covariance in individual interferograms has been well studied. (Hanssen, 2001)
- Extend the noise model to networks of interferograms – time-series applications.
- Develop uncertainty measures for time-series InSAR products.
- Build an InSAR noise covariance model from first principles.

Independent Observations ?

Simplest interferogram network





Simple network of coherent pixels in a single interferogram.

- Is information contained in interferogram BC different from information in AB and AC?
- Is information at X,Y and Z pixels independent?

- <u>Conventionally any covariances are ignored.</u>
  - Simplicity.
  - Ease of implementing algorithms.

Single pixel phase model

$$\Delta\phi_{ifg}^{x,i,j} = \Delta\phi_{defo}^{x,i,j} + \underbrace{\Delta\phi_{aps}^{x,i,j} + \Delta\phi_{decor}^{x,i,j} + \phi_{n}^{x,i,j}}_{\text{Noise terms}}$$

$\Delta\phi_{aps}^{x,i,j}$	➡	Atmospheric Phase Screen	➡	Spatially correlated. Correlated in pairs with common scene.
$\Delta\phi_{decor}^{x,i,j}$	➡	Decorrelation phase noise	➡	Spatially independent. Temporally correlated.
$\phi_{n}^{x,i,j}$	➡	Phase noise from uncorrelated sources	➡	Spatially and Temporally independent.

Pixel *x* in interferogram *(i,j)*

i and *j* are indices of SAR acquisitions.

Vector Phase model

$$\begin{bmatrix} \Delta\phi_{ifg}^{1,i_1,j_1} \\ \vdots \\ \Delta\phi_{ifg}^{1,i_M,j_M} \\ \vdots \\ \Delta\phi_{ifg}^{P,i_1,j_1} \\ \vdots \\ \Delta\phi_{ifg}^{P,i_M,j_M} \end{bmatrix} = \begin{bmatrix} A & \cdots & 0 \\ \vdots & \vdots & \vdots \\ 0 & \cdots & A \end{bmatrix} \cdot \begin{bmatrix} \phi_{sar}^{1,1} \\ \vdots \\ \phi_{sar}^{1,N} \\ \vdots \\ \phi_{sar}^{P,1} \\ \vdots \\ \phi_{sar}^{P,N} \end{bmatrix} + \begin{bmatrix} \phi_{n}^{1,i_1,j_1} \\ \vdots \\ \phi_{n}^{1,i_M,j_M} \\ \vdots \\ \phi_{n}^{P,i_1,j_1} \\ \vdots \\ \phi_{n}^{P,i_M,j_M} \end{bmatrix}$$

M Interferograms, *N* SAR scenes, *P* pixels
A represents the incidence matrix [+1,0,-1].

Pixel-by-pixel stack of phase observations.

Phase Covariance model

$$\phi_{sar}^{x,i} = \phi_{defo}^{x,i} + \phi_{aps}^{x,i} + \phi_{decor}^{x,i}$$

In absence of deformation, our noise estimate is

$$\overline{\Delta\phi_{ifg}} = \overline{A} \cdot \overline{\phi_{aps}} + \overline{A} \cdot \overline{\phi_{decor}} + \overline{\phi_n}$$

$$\Sigma_{ifg} = \Sigma_{aps} + \Sigma_{decor} + \Sigma_n$$

Assuming statistical independence of the noise terms.

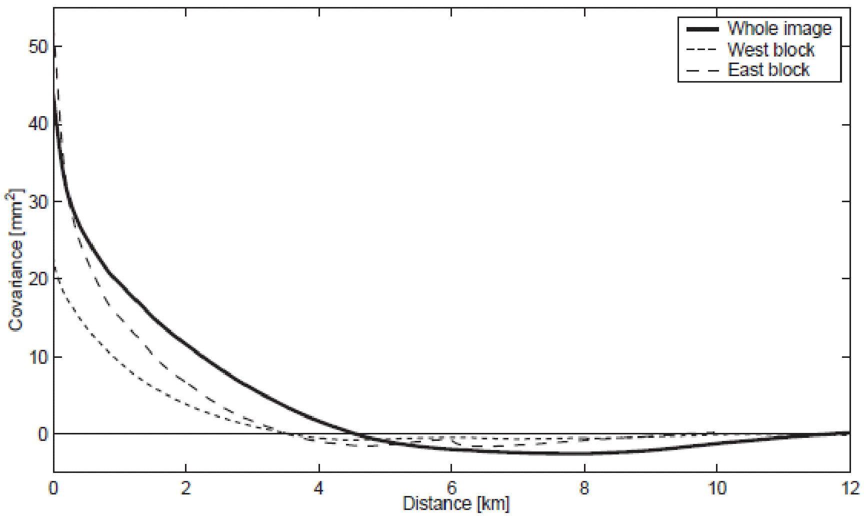
Total covariance matrix is a combination of three models.

