A methodology to use terrestrial gravity data sets for regional validation of GOCE products in Central Europe

Motivation / Introduction
The comparison/validation of GOCE products with existing gravity data sets is of topical interest.
- revised and newly-derived terrestrial gravity data sets form the basis for the regional validation of GOCE-level-II products in Central Europe
- methodologically the comparison/validation can be realized by means of terrestrial-only regional analytical gravity field models in frame of the so-called “Space-value problem” approach
- in absence of real GOCE data the approach is outlined using simulated data from the satellite-only EIGEN-5S spherical harmonic model

Data description
The terrestrial data sets used for regional validation of GOCE products in Central Europe are presented in figures 1 to 3. For this study we used:
- more than 530 000 gravity points in the area of investigation (46.2–56.0 N; 4.3 – 16.6 E), see fig. 3.
- about 900 GPS/leveling points in Germany and about 1400 GPS/leveling points in the surrounding area from the EU5-DA project (fig. 1).
- The gravity data base consists of data of quite different origin (terrestrial, airborne, marine, sea-bottom, altimetric-derived data, etc.). To reject suspicious points from the data base we performed:
  - subdivision of the data sets into nine subclasses according to information about the data quality
  - internal consistency checks of the terrestrial data sets by starting with the most reliable designed subclasses of absolute gravity measurements (fig. 2)
  - consistency check of the physical heights of gravity points against a 25 m digital terrain model.

Due to the lack of real GOCE data for this study we used the satellite-only spherical harmonic model EIGEN-5S of degree/order $L = 150$ to simulate GOCE data at the current GOCE altitude 254.9 km. Fig. 4 shows exemplary the gravity disturbances of the EIGEN-7S model.

In this study we apply the following regional analytical representations in frame of the SVP approach:
- spherical harmonic models (SH)
- Strakhov’s New Approximation (SNAP) (Strakhov et al., 1995)
- Single-layer integral representation (Slintax) (Strakhov et al., 2003)

Regardless of the used analytical representation the SVP approach leads to systems of linear equations (SLE) with full design matrices. Since the SLAE might become large (depending on the input data and number of unknowns) their rigorous solution comes to be demanding from the numerical point of view, requiring multi-processor computing systems.

Methodology
The determination of regional analytical models is methodologically based on posing a “Space-value problem” (SVP) instead of a “Boundary-value problem” (BVP).

SVP means, not to change the 3D positions of observed gravity functionals, i.e. not applying any kind of transformation, up- or downward continuation to a certain reference surface, etc.: just leave the values in 3D space as they are! This avoids implicit assumptions about the real gravity field in the prior data processing, i.e. about the gravity field one is going to determine. SVP directly accounts, that even “terrestrial” data might be spatially scattered over quite a range of altitudes (e.g. sea-bottom versus high mountains or airborne data). Moreover, such approach will be valuable when one is combining terrestrial and satellite data.

In this study we computed several terrestrial-only spherical harmonic models (TOM) Slintax, SNAP and SH up to degree/order $L = 200$.

The corresponding full systems of linear algebraic equations (SLE) were solved by the Successive Polynomial Multiplication method (SPM). In case of underdetermined system a minimum norm solution has been obtained. These TOMs have been used to “upward synthesize” various gravitational functionals (e.g. gravity disturbances gl or 2n derivatives as e.g. $\bar{\nabla}^2$) at GOCE altitude of 254.9 km, where they are compared with the “simulated” GOCE data provided by the EIGEN-7S model. Some results are given in fig. 5 to 7. The general pattern of functionals obtained from different regional representations are similar. Nevertheless, there remain certain differences between them, see Table I.

Conclusions – Outlook
The terrestrial-only regional analytical models derived within the presented methodology produce meaningful gravitational functionals at GOCE orbit altitudes. However, before they might be used for the validation of GOCE products in the orbit one has to clarify the reasons for slight differences in the synthesized gravitational functionals at GOCE orbit altitudes.

In future the developed analytical tools will be applied to regional gravity modeling combining directly terrestrial and GOCE gravity data.

Table I: Comparison of gravity disturbances in S3M0 points @ 254.9 km between various regional SH, SNAP, Slintax models and the satellite-only model EIGEN-5S (r.m.s., m)

<table>
<thead>
<tr>
<th>Method</th>
<th>$L = 200$</th>
<th>$L = 200$</th>
<th>$L = 200$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>-4.4</td>
<td>-0.9</td>
<td>2.7</td>
</tr>
<tr>
<td>RMS</td>
<td>1.7</td>
<td>2.9</td>
<td>1.1</td>
</tr>
<tr>
<td>Min. dec.</td>
<td>0.5</td>
<td>6.9</td>
<td>0.3</td>
</tr>
<tr>
<td>Max. dec.</td>
<td>4.0</td>
<td>6.6</td>
<td>5.8</td>
</tr>
</tbody>
</table>

Fig. 1. Area of investigation (red rectangle) with more than 530 000 gravity points in Europe acquired within the EU5-DA project (Kenyeres et al., 2010)

Fig. 2. Absolute gravity points in Germany (blue network 1996 (yellow points); F. GOCE-GRAVITI network 2008 (red dots) occupied by FG5 and/or AI0 gravimeters; D. Germany network 1996 (green points)

Fig. 3. Gravity point data in the BKG gravity data base (plotted data base only for the Czech Republic and Poland)

Fig. 4. Gravity disturbances @ GOCE altitude 254.9 km synthesized by the satellite-only EIGEN-5S model

Fig. 5. Gravity disturbances @ GOCE altitude 254.9 km computed from the regional terrestrial-only Slintax model (red = observed terrestrial data points); cf. also Table I.

Fig. 6. Difference of gravity disturbances @ GOCE altitude 254.9 km between the Slintax model (fig. 3) and the EIGEN-5S model (fig. 4).

Fig. 7. Difference of the 2nd derivative $\bar{\nabla}^2$ of the gravity disturbances @ GOCE altitude 254.9 km between the Slintax model and the EIGEN-5S model.