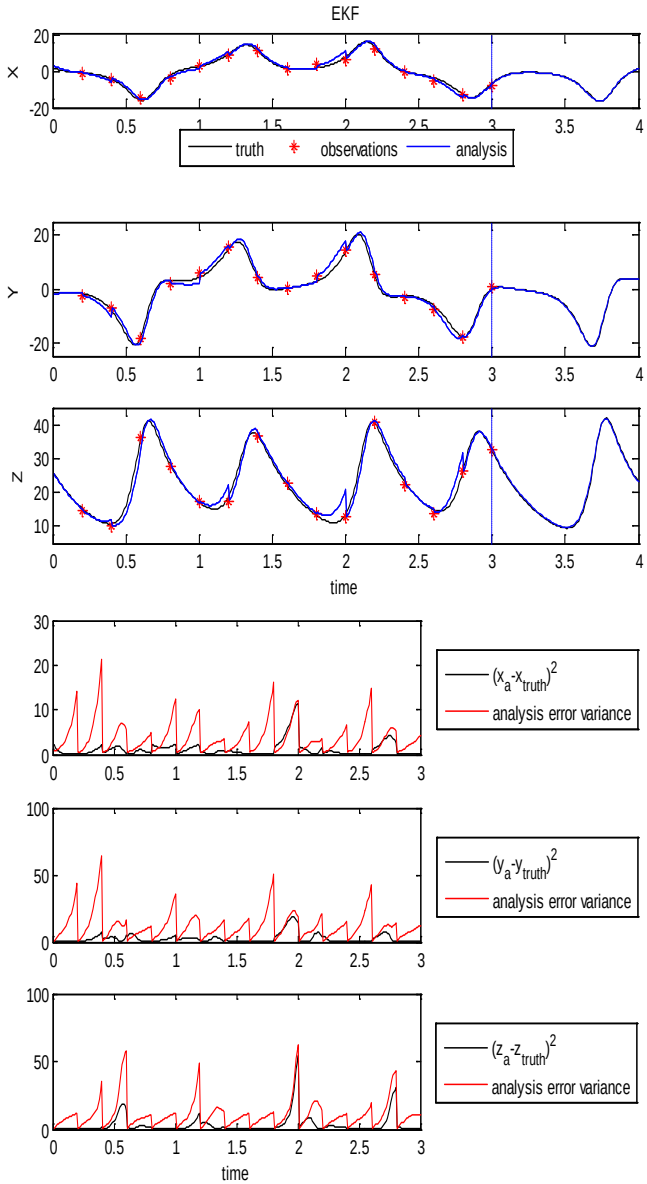
Effect of Correlated Observation Error with Extended Kalman Filter

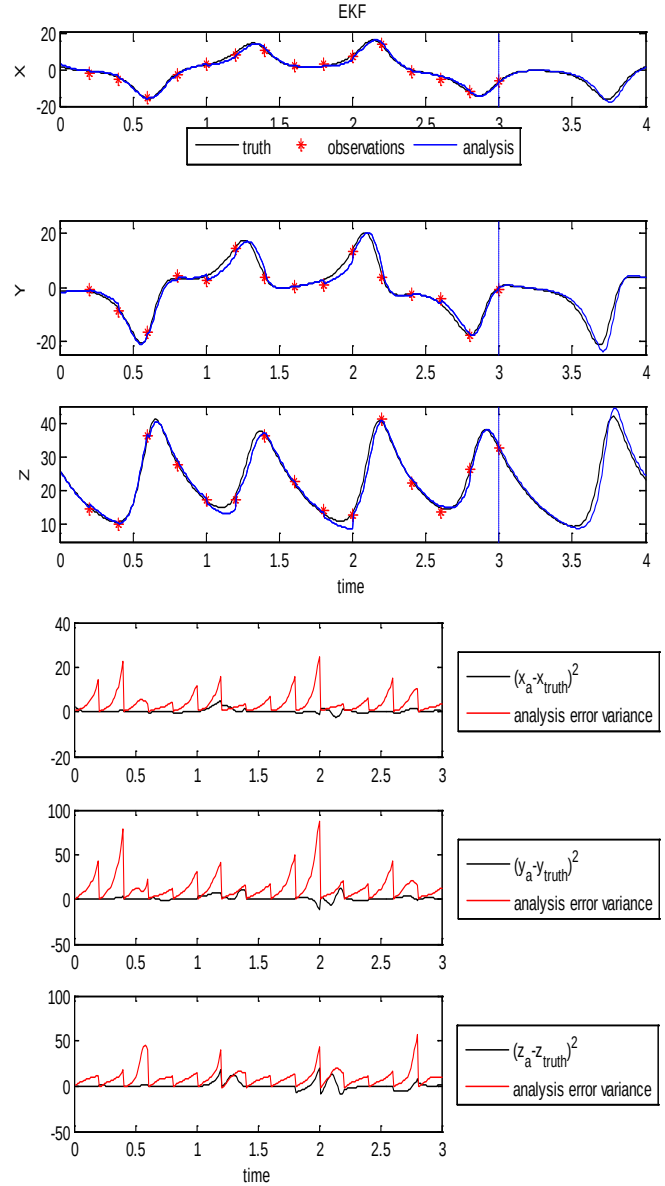
Observation Error Correlation

Assimilation Scheme Error Correlation	None	2	10
None			
2			
10			



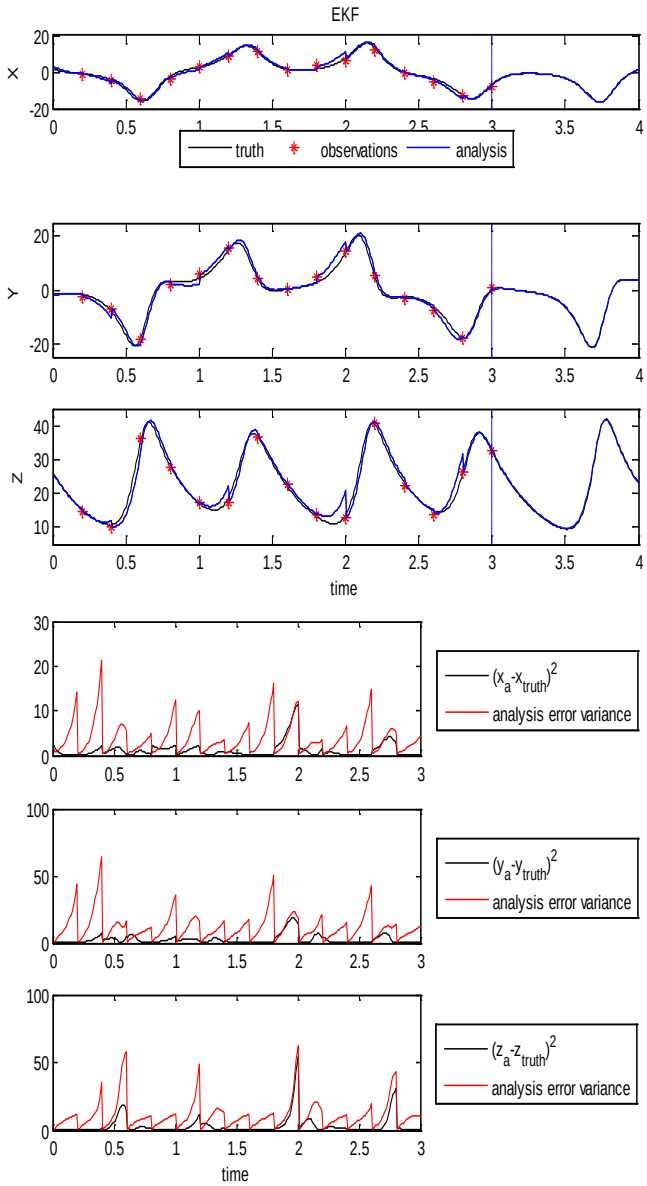
Observation error correlation = 0

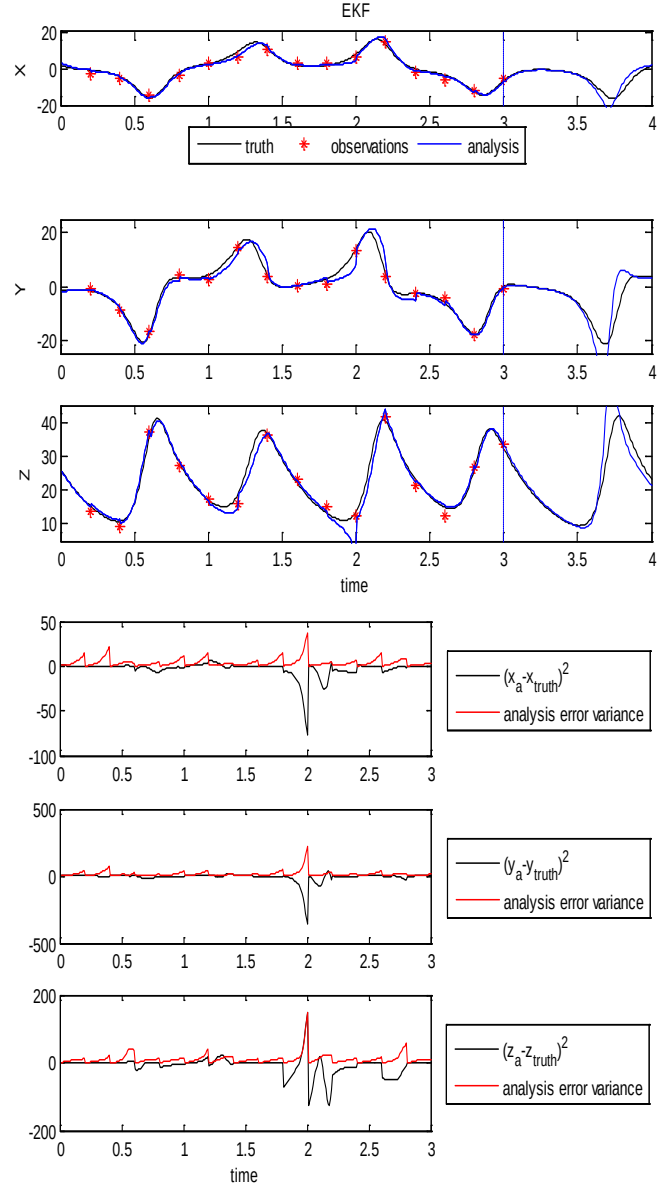
Assimilation error correlation = 0



Observation error correlation = 2

Assimilation error correlation = 2



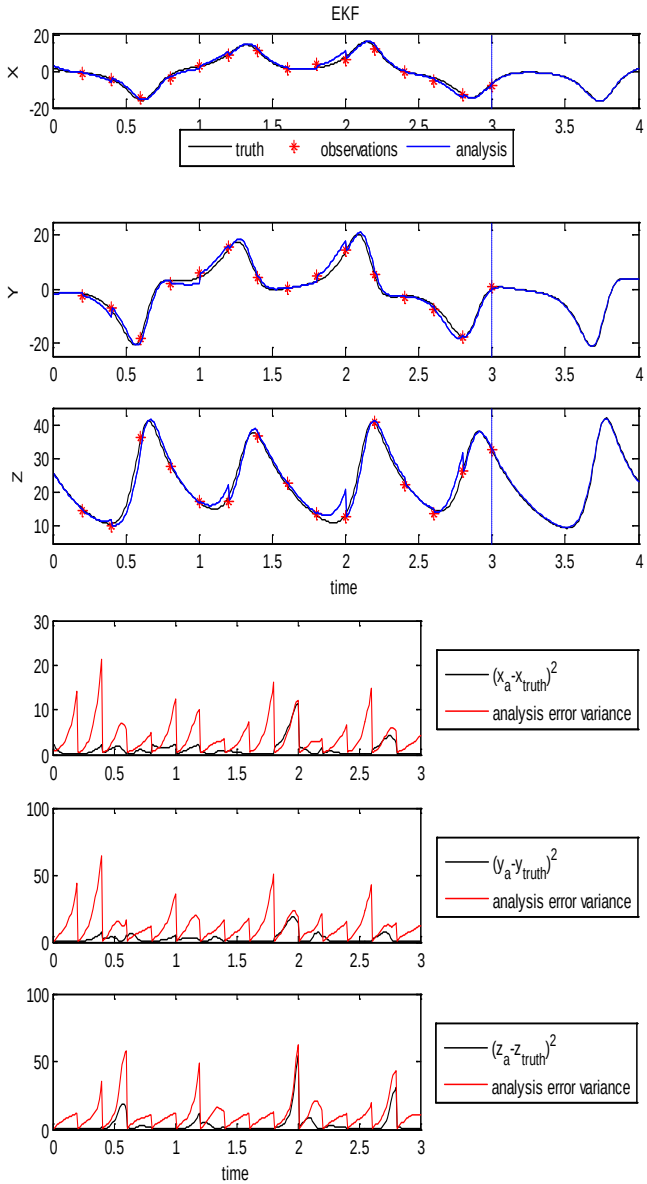


Observation error correlation = 0

Assimilation error correlation = 0

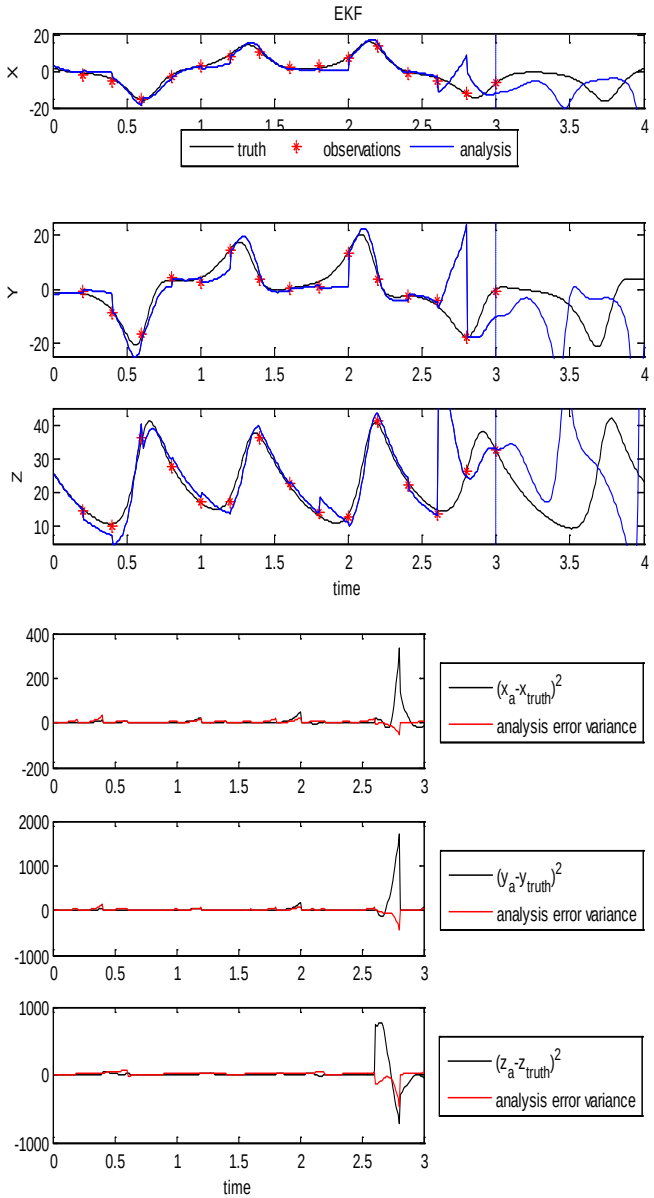
Observation error correlation = 10

Assimilation error correlation = 2



Observation error correlation = 0

Assimilation error correlation = 0



Observation error correlation = 2

Assimilation error correlation = 10

Summary

- Initial results demonstrate that contrast in observed and assumed error correlation leads to breakdown in forecast.
- Increased correlation in observation errors suggests to the assimilation scheme that observations are not independent
- For example: three different meteorological instruments (x,y,z) in a remote environment share a power source (e.g. solar).

DA Project 2

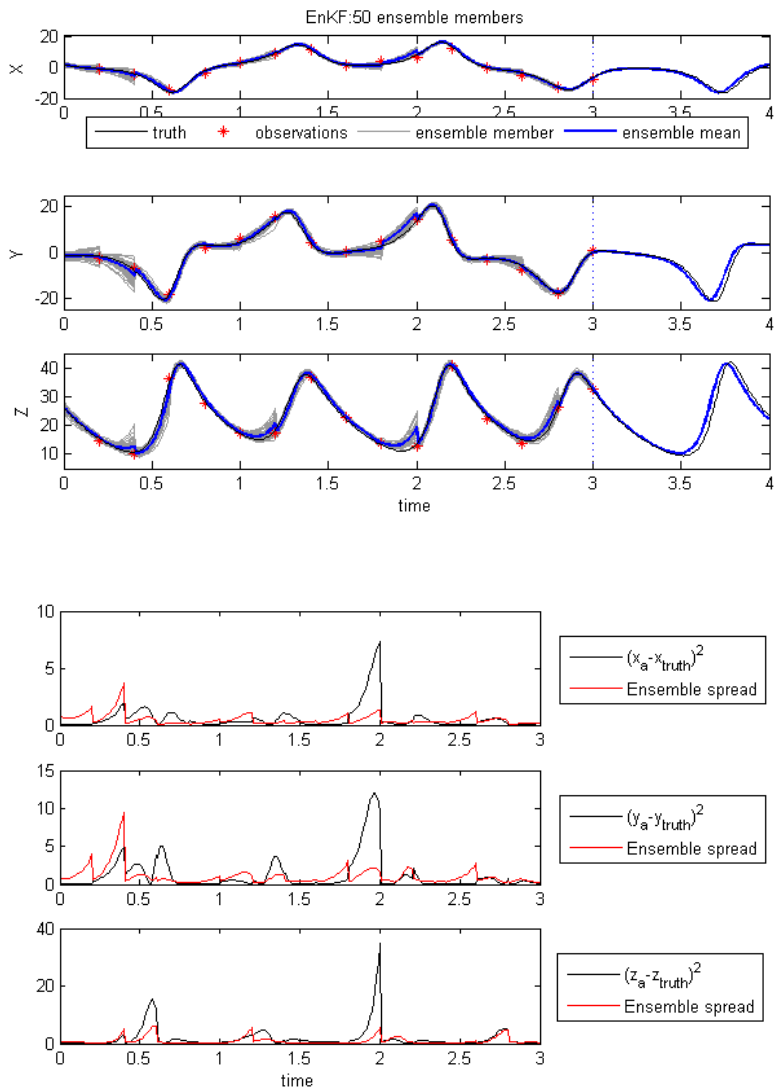
Robin Wilson
Sebastian Hahn
Ling Chang

DA Project 2

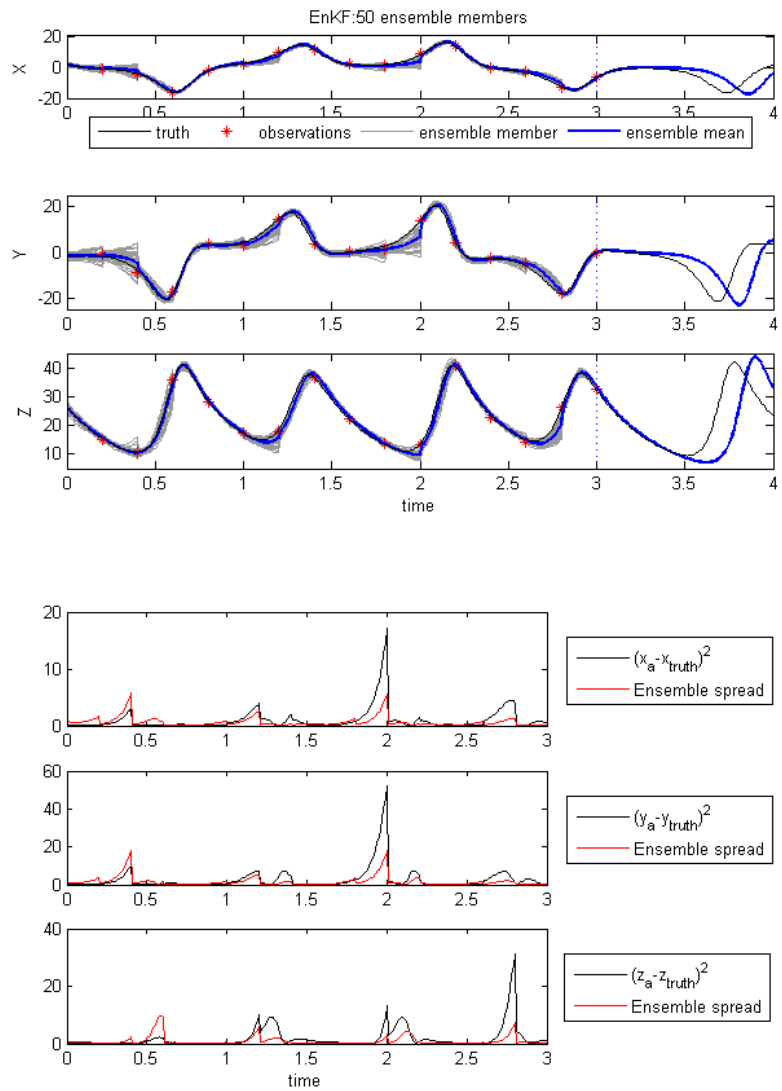
Investigate the effect of correlated observation error in the ensemble Kalman filter (EnsKF).

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad R = \begin{bmatrix} 1 & 0.5 & 0.5 \\ 0.5 & 1 & 0 \\ 0.5 & 0 & 1 \end{bmatrix}$$

Default values

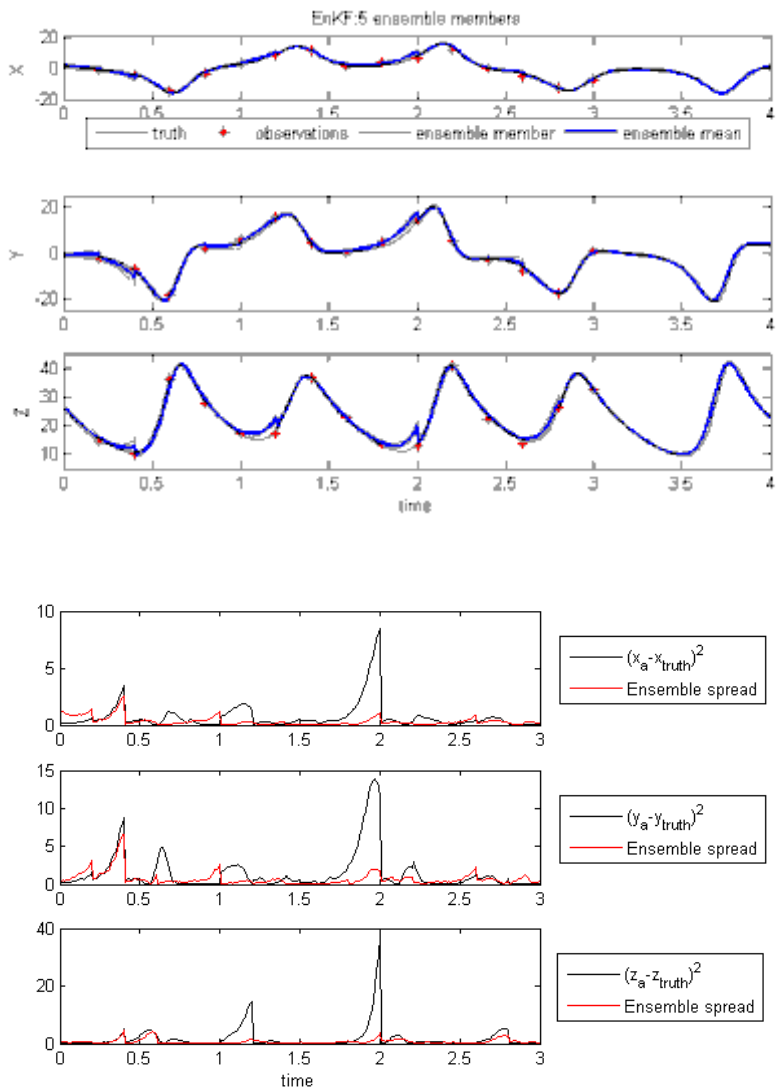


uncorrelated

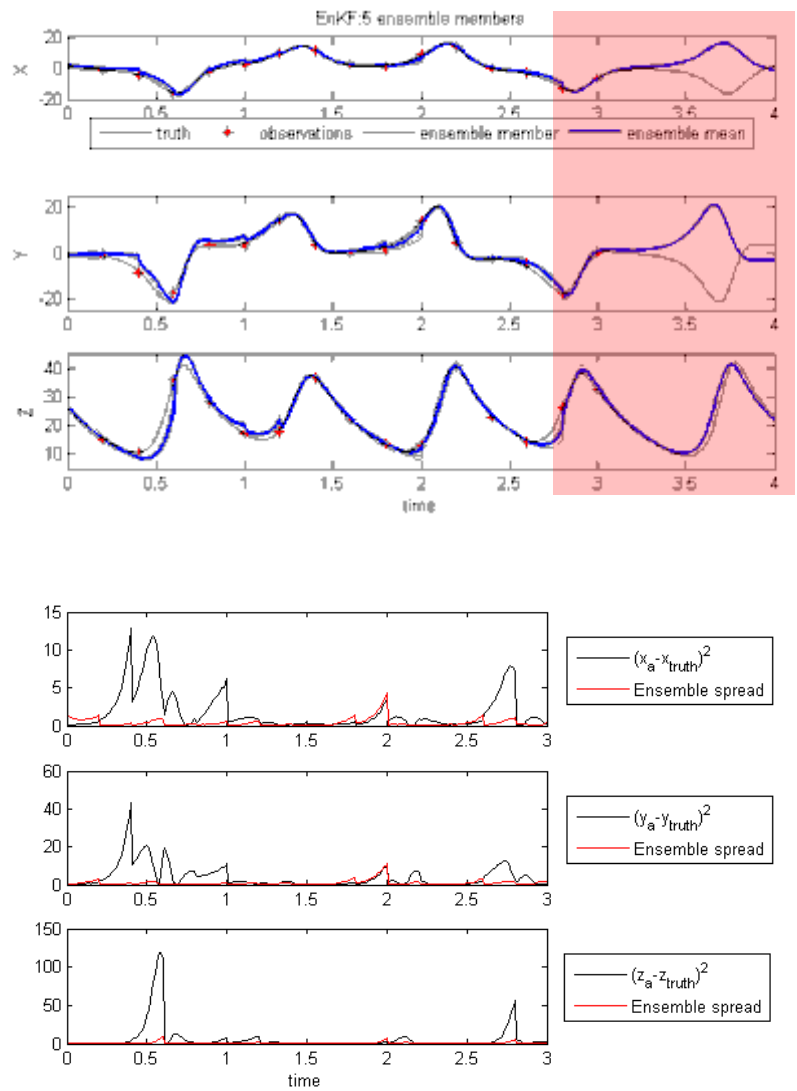


correlated

Small number of Ensemble Members (5)

