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Ocean Clutter Modeling for Ship Detection

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Ocean Clutter Statistics

$$\boldsymbol{s} = [s_{hh}\ s_{hv}\ s_{vh}\ s_{vv}]' = \sqrt{\tau}\boldsymbol{x}$$

Radarsat-2 quad-pol SAR imagery (PauliRGB), 30th Apr 2010 and 27th Oct 2011.

Covariance Matrix Estimation

$$\hat{C} = \frac{1}{n}\sum_{k=1}^{n} w \cdot \boldsymbol{s}_k \boldsymbol{s}_k^{\dagger}$$

Sample Mean covariance matrix estimator

$$w = 1$$

$$\hat{\boldsymbol{C}}_{SM} = \frac{1}{n}\sum_{k=1}^{n} \boldsymbol{s}_k \boldsymbol{s}_k^{\dagger}$$

Fixed-Point covariance matrix estimator

$$w = \frac{d}{\boldsymbol{s}_k^\dagger \hat{C}^{-1} \boldsymbol{s}_k}$$

$$\hat{C}_{FP}(i+1) = \frac{1}{n} \sum_{k=1}^{n} \frac{d}{\boldsymbol{s}_k^\dagger \hat{C}_{FP}^{-1}(i) \boldsymbol{s}_k} \cdot \boldsymbol{s}_k \boldsymbol{s}_k^\dagger$$

Maximum Likelihood covariance matrix estimator

$$w = \frac{h_{d+1}(\boldsymbol{s}_k^\dagger \hat{\boldsymbol{C}}^{-1} \boldsymbol{s}_k)}{h_d(\boldsymbol{s}_k^\dagger \hat{\boldsymbol{C}}^{-1} \boldsymbol{s}_k)}$$

$$h_d(t) = \int_0^{+\infty} \tau^{-d} \exp(-t/\tau) p_\tau(\tau) d\tau$$

$$w(t) = \sqrt{\frac{\alpha}{\mu t}} \frac{K_{\alpha-d-1}(\sqrt{4\alpha t/\mu})}{K_{\alpha-d}(\sqrt{4\alpha t/\mu})}$$

Approximate ML covariance matrix estimator

$$w(t) = \sqrt{\frac{\alpha}{\mu t}} \frac{K_{\alpha-d-1}(\sqrt{4\alpha t/\mu})}{K_{\alpha-d}(\sqrt{4\alpha t/\mu})}$$

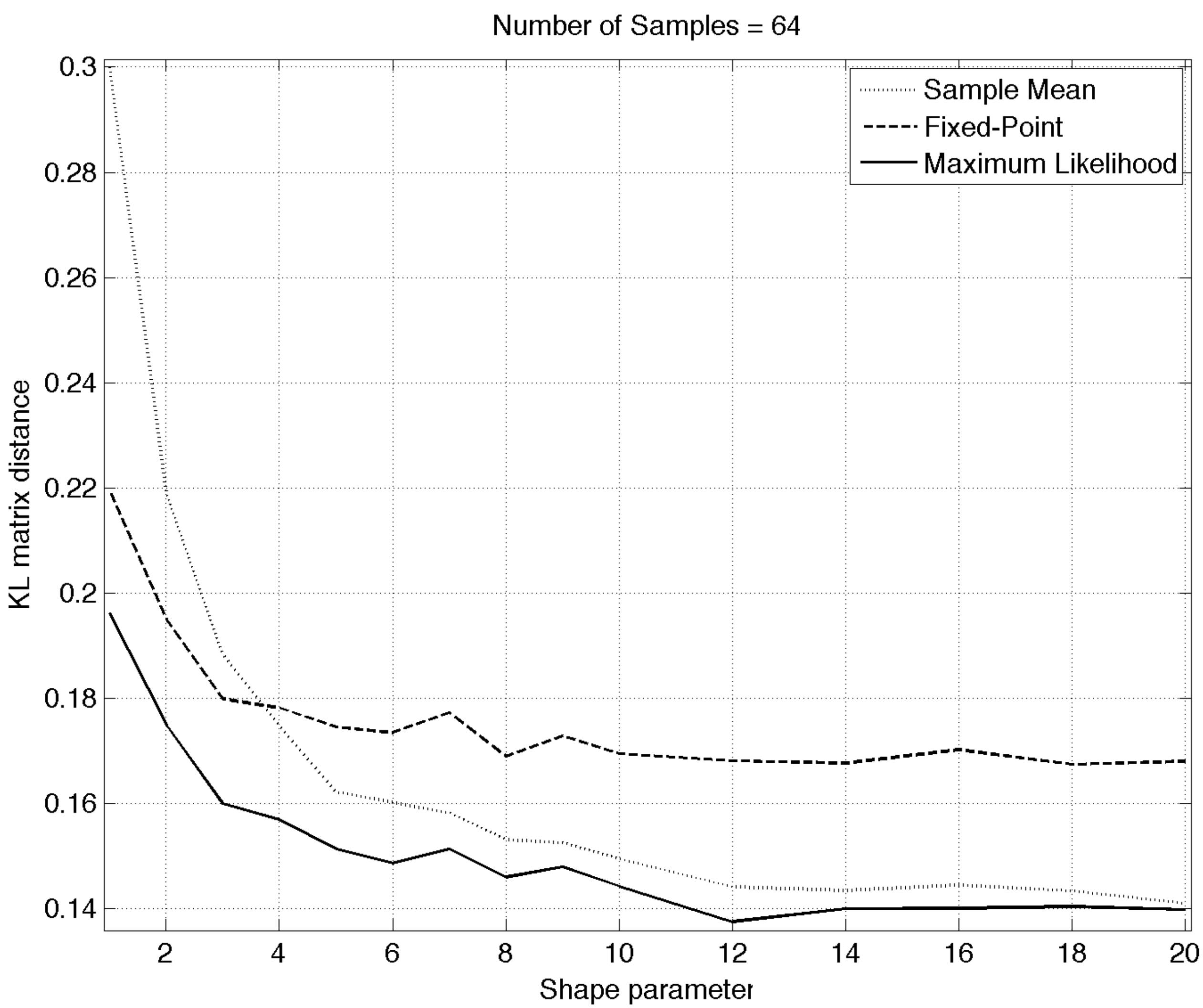
$$0 < L_\nu^{(0)}(t) < \frac{1}{t} \frac{K_\nu(t)}{K_{\nu+1}(t)} < U_\nu^{(0)}(t)$$

