

# EXTERNAL GEOPHYSICAL VALIDATION AND CALIBRATION OF AN ORBITING GRAVITY GRADIOMETER

Ernst J.O. Schrama <sup>(1)</sup>

<sup>(1)</sup> *Delft University of Technology, Department of Geodesy,  
Thijsseweg 11, 2629 JA Delft, The Netherlands  
Email: [schrama@geo.tudelft.nl](mailto:schrama@geo.tudelft.nl)*

## ABSTRACT

In most satellite based Earth observation programs ground truth verification of a sensor is normally considered to be a part of the activities. In the case of GOCE, cf. [3], a similar task can be expected. The level 2 product of this mission is a spherical harmonic coefficient set of the Earth's gravity field. The solution will be obtained by combining the observation equations from precise orbit determination (POD) and satellite gravity gradiometry (SGG). In this paper the problem of in flight performance monitoring of the GOCE measurements is considered using a collinear track technique in analogy to a data reduction method employed in satellite altimetry. We discuss the results of a new algorithm designed for a sun-synchronous frozen repeat orbit and treat two extreme scenarios. For the worst case scenario our conclusion is that it is possible to monitor the stability of the instrument within the measurement band to around  $10^{-3}$  E/sqrt(Hz) whereas our best case scenario shows that a factor 10 improvement is feasible. The temporal gravity effect caused by atmosphere and tides is considered separately; its magnitude, relative accuracy, spectral characteristics and relevance are discussed.

## INTRODUCTION

The problem of in-orbit calibration and validation of the gravity gradiometer proposed for the GOCE mission, cf. [3], is the main motivation for writing this paper. The GOCE gradiometer relies on observing differential accelerations that are obtained from 6 electrostatically suspended proofmasses while it flies in a low altitude dawn-dusk orbit. The goal is to realize an Earth pointing gradiometer with an accuracy of about 0.003 E/sqrt(Hz) in the measurement band between 0.005 and 0.1 Hz; this performance level holds for the twice radial component  $\Gamma_{zz}$ . The full gravity gradient tensor will not be observed by GOCE, ie. some components perform better than others, details can be found in [3]. The performance of the GOCE gradiometer depends on several factors, important ones are in our opinion: 1) common mode rejection of the gradiometer 2) decoupling of rotational accelerations, 3) pointing accuracy of the instrument and 4) the noise level of the accelerometers. At the moment of writing experience is gained with the CHAMP mission which carries prototype accelerometers built by the ONERA company, see also [1].

## CALIBRATION AND VALIDATION

Normally calibration is the process where the output of an instrument is translated into a reproducible quantity that only depends on known physical constants. Calibration is usually an experiment in the laboratory under well reproducible conditions. In principle validation could rely on a similar procedure except that the process is repeated in an operational environment usually with the intend to verify whether the instrument is still performing as specified. During calibration and validation it is not uncommon to rely on well defined external source signals.

This generic tutorial in metrology applies to some extent to gravimeters and gradiometers. From experience we know that it is impractical to find a gravitational source in the neighborhood of an instrument that can serve as an accurate reference. Other methods are therefor used during calibration and validation. If the procedure in gravimetric surveying is taken as an example then point of reference is set of absolute gravimetric stations where gravity is accurately measured. A level of 15  $\mu\text{Gal}/\sqrt{\text{Hz}}$  can be obtained, ie. 0.1  $\mu\text{Gal}$  is feasible after 6.25 hours, with the FG5 instrument, cf. [5]. The remainder of the network is then determined by relative gravimeters whereby drift and offset constants are reduced by internal looping methods and differencing to the absolute network. The FG5 absolute gravimeter surveys are based on an interference range measurement to a falling proofmass. The measurement of vertical acceleration thus depends on the definition of length and time. The estimated gravitational acceleration also partly depends on a priori knowledge of the vertical gravity gradient at the point of observation and the quality of geophysical corrections for instance to eliminate the gravitational effect of variations in the ground water level and Earth rotation.

In the case of gradiometry on GOCE the required differential accelerations are obtained from the feedback voltages that position the six electrostatically suspended proofmasses. In [3] it is mentioned that one needs to line up the sensitive axes of all accelerometers and to match the scale factors within the measurement band. Part of the calibration task will be initiated on ground and relies on pendulum bench tests. The remaining part will be repeated during the flight and deals with the interaction of the gradiometer, the spacecraft attitude and control system, the drag free control system and POD. Outcome after calibration will be an assessment of all parameters that translate the proofmass feedback voltages into gravity gradient observations. Scientific users will finally obtain data that is freed from rotational and pointing effects and optimized for common mode rejection along all axes.