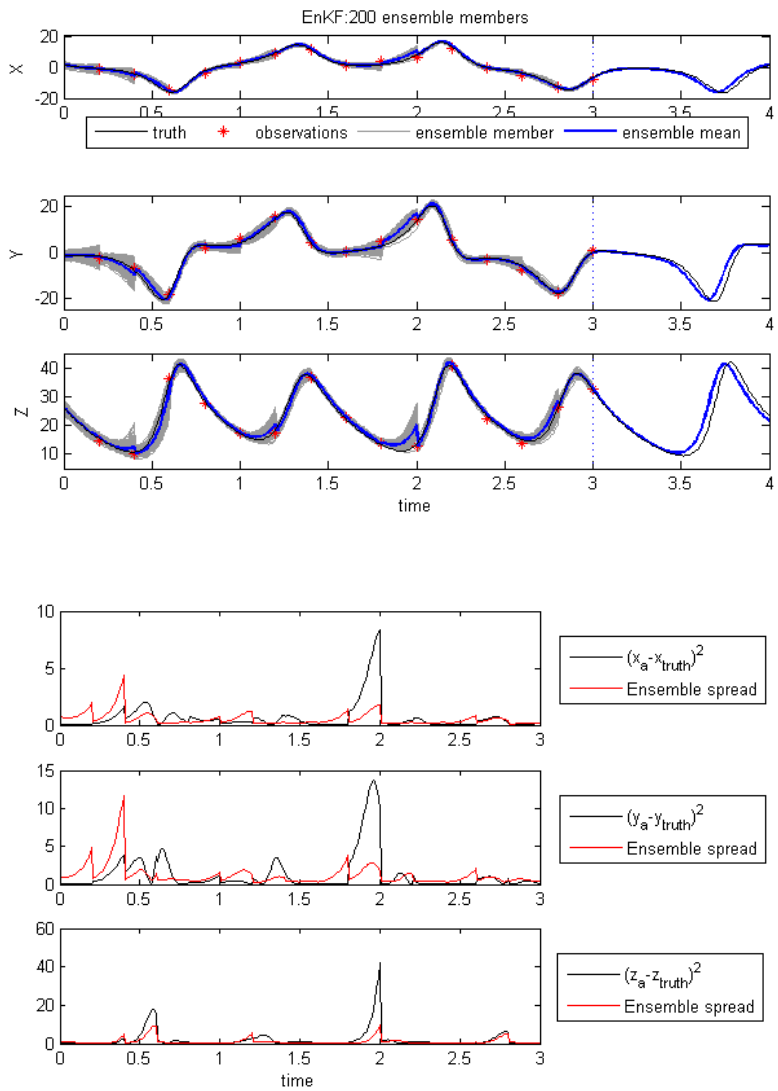


uncorrelated

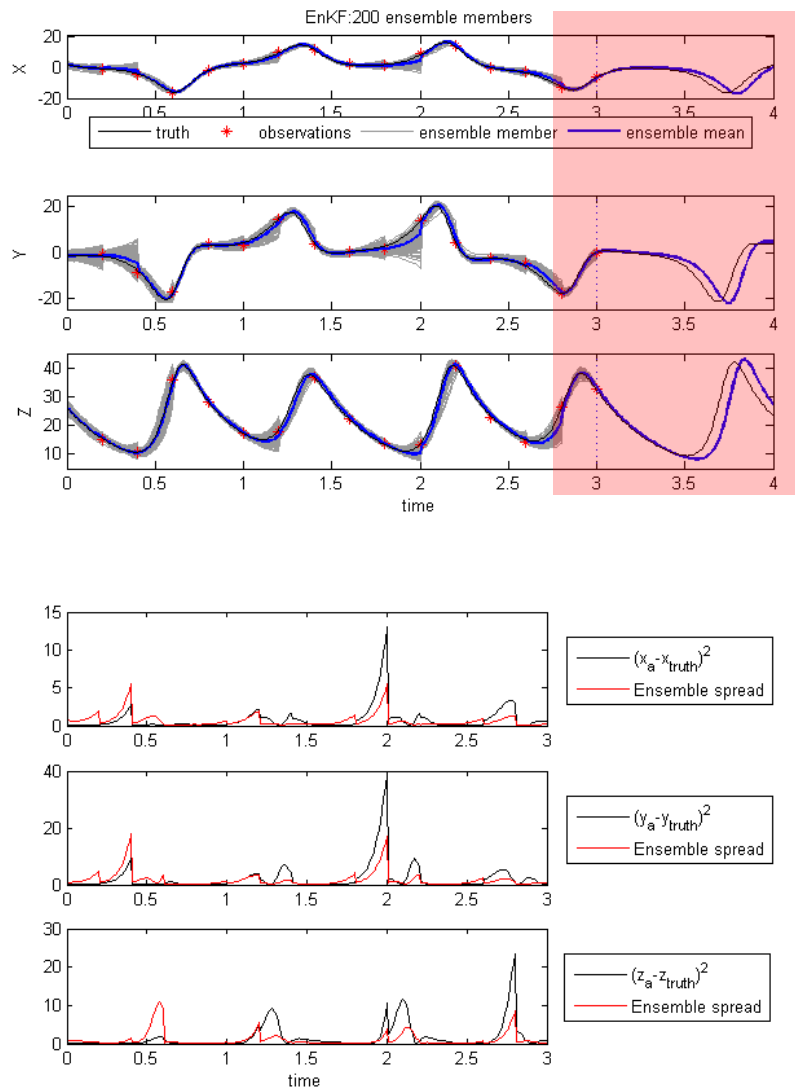


correlated

High number of Ensemble Members (200)

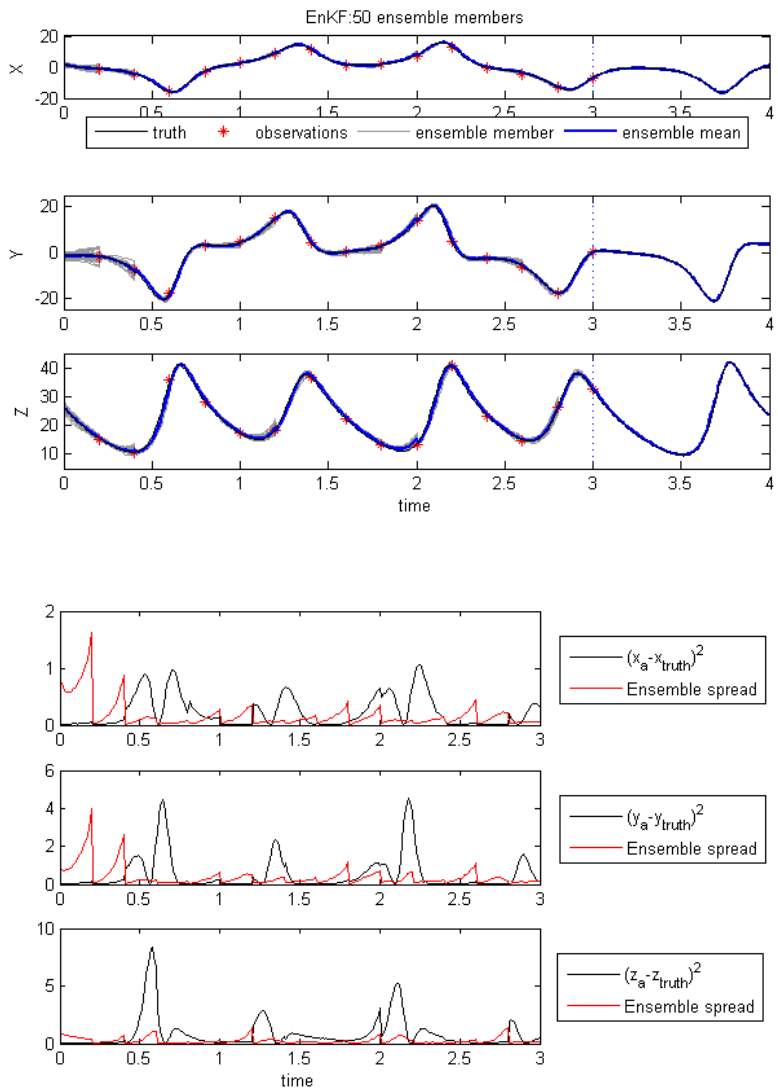


uncorrelated

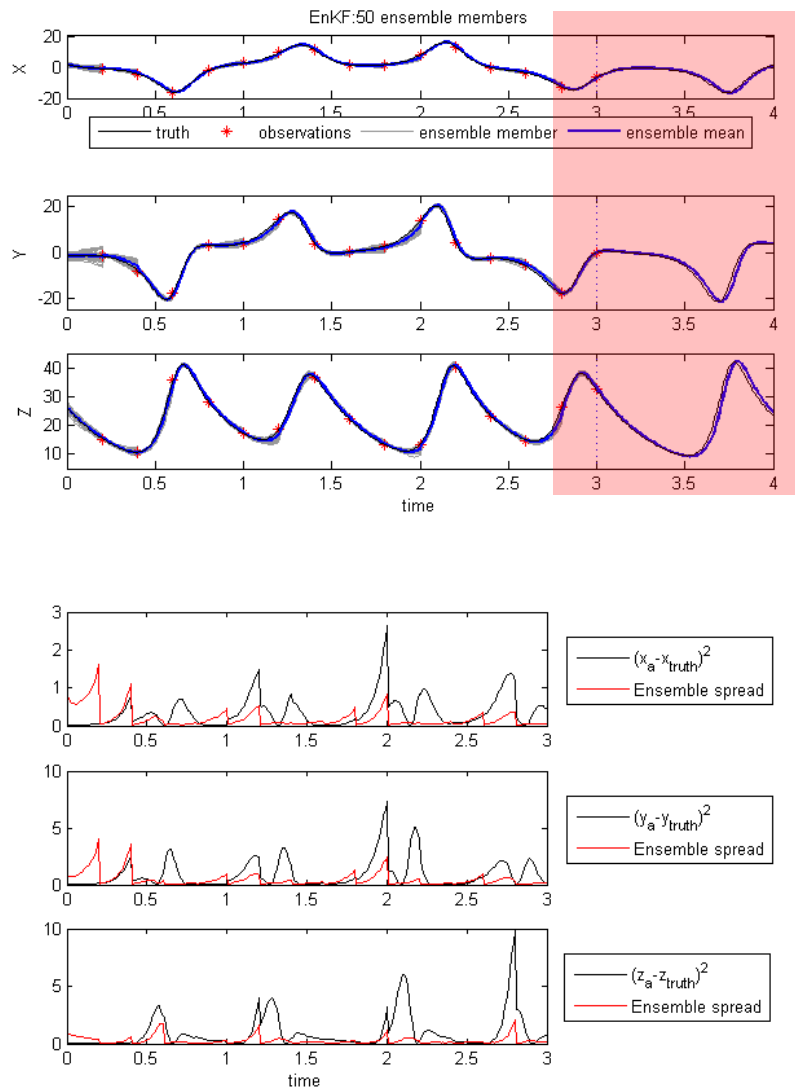


correlated

Small observation error (0.1)

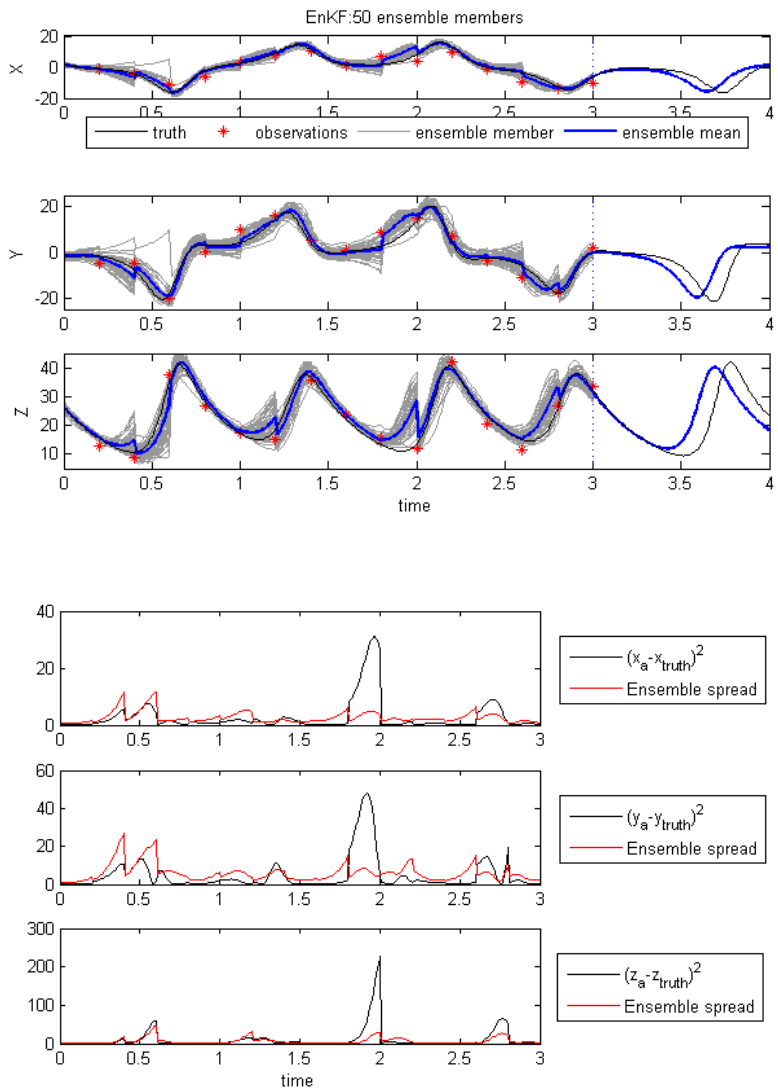


uncorrelated

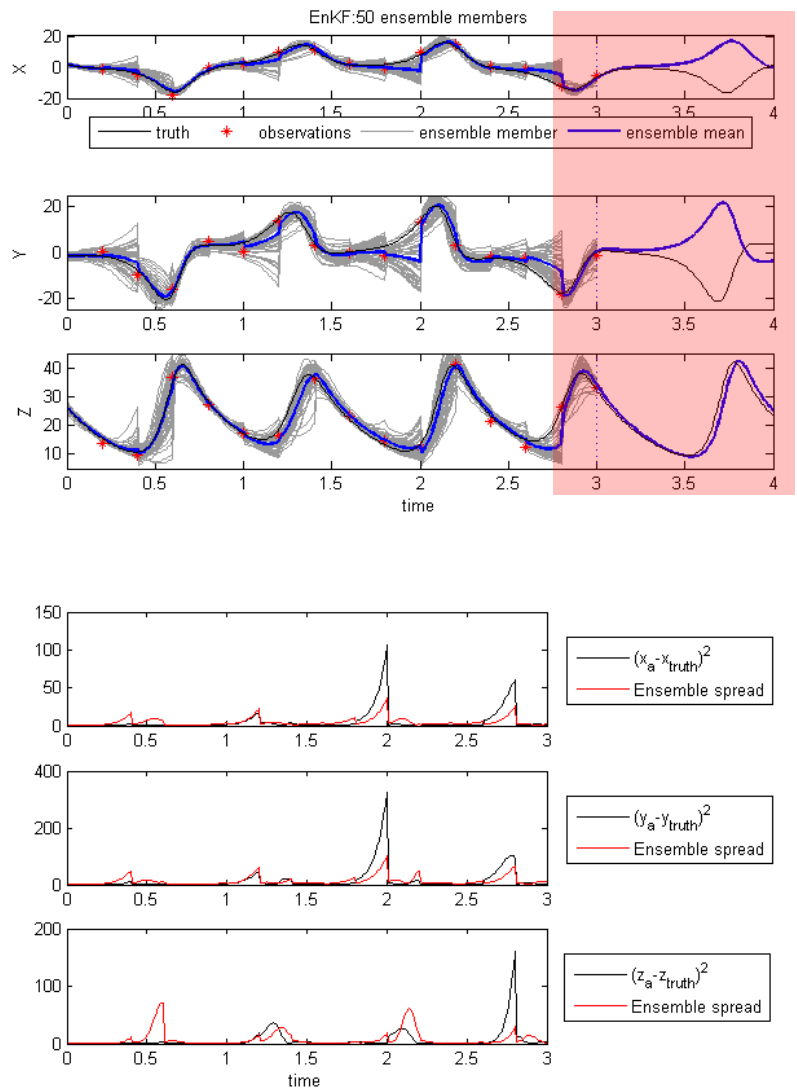


correlated

High observation error (10)

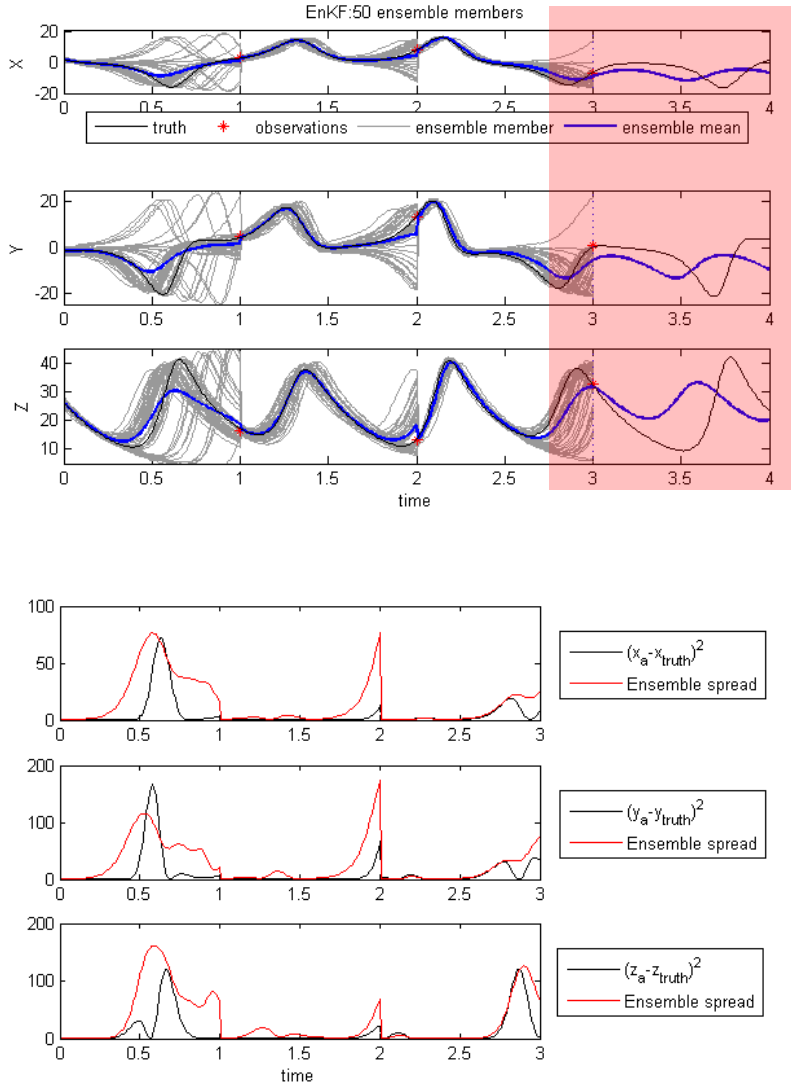


uncorrelated

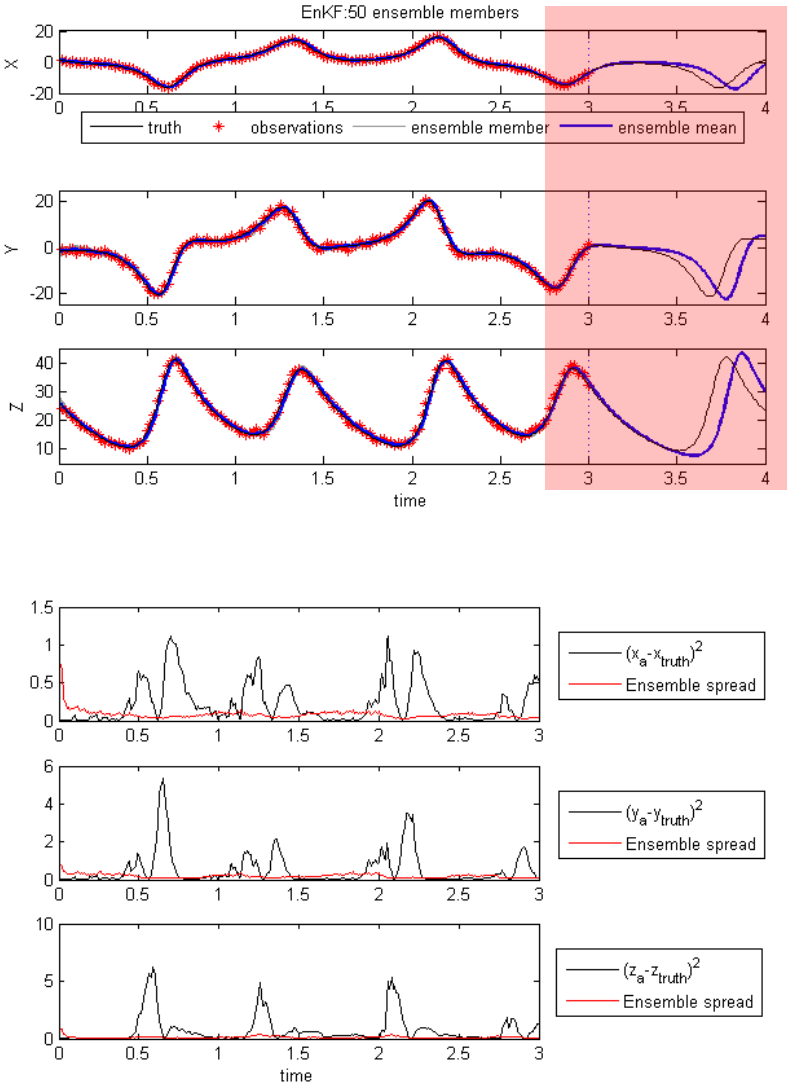


correlated

Less vs. many observations



correlated



correlated

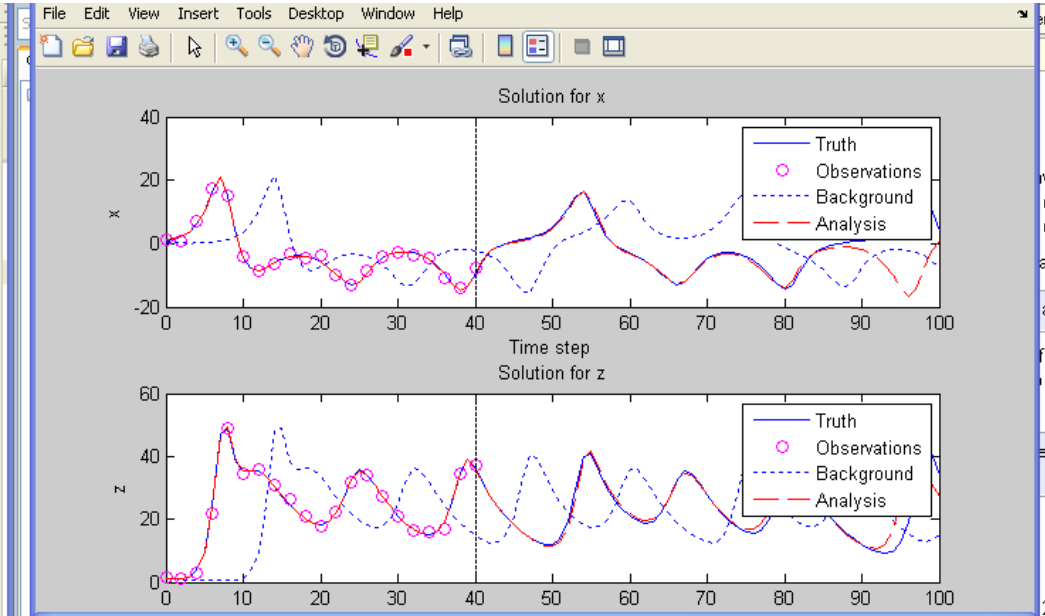
Conclusion

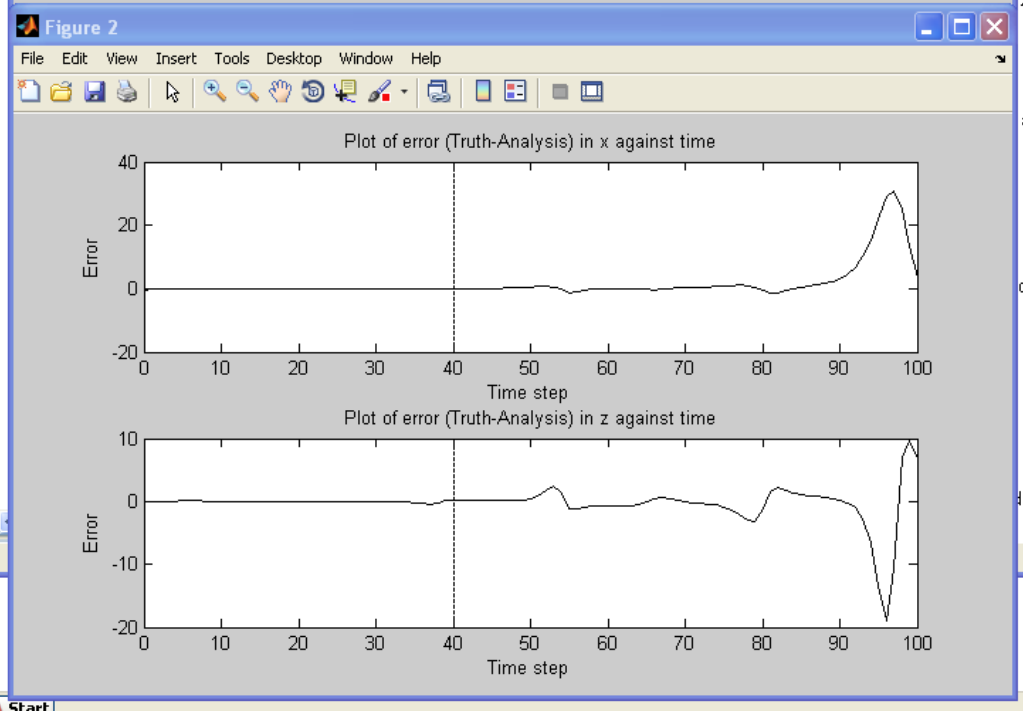
- More ensemble members give a better forecast in case of corr. obs. errors (the other way around in the uncorrelated case)
- If the observation error is high, then the forecast is less confidence
- More measurements give a better forecast in case of corr. obs. errors
- Background error has a similar impact in both cases, correlated and uncorrelated obs. errors.