- Atmosphere
- Scattering / decorrelation
- System noise

Σ_{aps}

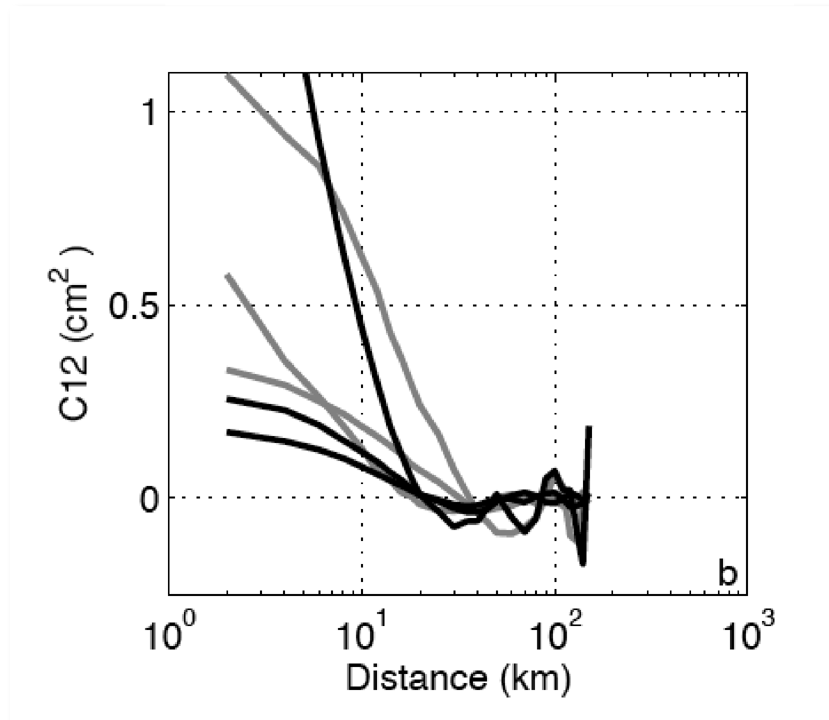
Atmospheric Phase Models

$$\sigma_{aps}^{x,y} = c \cdot L_{x,y}^{\alpha} + k \cdot H_{x,y}$$



S. Jonsson (PhD thesis)

Derived from the data itself using radon transform.



Hanssen (2001)
Emardson et al. (2003)
Lohman & Simons (2005)

Σ_{aps}

APS Covariance model

1) Spatial covariance for a SAR scene.

$$\eta_{aps}^{x,y} = \frac{1}{2} \cdot \left[\left(\sigma_{aps}^{x,ref}\right)^2 + \left(\sigma_{aps}^{y,ref}\right)^2 - \left(\sigma_{aps}^{x,y}\right)^2 \right]$$

2) Covariance of all pixels in one SAR scene.

$$\Sigma_{aps}^{sar} = \begin{bmatrix} \eta_{aps}^{1,1} & \cdots & \eta_{aps}^{1,P} \\ \vdots & \eta_{aps}^{x,y} & \vdots \\ \eta_{aps}^{P,1} & \cdots & \eta_{aps}^{P,P} \end{bmatrix}$$

3) Covariance of all pixels in all SAR scenes.

$$\left[\Sigma_{aps}^{sar} \otimes I_{P,P}\right]$$

4) Covariance of all pixels in all IFGs.

$$\Sigma_{aps} = \overline{A} \cdot \left[\Sigma_{aps}^{sar} \otimes I_{P,P}\right] \cdot \overline{A}^T$$