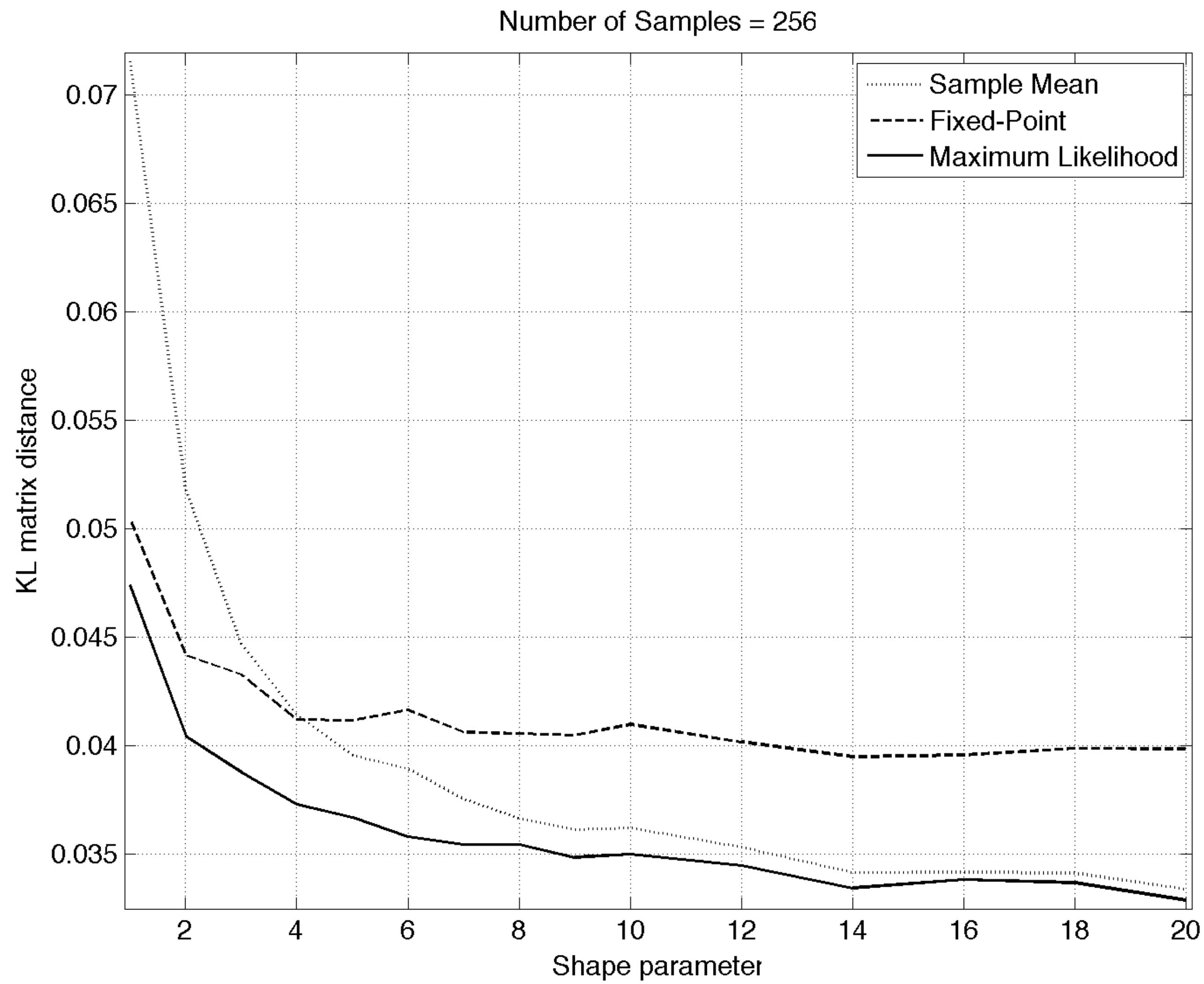
J. Segura, "Bounds for ratios of modified Bessel functions and associated Tura ́n-type inequalities," *Journal of Mathematical Analysis and Applications,* vol. 374, no. 2, pp. 516–528, 2011.