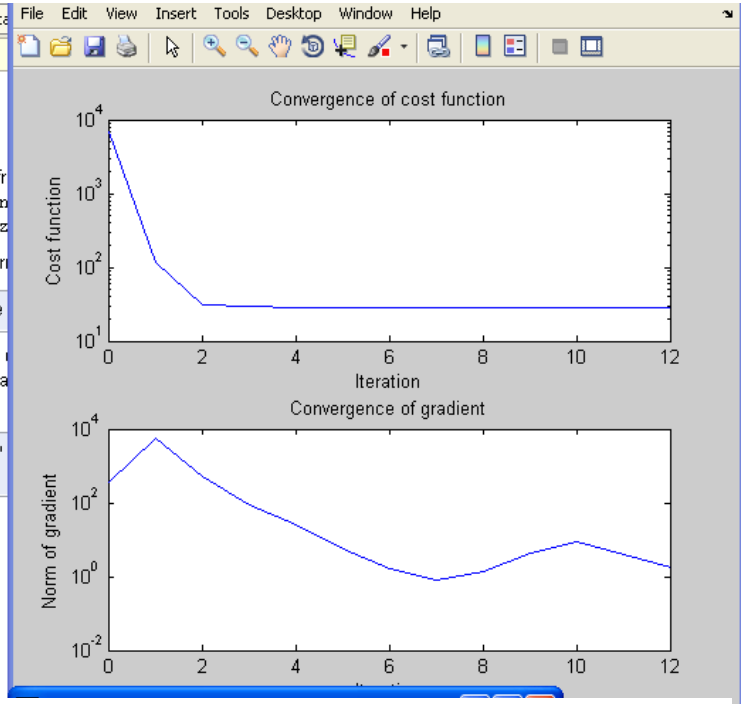
- Outlook J
  - Influence of changing the magnitude of correlation
  - Influence of changing the starting point
  - Quantitative investigation

PROJECT #3

INVESTIGATE THE EFFECT OF CORRELATED OBSERVATION ERROR IN 4D-VAR

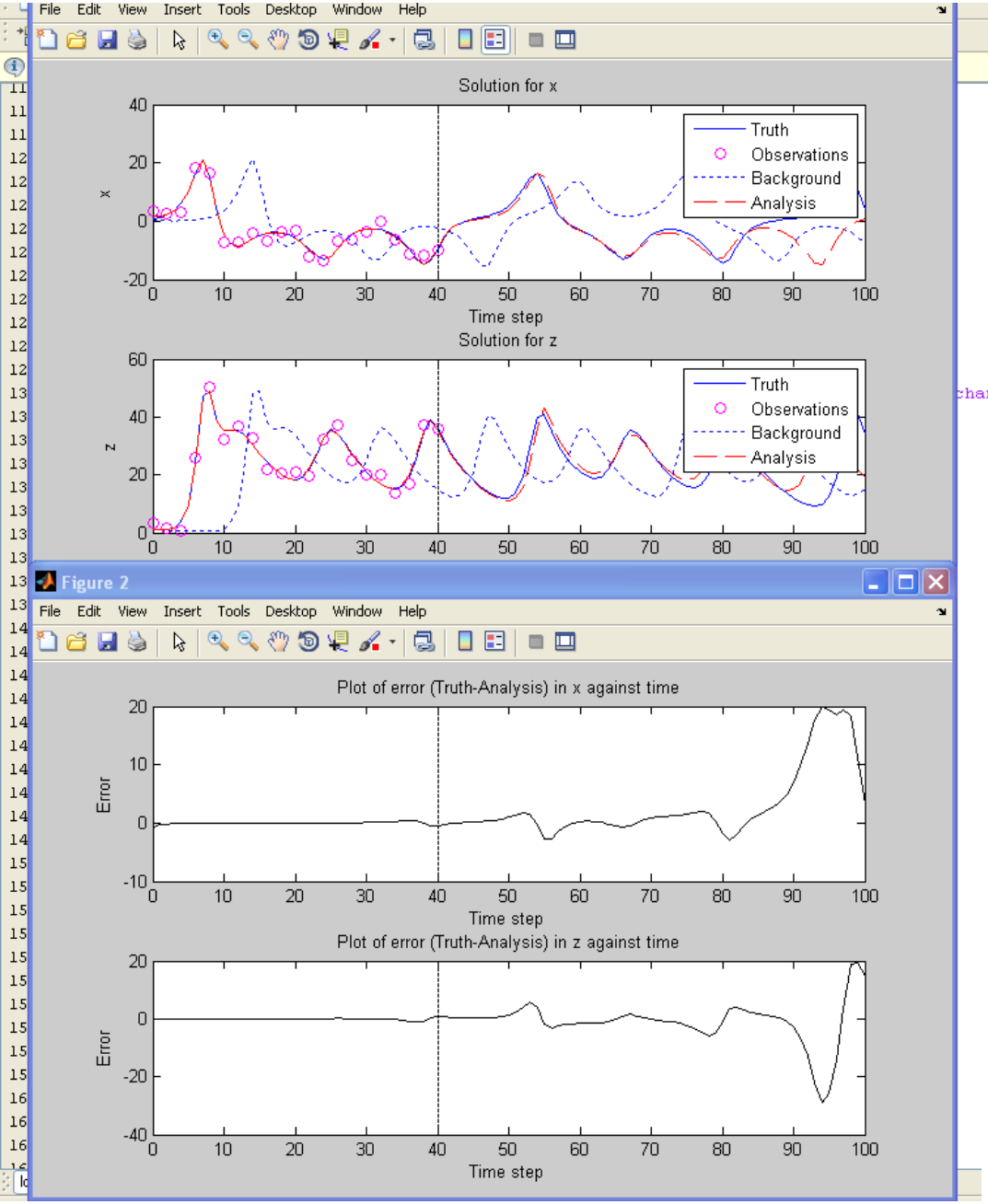


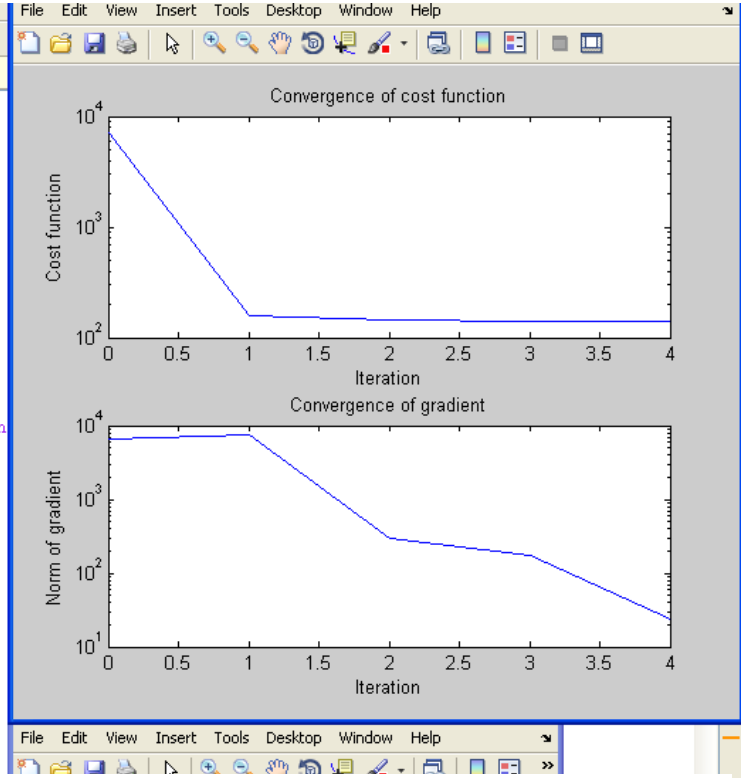




CASE 0 :

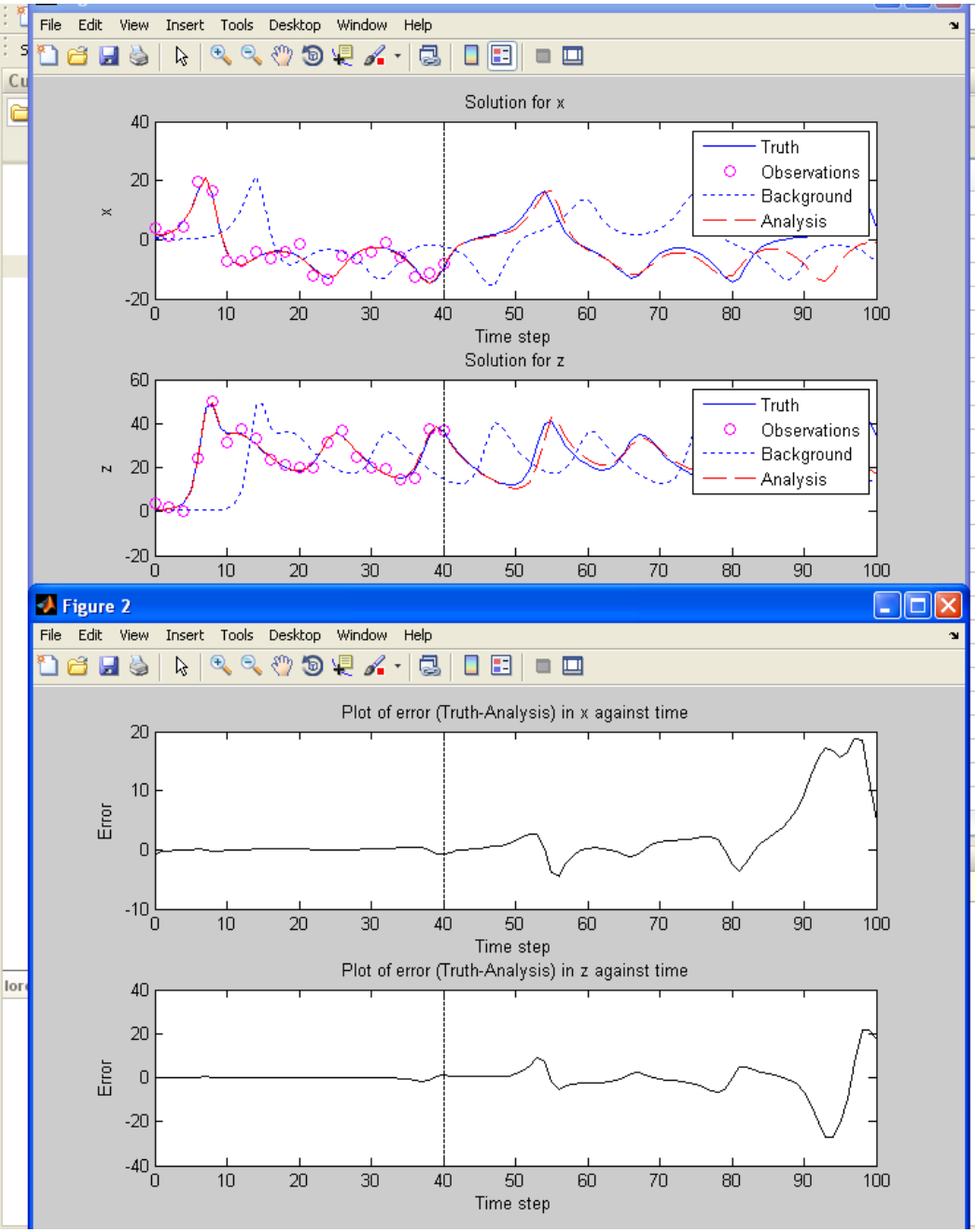
No correlation in the obs. Errors;

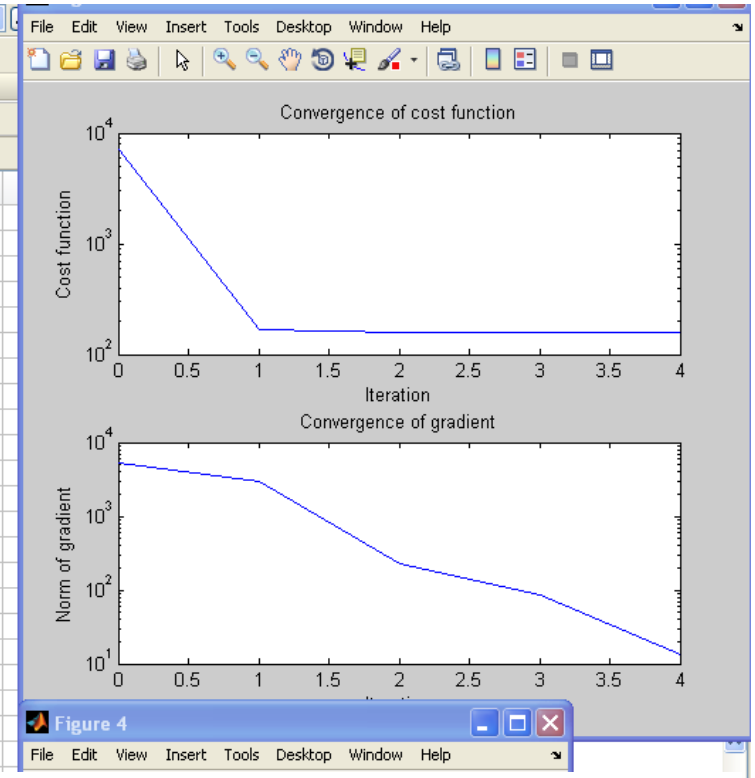




CASE 1:

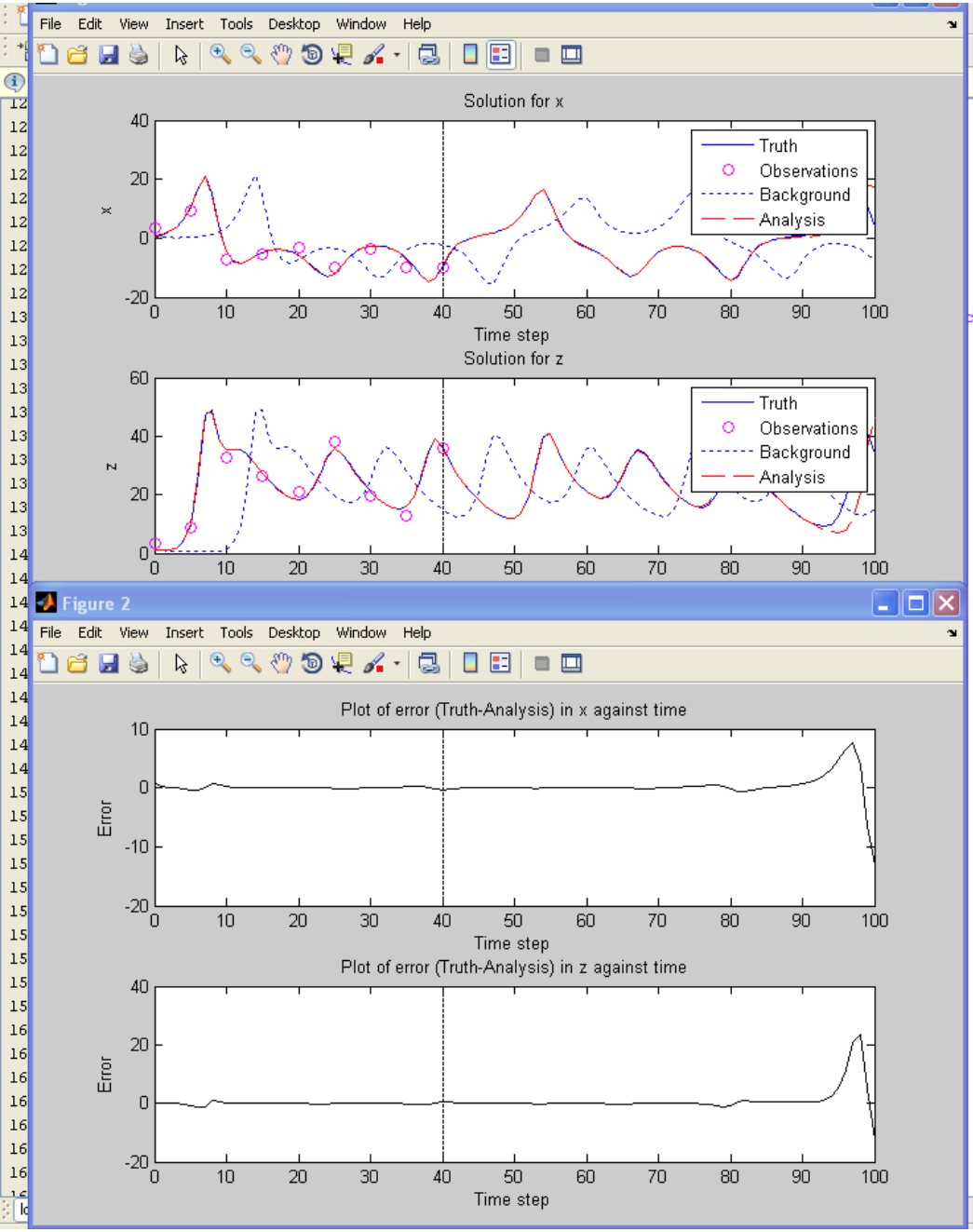
correlation in the obs. Errors is 100 %

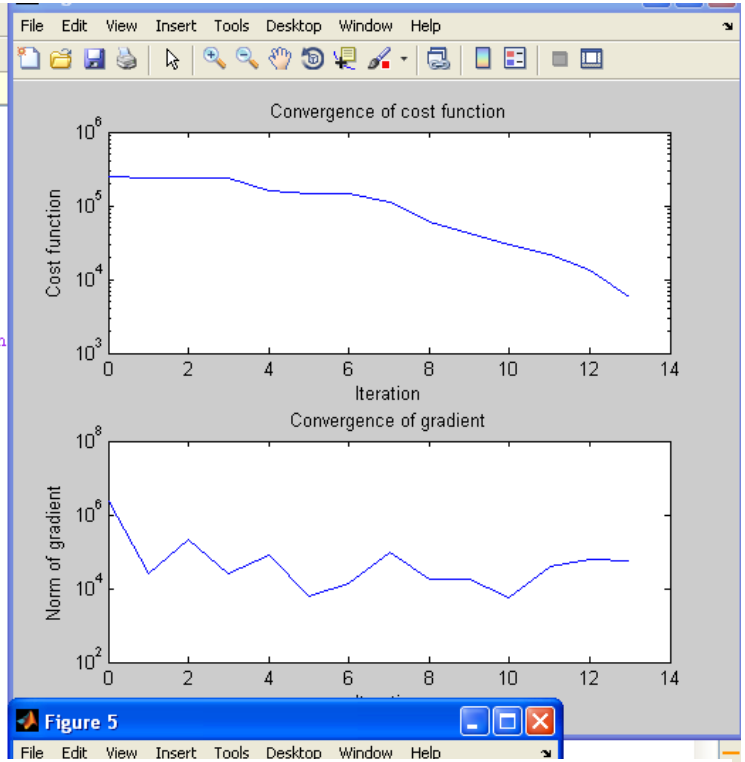




CASE 2:

Random + temporal dependent correlation error [+ 3sin(t)]





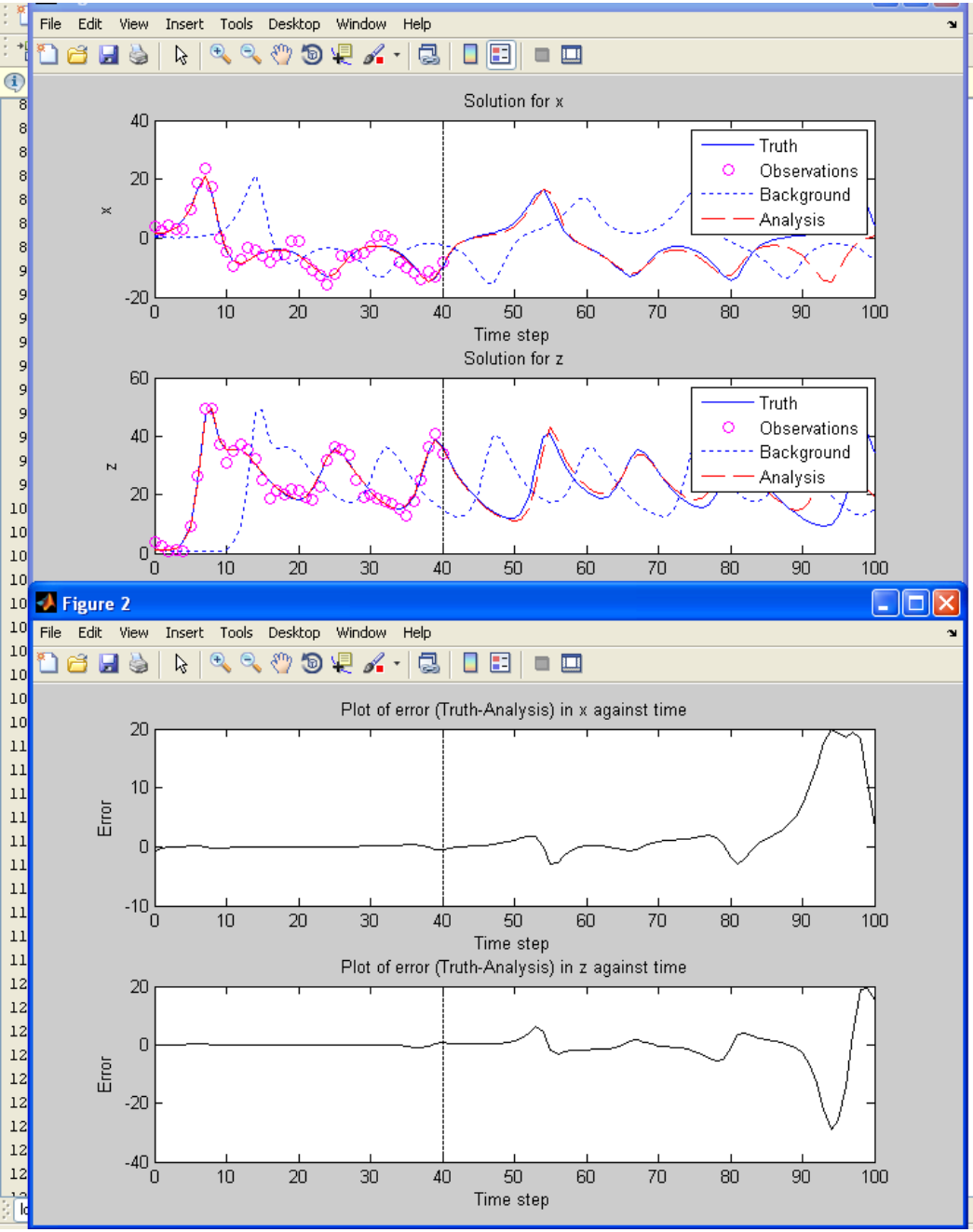
CASE 3:

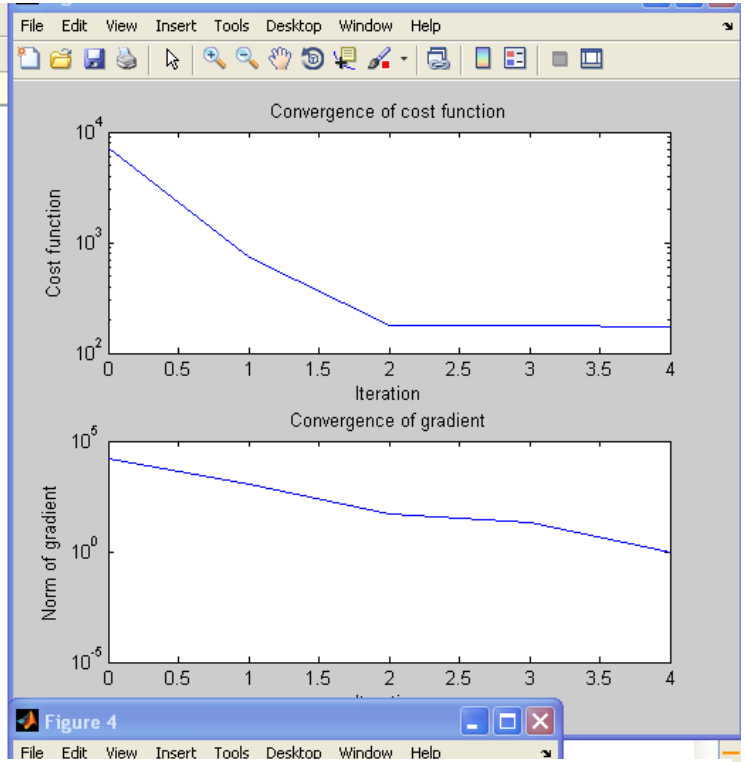
Random + temporal dependent correlation error [+ 3sin(t)]

Frequency of obs = 5 time-steps

Obs. Error = 0.01 [very accurate obs!!]

-Worst results!!!