The proposed calibration methods in [3] are only performed in certain time windows. Under ideal conditions we strive toward a method that enables us to continuously monitor the gradiometer performance while in flight. This concept motivates to study a simulation experiment whereby gradiometer measurements are repeated along the same ground tracks when viewed in an Earth fixed coordinate system. We use the property that a perfect gradiometer would reproduce the same tensor components when revisiting the same geographical location in the same orientation. This technique is referred to as a collinear track method in analogy to satellite altimetry whereby height measurements are differenced between the same ascending or descending ground tracks yet advanced by one repeat cycle. Similar methods are nowadays routinely applied for ERS-1/2 and TOPEX/Poseidon to monitor the relative altimeter bias in time. The applicability of the same method for GOCE is now a subject of discussion. In the following several aspects of this method are described: it deals with the generation of the simulation file and properties of the trajectory to fly, the results of our method are discussed and geophysical signal contamination is mentioned.

## SETUP OF THE EXPERIMENT

### Orbit characteristics

In [3] the basic outline of the mission is shown. It mentions that GOCE will fly in a sun-synchronous trajectory whereby the ascending node is chosen such that the orbital plane is facing the Sun. GOCE will go through eclipsing periods, furthermore it is expected that the drag-free control system on GOCE will result in an trajectory that is free from skin forces such as caused by the atmospheric drag and solar radiation. Whether the drag-free control system only relies on the observed accelerations or that feedback from POD is required is still an open question.

For the simulation experiment it is assumed that we deal with a sun-synchronous frozen repeat trajectory. The repeat length to fly should be sufficiently long to avoid undersampling of the gravity field. This is possible by a minimal repeat length counted in orbital revolutions ( $N_r$ ) which is at least greater than twice times  $l_{max}$  of the gravity field to observe. The corresponding number of nodal days ( $N_d$ ) should be unambiguous, ie.  $N_r$  and  $N_d$  are integers not sharing a common prime factor. For our experiment we have adopted  $N_r = 495$  and  $N_d = -31$  allowing an unambiguous recovery to spherical harmonic degree and order 240 which is the maximum to expect for GOCE.

The class of orbits that fulfills the sun-synchronous condition and the repeat condition constrain the nominal values of inclination and semi-major axis. We ignore the statement that frozen orbits are discarded in [3] and propose such an trajectory for a number of reasons to be discussed later on. According to [8] a frozen trajectory implies that the mean orbital eccentricity ( $e$ ) is fixed at a value of  $C/k$  while the mean argument of perigee ( $\omega$ ) is fixed at 90 degrees. In this case  $C$  is a function that depends on the odd zonal coefficients of the gravity field in question whereas  $k$  corresponds to the mean rate of change of the argument of perigee due to  $J_2$  drift perturbations.

The numerical orbit determination software that was used to generate the state vectors for the simulation experiment uses the EGM96 gravity model described in [4] with spherical harmonic coefficients up to degree and order 70. Tidal accelerations were considered in the process, atmospheric drag and solar radiation were not taken into account because of the drag-free control system on GOCE. The initial state vector was obtained by a combination of analytical predictions and a numerical iteration scheme. The sun-synchronous repeat condition was verified by evaluating the longitudes at the equator transitions of the ascending ground tracks. The frozen condition was obtained by inspection of the daily mean non-singular orbital elements  $u = e \cos(\omega)$  and  $v = e \sin(\omega)$ .

The obtained trajectory has the property that orbit height variations are minimized. These variations are of the order of 10 km and are caused by the combined effect of gravitational flattening (the  $J_2$  term), the orbit eccentricity ( $e$ ) and the higher degree zonal terms ( $J_3$ ,  $J_5$  and onwards). Moreover there is a relation between orbit altitude and latitude. Within

the context of a calibration experiment these conditions are favorable compared to non-frozen orbits which in general cause larger height variations that vary in time, see also [3]. Figure 1 shows two cases where on the geocentric height variations against latitude are shown for frozen and non-frozen orbits.