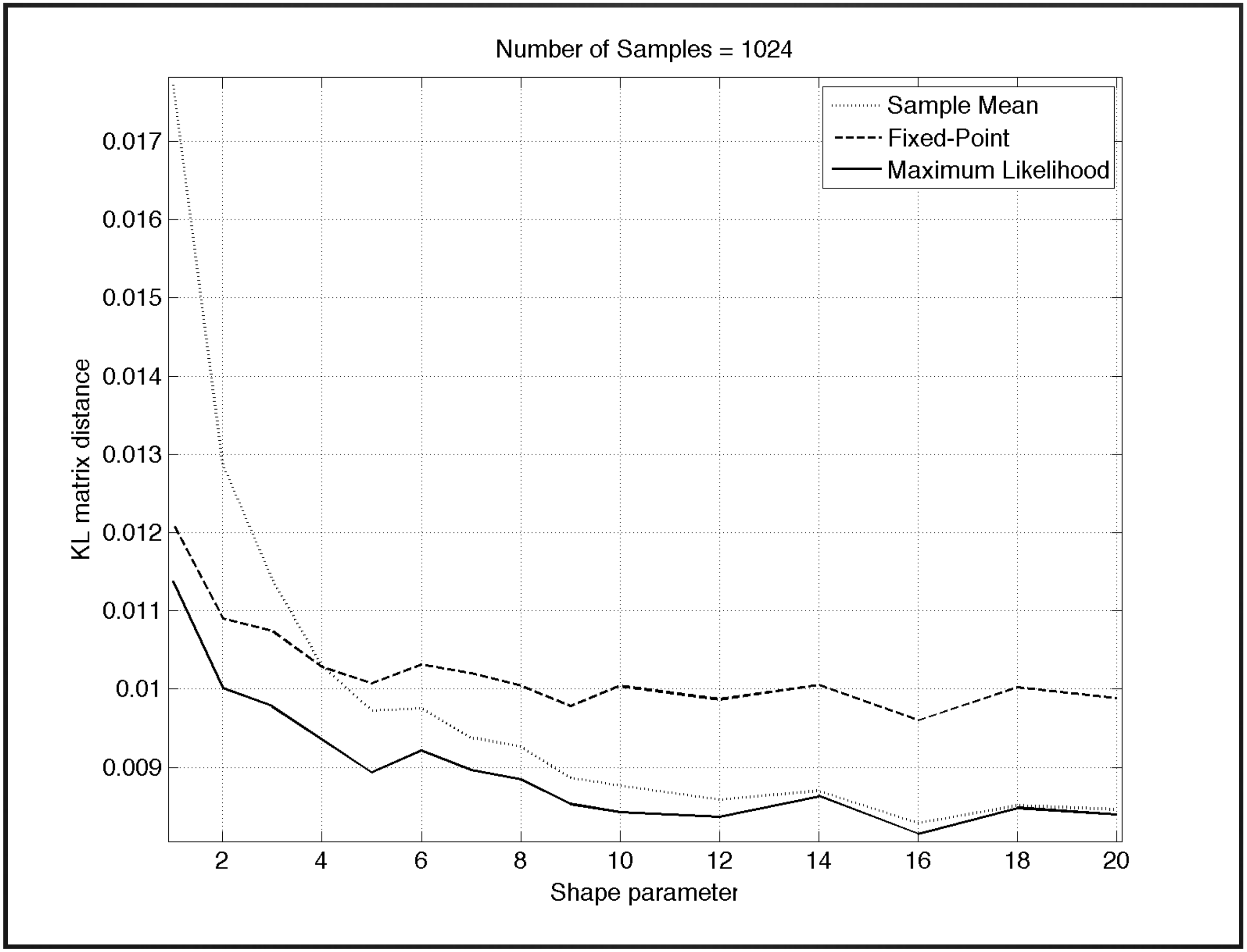
Estimation Accuracy

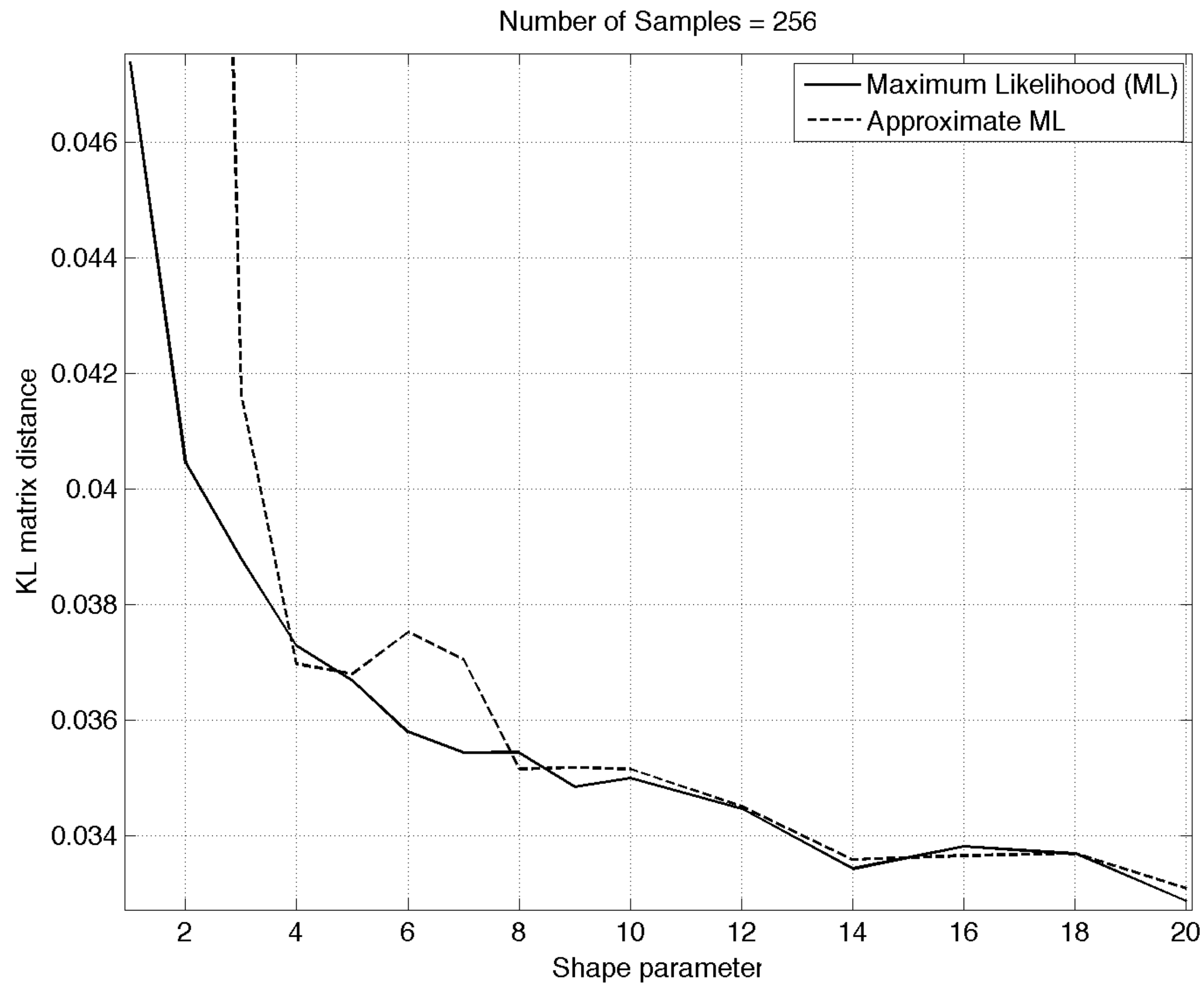
in terms of the Kullback-Leibler (KL) matrix distance

$$D_{KL} = \frac{tr(\boldsymbol{C}^{-1}\hat{\boldsymbol{C}}) + tr(\hat{\boldsymbol{C}}^{-1}\boldsymbol{C})}{2} - d$$


Number of Samples = 64
0.3
0.28
0.26
0.24
0.22
0.2
0.18
0.16
0.14
KL matrix distance
2
4
6
8
10
12
14
16
18
20
Shape parameter
Sample Mean
Fixed-Point
Maximum Likelihood


Number of Samples = 256
Sample Mean
Fixed-Point
Maximum Likelihood
0.07
0.065
0.06
0.055
0.05
0.045
0.04
0.035
KL matrix distance
2
4
6
8
10
12
14
16
18
20
Shape parameter


Number of Samples = 1024
Sample Mean
Fixed-Point
Maximum Likelihood
KL matrix distance
0.017
0.016
0.015
0.014
0.013
0.012
0.011
0.01
0.009
2
4
6
8
10
12
14
16
18
20
Shape parameter