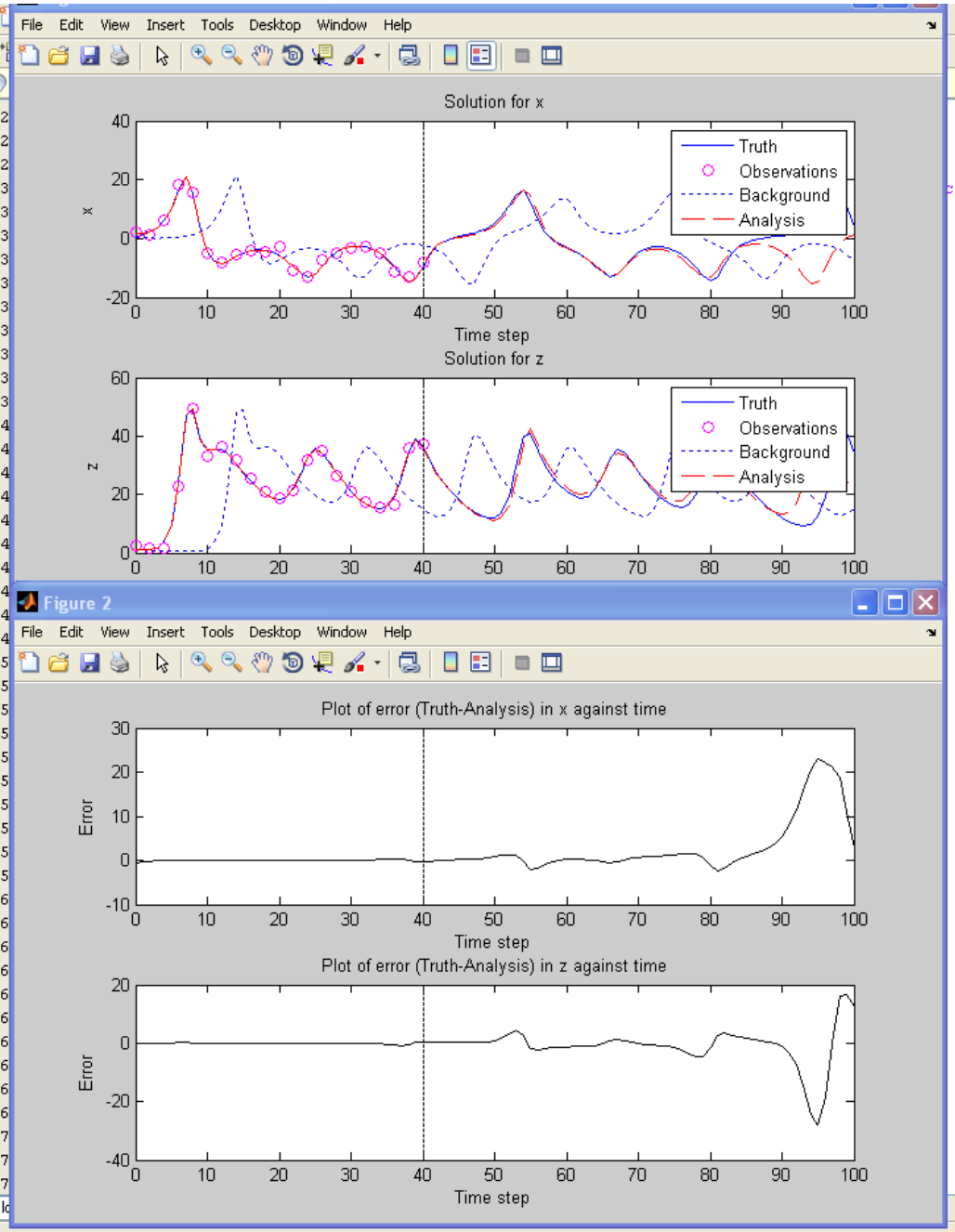
CASE 4: more errors in the obs.

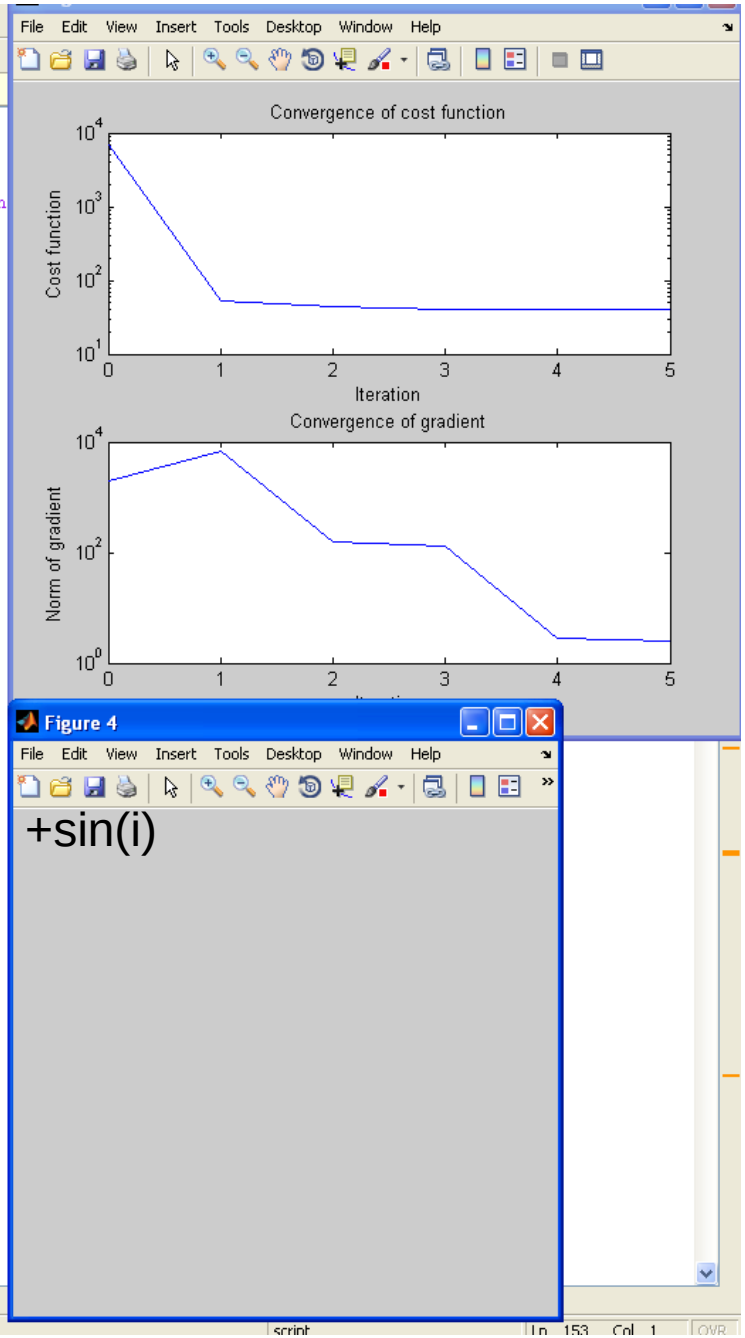
Random + temporal dependent correlation error [+ 3sin(t)]

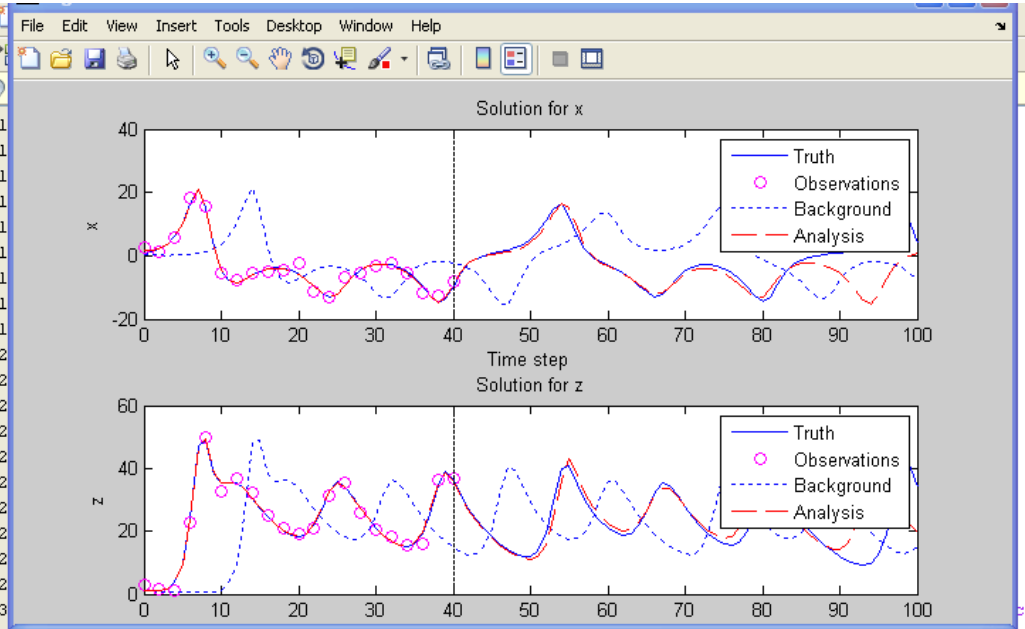
Frequency of obs = 1 time-steps

Obs. Error = 2.0 [noisy obs!!]

- Best results!!!


File Edit View Insert Tools Desktop Window Help
Solution for x
Truth
Observations
Background
Analysis
x
Time step
Solution for z
z
Figure 2
Plot of error (Truth-Analysis) in x against time
Error
Plot of error (Truth-Analysis) in z against time


File Edit View Insert Tools Desktop Window Help
Convergence of cost function
Cost function
Iteration
Convergence of gradient
Norm of gradient
Figure 4
+sin(i)


File Edit View Insert Tools Desktop Window Help
Solution for x
x
Time step
Truth
Observations
Background
Analysis
Solution for z
z
Time step
Truth
Observations
Background
Analysis

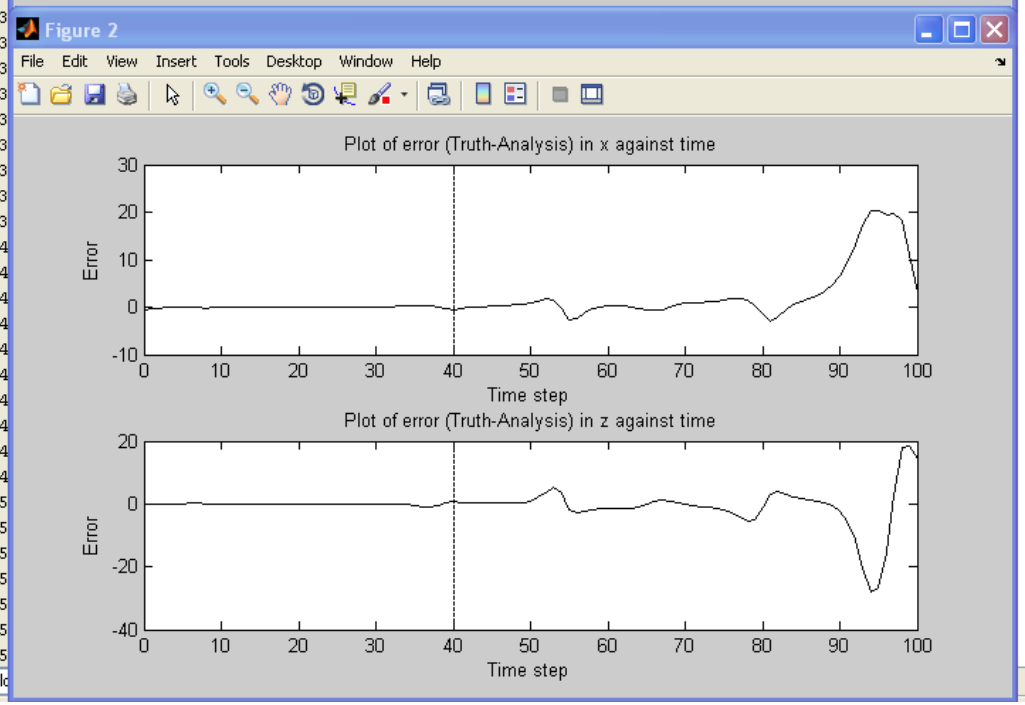
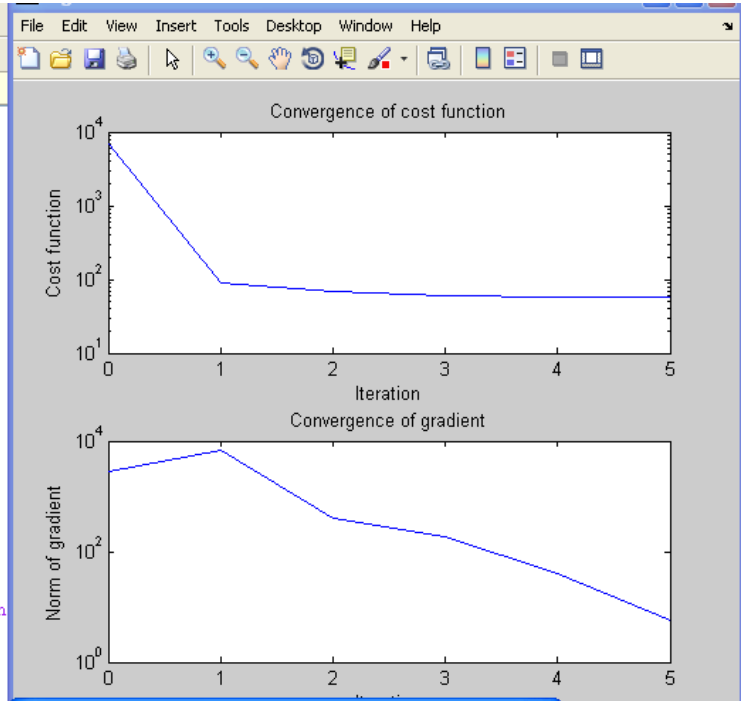


Figure 2
File Edit View Insert Tools Desktop Window Help
Plot of error (Truth-Analysis) in x against time
Error
Time step
Plot of error (Truth-Analysis) in z against time
Error
Time step


File Edit View Insert Tools Desktop Window Help
Convergence of cost function
Cost function
Iteration
Convergence of gradient
Norm of gradient

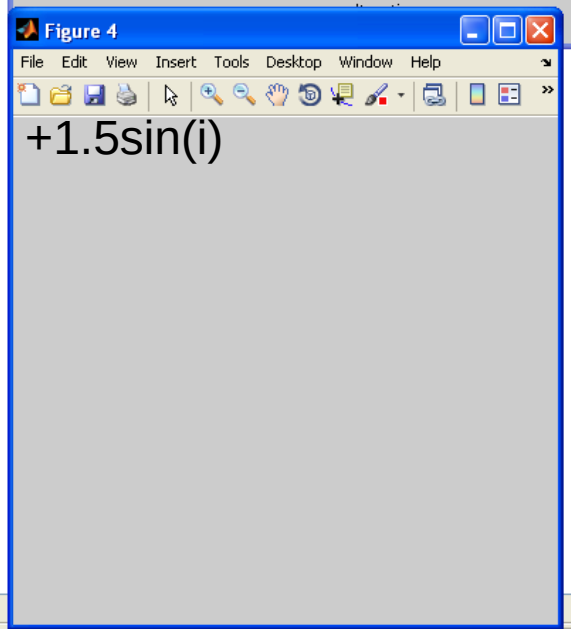


Figure 4
File Edit View Insert Tools Desktop Window Help
+1.5sin(i)

Extended Kalman filter:

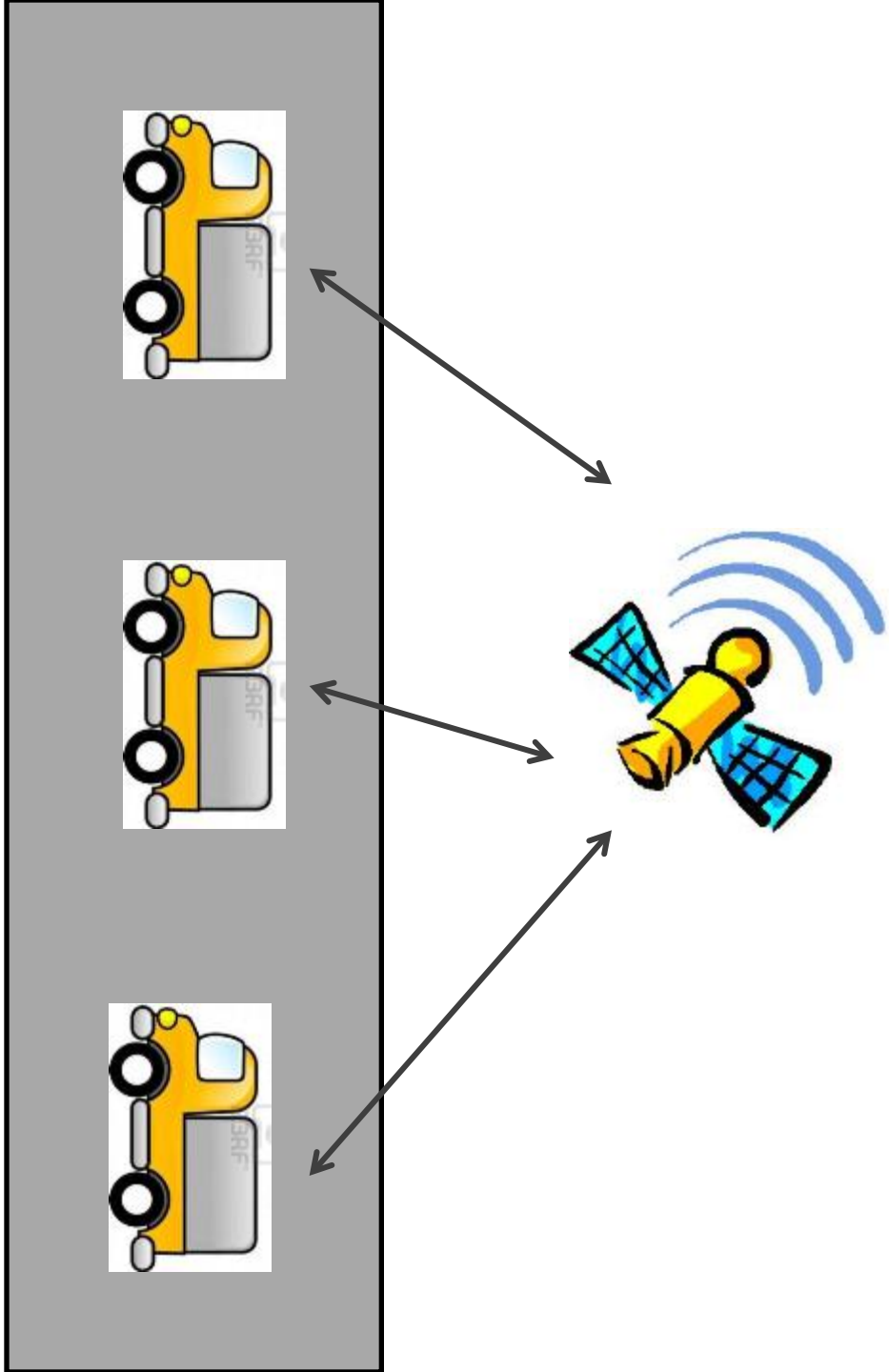
The effect of time correlated observation error

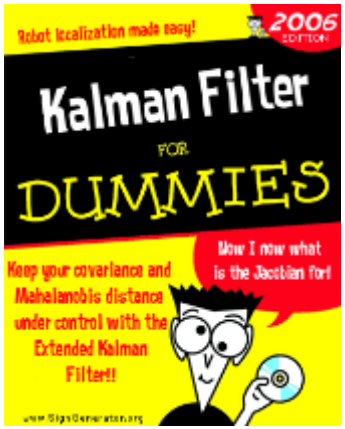
Ksenia, Joachim, Florian, Philipp

Extended Kalman filter

- Is used to remove disturbances in observations
- Algorithm to guess the state of linear and nonlinear systems.
- Applications: guidance, navigation, control of vehicles, particularly aircraft and spacecraft, time series analysis (meteorolgical estimations), oceanography, data assimilation and also in signal processing and econometrics.

Extended Kalman filter

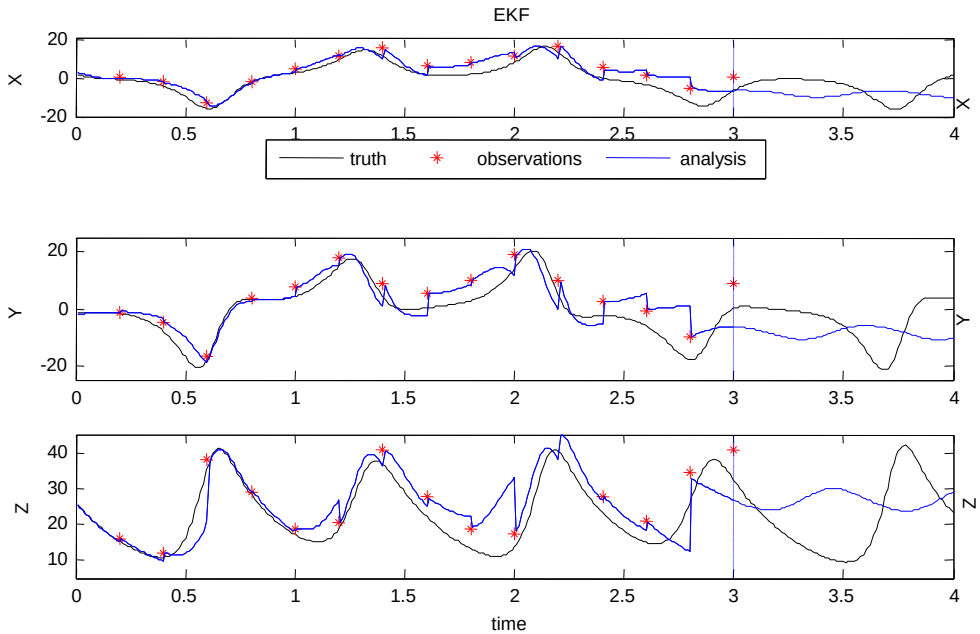


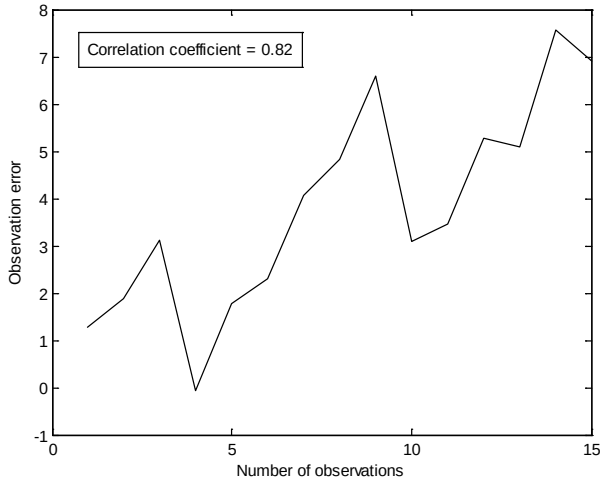


Several measurements

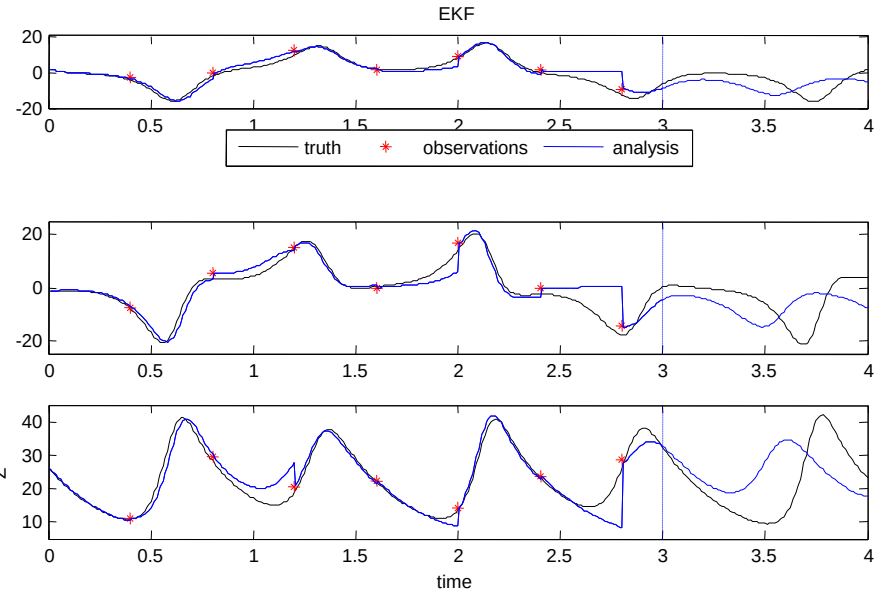
GPS measurements keeps the truck on the road

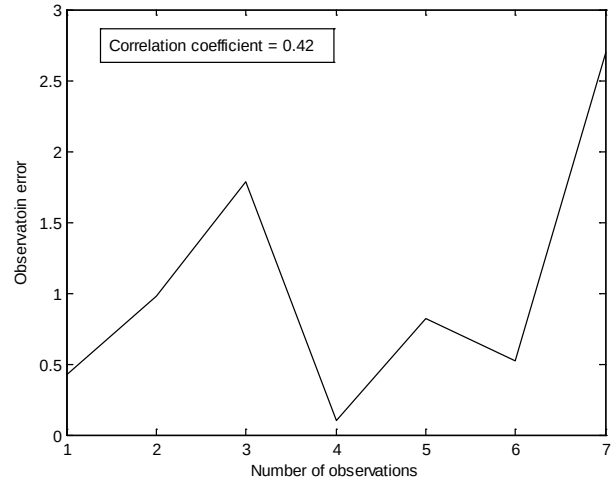
More observations with higher correlation (“worse observations”)





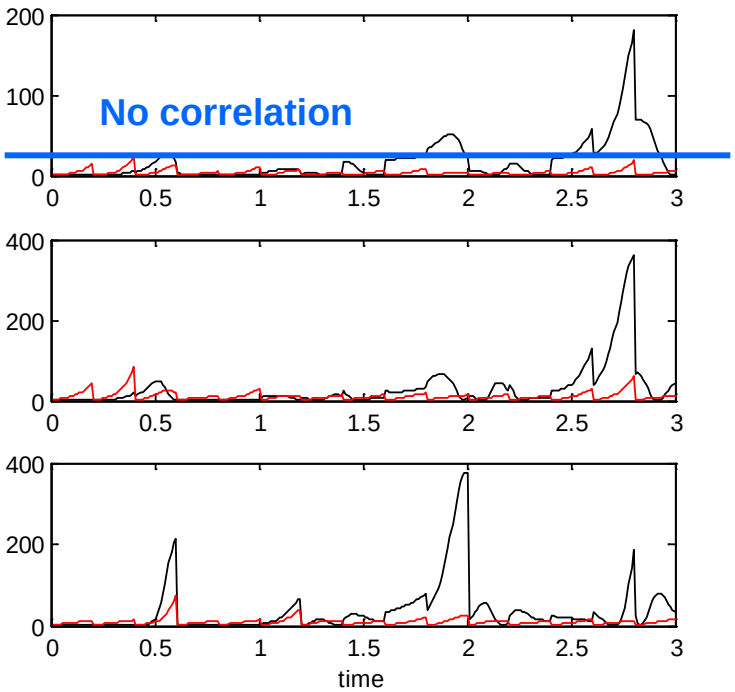
Less observations with lower correlation (“better observations”)