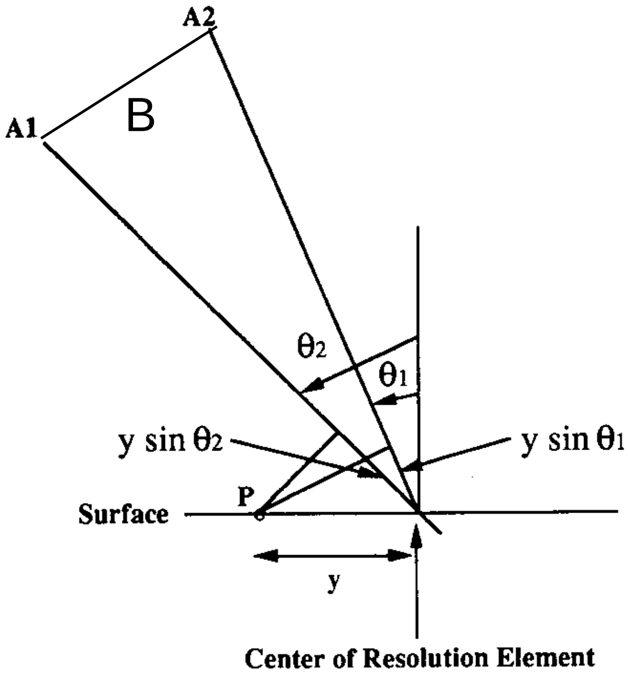
Σ_{decor}

Decorrelation model

$$\rho_{spatial} = 1 - \frac{2|B|R_y \cos^2\theta}{\lambda r}$$

$$\rho_{rotation} = 1 - \frac{2\sin\theta|d\phi|R_x}{\lambda}$$

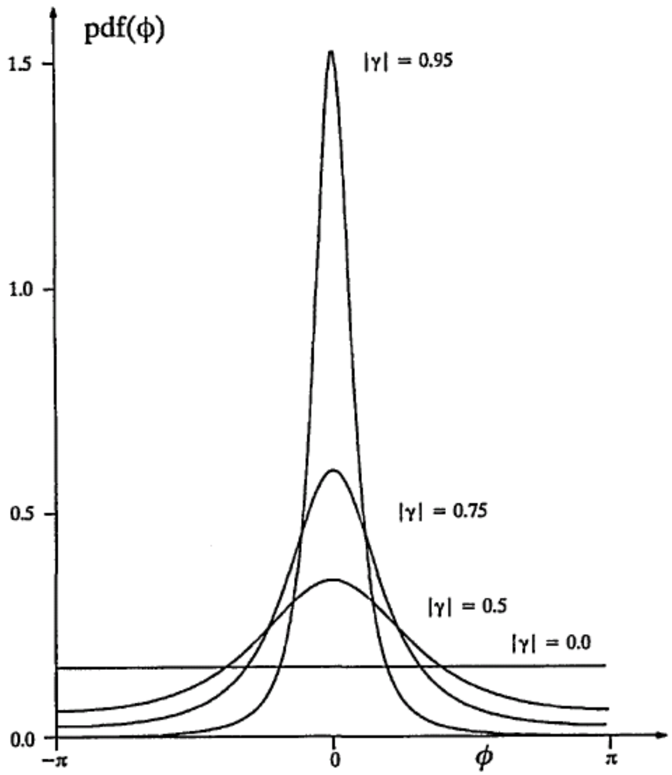
$$\rho_{temporal} = \exp\{-\frac{1}{2}(\frac{4\pi}{\lambda})^2(\sigma_y^2\sin^2\theta + \sigma_z^2\cos^2\theta)\}$$



- Zebker and Villasenor (1992), Bamler and Just (1993).
- SAR pixel made up of infinite Gaussian scatterers.

Σ_{decor}

Observable: InSAR coherence



$$\gamma^{x,i,j} = \frac{\mathrm{E}\left(z_{x,i} \cdot z_{x,j}^{*}\right)}{\sqrt{\mathrm{E}\left(|z_{x,i}|^{2}\right) \cdot \mathrm{E}\left(|z_{x,j}|^{2}\right)}}$$

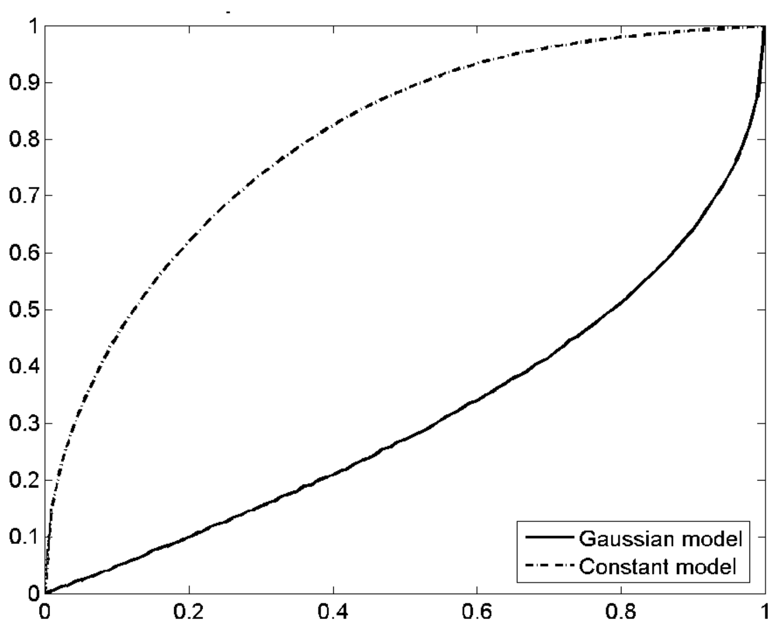
Just and Bamler (1994)
Gaussian signal model

- InSAR coherence is a function of phase noise.
- We derive a model to relate coherence directly to noise covariance matrix.
- Actual temporal behavior of scatterers not known.
- We use a signal model instead.

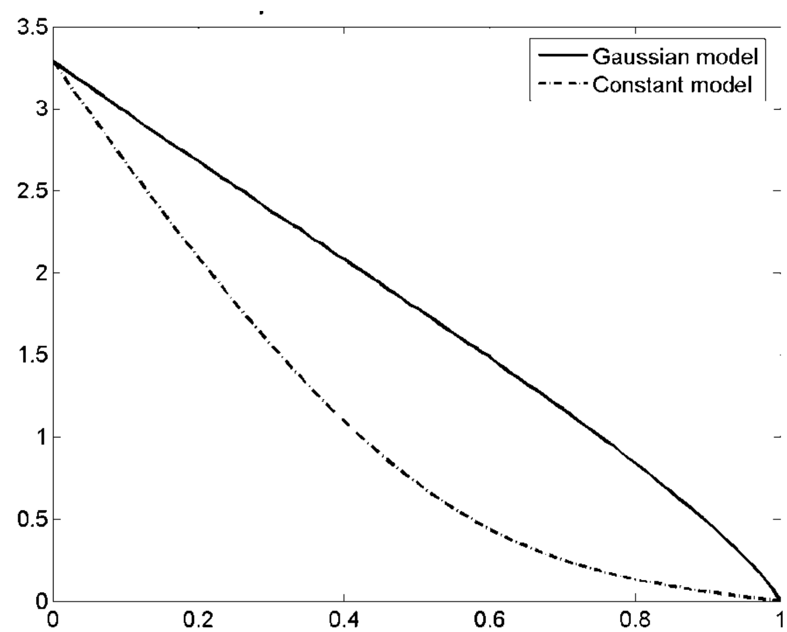
Σ_{decor}

Coherence => Signal Model => Stats

SAR phase correlation



InSAR phase variance



Observed InSAR coherence

Number of looks taken into account for the mapping.

