COMPARATIVE ANALYSIS OF THE GOCE SIMULATED DATA.

Janusz B.Zielinski[1], **Jan Latka**[1], **Adam Lyszkowicz**[1],
Margarita S. Petrovskaya[2]

(1)*Space Research Centre, Polish Academy of Sciences,*
Ul.Bartycka 18A, 00-716 Warsaw, Poland
Email: jbz@cbk.waw.pl
(2) *Central Astronomical Observatory of Russian Academy of Sciences,*
Pulkovskoe Shosse 65, St.Petersburg, Russia
Email: petrovsk@gao.spb.ru

INTRODUCTION

The group from the Department of Planetary Geodesy of the Space Research Centre in Warsaw was involved for a long time in the satellite gradiometry problems. Publications by Krynski [13] and Latka [3] solved some important theoretical problems before any technical attempts have been made. In the beginning of 1980-ties the project DIDEX was proposed to the Intercosmos Program [14] based on the satellite to satellite tracking technique. In last years the effort was directed towards the Free Falling Gradiometer Project [14] and the development of the analytical expressions reflecting various aspects of the problem of gradiometry [9]. Therefore, we received the final decision of ESA about realisation of the GOCE project with great interest and we started immediately to work along the lines of GOCE design. We suppose that some interesting contribution may be presented from our side as a part of the joint European effort.

FREE FALLING GRADIOMETER PROJECT

In CNR in Frascati (Italy) the new instrument for gradiometry has been developed which can be applied in a range of missions. It consist of a cluster of accelerometers which connected to perpendicular axes yield the gravity gradient tensor components. An appropriate geometrical arrangement of accelerometers and baselines allows for the measurement of all elements of the gravity gradient tensor, in principle. The precision of the measurement is of the level of 0.01 EU (1Etvos Unit = 10^{-9}m s^{-2}/m). The first mission, which is proposed for this system, is the free fall drop from the stratospheric balloon. Such implementation was proposed by the Italian Space Agency ASI as a project called G-zero or GIZERO [2]. Essential for this project is a payload that allows achieving zero gravity with residual noises 10^{-13}g√Hz for about 25-30 seconds. These circumstances became an inspiration for the study by Zielinski and Lorezini [14], with the main task to consider the possibility of the application of the Frascati accelerometr system inside the GIZERO capsule to the Earth gravity field measurement. This project is called Free Falling Gradiometer.
GIZERO is composed of a capsule with a vacuum chamber and tracking, control and telemetry systems. The capsule is lifted to the altitude 40-45 km by the balloon. The time of ascent is about 3 hours. When the pre-programmed altitude is achieved the capsule is released and the whole probe starts its downfall. The same moment the payload inside the vacuum chamber is released from the locking device at the ceiling of the chamber. Because of the small difference in velocity of the capsule, which is moving in the atmosphere and of the payload, which is moving in the high vacuum, after 25-30 seconds the payload reaches the floor of the chamber and the experiment terminates. By the time the payload released inside the capsule reaches the capsule's floor, the capsule itself is fallen ca. 4.5 km.
Afterwards, the capsule is going to land. The payload is retained by an appropriate mechanism and the capsule recovery phase starts with the sequence of opening of two progressive bigger parachutes. It is expected that the capsule will splash down in the sea, than the rescue crew recovers it and it can be used for another mission.
Let us assume that the scientific payload consists of the three-axis gradiometer as described in [2] and assume that the gradiometer is free falling (e.g. inside GIZERO). We need an information about the attitude of the system and, in addition, about the position of the gradiometer in space referred to the Earth fixed coordinate system. To accomplish this, two additional systems are necessary:

1. The tracking system of the entire capsule by terrestrial stations;
2. The internal tracking of the payload position inside of the chamber.

Simulations of this system have been done with the software package FREEFALL and GGTENSOR.

ANALYTICAL METHOD OF THE GRAVITY FIELD DETERMINATION FROM GRADIOMETRY

The studies have been undertaken devoted to the development of the analytical theory for orbital missions. In a work by Petrovskaya [8] a new approach is developed for solving the spatial boundary value problem. One unique expression is derived for the improvement of the potential coefficients $\underline{C}_{nm}$, from the gravity gradient disturbance ΔΓ. This theory was later applied to the case of the free falling body along the vertical line.