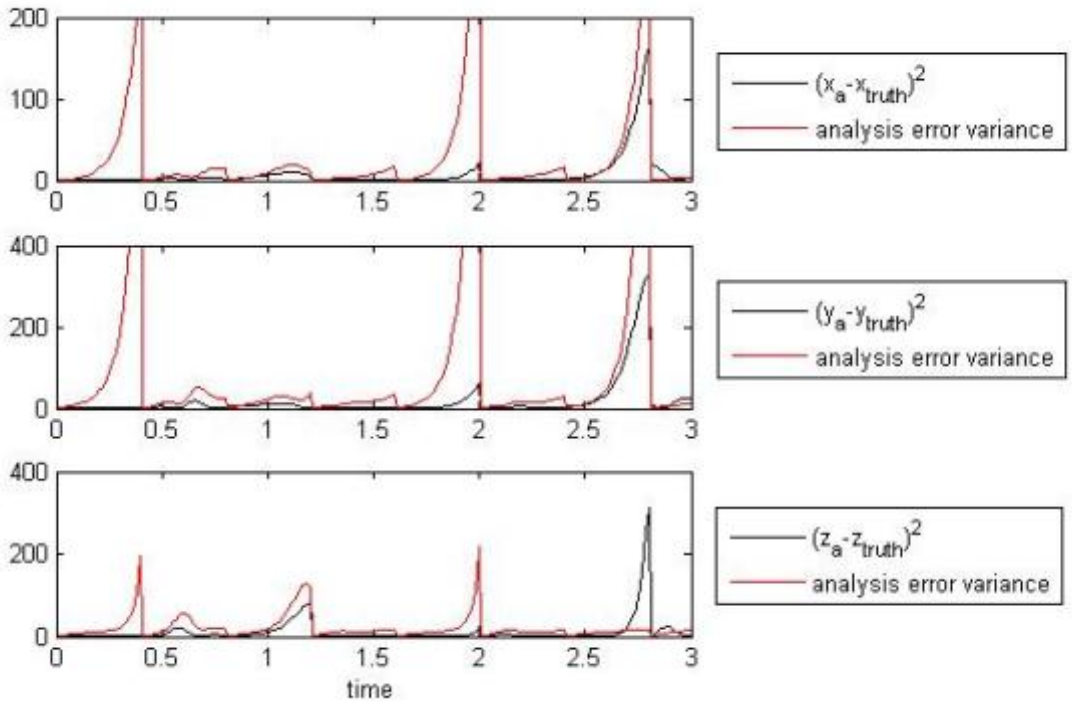
More observations with higher correlation

(“worse observations”)



Less observations with lower correlation

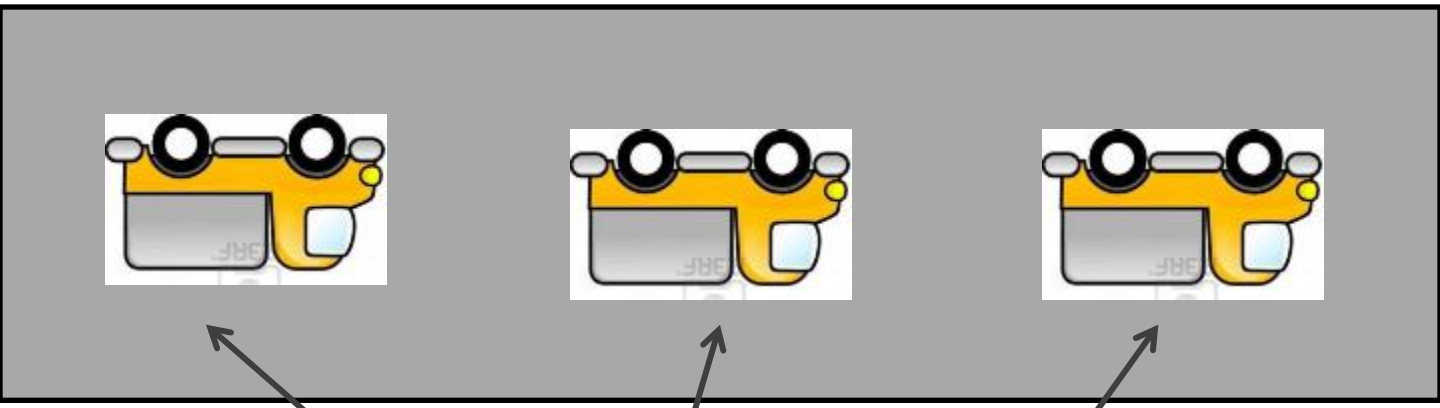
(“better observations”)



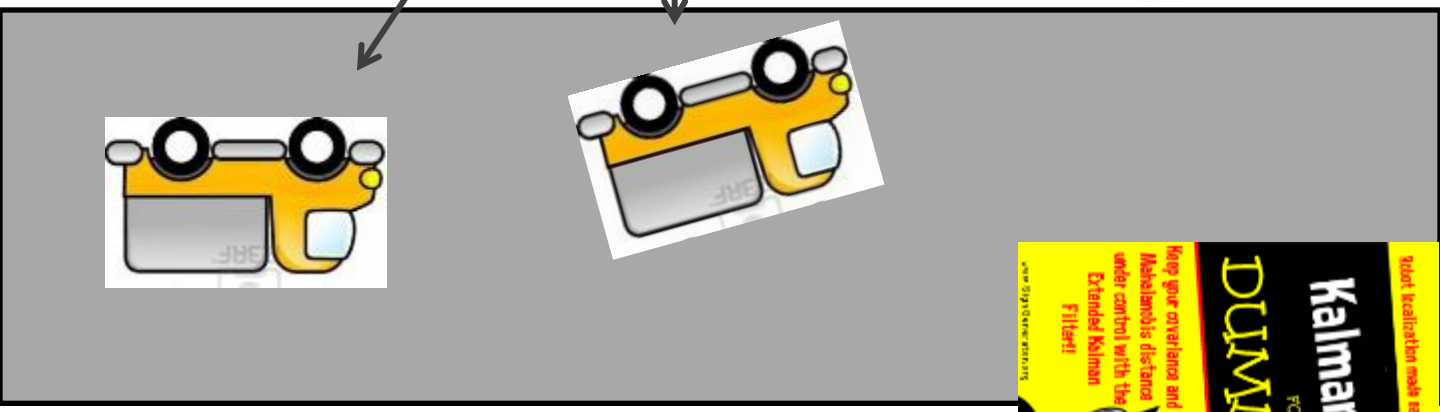
Time correlated observation errors are a major problem!

It is better to have less but “better” observations!

Extended Kalman filter



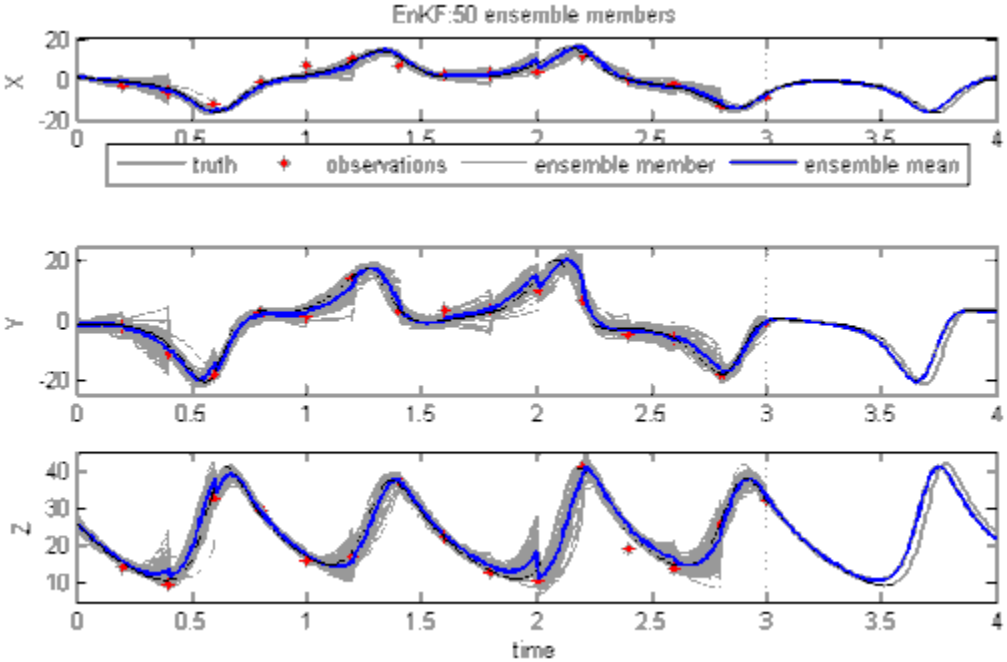
Extended Kalman filter with with time correlated observation error

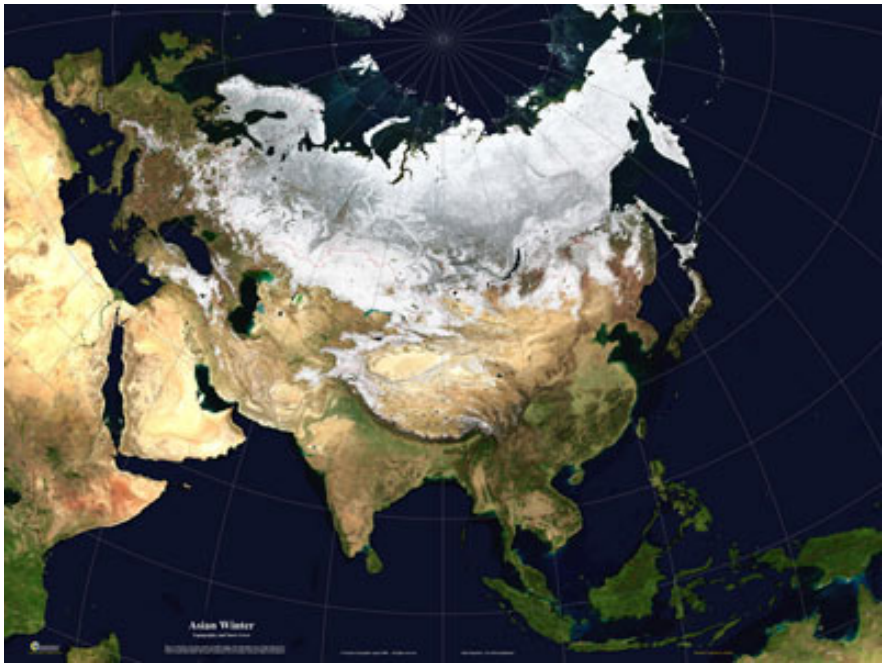


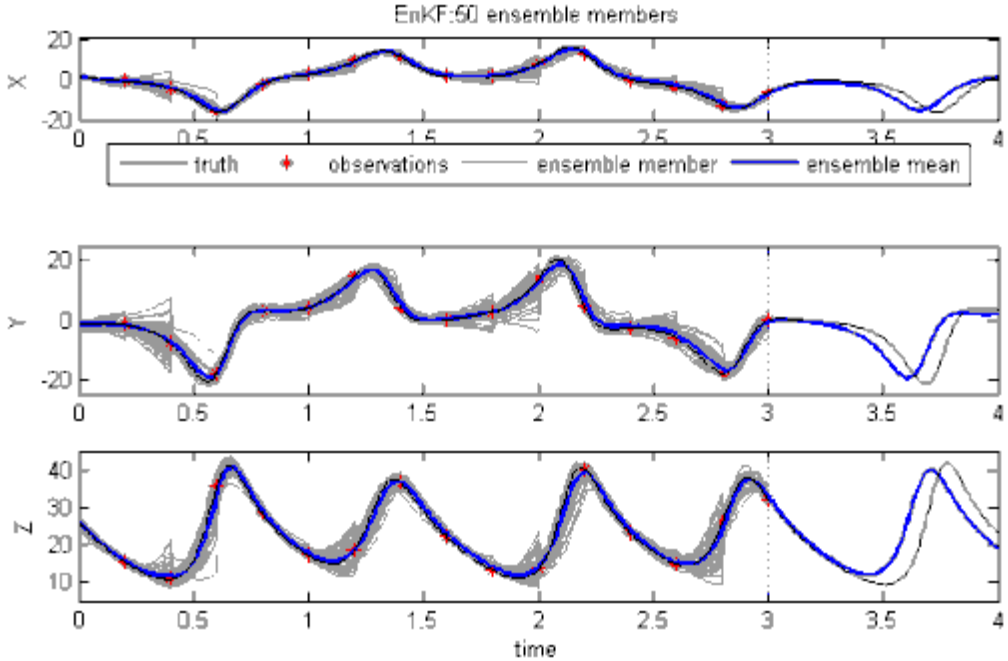
Project 5

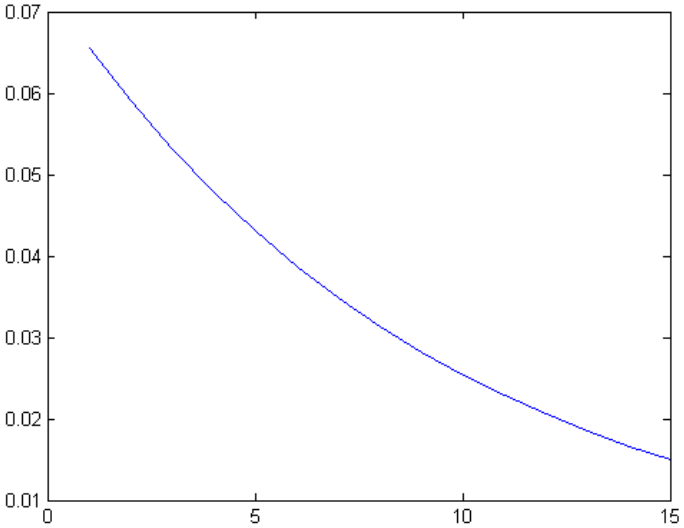
Effects of temporal correlation in observation errors using ensemble Kalman filtering

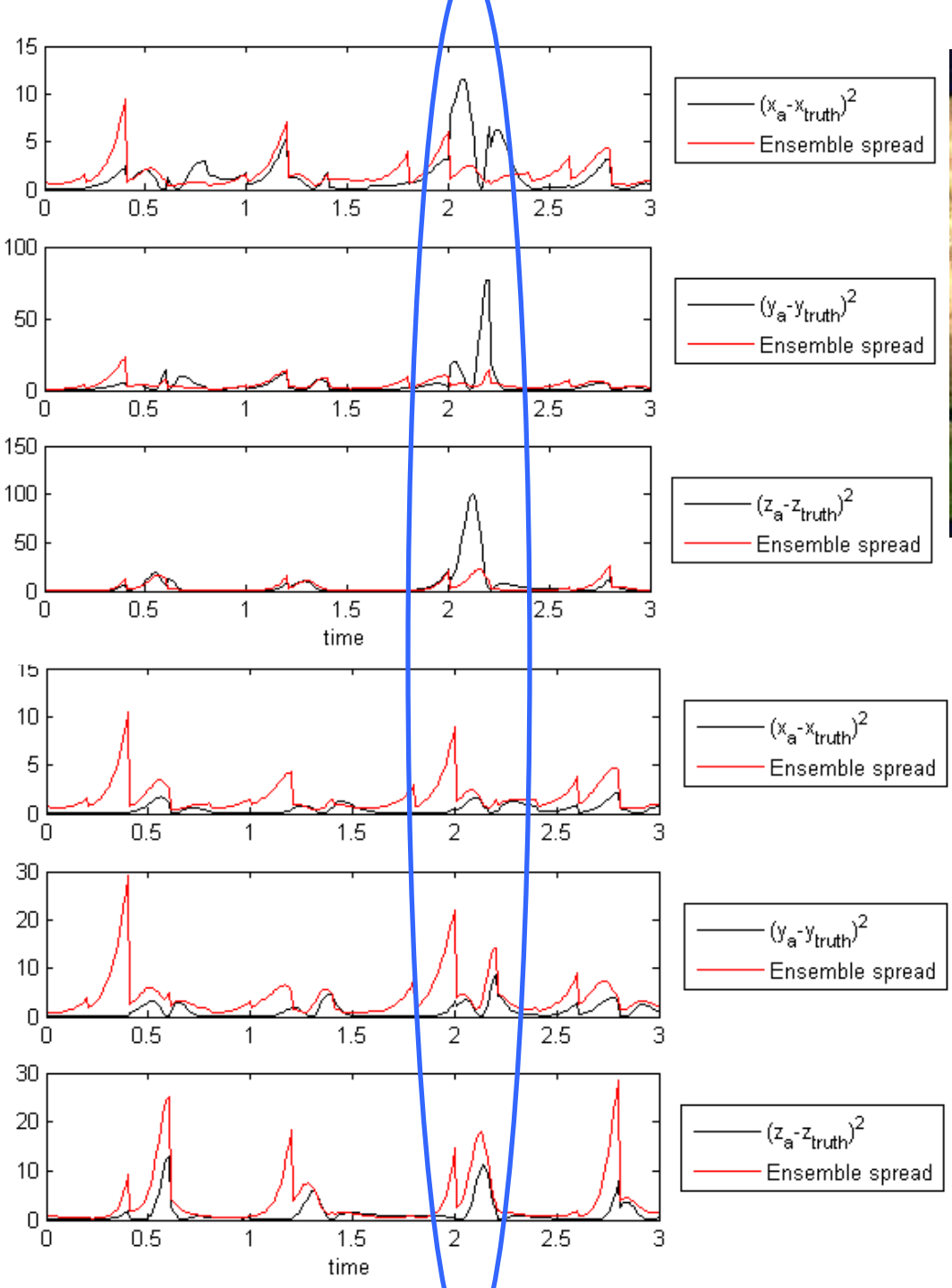
Jinfeng CHANG
Erin HESTIR
Jana KOLASSA
Paolo VILLA

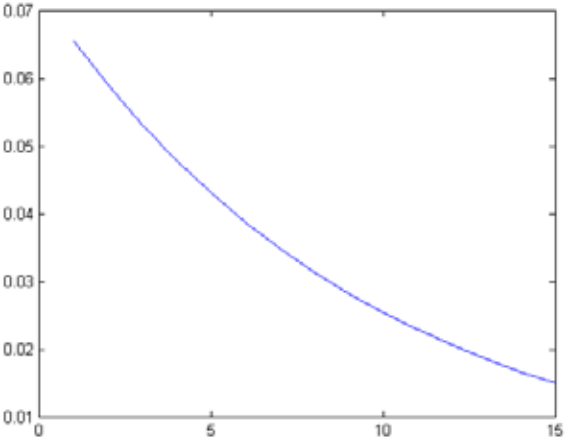

EnKF:50 ensemble members
X
Y
Z
time
truth
observations
ensemble member
ensemble mean




EnKF:50 ensemble members
X
Y
Z
time
truth
observations
ensemble member
ensemble mean

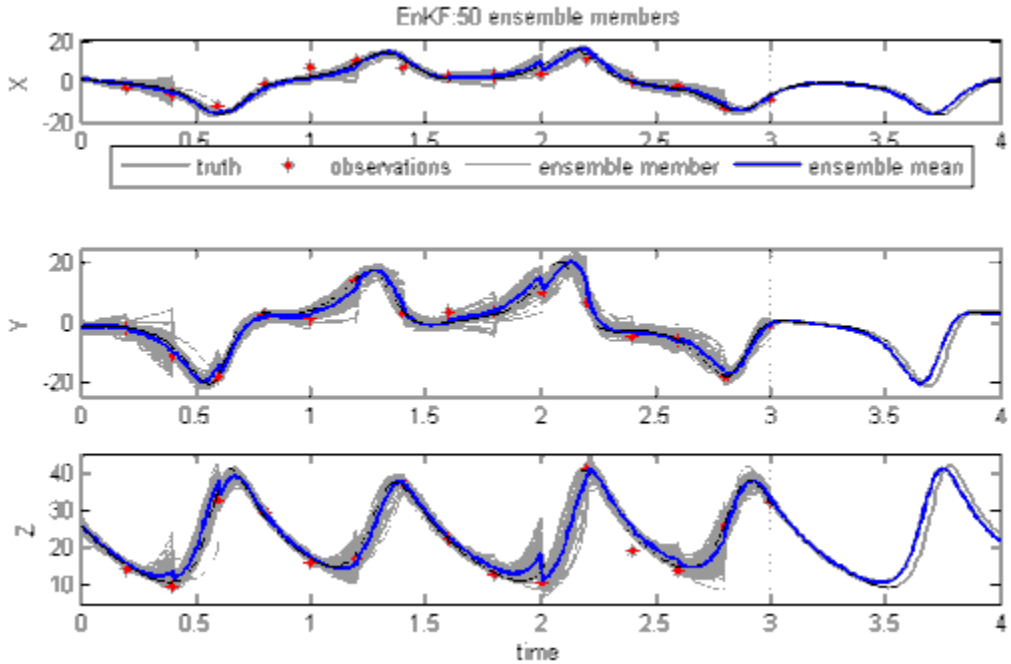


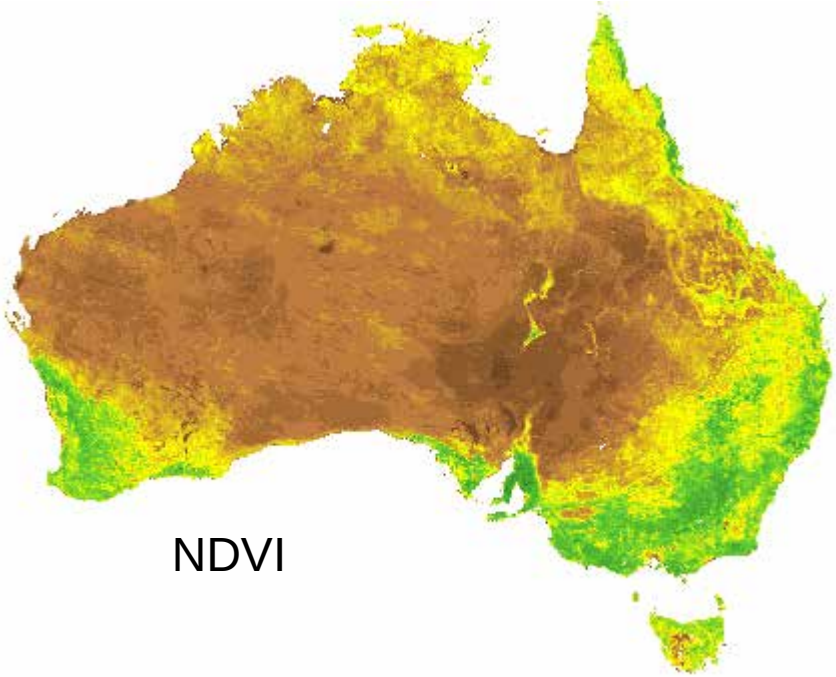

$(x_a-x_{truth})^2$
Ensemble spread
$(y_a-y_{truth})^2$
Ensemble spread
$(z_a-z_{truth})^2$
Ensemble spread
time
$(x_a-x_{truth})^2$
Ensemble spread
$(y_a-y_{truth})^2$
Ensemble spread
$(z_a-z_{truth})^2$
Ensemble spread
time

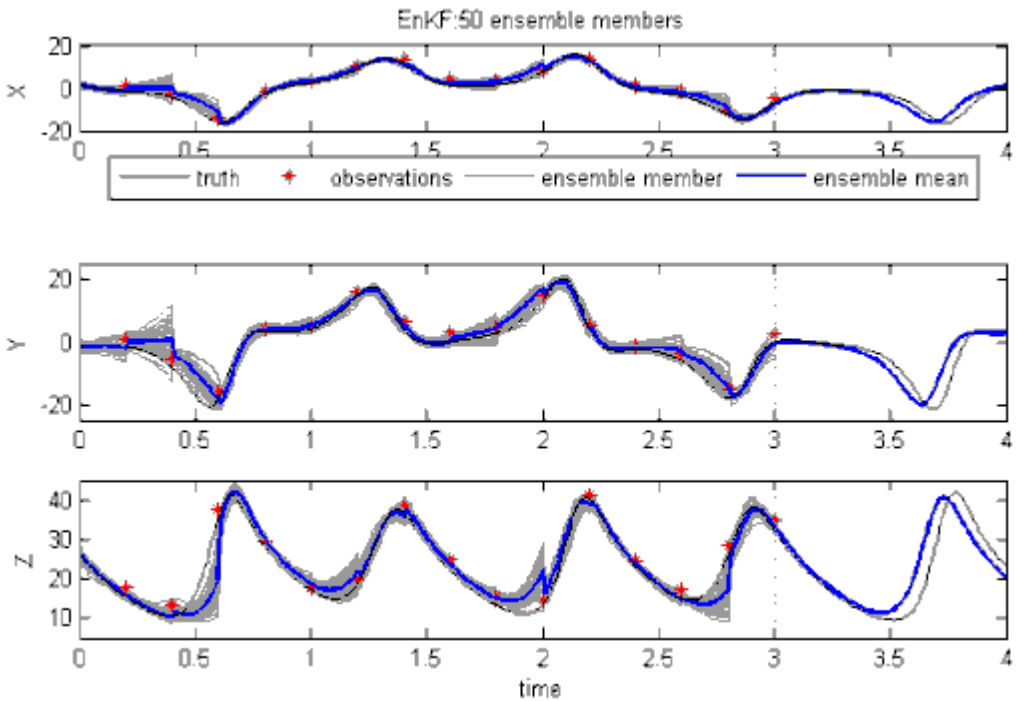


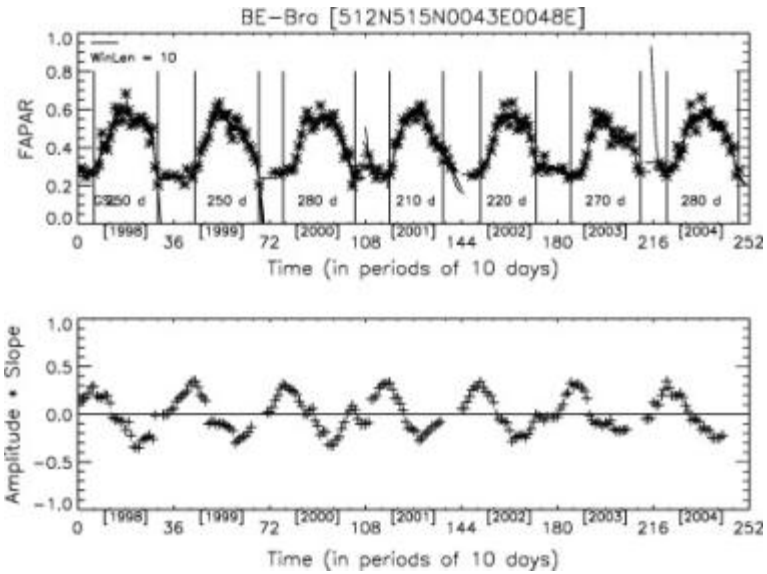
Conclusions

- Temporally correlated observation errors improved analysis
  - Observations provided constraints
- May be useful when forecasting discontinuities


EnKF:50 ensemble members
X
Y
Z
time
truth
observations
ensemble member
ensemble mean