Σ_{decor}

Decorrelation covariance model

1) SAR phase correlation of one pixel from InSAR coherence.

$$\Omega_x^{sar} = \begin{bmatrix} 1 & \rho_{sar}\left(\gamma_{x,1,2}\right) & \cdots \\ \rho_{sar}\left(\gamma_{x,i,j}\right) & 1 & \cdots \\ \vdots & \vdots & 1 \end{bmatrix}$$

2) Pseudo-correlation of one pixel in all interferograms.

$$\Omega_x^{ifg} = A \cdot \Omega_x^{sar} \cdot A^T$$

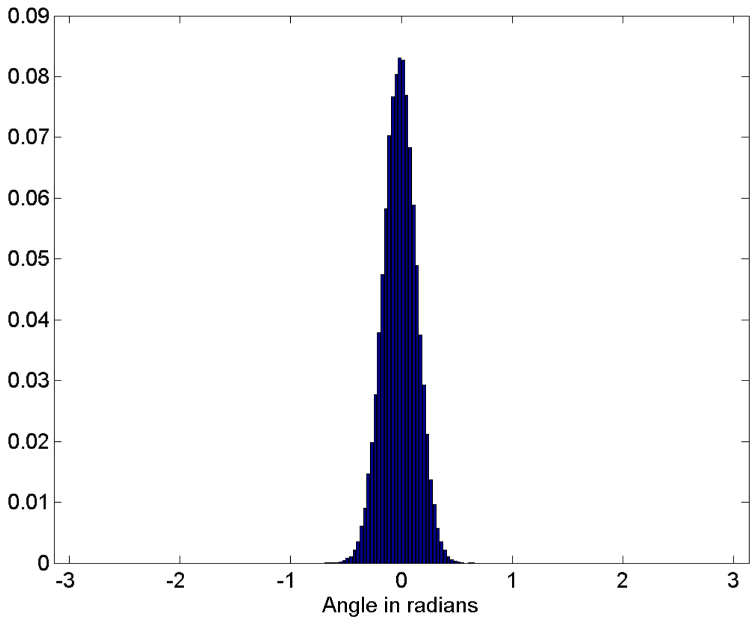
3) Covariance of one pixel in all IFGs. (Pixels are uncorrelated with each other.)

$$\begin{aligned} \Sigma_x^{decor} &= D \cdot \Omega_x^{ifg} \cdot D \\ D &= \begin{bmatrix} \frac{\sigma_{ph}\left(\gamma_{x,i_1,j_1}\right)}{\sqrt{\Omega_x^{ifg}(1,1)}} & 0 & 0 \\ \vdots & \frac{\sigma_{ph}\left(\gamma_{x,i_k,j_k}\right)}{\sqrt{\Omega_x^{ifg}(k,k)}} & \vdots \\ 0 & 0 & \frac{\sigma_{ph}\left(\gamma_{x,i_M,j_M}\right)}{\sqrt{\Omega_x^{ifg}(M,M)}} \end{bmatrix} \end{aligned}$$

4) Repeat for every pixel.