In another work [9] the theoretical study has been conducted devoted to the derivation of simple explicit analytical formulae for evaluating the Earth's potential coefficients. The new approach, instead of a modified orthogonal basis, the concept of modified observable is developed, with the standard basis of the solid spherical functions. A boundary value equation is derived which represents a linear relation between the unknown potential coefficients $\underline{C}_{nm}$ and a slightly modified gradio observable. This relation is used for deriving very simple quadrature formulae that determine $\underline{C}_{nm}$ successively and uniquely up to arbitrary degree and order.

Initially the general problem of the gravity field determination from the gradiometry measurement have been considered and some generalized approach was applied for the solution of the spatial boundary value problem with gradiometry data [10] looking for the solution in terms of the potential coefficients $\underline{C}_{nm}$. Later on, as the boundary values to be determined the gravity anomaly Δg is accepted as well, because it is the most widely used parameter in the local gravity field presentation and is applicable as well for global modeling. As an observable there is a gravity gradient tensor used, splinted in two parts:

$$\Gamma = \Gamma_0 + \Delta\Gamma \tag{1}$$

where Γ_0 refers to the normal field and ΔΓ is the anomalous part.

Different options concerning the number and the kind of components are considered.

The first case is discussed for the full gravity gradient tensor observed. The method implies that the approximate field is known and we are looking for some correction parameters to the normal model [10]. Correspondingly, the unique spatial boundary value relation is derived

$$\Delta\Gamma = \Delta\tilde{\Gamma}_0^{(N)} \tag{2}$$

which is the truncated to the degree N spherical harmonics series with the unknown global potential coefficients $\underline{C}nm$. Basing on this relation the potential coefficients can be derived by the least square algorithm. Proceeding from the expression for ΔΓ in terms of spherical harmonics, the same quantity is presented as a linear combination of the partial derivatives of the disturbing potential T with respect to the geodetic coordinates $\{r, \varphi, \lambda\}$ and then the corresponding local Cartesian reference set $\{x,y,z\}$ directly connected with the trajectory of the free falling chamber containing the gradiometer. The above mentioned relation can be used for the point-wise testing of a global geopotential model. Moreover, an explicit integral formula is derived for $\underline{C}nm$:

$$\underline{C}_{nm} = \frac{r^3}{4\pi\mu(n+1)(n+2)}\left(\frac{r}{\mathrm{R}}\right)^n \int_\sigma \Delta\tilde{\Gamma}_0^{(N)}\overline{Y}_{nm}(\theta,\lambda)\,d\sigma \tag{3}$$

As a definite solution, the expression is derived allowing to evaluate the regional surface gravity anomaly Δg. For solving this problem another basic relation is derived by means of downward continuation of ΔΓ from a spatial observation domain to its projection onto a terrain region of interest. The formula (4) represents Δg in form of a sum of a Stokes integral, depending on ΔΓ, and correction terms due to the downward continuation and the Earth's non-sphericity

$$\Delta g = \frac{a}{4\pi}\int_\sigma \frac{\Delta\Gamma^*}{\sin\psi/2}\,d\sigma + a\left(\frac{e^2}{2}\cos^2\theta - \frac{\mathrm{H}}{a} + \frac{h_2 + h_1 - 2h_0}{2a}\right)\Delta\Gamma^* \tag{4}$$

This expression allows to recover the regional surface gravity anomaly Δg on the basis of spatial gradiometry data. The principal part of this formula is a reduced Stokes integral over a segment σ_0 (a spherical cap or a spherical rectangle) of the unit sphere of integration, upon which the domain of gradiometry observations is projected. The integral depends on the observable ΔΓ minus the truncated expansion $\Delta\Gamma^{(N)}$ induced by global geopotential model with coefficients $\underline{C}nm$.