NDVI


EnKF:50 ensemble members
X
Y
Z
time
truth
observations
ensemble member
ensemble mean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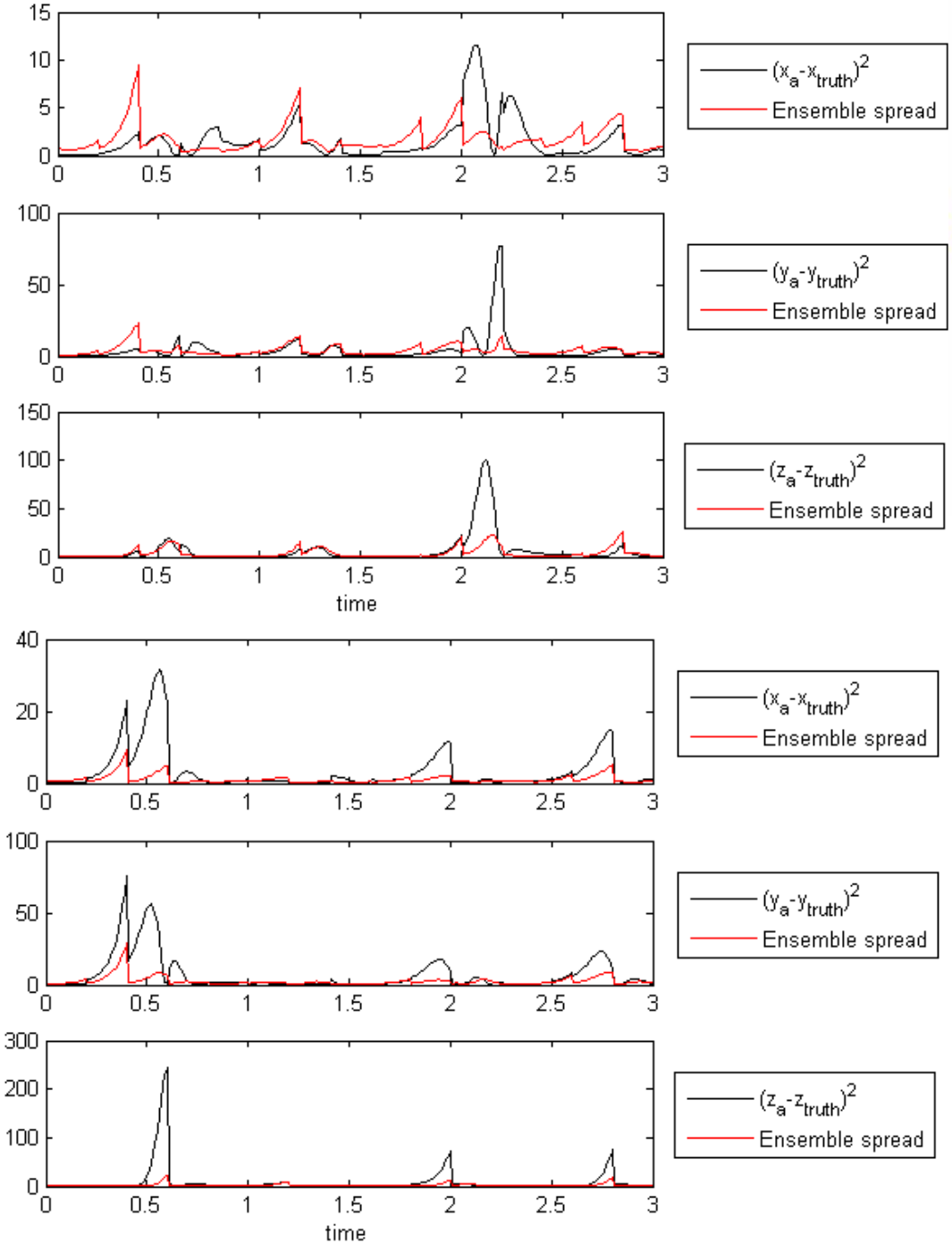
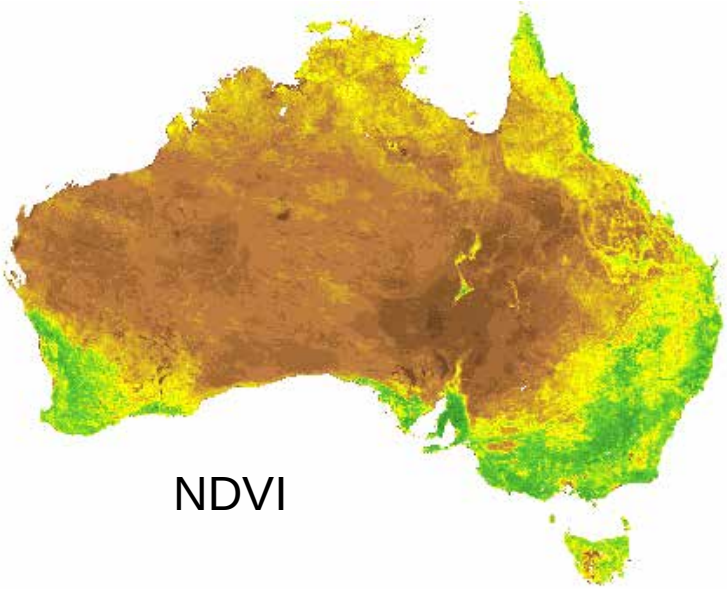
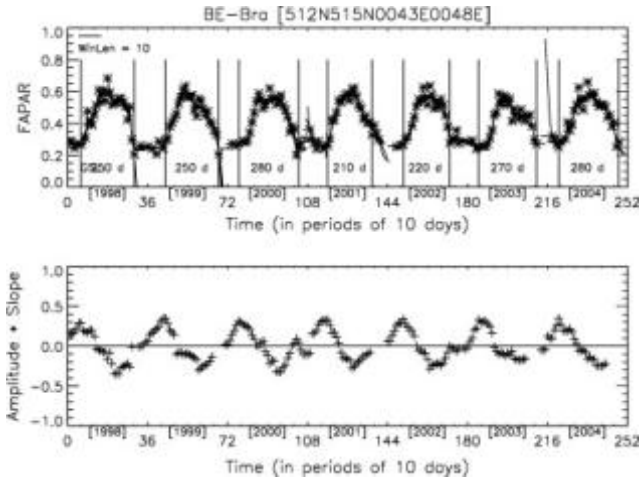
WinLen = 10
FAPAR
Time (in periods of 10 days)
Amplitude * Slope
Time (in periods of 10 days)


$(x_a-x_{truth})^2$
Ensemble spread
$(y_a-y_{truth})^2$
Ensemble spread
$(z_a-z_{truth})^2$
Ensemble spread
time
$(x_a-x_{truth})^2$
Ensemble spread
$(y_a-y_{truth})^2$
Ensemble spread
$(z_a-z_{truth})^2$
Ensemble spread
time


NDVI

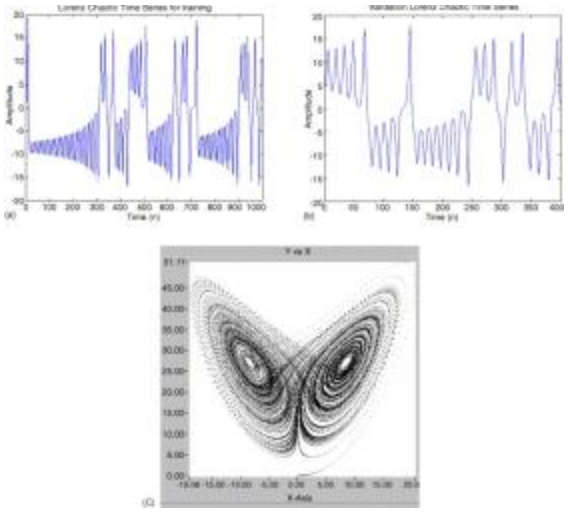

BE-Bra [512N515N0043E0048E]
FAPAR
Time (in periods of 10 days)
Amplitude * Slope
Time (in periods of 10 days)

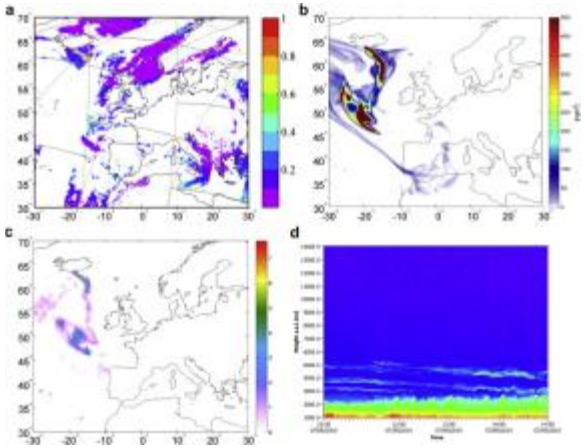
Conclusions

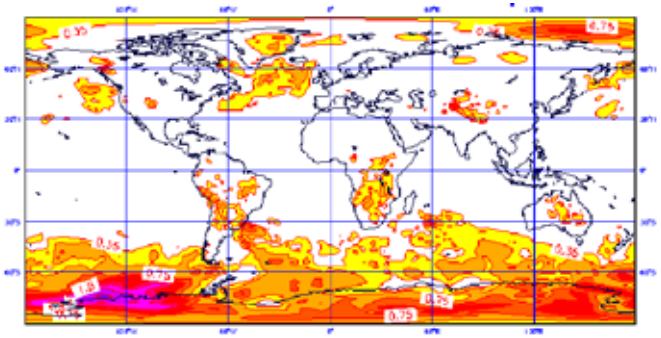
- Ensemble spread is reduced with <u>high</u> temporally correlated observation errors
- Temporally correlated errors do not improve longer forecasting

Investigate the effect of time correlate observation error in 4D-VAR

Jana Souckova, Jana Slacikova, Ruslan Zhuravlev & Cristian Mattar







Introduction

- Time Correlated observations are one of the most difficult and key variables in forecast modelling and numerical wethear prediction models

- Time Correlated observations are often related to Atmospheric profile data assimiliation which can be assumed non correlated.

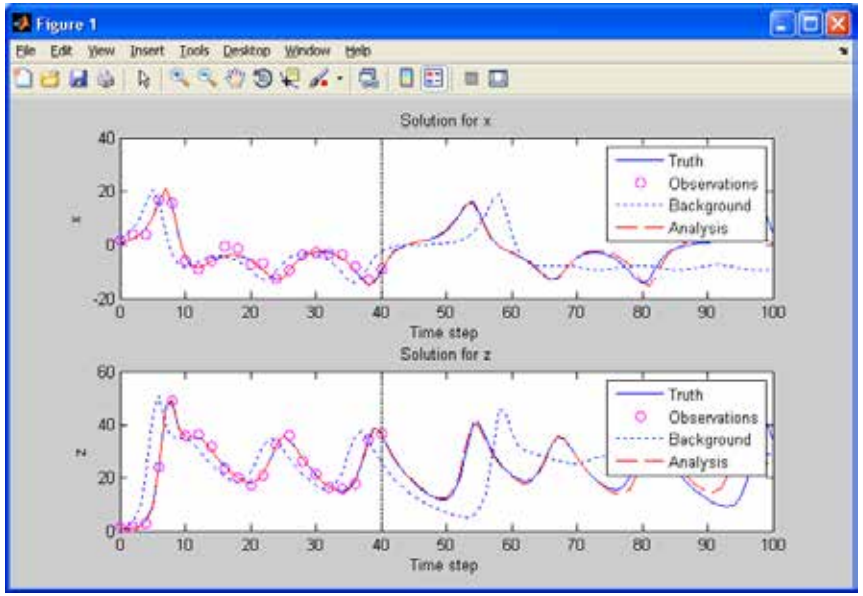
- Error correlation in 4D data base are analyzed in Weston (2011, Metoffice Report).

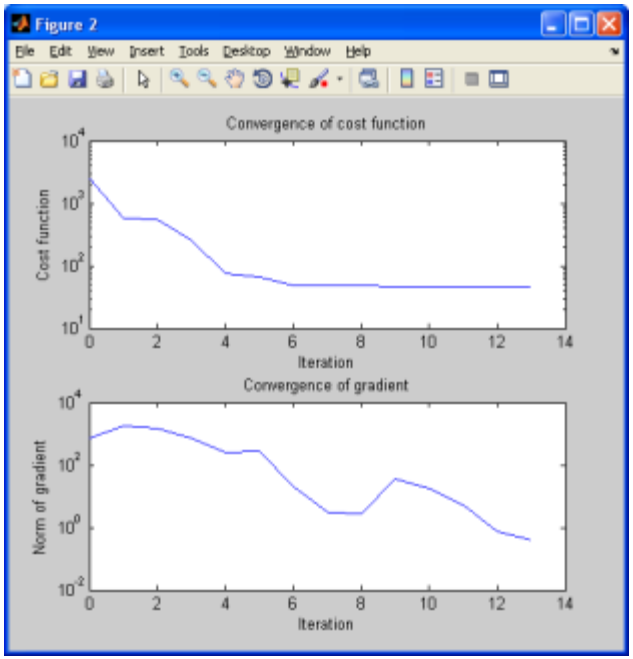
Method

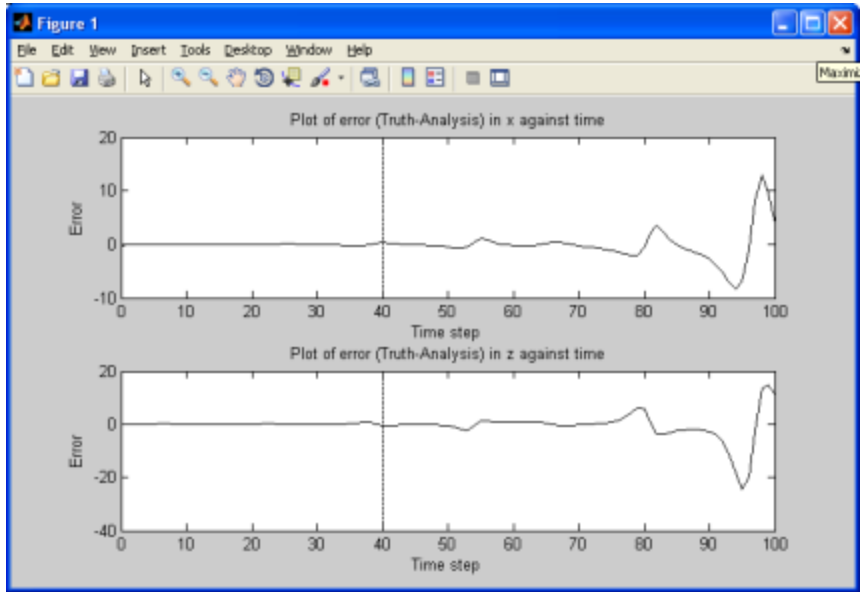
- Generate the real state of the simulation by using initial parameters
- Insert a time correlation in the model by using the following fixed equations.
    - Random
    - Fixed Sinusoidal function
    - Fixed Cosin function
    - Fixed in 1.5
    - x=1.5 y= 0.5 z = 1.0