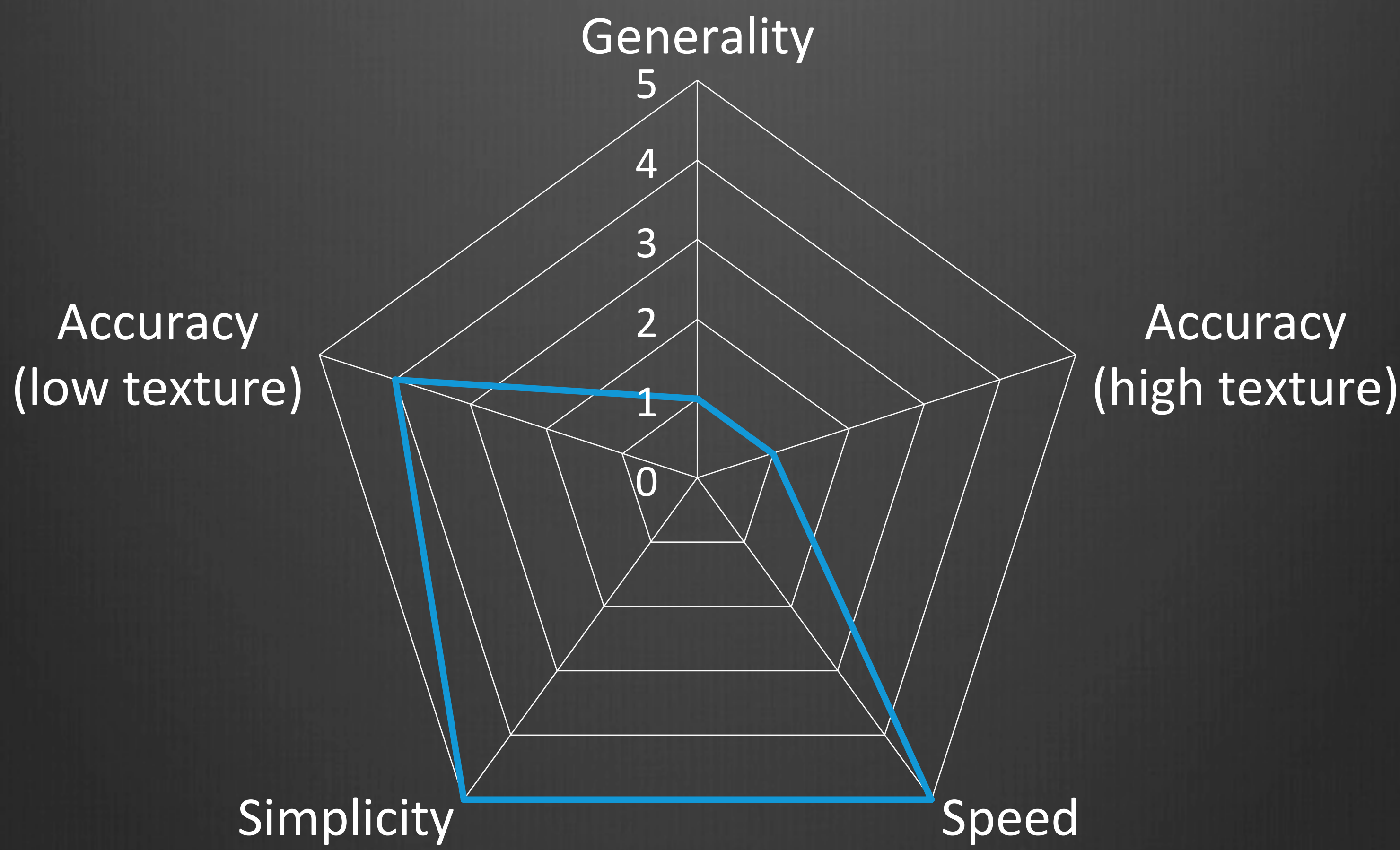

Number of Samples = 256
Maximum Likelihood (ML)
Approximate ML
KL matrix distance
0.046
0.044
0.042
0.04
0.038
0.036
0.034
2
4
6
8
10
12
14
16
18
20
Shape parameter