In (4) a mean value is applied after averaging ΔΓ over observation altitudes of the free falling gradiometer

$$\Delta\Gamma^* = \frac{1}{h_2 - h_1}\int_{h_1}^{h_2} \Delta\Gamma\,dh \tag{5}$$

The third basic relation is derived between the observable ΔΓ at the height h and the quantity Δg derived by means of upward continuation of the surface gravity anomaly to the altitude h

$$\Delta\Gamma = -\frac{a^2}{4\pi r^2}\int_{\sigma_0} \Delta g K(r',\theta',\lambda')\,d\sigma + \frac{6}{r}T(r,\theta,\lambda) \tag{6}$$

where

$$K(r',\theta',\lambda') = \frac{2R^2}{l^3} - \frac{5r(r-R)}{l^3} - \frac{6R(r-R)^2}{l^5} \tag{7}$$

The principle term in this expression is the radial derivative of the Poisson integral, depending on Δg. Due to rapid decreasing of the kernel function, the domain of integration can be reduced to the local area and this formula can be very efficiently applied in the free falling gradiometer problem [10]**.**

Another work concerns the case when only diagonal components of the gravity gradient tensor Γxx, Γyy, Γzz are measured [11]. The theory was extended also to the situation when the measurements are done on board of the drifting balloon or even an airplane what is typical for airborne gradiometry. Several forms of the solution have been derived, both by downward and upward continuations. The first solution, given by formula (8), refers to the Earth's surface. It has a form of a Stokes-type integral, depending on the observable $\Delta\Gamma$:

$$\Delta g = \frac{a}{4\pi}\int_{\sigma}\frac{\Delta\Gamma}{\sin \psi/2}\, d\sigma + (h-H)\Delta\Gamma \tag{8}$$

where h is the mean observation altitude and H is the terrain height, and

$$\Delta\Gamma = (2T_{zz} - T_{xx} - T_{yy})/3 \tag{9}$$

Two modifications of this solution are proposed, by formulas (10), (11) and (12), (13). They contain a correction term for its downward continuation to the Earth surface, taking into account the Earth ellipsoidal shape and surface topography.

$$\Delta g = \frac{a}{4\pi}\int_{\sigma_0}\frac{\Delta\Gamma}{\sin \psi/2}\, d\sigma + (h-H)\Delta\Gamma + \delta g^{(N)} \tag{10}$$

$$\delta g^{(N)} = \frac{\mu}{a^2}\sum_{n=2}^{N}\sum_{m=-n}^{n}(n-1)c_n(\psi_0)\left(\frac{a}{r}\right)^2 C_{nm}Y_{nm}(\theta,\lambda) \tag{11}$$

$$\Delta g = \frac{a}{4\pi}\int_{\sigma_0}\frac{\Delta\Gamma - \Delta\Gamma^{(N)}}{\sin \psi/2}\, d\sigma + \left(\frac{ae^2\cos^2\theta - H}{2}\right)\Delta\Gamma + \tilde{\Delta} g^{(N)} \tag{12}$$

$$\tilde{\delta} g^{(N)} = \frac{\mu}{a^2}\sum_{n=2}^{N}\sum_{m=-n}^{n}(n-1)C_{n,m}Y_{n,m}(\theta,\lambda); \tag{13}$$

The spatial boundary value equation (14) has been derived referring to the mean altitude of gradiometry measurements during a balloon mission. By restricting to the inner zones in the integrals in (14) one can obtain the numerical relation between the gradiometry data and the surface anomaly. Such relation can be used for testing either the space ($\Delta\Gamma$) or the surface (Δg) data, depending on the problem under consideration.

It can be noted that the diagonal components, which are used as observables, contain more information of the field than skew components. Their application allows easily summarizing the infinite series and deriving the solution for the gravity anomaly in a closed form.

$$\int_{\sigma}\frac{\Delta\Gamma}{\sin \psi/2}\, d\sigma = \int_{\sigma}\left[\Delta g - (h_0 - H)\Delta\Gamma\right]\frac{(r^2 - R^2)}{l^3}\, d\sigma \tag{14}$$

The derived solutions can be further refined by taking into account some correction terms of higher order with the aid of iteration procedures.