Results

- Random

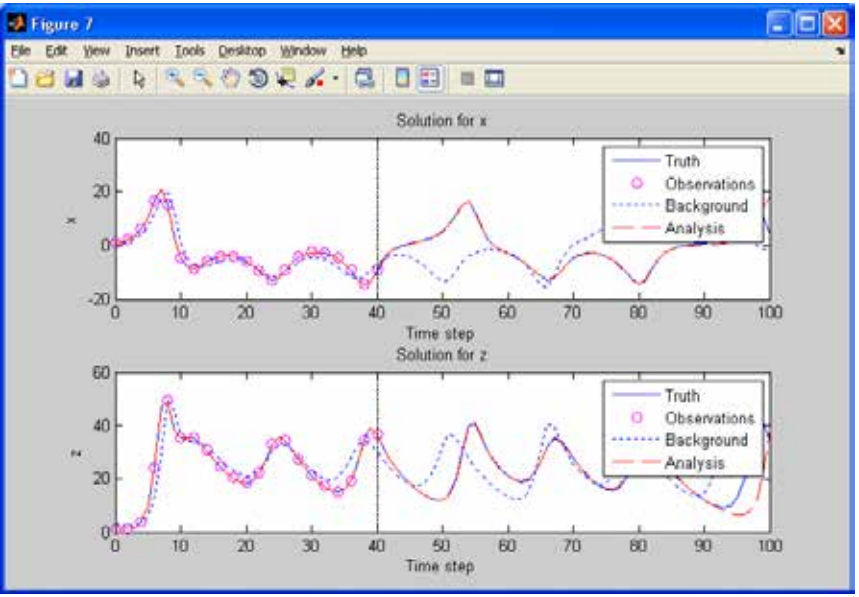


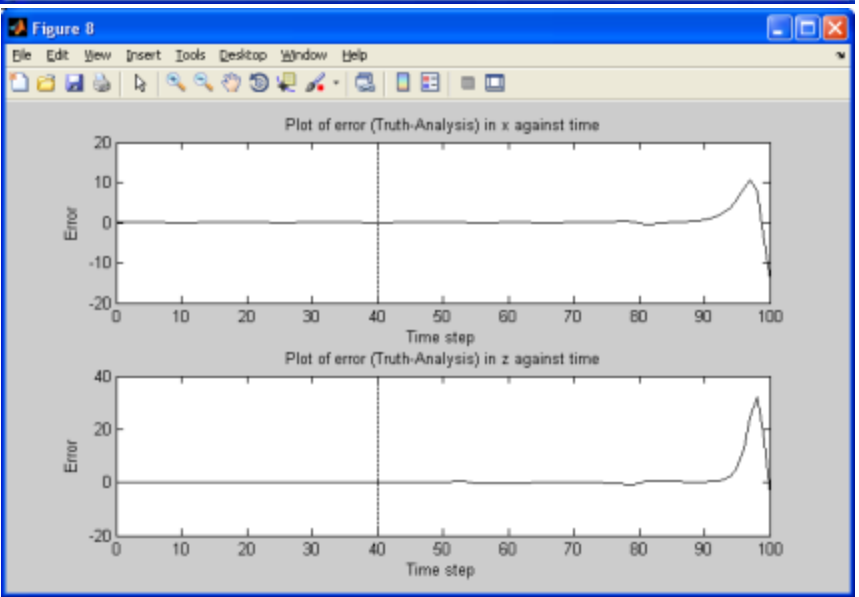


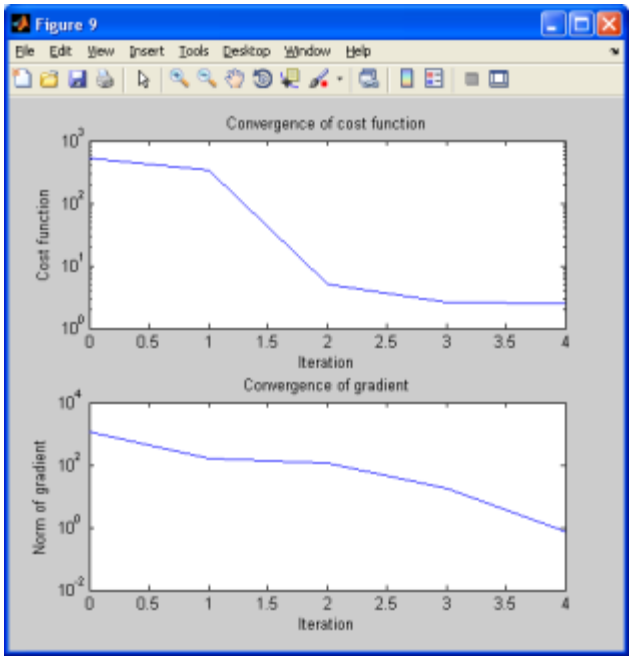


Results

- Fixed Sinusoidal function

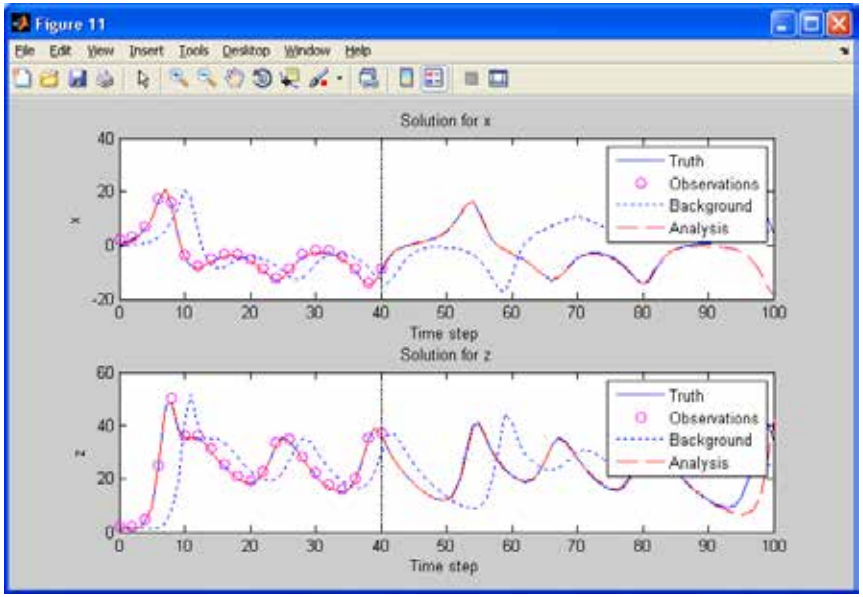


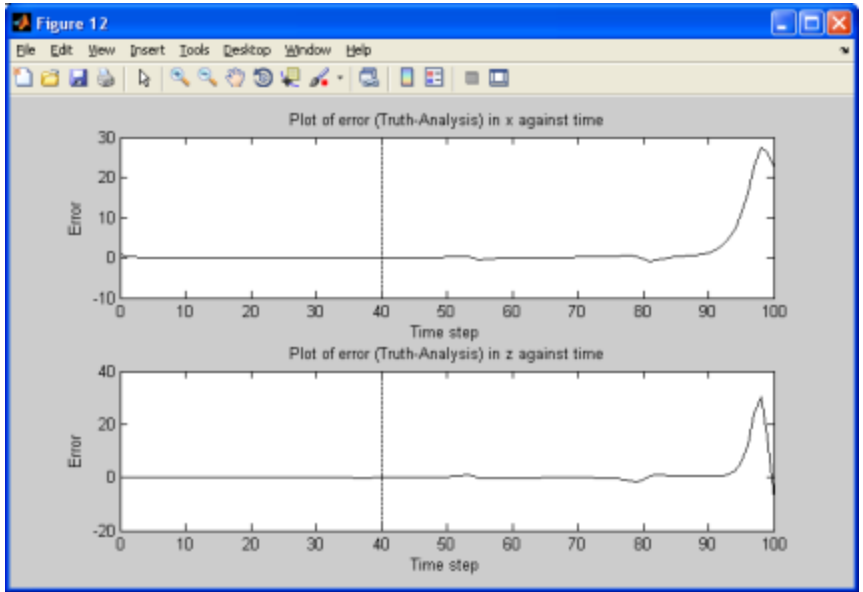


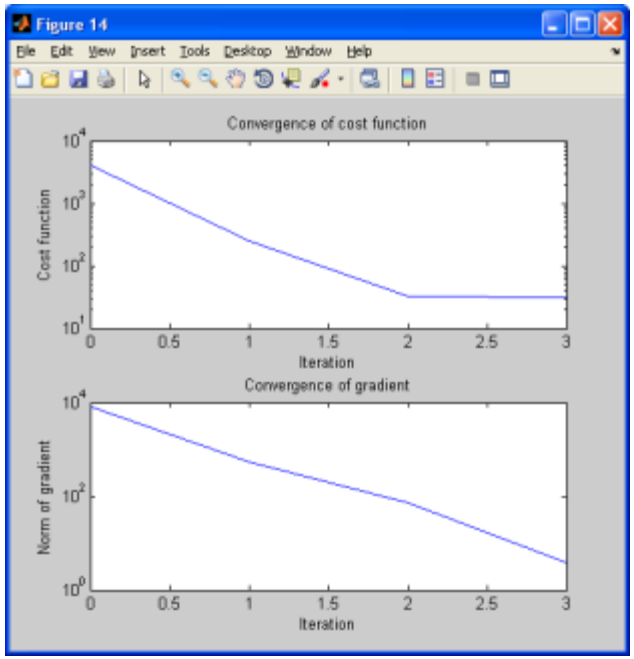


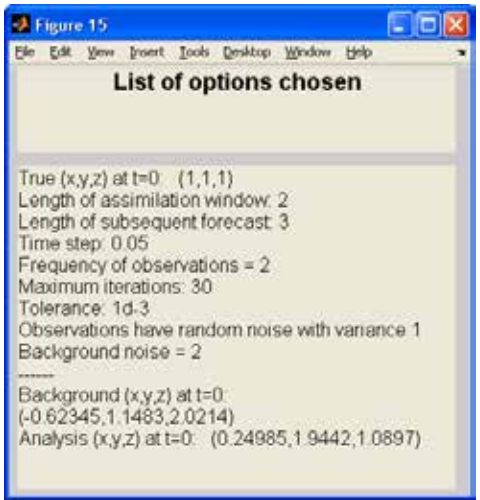
Results

- Fixed Cosin function



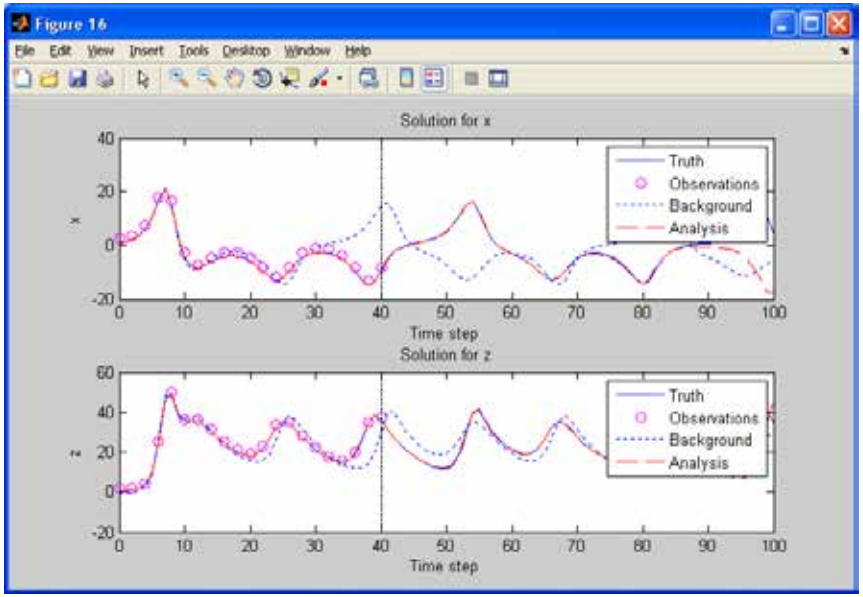


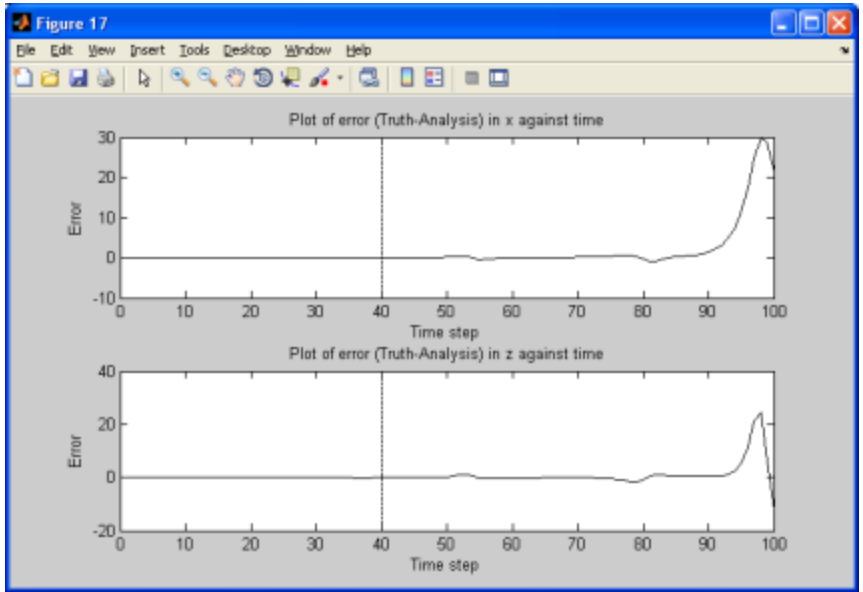


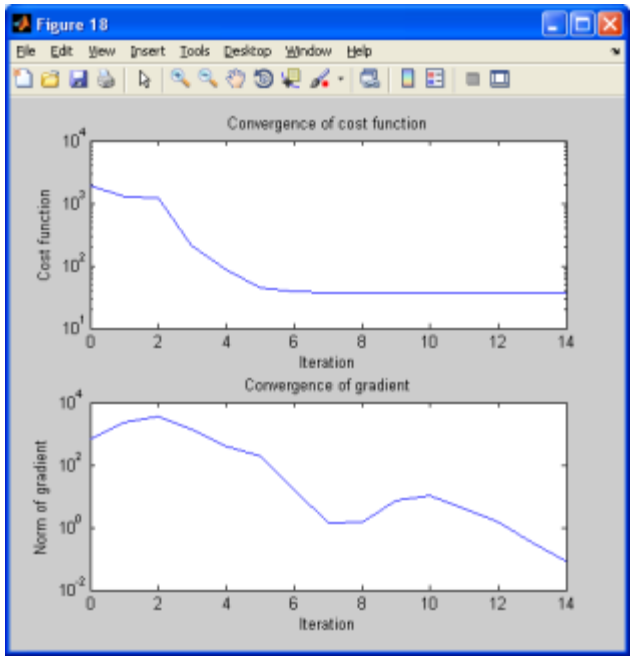


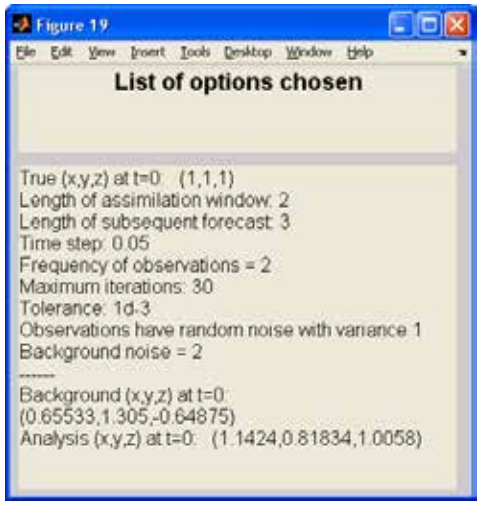
Results

- Fixed in 1.5



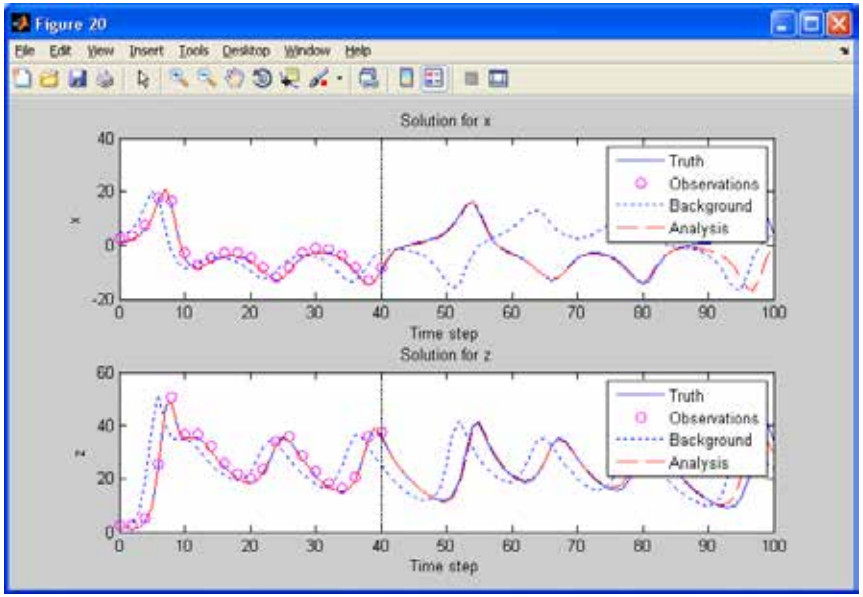


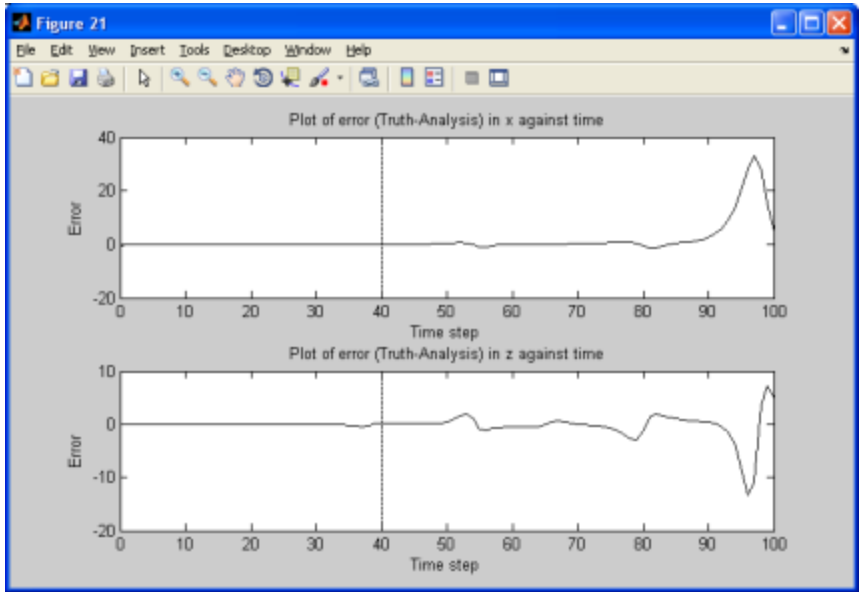


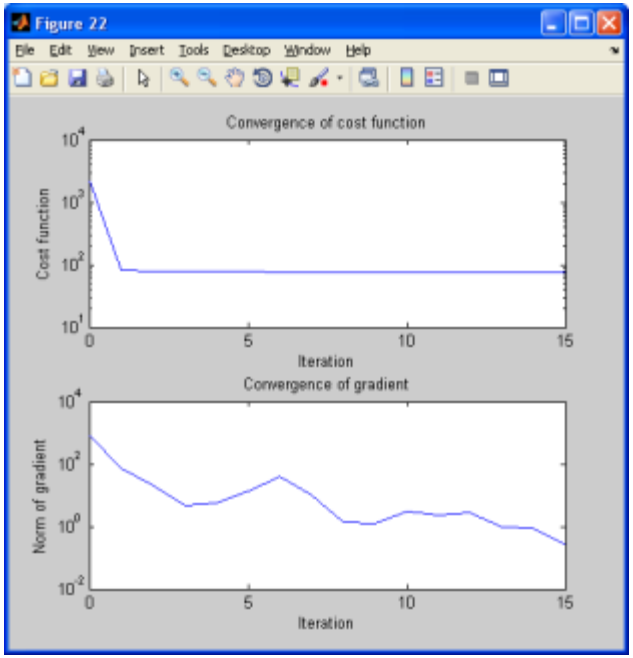


Results

- x=1.5 y= 0.5 z = 1.

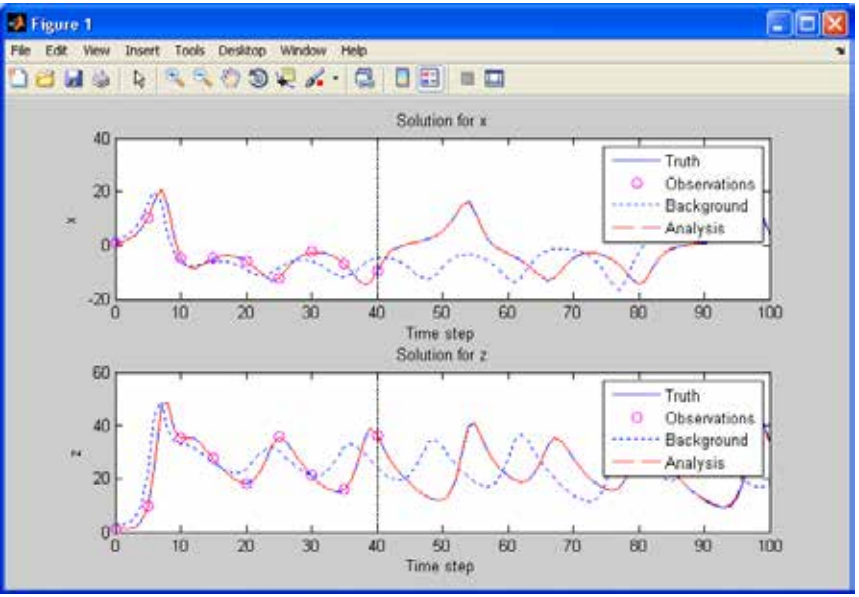


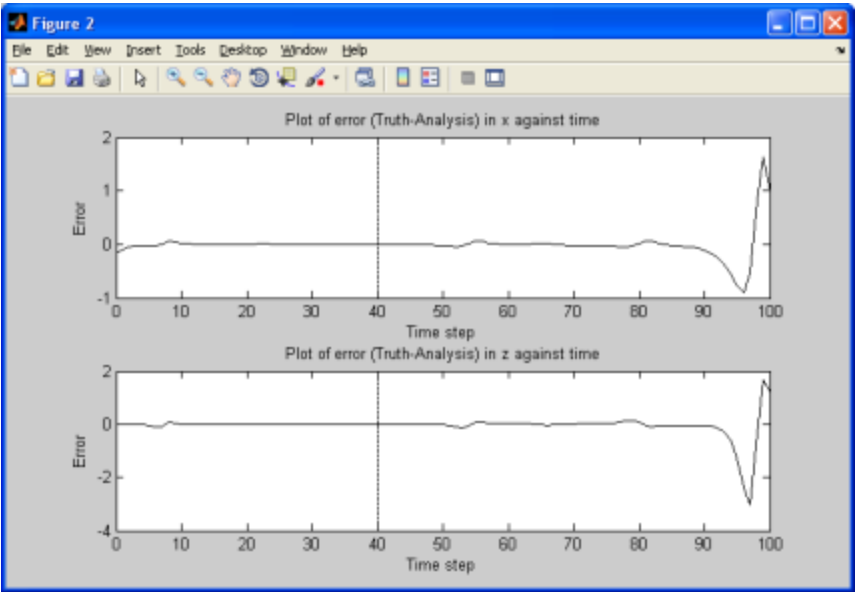


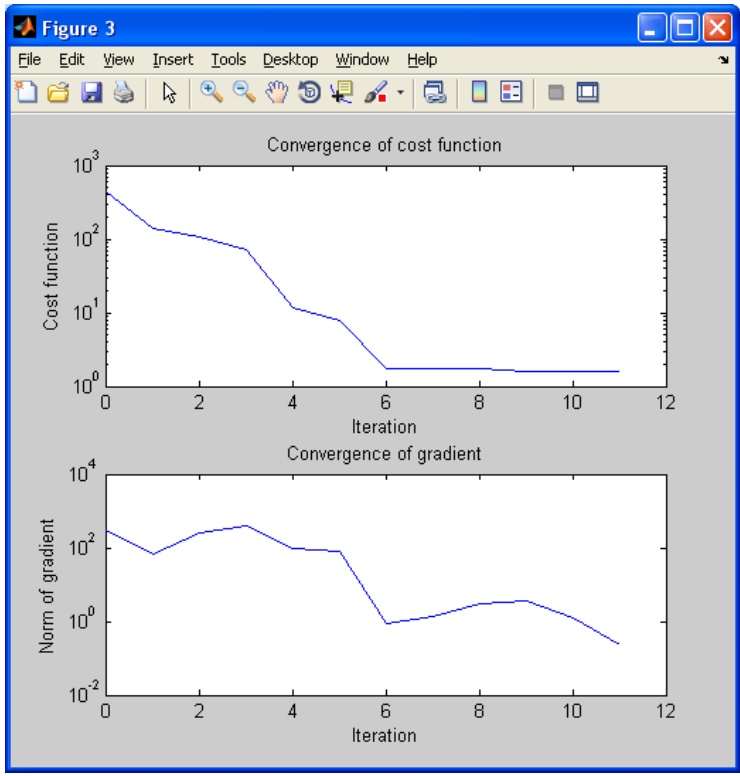


Results

- Sinusoidal function (different Num. obs)

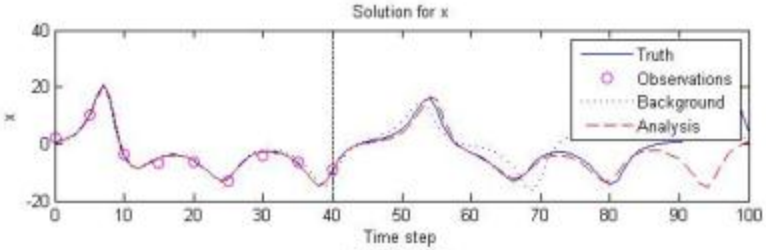


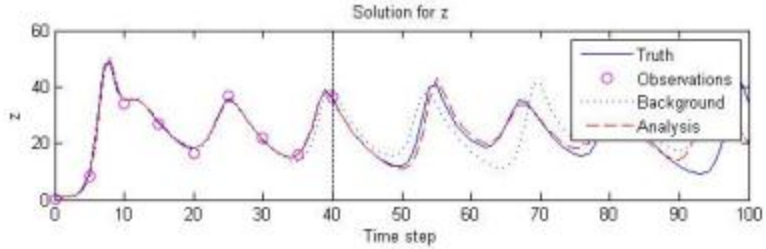


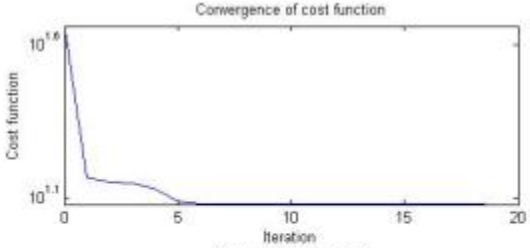


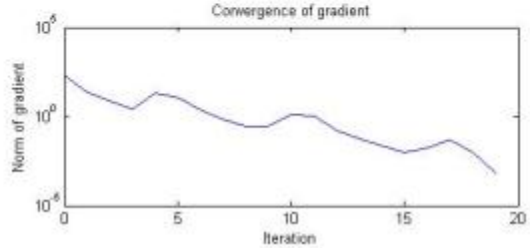
Results

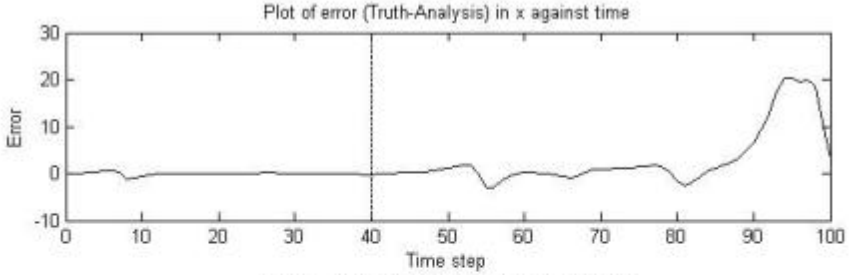
- Random (different Num. obs)

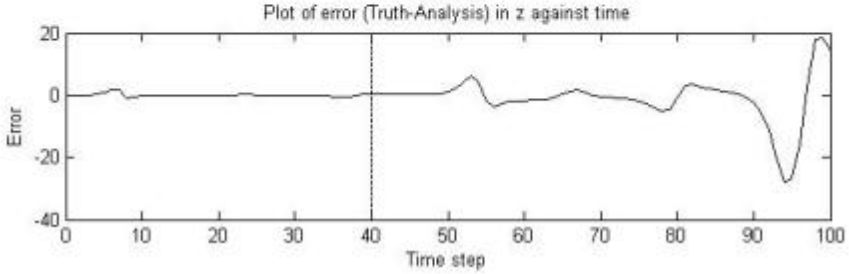












List of options chosen

True (x,y,z) at t=0: (1,1,1)
Length of assimilation window: 2
Length of subsequent forecast: 3
Time step: 0.05
Frequency of observations = 2
Maximum iterations: 30
Tolerance: 1d-3
Observations have random noise with variance 1
Background noise = 2

Background (x,y,z) at t=0:
(-0.12877,2.4406,0.8116)
Analysis (x,y,z) at t=0: (0.86974,1.0592,1.0397)

Conclusions and further research

- Based on the simulations, the information about time correlation in data assimilation systems could be better than non time correlated in some particular cases.
- However, time correlation could not be easy to apply in physical models and in most of cases for 4-D var models.
- Therefore, time correlated observations errors have to be excluded before the data assimilation.
- Finally, further research have to be carried out in order to develop robuts analysis in time correlated observations error and random assumptions.

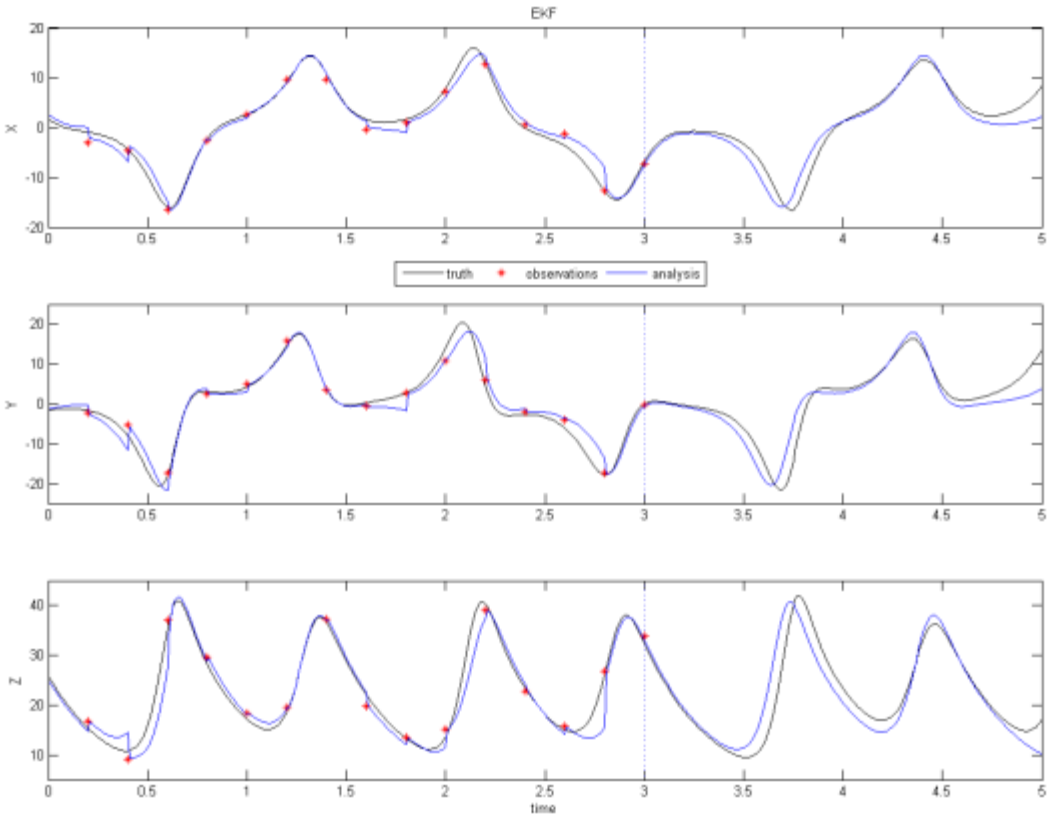
Project 7: Investigate the effect of biased observations in the extended Kalman filter.

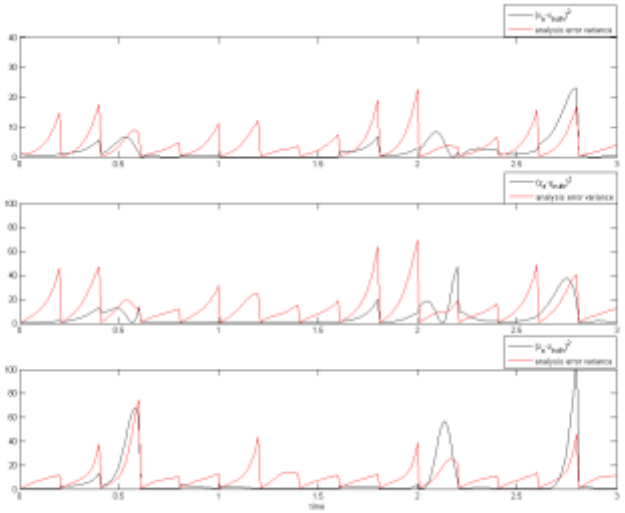
Susanne Schnitzer, Alka Singh,
Christopher Allen, Christopher Straif

Outline

- Reference – no bias
- Introduce a bias that is constant in time
- Introduce a bias that increases linearly over time
- Increase the observation sampling rate

Reference – no bias

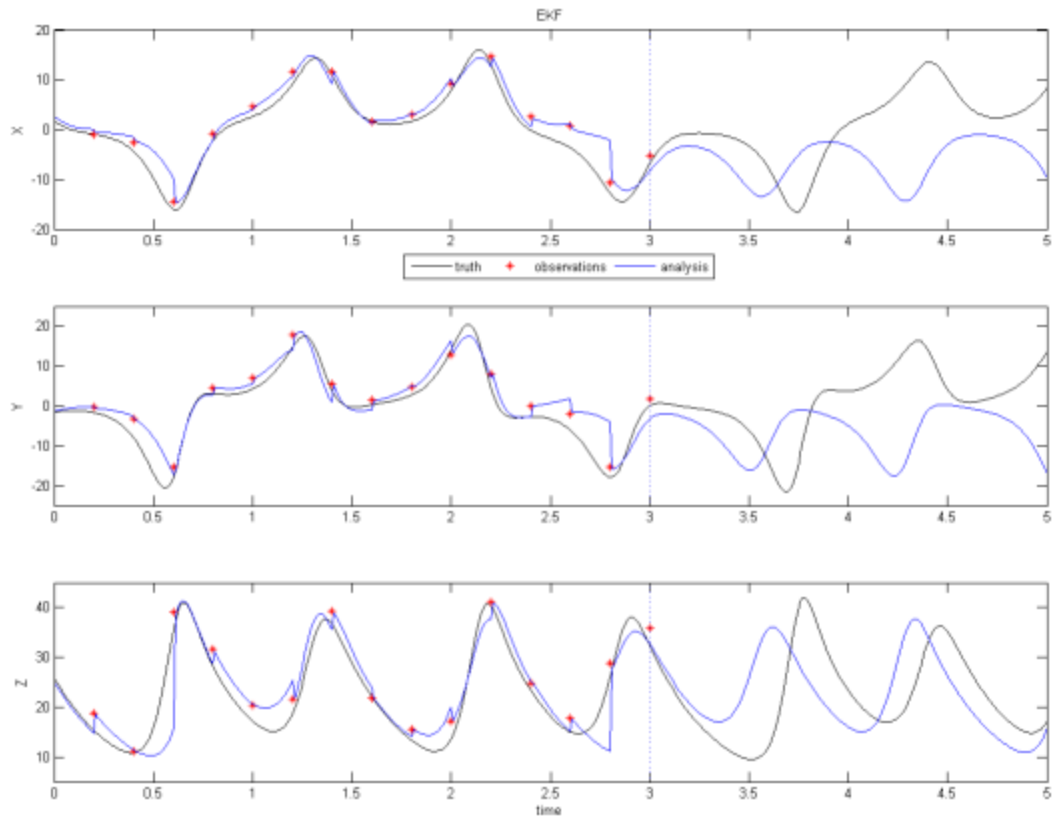


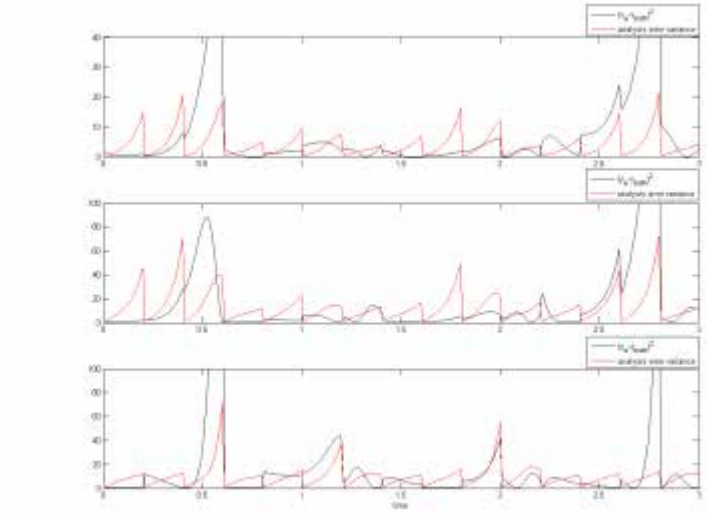


List of options chosen for the Extended Kalman filter

True (x,y,z) at t=0: (1.5,-1.5,25.5)
Length of assimilation window: 300
Length of subsequent forecast: 200
Time step: 0.01
Frequency of observations = 20
Observations have random noise with variance 1
Background noise = 1