Computational Efficiency

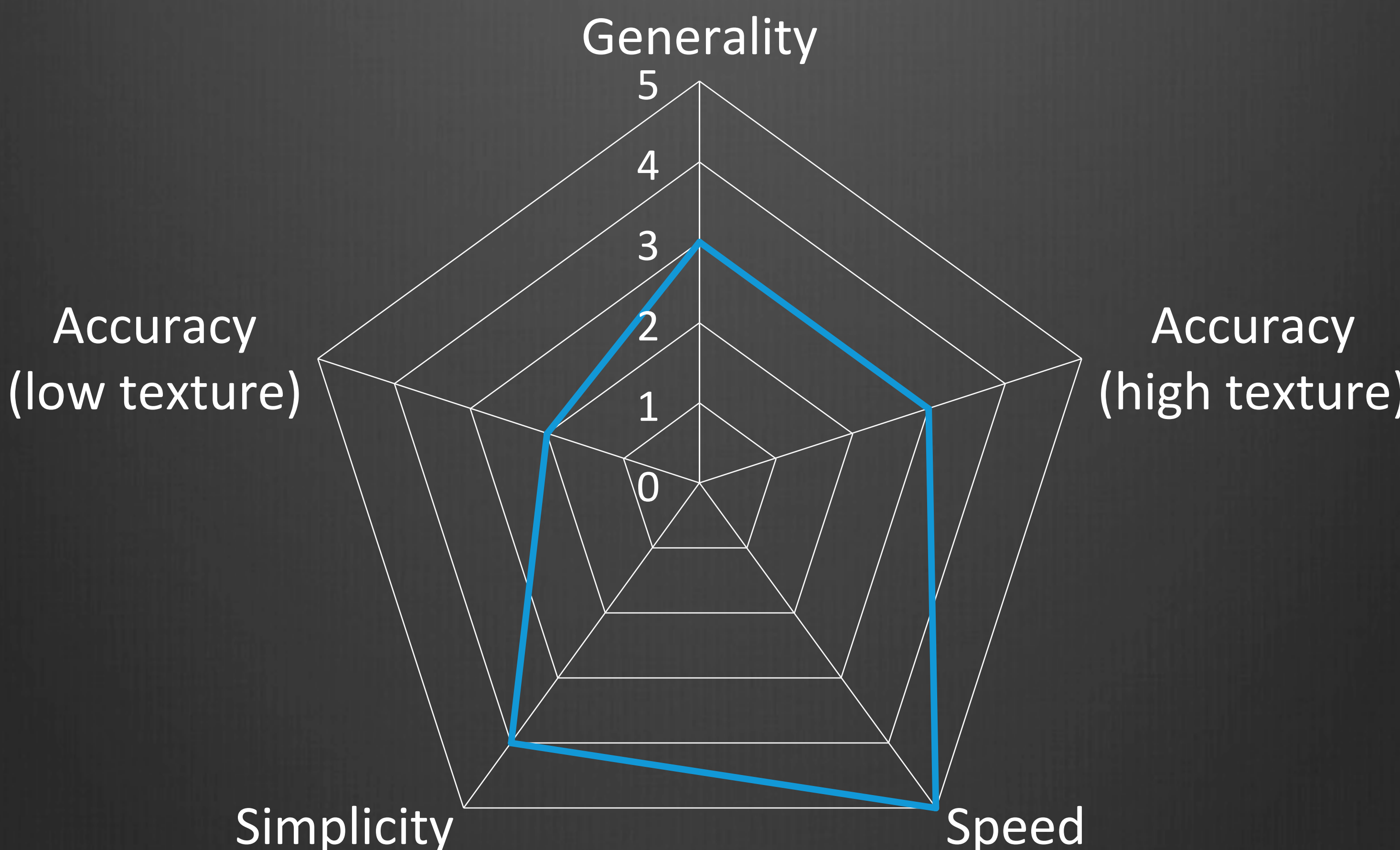
	$p_\tau(\tau)$	Iterative	Computation time under different texture conditions [ms]			
			high (α = 1)	moderate (α = 5)	low (α = 10)	very low (α = 20)
$\hat{C}_{SM}$	✖	✖	1.36	1.36	1.37	1.38
$\hat{C}_{FP}$	✖	✔	6.24	5.96	5.80	5.69
$\hat{C}_{ML}$	✔	✔	24128.71	7935.55	5421.32	5072.61
$\hat{C}_{AML}$	✔	✔	77.97	17.90	13.22	9.99