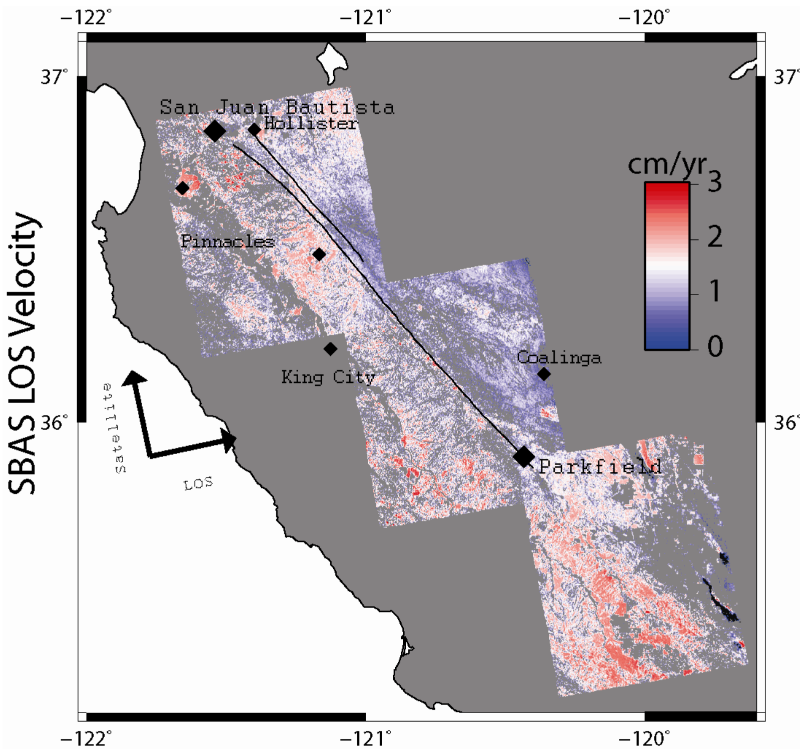
System (processor) noise

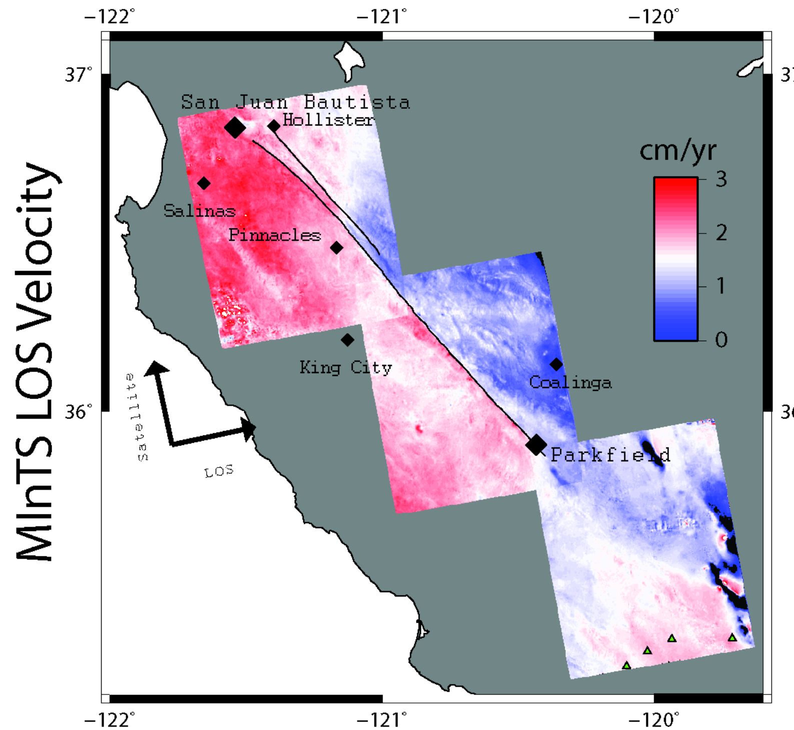
- Two SAR scenes – A and B.
- Form two interferograms AB, BA.
- AB * BA = 0 (theoretically).
- We observe a noise pattern.



- Noise levels for ALOS => 0.15 rads or 8 degs.
- Estimates using Stanford mocomp processor over Parkfield, CA.
- Diagonal structure.

Spatial structure: Implication





- SBAS => pixel-by-pixel inversion.
- MInTS => wavelet coefficient-by-coefficient.

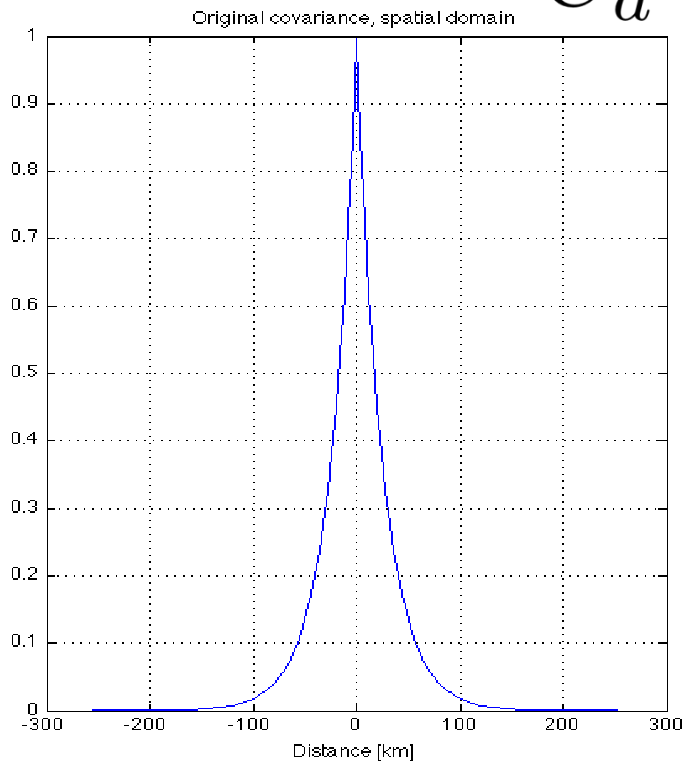
<u>Poster Today: Multi-scale InSAR Time Series (MInTS) analysis of the creeping section of the San Andreas Fault</u>

What does MInTS do?