One can observe that the expressions obtained are characteristic for physical geodesy. They do not demand new numerical procedures and can be treated by conventional numerical techniques applied in geodesy. By a similar approach the solutions can be found for the geoid undulation N and vertical deflections. It is only necessary that the observations will cover some area above the chosen terrain region.

In the third work by Petrovskaya and Zieliński [12] the analysis is continued for determination of the height anomaly and gravity anomaly but from the airborne missions, allowing balloon as well as aircraft gradient measurements. This approach is focused on more practical application of the gradiometry, going beyond the Free Falling Gradiometer scheme, but this is a logic extension of the previously developed theory.

As before, the observable $\Delta\Gamma$ is applied, which can be considered as the anomaly of the full gravity gradient tensor or the diagonal components only. Completely new results are derived in form of expressions:

For the height anomaly

$$\zeta = \frac{a^2}{4\pi\gamma}\int_\sigma \Delta\Gamma\, K_\zeta(\psi)\, d\sigma + \frac{a(h-\mathrm{H})}{4\pi\gamma}\int_\sigma \frac{\Delta\Gamma}{\sin \psi/2}\, d\sigma \qquad (15)$$

where the kernel function is

$$K_\zeta(\psi) = (1 - 3\cos\psi)\ln\frac{\sin \psi/2}{1+\sin \psi/2} - 3 + 6\sin \psi/2 \qquad (16)$$

For the gravity anomaly the solution is

$$\Delta g = \frac{a}{4\pi}\int_\sigma \Delta\Gamma\; K_{\Delta g}(\psi)\, d\sigma + (h - \mathrm{H})\,\Delta\Gamma \qquad (17)$$

where

$$K_{\Delta g}(\psi) = \frac{1}{\sin \psi/2} + (9\cos\psi - 2)\ln\frac{\sin \psi/2}{1+\sin \psi/2} + 9 - 18\sin \psi/2 \qquad (18)$$

NUMERICAL SIMULATIONS

The results presented above inspired authors to submit the proposal for participation in the preparation of the GOCE project. We intend to use our analytical theory as well as the numerical procedures to the solution basing on the simulated data. It is assumed that these data can be obtained from the SID consortium, the group of German and Dutch institutes working on GOCE.

The theory developed above based on the geodetic approach to the Earth gravity field description was suitable for setting up the software simulating the gravitational effects with the required precision. However this numerical modeling is restricted, it does not take into account all forces which appear during the flight, like atmospheric drag, magnetic forces etc.

Two separate programs have been prepared:

1. Program **FREEFALL** calculates the trajectory of the free falling body in the assumed gravity field for the initial position vector and velocity vector. Program accepts a 40×40 gravity field model. Runge-Kutta numerical integration procedure is applied, no atmospheric drag is taken into account. The program gives position and velocity of the falling body every *n* second.

The input of the program is as following: geographic coordinates and the altitude of the initial point of the free fall as well as the direction and magnitude of the initial velocity and in addition coordinates of the point of observation.

Actually, **FREEFALL** prepares input data for the next calculations with **GGTENSOR** program.

2. Program **GGTENSOR** calculates six components of the second gradient of the perturbing potential **T**, for positions calculated by the program **FREEFALL.** Potential **T** is calculated with the help of the Stokes formula and gravity anomalies, which are given as follows:

a/ inner circle, integration with the anomaly block size 5’×5’;

b/ middle circle, anomaly block size 15’×15’;

c/ external square, anomaly block size $1^0\times1^0$;

Gravity anomalies can be taken from the International Gravity Bureau data base or from other sources. Center of the circles is always at the sub-satellite point, the radius of each circle can be defined optionally.

With the above software package calculations have been performed for simulation of the FFG project. Simulations were made for the ASI stratospheric balloon range Trapani-Milo, Sicilly,($\varphi=38^0 02'05''15$, $\lambda=12^0 37'19''5$). Trajectory of the probe was calculated with different initial conditions: height of the drop and initial velocity depending on the wind speed. As the gravity field model the GRIM3 was used, truncated to 12×12 degree and order. Along this trajectory six gravity gradients were calculated with the anomalous gravity field modeled according to the International Gravity Bureau data base containing $1^0\times1^0$ gravity anomalies.