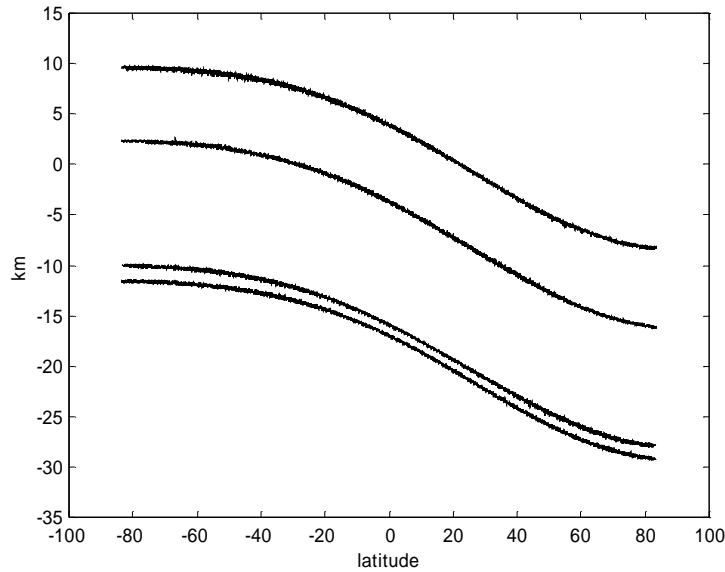


Figure 1: Relation between geocentric latitude and orbit height relative to 6678137 [m] (read: 300 km) for frozen and non frozen orbits. The lower two curves show the extreme altitudes of the frozen orbit used in the simulation experiment. The upper two curves show the same for an arbitrary non-frozen orbit.

Height variations of the orbit are a critical design parameter that needs to be controlled with care for an orbit at about 270 km altitude. If one would not have the availability of a drag-free control system then atmospheric drag would cause a natural decay of the semi-major axis of several kilometers per day depending on solar activity. The operational consequences with regard to the maintenance of this type of orbit is of no direct concern to us. Instead we assume that there is a part of the mission where it is possible to fly the specified trajectory. Maintenance of this trajectory will most likely require an interaction between navigation of the spacecraft and the drag-free control system.

For our experiment the simulated orbit file spans 719 orbital revolutions allowing to compare ground tracks that repeat in time. The optimization procedure to create the repeating ground track was not optimized in the sense that there is a perfect match between the equator transit longitudes at track number  $i$  and track number  $i+990$  which is mapped 495 revolutions later on the Earth's surface. We will use this property to demonstrate the consequence of non perfect track alignment in our method.

### Signal properties

The gradiometer tensor components are computed in Earth center fixed coordinates at 10 second intervals using the EGM96 reference gravity model, cf. [4]. The tensor is rotated to the local satellite frame assuming that the gradiometer instrument axes exactly line up with the radial direction ( $z$ ), and the perpendicular to orbit or cross track direction ( $y$ ). The instrument axis in the direction of flight ( $x$ ) is approximated in this procedure, ie. there is a residual pitch angle of a few tenth of a degree between the instrument axis and the true flight velocity vector resulting from orbit dynamics.

Statistics of the simulated tensor signal computed between time index 0 and 10000 seconds are shown in table 1. The mean constant value of the diagonal terms can be attributed to the  $\mu/r$  term of the gravity field. The large excursions relative to these mean values occur predominantly at once and twice per orbital revolution. They are on one hand due to orbital dynamics, on the other hand the direct result of the  $J_2$  effect itself which is about 1000 times larger than all other higher degree and order potential coefficients.

Table 1: Gravity gradient statistics in Eötvös units.

	$\Gamma_{zz}$	$\Gamma_{xz}$	$\Gamma_{yz}$	$\Gamma_{xx}$	$\Gamma_{xy}$	$\Gamma_{yy}$
Mean	2694.2	0.3	0.1	-1348.0	0.0	-1346.2
Min.	-20.4	-8.3	-2.3	-7.2	-1.0	-5.6
Max.	12.7	7.8	2.1	11.3	0.8	9.2
Std.Dev.	10.7	5.6	1.5	5.9	0.5	4.9

The above mentioned constant and once and twice per revolution signals have been considered as a calibration source for STEP, cf. [9]. The applicability of these signals for calibration of the GOCE gradiometer is limited since these signals occur in the  $1/f$  part of the gradiometer performance spectrum which flattens after 5 mHz or 27 cycles per revolution. Nevertheless we shall use both effects in the collinear track method described in the next section.

## COLLINEAR TRACK CALIBRATION

The method used in this study relies on differencing the observed and level 1 corrected tensor components  $\Gamma(P)$  and  $\Gamma(Q)$  where  $P$  and  $Q$  are points on the orbit that differ by one repeat cycle. Search and interpolation techniques are used to locate both points in the simulation file; the criterion is that a minimum distance is obtained. After both points have been located it is necessary to free the difference signal  $\Delta\Gamma(P,Q) = \Gamma(P) - \Gamma(Q)$  from displacement effects in the  $x$ ,  $y$  and  $z$  directions. In [6] we use the first-order terms in this process due to the central term of the Earth's gravity field. Due to the distances involved and the inherent truncation effects this approach is not sufficiently accurate to explain the gravity gradient displacement effect. A more exact approach is that  $\Delta\Gamma(P,Q)$  is corrected by assuming an existing geopotential coefficient solution to low degree and order. The simulated calibration signal therefore becomes

$$\Delta\Gamma(P,Q) = \Gamma(P) - \Gamma(Q) - \Gamma'(P) + \Gamma'(Q) \quad (1)$$

where in our simulation experiment  $\Gamma'(x)$  is a function complete till degree and order 2.