$C_d = A \exp(L/L_c)$

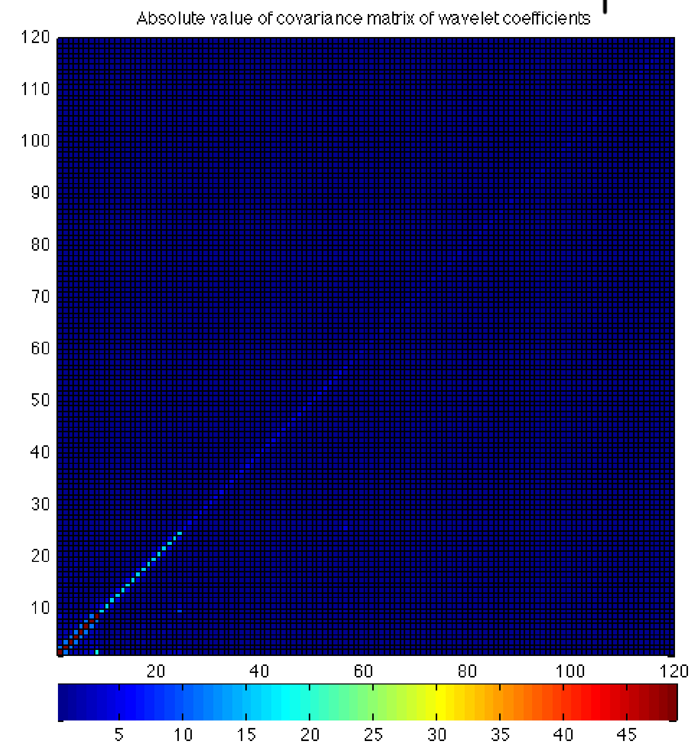
$L_c = 25$ km

$$C_d$$



Corresponding wavelets coefficients nearly uncorrelated

$$|C_w|$$



$$C_w = W C_d W' \text{ where } W \text{ is the wavelet transform matrix}$$

Temporal correlation: Implication

Hanssen (2001)

- Σ_{aps} is the only correlated noise component in a network of IFGs.

- $\Sigma_{decor} + \Sigma_n$ => diagonal covariance matrix.

Our model

- Consistent with decorrelation models.

- Even if the atmospheric phase signal could be perfectly corrected, scatterer noise would still be correlated in time.

- Weighting scheme for InSAR observations in time domain. (Not in MInTS yet)

Thus far …

- We have a presented a covariance model in space and time

- Accounts for
  - ✓ System noise
  - ✓ Interferograms with common scene
  - ✓ Decorrelation phase model
  - ✓ Atmospheric propagation delays

- Does not yet account for
  - ❑ Orbit errors
  - ❑ Tropostatic delays
  - ❑ Ionospheric delays

Application: Optimal IFG networks

Questions of Interest for data centers

1. Given *M* SAR scenes, what is the optimal IFG network of size *N?*

2. Given a network of IFGs, how do I augment it optimally with new SAR scenes?

$$\overline{\Delta\phi_{ifg}} = \Theta \cdot \nu + \overline{e}$$

$$C_{\hat{\nu}} = \left(\Theta^T \cdot \Sigma_{ifg}^{-1} \cdot \Theta\right)^{-1}$$

- Θ is the temporal baseline matrix.
- υ is the LOS velocity.
- C_{υ} is the uncertainty in estimated velocity.

Algorithms: Subset of Observations

- M SAR scenes = $2^{(M^2/2)}$ IFG networks. Impossible to evaluate all networks.

- Subset of observations (Reeves and Zhe, 1999).

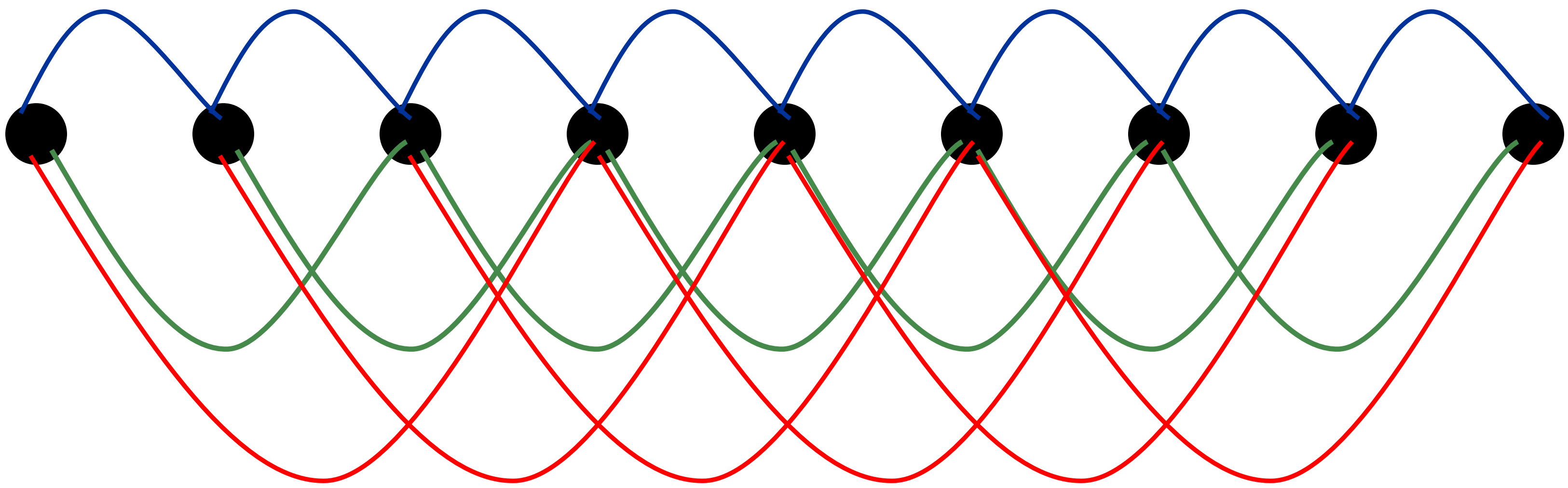
- Sequential Backward Selection (SBS) – Remove IFG that contributes the least at each step.

- Sequential Hybrid Selection (SHS) – Allows IFG already removed to re-enter optimal solution at later stage.

- Any reasonable metric chosen to optimize network – we choose LOS velocity uncertainty for this work.