The similar procedure can be used for the GOCE satellite. The program **GGTENSOR** may be useful for calculation of the O–C values in observation equations when we use the approximate gravity field model – but no anomalies -which we want to improve with the gradio measurements.

INFLUENCE OF THE DATA DISTRIBUTION

The numerical experiment, on arbitrary simulated gravity gradients at orbital height [3], using least square collocation, gave interesting result confirming the possibility of accurate determination of gravity anomaly at the Earth surface. We would like to continue this investigations using more realistic though still simulated data.

We are planning a numerical test using the simulated GOCE data in a regional frame. Our aim is to investigate the influence of data configuration and its spatial distribution on the accuracy of gravity anomaly determination at the Earth surface. An other task it is the determination of the region optimal from point of view of feasibility taking in consideration processing time and hardware limitations. Finally it would be interesting to estimate the possibility of joining the results from neighboring regions.

In the case of GOCE the order of magnitude of the components of the gradient tensor at satellite altitude is Γzz = 2800EU, $\Gamma xx = \Gamma_{yy}$ = 1400 EU, Γxz = 10 EU, $\Gamma_{yz} = \Gamma_{xy} = 0$. The simulated components of the gradient tensor Γxx, Γ_{yy}, Γ_{zz} presented in a local frame x, y, z with x-axis pointing to the east y-axis pointing to the north and z-axis pointing radial outwards can be readily transformed to the frame of spherical co-ordinates. It will facilitate the use of the spherical harmonics developments of gravity models.

$$T_{xx} = \frac{1}{r^2} T_{FF} + \frac{1}{r} T_r \; ; \qquad T_{zz} = T_{rr}$$

$$T_{yy} = \frac{1}{r^2 \cos F} T_{LL} - \frac{\operatorname{tg} F}{r^2} T_F + \frac{1}{r} T_r \tag{19}$$

$$T_r = \frac{\partial T}{\partial r} \; , \; T_{rr} = \frac{\partial^2 T}{\partial r^2} \qquad \text{- for radial components}$$

Analogously $T_F \;\; T_{FF} \;\; T_L \;\; T_{LL}$ for latitudinal and longitudinal components respectively.

One can easily represent above values in spherical harmonics development. For instance the radial component has following form:

$$T_{zz} = \frac{1}{r}\left(\frac{R}{r}\right)^{n+1} (n+1)(n+2)\frac{GM}{R}\sum_{m=0}^{n} \overline{P}_{nm}(\cos\theta)[\overline{C}_{nm}\cos m\lambda + \overline{S}_{nm}\sin m\lambda] \tag{20}$$

For our investigations the method of least squares collocation will be used additionally for comparisons a numerical integration should be used. The covariance function K can be determined on the basis of global gravity field models

$$K(\psi) = \sum_{n=0} k_n P_n(\cos\psi) \, , \qquad k_n = \sum_{n=0}^{n} (\overline{a}_{nm}^2 + \overline{b}_{nm}^2) \tag{21}$$

P_n - Legendre polynomial, ψ - spherical distance, a, b - fully normalized coefficients of spherical development.

Eventually certain empirical models for given region ought to be taken in consideration. The law of covariance propagation gives us possibility to merge different elements of gravity field such as gradients, gravity anomaly, terrestrial gravimetric data and so on.