In the case where actual gradiometer measurements are available the proposed method would be equivalent to correcting the observed tensors  $\Gamma(P)$  and  $\Gamma(Q)$  with a given low degree reference model. In this case the left hand side of (1) should be nulled. Excursions will reveal the earlier mentioned effects such as noise due to instrumental effects which violate the common mode rejection of the gradiometer and systematic noise such as rotational effects and mispointing that were not adequately corrected for and noise introduced by the spacecraft such as self gravity and structural effects. In this case we will also see residuals due to our inability to create an ideal repeating trajectory and residuals due to temporal changes in the gravity field as a result of geophysical effects. The next section will deal with the latter point where we will assess the effect of known contributions from a geophysical origin. The remainder of this section will deal with "calibration noise effect" due to our inability to fly a perfect frozen repeat trajectory.

We investigate the spectral properties of  $\Delta\Gamma(P,Q)$  in (1) relative to the signal  $\Gamma(P)$ . The quality of the proposed method will depend on the magnitude of the residual distance  $d=|P-Q|$ ; ie. the smaller the distance  $d$  the better we will be able to compare previously observed tensor component to new observations. A worst and best case scenario obtained from the simulation file are shown in figure 2. In the left part of this figure we show the mean signal spectrum and the corresponding noise spectrum obtained by differencing track numbers  $10+i$  and  $1000+i$  for  $i$  running between 0 and 15. This case is a worst case scenario where the mean distance between  $P$  and  $Q$  in (1) varies between respectively  $-10$  km and  $10$  km in the  $y$  direction and  $-500$  to  $500$  meter in the  $z$  direction. In the along track direction  $x$  significantly smaller differences remain due to our search and interpolation method. Figure 2 shows that a signal to noise ratio around 100 can be achieved at both ends of the measurement band. The best case scenario is shown in the right part of figure 2. Here we show a similar analysis now based upon differencing track numbers  $209+i$  and  $1199+i$  for  $i$  running between 0 and 15. The distances between  $P$  and  $Q$  are now reduced because of a more favorable regime in the simulation file where the mean differences in the  $y$  and the  $z$  direction remain below approximately 500 meter. The right part of figure 2 shows that the calibration noise is almost reduced by a factor of 10 compared to worst case scenario. The conclusion is that an optimized frozen repeat trajectory will reduce the calibration noise to about  $10^{-4}/\sqrt{\text{Hz}}$  in the measurement band.

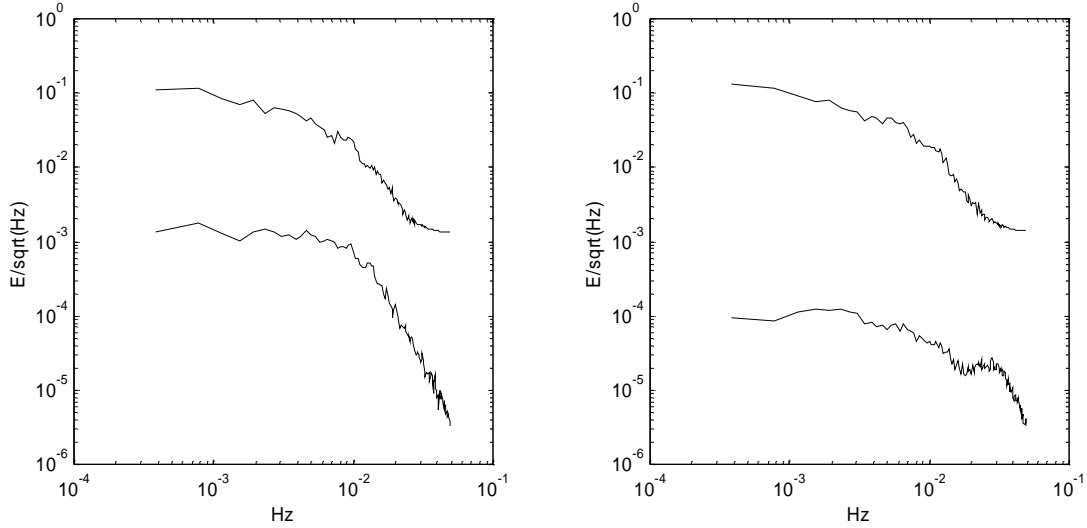


Figure 2: An example of two extreme scenarios where mean spectra of the  $\Gamma_{zz}$  gravity gradient tensor component including the corresponding values of  $\Delta\Gamma_{zz}$  in a collinear track set-up are shown.