Example: 9 SAR scenes

Monthly acquisitions with zero geometric baseline.
Decorrelation time constant of 0.36 yr and 1 cm atmosphere.

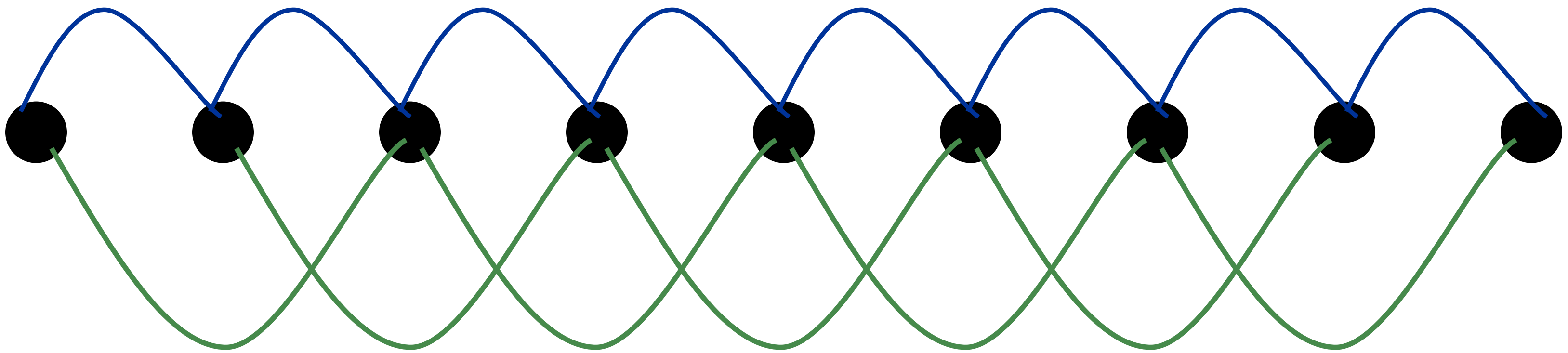


Total number of viable IFGS = 21

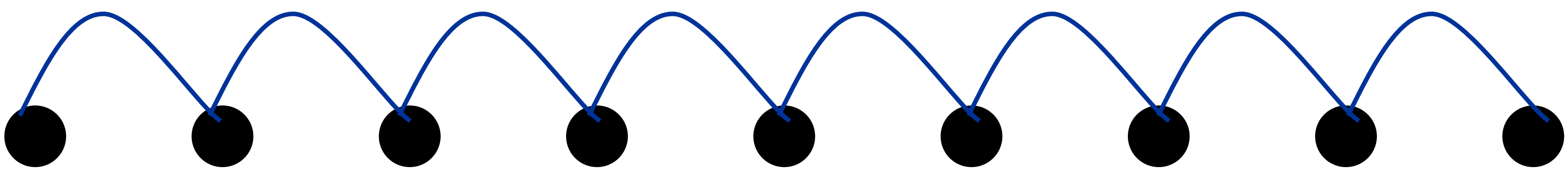
- 8 one hop IFGs (blue)
- 7 two hop IFGs (green)
- 6 three hop IFGs (red)