GEOID RECOVERY

The first gravimetric geoid for the territory of Poland computed by the collocation-integral method [10] comprised about 6000 mean free-air gravity anomalies [9]. In such geoid determination, a number of error sources limits the accuracy of the predicted geoid undulations. The most significant of these errors are due to the noisy data used in the conventional geoid estimation: the geopotential model (GM), local gravity anomalies Δg and the heights. In [10] some simple approaches for minimising truncation error for geoid height due to application of modified Stokes' integral have been considered.
Therefore, it becomes apparent that the refinement of geoid determination methods can be achieved through analysis of the properties of gravity field signals and errors provided by covariance functions, as well as through the use of optimal data combination techniques. Two methods will be used to extract important properties of gravity field signals and errors. Both the space domain method and frequency domain methods, which utilise the fast Fourier transform (FFT) algorithm. The two methods have their advantages and disadvantages. In recent times, the spectral (frequency domain) technique has been more popular due to its computational ease. The space domain method was used to estimate empirical covariance functions of gravity anomalies for selected areas in Poland [8]. The resulting empirical covariance functions were then modelled by modifying global covariance models.
Spectral techniques provide excellent means of extracting gravity field information contained in each of the gravity field data with the view of determining the contribution to the gravity spectrum of each data type. Analysis of the gravity field spectrum should involve critical examination of the spectral information from each data set and from a combination of geopotential model (GM), local gravity data and heights, which respectively provide long, medium and short wavelengths information of the gravity field. Such analysis would provide the necessary gravity field signal and error covariance or power spectral density (PSD) functions required for geoid prediction techniques. In addition, estimates of data sampling density derived from degree variances of the gravity signal would give a better picture of the data requirement for geoid estimation with sub-decimetre accuracy.
Both the space domain and frequency domain methods will be critically examined to determine which method will be better in estimating the covariance functions for a local area. It will also be necessary to know how the two methods could be combined to achieve the best results in extracting spectral properties of the gravity field signals. In addition, estimated PSD functions and degree variances should be thoroughly exploited in various spectral bands to determine the geoid power contribution in various spectral bands, as well as to estimate the data resolution required for centimetre to decimetre level of geoid accuracy.
Investigation of the contribution to geoid undulations of the GM coefficients and errors using the simulated coefficient errors estimates for GOCE, in combination with local gravity in Stokes's integral, would provide the means of optimally combining the two data types. Consequently, the effects of truncating the degree of the spherical harmonic expansion of the GM in favour of using larger cap size to evaluate Stokes's integral need to be examined. In similar studies, an increased area of integration would show improvement of the results for geoid estimates. The limit to which the cap size should be increased should be investigated in order to derive an optimal cap size for Stokes's integral solution.
Critical analysis of the signal information from the short wavelength part of the gravity field spectrum in which the topographic terrain corrections have a dominant role is required in order for the centimetre geoid to be achievable. Li and Sideris established that the discrepancy between the gravimetric geoid and GPS/levelling derived geoid is correlated with the roughness of the topography. Therefore, rigorous modelling of the effect of topography and density variations especially in the mountainous areas would provide information on the recoverable geoid power in the very high spectral band.
Proper description of the behaviour of data source errors are provided by suitable covariance function models. While the error covariance function from the GM can be easily derived from the error degree variances of the coefficients, empirical error covariance models have to be derived for the gravity and height data that will represent the actual error behaviour for the local area. Barlik [2], Łyszkowicz [8] documented covariance models of gravity for area of Poland. In order to derive error models for gravity and heights that will actually represent the local area, models for areas with different topography should be derived separately, and the overall geoid error from the data errors should be estimated and modelled. In addition, a thorough analysis of errors from each of the data sources and combination of the data would provide a good picture of internal geoid accuracy.
Therefore, it can be expected that spectral analysis of the gravity field data can provide the information required for refining geoid estimation methods in order to obtain geoid with a cm level of accuracy in a local area. Information contained in the spectrum of different data types would provide the geoid rms power from each of the data types and

their combination. Such information will be useful in determining the optimal procedures for combining the different data for the geoid prediction methods.

CONCLUSIONS

The theory developed in [7] and [9] provides the proof that the regional solution for the gravity field mapping from the satellite gradiometry is possible. Advantage of such a solution can be seen when the problem arise to fill territorial gaps, poorly covered by the terrestrial measurements. A good example of such a territory is the Mediterranean Sea. This is an extremely important place from the point of view of plate tectonic theory and the earthquake prediction research but has no uniform and dense gravimetry coverage. Application of the satellite method combined with balloon- or airplane-borne gradiometry measurement of this region as well as terrain topography and gravity anomalies could solve the problem in the very effective way.