## GEOPHYSICAL SIGNALS

Temporal variations in the Earth gravity field will add as calibration noise (whereas the temporal variations themselves are of course of scientific interest). The question is now to what level this occurs. There are several well known signals that must be modeled in the generation of the level 1 product. In [2] we showed that all direct and most indirect tide effects can be reduced under the 0.001 Eötvös level. In [7] the temporal gravitational effects relevant to the GRACE mission are summarized. Known examples are the gravitational effect as a result of atmospheric pressure variations which require the presence of an atmospheric pressure model. The signal spectrum as a result of air pressure variations in the spherical harmonic degree representation is shown in figure 3. Other examples are gravitational effects caused by continental water storage changes and temporal gravity effects as a result of mass changes in the oceans.

## CONCLUSIONS

The discussion started with a tutorial from metrology with regard to calibration and validation of instruments. The case of relative stability monitoring is discussed and an algorithm is presented that relies on the least possible assumptions with regard to a priori gravity information. We explore the possibilities to assess the in-flight performance of the GOCE gradiometer system using the technique of collinear differencing. The success of this algorithm depends on the ability to fly in the neighborhood of trajectories that are flown  $N_r$  revolutions prior to the current cycle. The case we studied is based upon a sun-synchronous frozen repeat orbit whereby the orbital plane is facing the sun. The algorithm itself only requires knowledge up to degree and order 2 of the gravity field. To actually implement this method a decision must be made whether it is possible to overfly previous trajectories to within 500 m in order to be able to repeatedly observe the same signal to within  $10^{-4}$  E/sqrt(Hz). If we relax this condition, ie. fly not-exactly repeating trajectories, then the calibration noise goes up by almost a factor of 10.

Temporal gravitational effects as a result of geophysical signals do directly follow from the collinear differences. A possible advantage of the proposed method could be that the observed temporal variations are independent from a data reduction method requiring a full inversion of the gravity normal matrix. We completely rely on the quality of independent methods to correct for instrument noise, systematic effects and spacecraft noise. The alternative for observing temporal gravity variations will be to focus on the POD experiment which will complement for most part the lower degree and order part of the spherical harmonic spectrum.

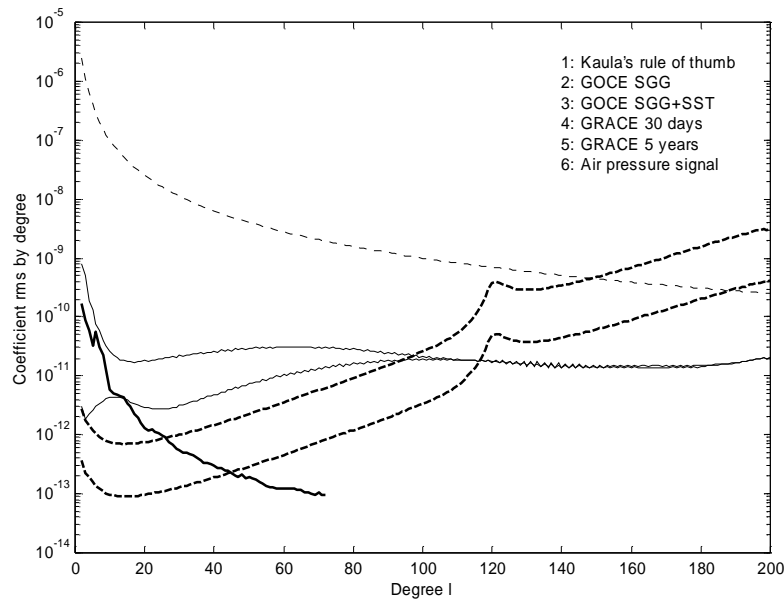


Figure 3: Performance curves of the GOCE and GRACE gravity missions overlaid on the gravitational signal that follows from mean air pressure variations. The reference numbers in the legend refer to vertical positions of curves: the thin dashed line is Kaula's rule of thumb, both thin solid lines reflect the GOCE performance curve, both thick dashed lines reflect the GRACE performance curves, the solid thick line is the air pressure signal.

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