Example: Pruning networks

Optimal 15/21 IFG network using SBS/ SHS.
Uncertainty ratio = 1.05

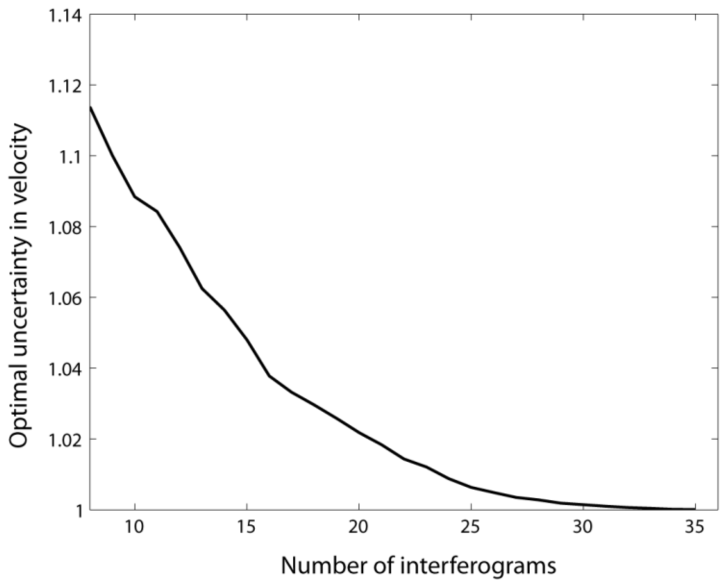


Optimal 8/21 IFG network using SBS/ SHS.
Uncertainty ratio = 1.12

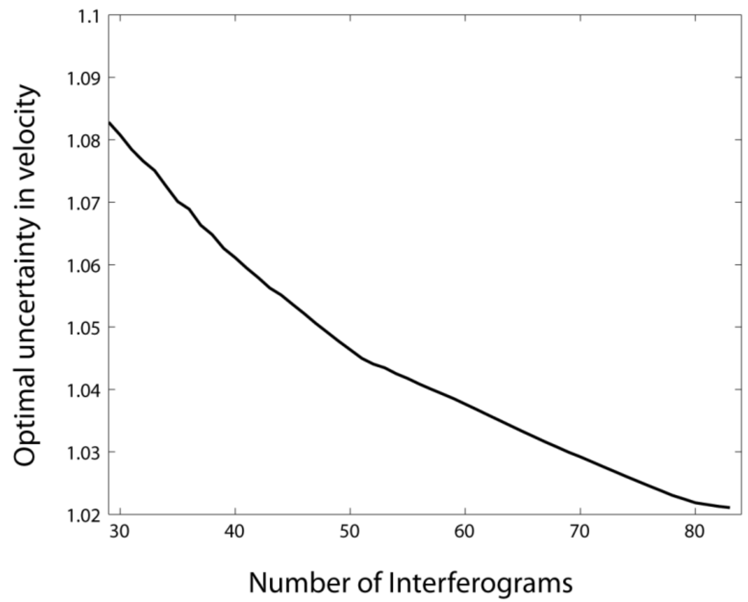


Network design

9 SAR scenes



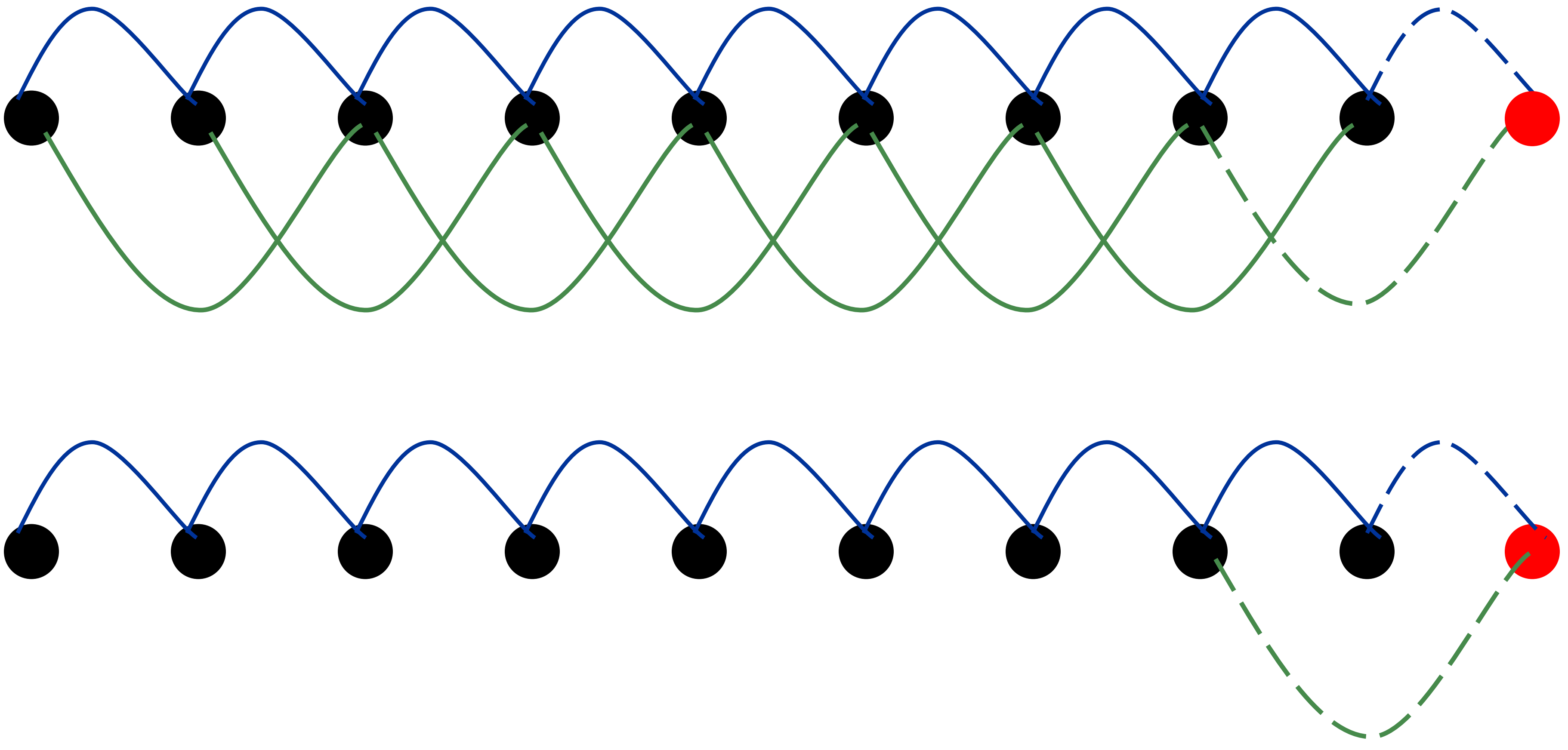
30 SAR scenes



1. SBS/ SHS allows us to design networks of any size.
2. Allows us to prioritize processing of IFGs.
3. Great for designing networks with large number of acquisitions.

Example: Augmenting networks

Adding 1 new SAR scenes and 2 IFGs to existing networks.



Conclusions

- InSAR phase observations are correlated over space and time - not independent observations.

- Accounting for spatial correlation significantly improves deformation estimates (e.g. MInTS).

- Observed InSAR coherence can be directly translated to a temporal covariance matrix using a model.

- A noise covariance model allows us to provide conservative error estimates with time-series products.