Conclusions

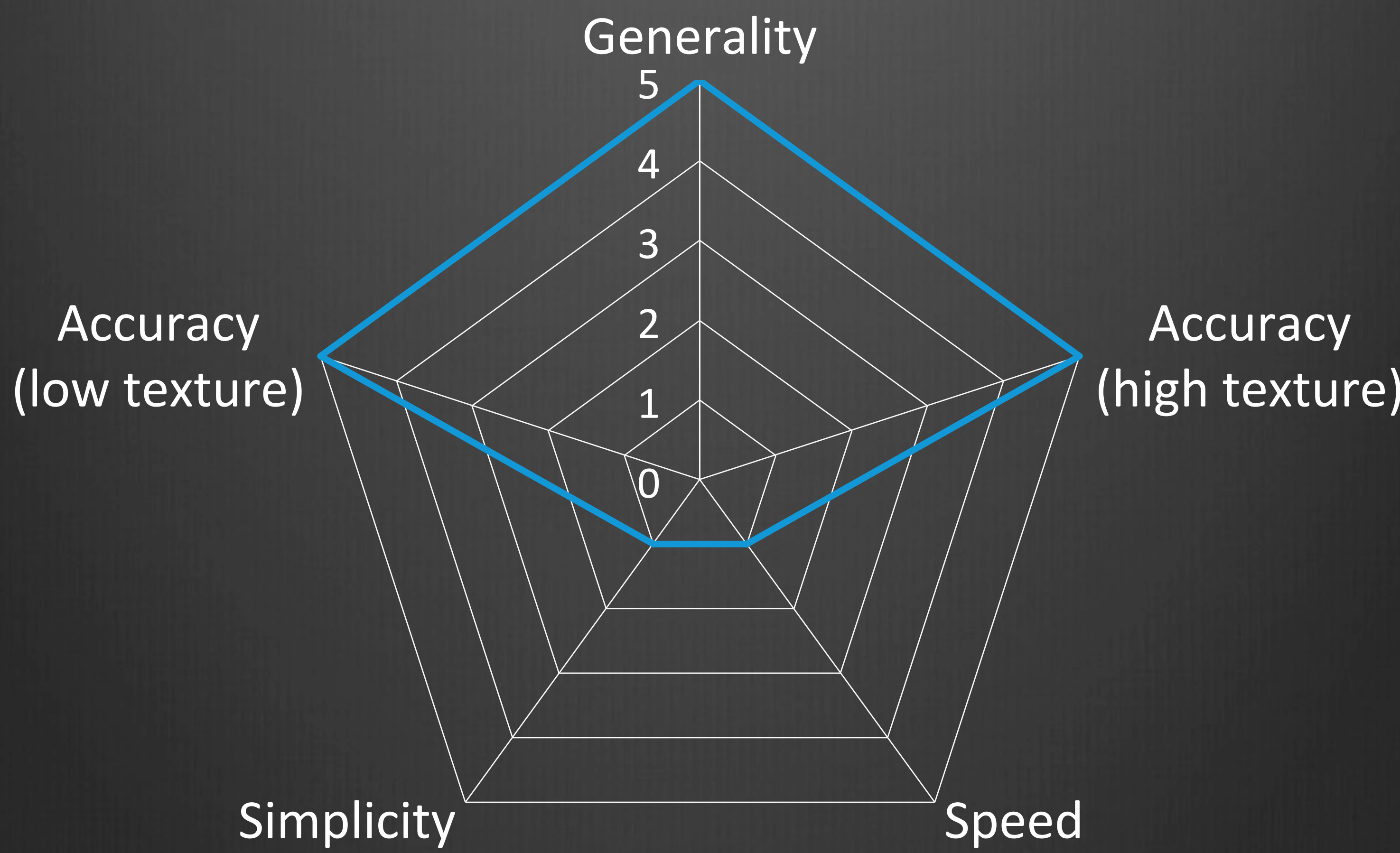
Sample Mean covariance matrix estimator



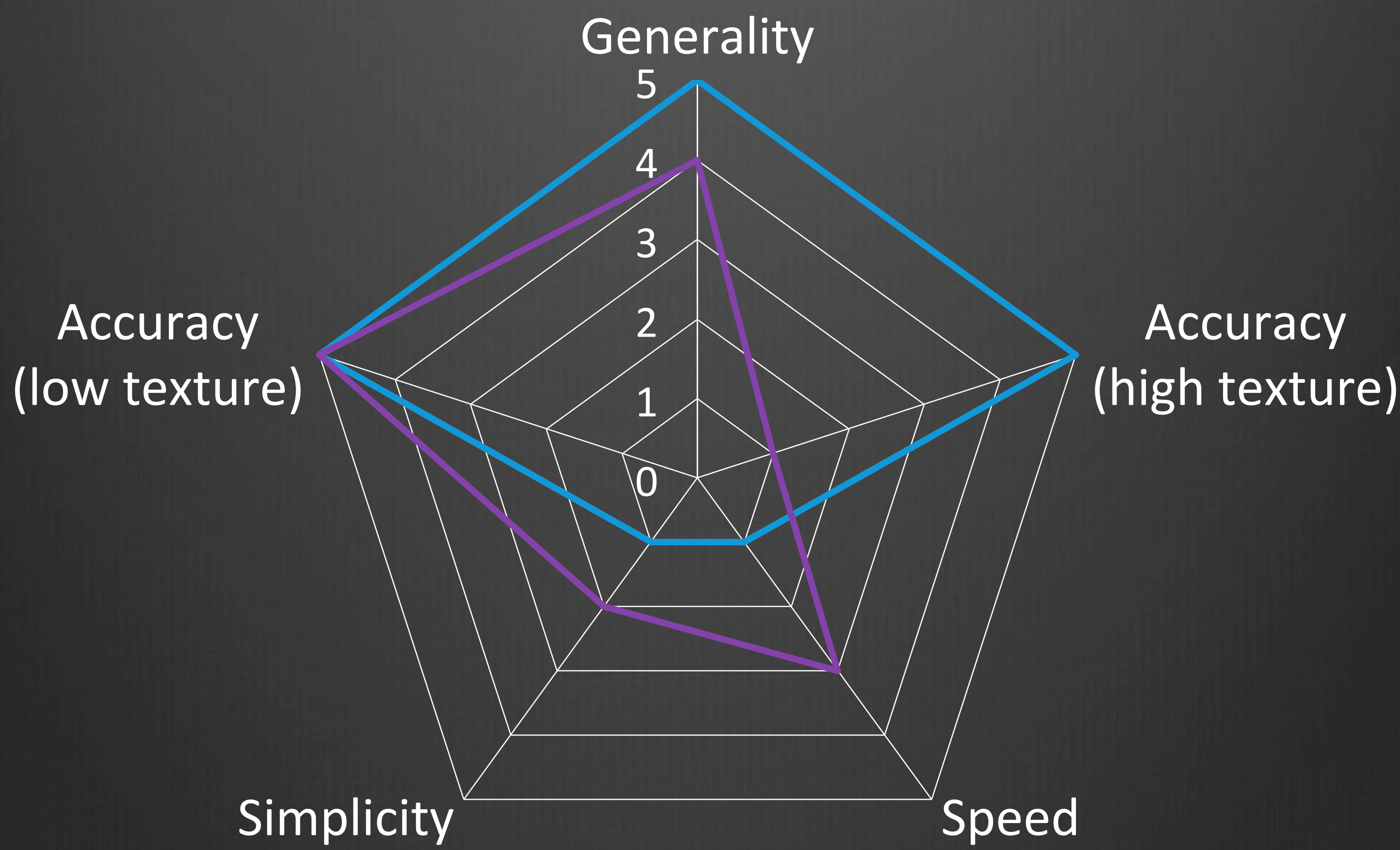
Fixed-Point covariance matrix estimator

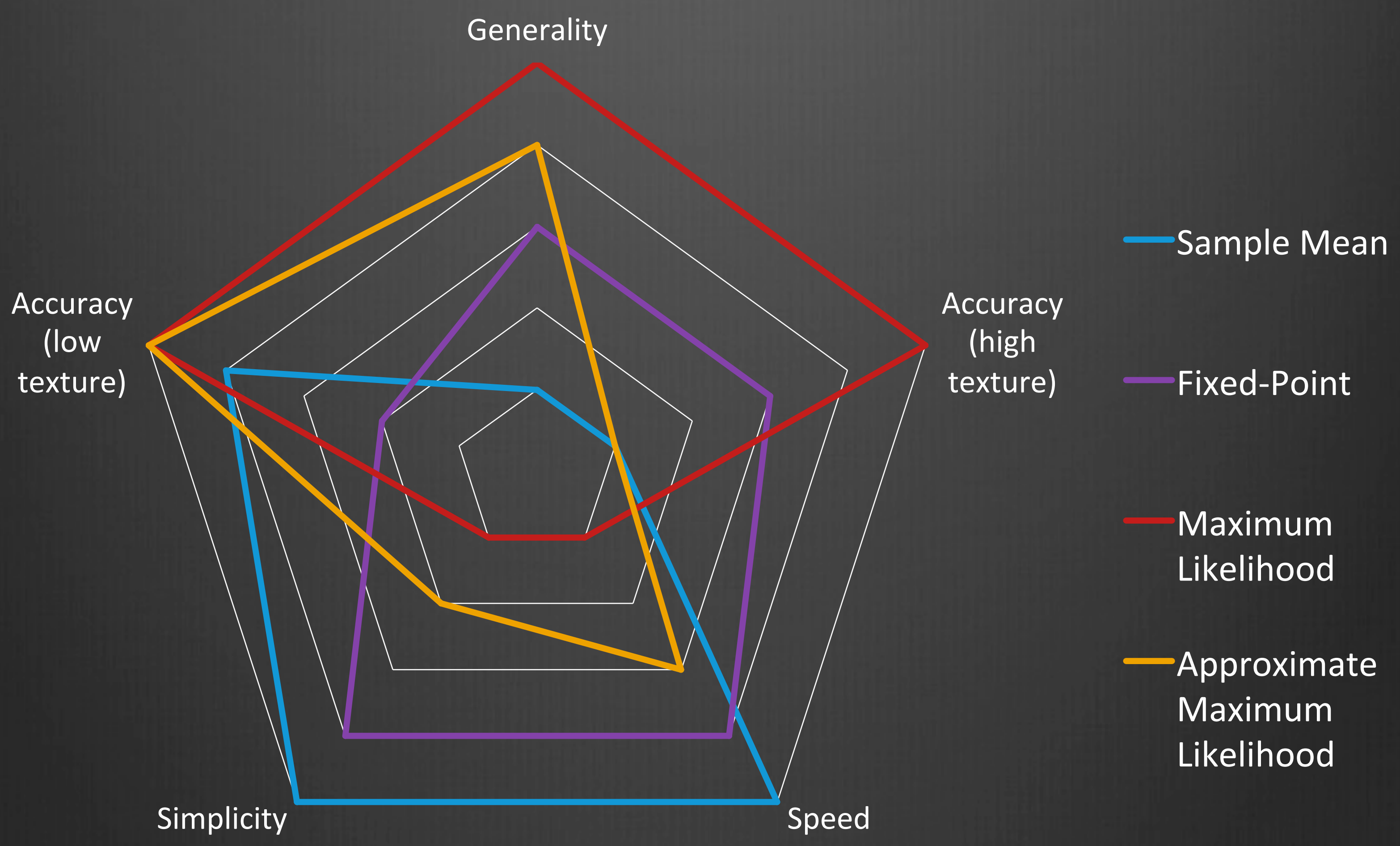


Maximum Likelihood covariance matrix estimator



Approximate Maximum Likelihood covariance matrix estimator




Generality
Accuracy (high texture)
Speed
Simplicity
Accuracy (low texture)
Sample Mean
Fixed-Point
Maximum Likelihood
Approximate Maximum Likelihood

Thank you very much!

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Maximum Likelihood covariance matrix estimator

$$\hat{C}_{ML}(i+1) = \frac{1}{n}\sum_{k=1}^{n} \frac{h_{d+1}(\boldsymbol{s}_k^{\dagger}\hat{C}_{ML}^{-1}(i)\boldsymbol{s}_k)}{h_d(\boldsymbol{s}_k^{\dagger}\hat{C}_{ML}^{-1}(i)\boldsymbol{s}_k)} \cdot \boldsymbol{s}_k\boldsymbol{s}_k^{\dagger}$$

$$\hat{C}_{ML}(i+1) = \frac{1}{n}\sum_{k=1}^{n} c_d(\boldsymbol{s}_k^{\dagger}\hat{C}_{ML}^{-1}(i)\boldsymbol{s}_k) \cdot \boldsymbol{s}_k\boldsymbol{s}_k^{\dagger}$$