There are several problems to address:

- ♦ The choice of numerical procedures optimal from the point of view of economy of calculation and accuracy;
- ♦ The investigation of optimal data distribution and its spatial configuration, taking in consideration determined area of interest and required accuracy.
- ♦ The influence of data distribution on stability of the results.
- ♦ The investigation of the possibility of connecting the neighboring regions with sufficient accuracy. An integration of neighboring regions could be an alternative for global treatment of data with enormous amount of data.

Authors propose to use existing software and experience to analyze the simulated GOCE data in order to derive regional solutions and to compare the results with solutions by other groups.

REFERENCES

[1] Barlik M., **Funkcje kowariancji niektorych elementow pola sily ciezkości na obszarze Polski,** *Geodezja i Kartografia*, vol. XXXV, z. 1-2, 1986.

[2] Iafolla V, E.Lorenzini, V.Miliukov, S.Nozolli **GIZERO: New Facility for Gravitational Experiments in Free Fall** *Gravitation and Cosmology,* Vol.3, pp.1-10, 1997

[3] Latka J.K. **The Use of Satellite Gradiometry for Determination of Gravity Anomaly.** *Beitrage zur Astr.Geodesie unf Geodynamik,* Inst. Fuir Physikalische Geodesie, TH Darmstadt, 1978.

[4] E.C.Lorenzini, F.Fuligni, J.B.Zielinski, M.L.Cosmo, M.D.Grossi, V.Iafolla, T Rothman **Baloon-Released Gravitation Experiments in Free Fall.** *Advances of Space Research,* Vol.14, No.2, pp.113-118, 1994

[5] Lyszkowicz A., **Estimation of the Local Covariance Function Using 12'x 20' Mean Free Air Gravity Anomalies for Poland Region**, *Artificial Satellites*, z.14, Warszawa, pp. 25-33, 1990.

[6] Lyszkowicz A., **Efforts Towards a Preliminary Gravimetric Geoid Computations in Poland Area**, *International Association of Geodesy Symposia, nr.106, Determination of the geoid - present and future, Spring-Verlag,* New York, Inc. pp.269-276, 1991

[7] Lyszkowicz A., **The Geoid for the Area of Poland,** *Artificial Satellites*, vol. 28, No 2, *Planetary Geodesy*, No 19, 1993

[8] Petrovskaya M.S. **Optimal Approach to the Investigations of the Earth's Gravitational Field by Means of Satellite Gradiometry** *Artificial Satellites,* Vol.31 Nr.1, pp 1-23,1996

[9] Petrovskaya,M.S., J.B.Zielinski **Determination of the Global and Regional Gravitational Fields from Satellite and Baloon Gradiometry Missions.** *Advances in Space Research,* Vol. 19,No.11 *„Orbit determination and Analysis"* ed.J.M.Dow, pp.1723-1728, 1997

[10] Petrovskaya M.S., J.B.Zielinski, **Evaluation of the Regional Gravity Anomaly from Baloon Gradiometry.** *Bolletino di Geodesia e Scienze Affini,* Vol.57,No.2, pp.141-164, 1998

[11] Petrovskaya M.S., J.B.Zielinski **Application of Spatial Gradiometry for Constructing Quasigeoid Model.** *Second Continental Workshop on the Geoid in Europe,* Budapest, March 10-14, 1998

[12] Petrovskaya M.S., J.B.Zielinski, **Solution of the Airborne Gradiometry Boundary Value Problem for the Height Anomaly and Gravity Anomaly.** *Bolletino di Geodesia e Scienze Affini*, Vol.58, No.1, pp.1-19, 1999

[13] Schwarz, K.P., Krynski J. **Improvement of the geoid in local areas by satellite gradiometry.** *Bulletin Geodesique.* Vol.51, No.3, 1977.

[14] Zielinski J.B. **Project DIDEX.** *Proc. of 4-th Symposium "Geodesy and Physics of the Earth*", Karl Marx-Stadt, 1980.

[15] Zielinski J.B., E.Lorenzini, **Study of the Free Falling Gravimeter Experiment.** *Final Report to Polish Academy of Sciences and Smithsonian Institution.* Warsaw, 1998.