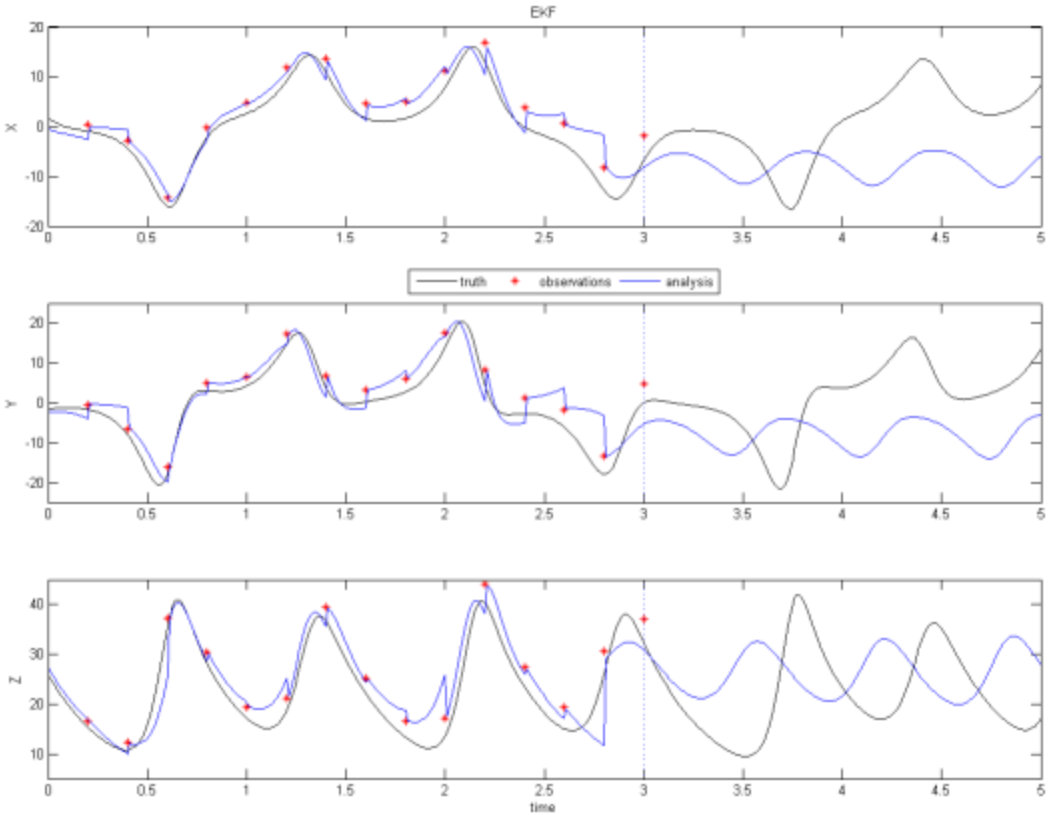
Introduce a bias that is constant in time

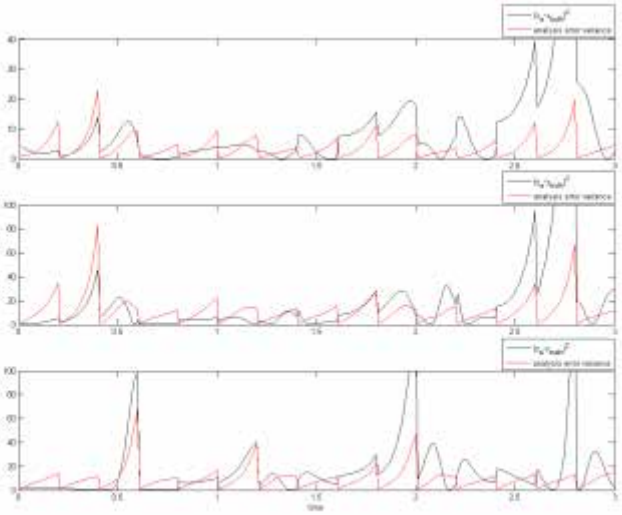




Biased +2

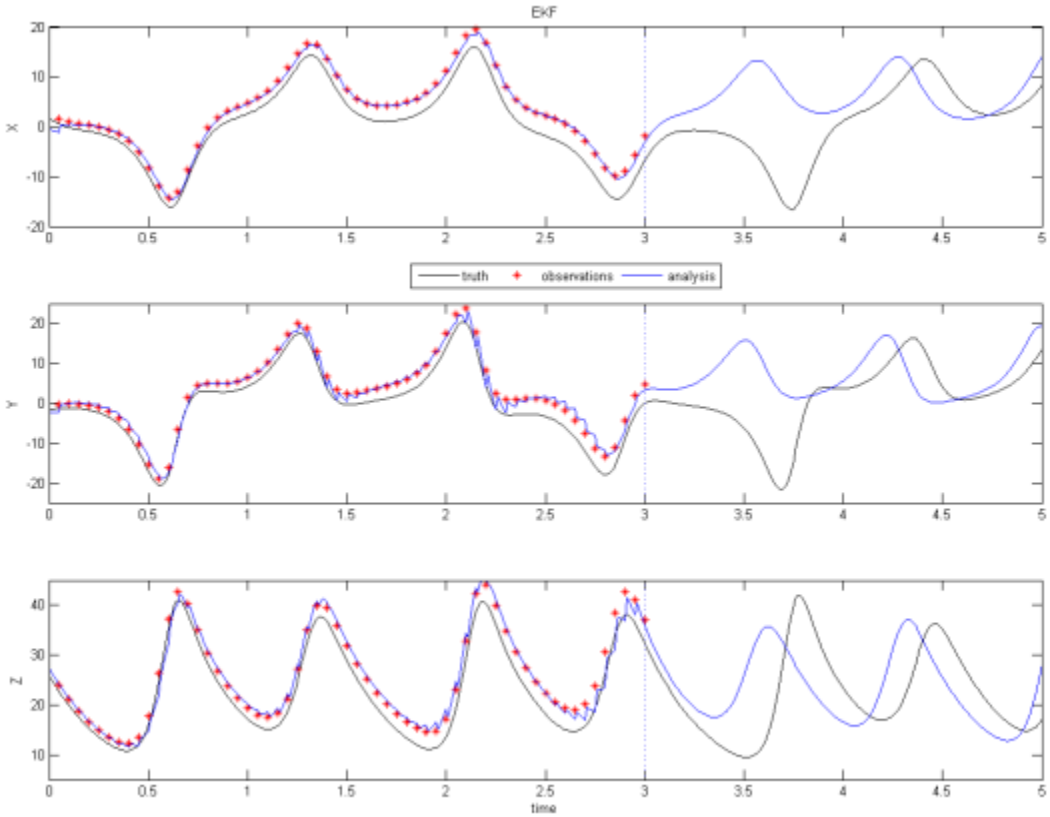
Introduce a bias that increases linearly over time

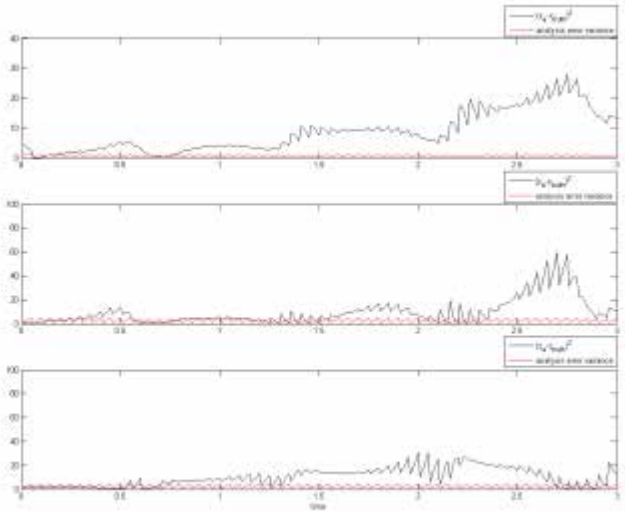




Linear biased +0.25 for every time step

Increase the observation sampling rate



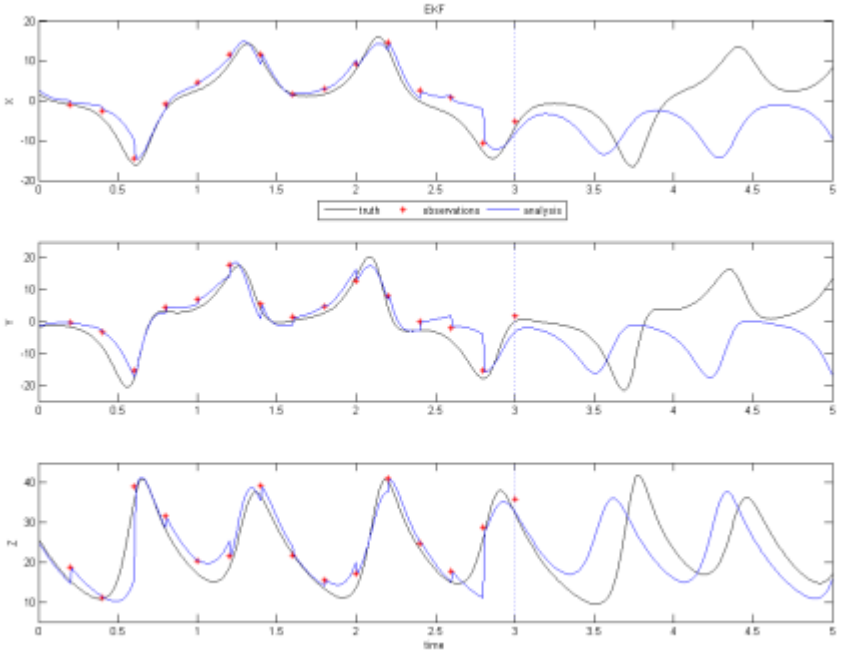


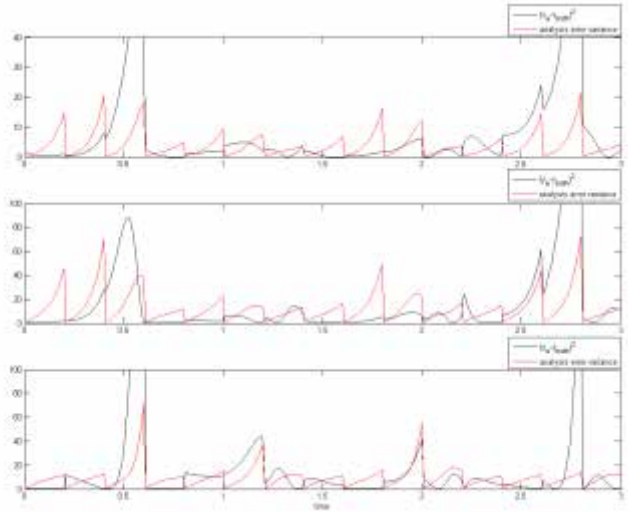
Linear biased +0.25 for every time step, 4x observations

Real world example

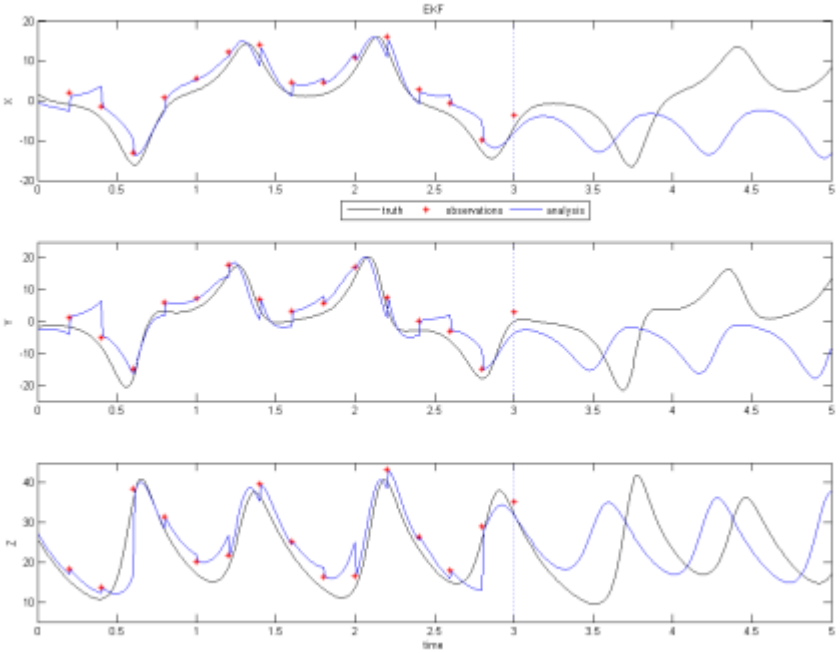
- Constant bias – calibration offset
- Linear bias – environmental degradation
- Harmonic bias – seasonal effects

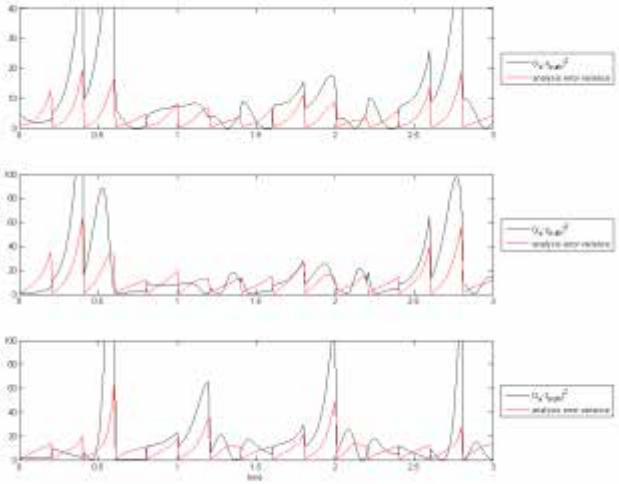
Random





Non random

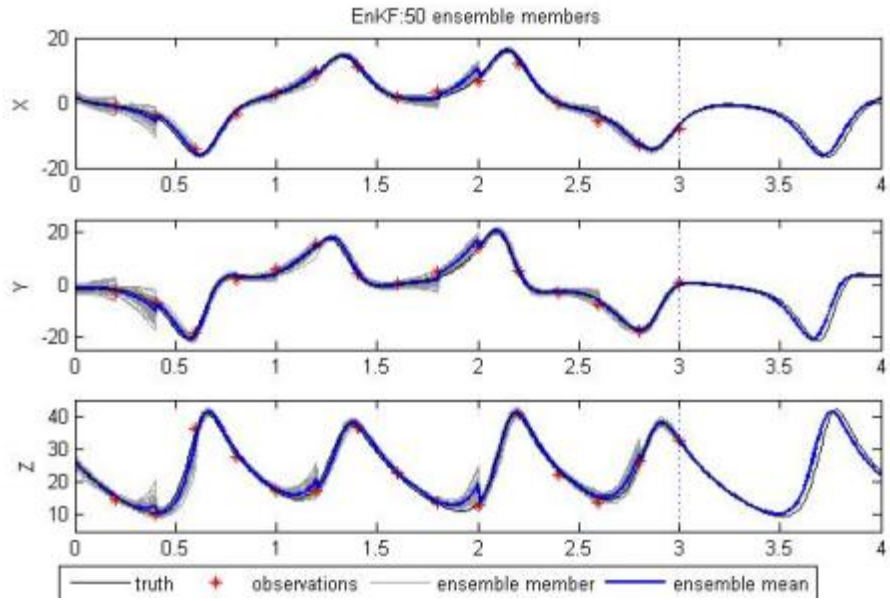


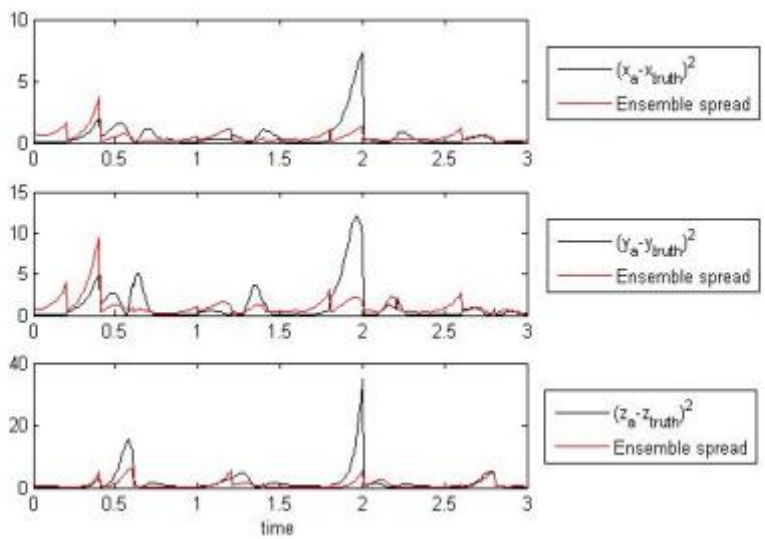


Project 8:
An investigation of biased observation error on the ensemble Kalman filter

Caroline H. Hardy
Sabrina Melchionna
Adam Chambers
Flavia Lenti

Unbiased Observations

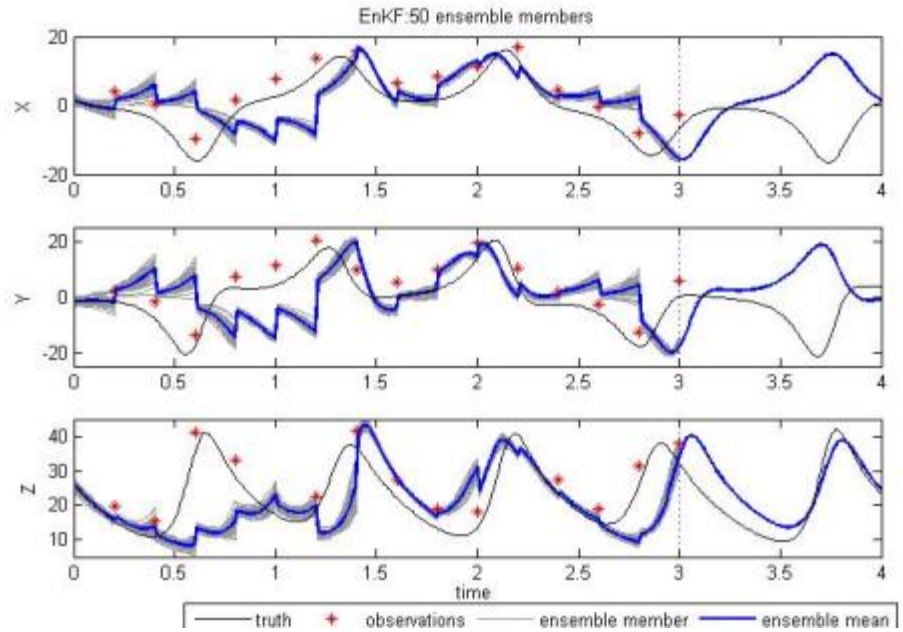


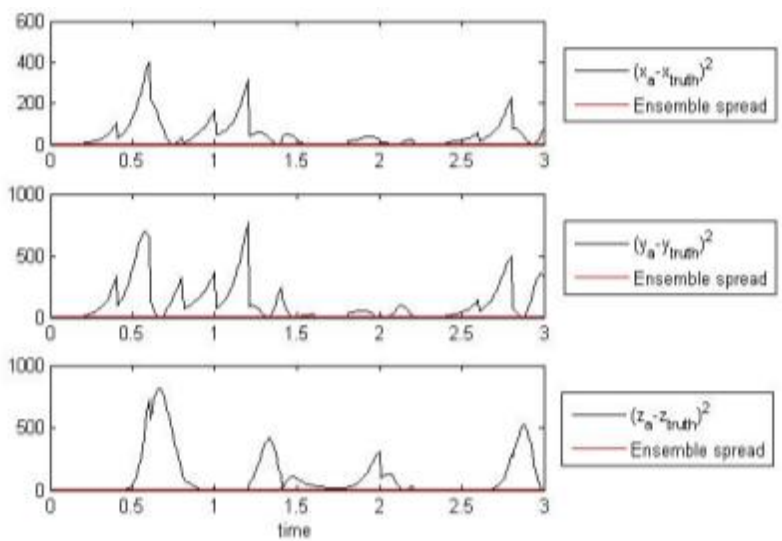


List of options chosen for the Ensemble Kalman filter

True (x,y,z) at t=0: (1.5,-1.5,25.5)
Length of assimilation window: 3
Length of subsequent forecast: 1
Time step: 0.01
Frequency of observations = 20
Ensemble size = 50
Observations have random noise with variance 1
Background noise = 1

Constant Bias [+5]

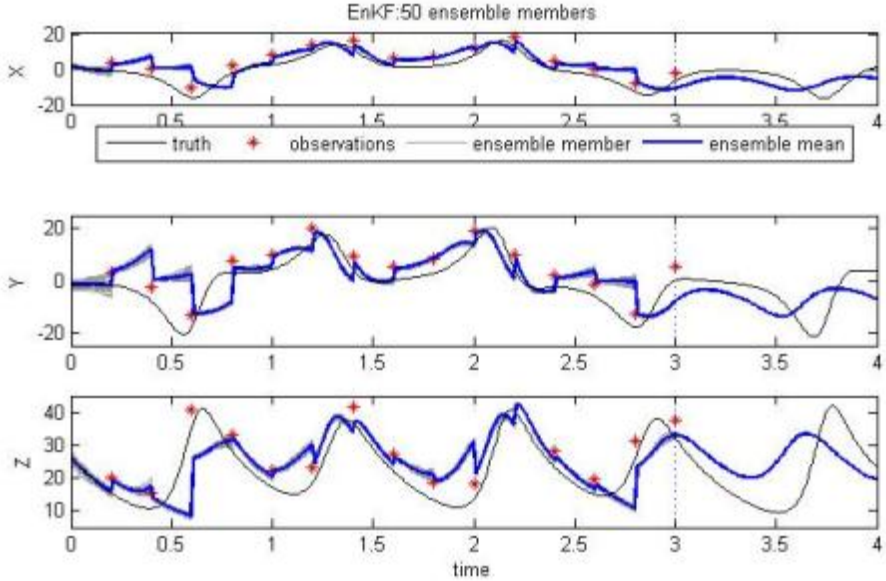


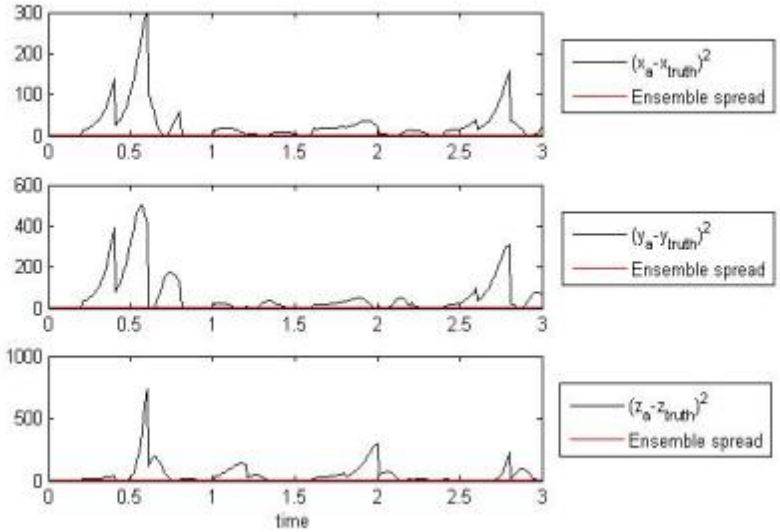


List of options chosen for the Ensemble Kalman filter

True (x,y,z) at t=0: (1.5,-1.5,25.5)
Length of assimilation window: 3
Length of subsequent forecast: 1
Time step: 0.01
Frequency of observations = 20
Ensemble size = 50
Observations have random noise with variance 1
Background noise = 1

Constant Bias [+5, obs var 0.1]

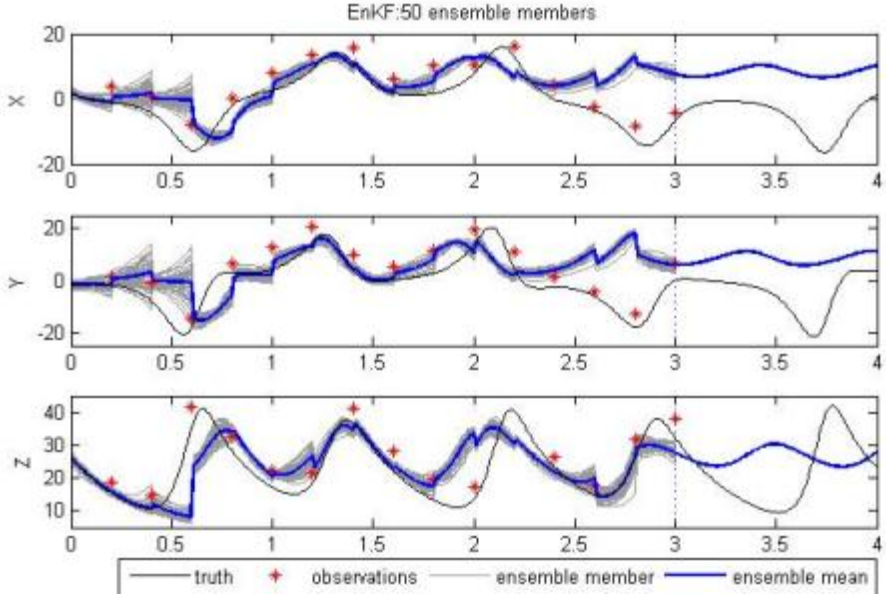


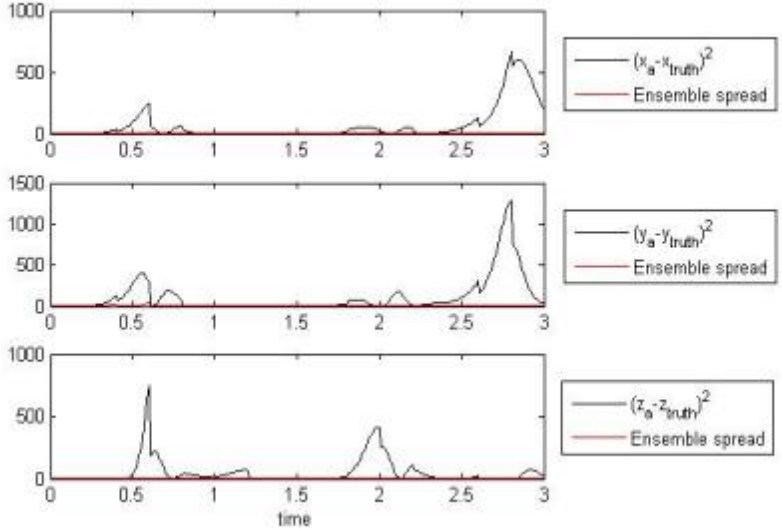


List of options chosen for the Ensemble Kalman filter

True (x,y,z) at t=0: (1.5,-1.5,25.5)
Length of assimilation window: 3
Length of subsequent forecast: 1
Time step: 0.01
Frequency of observations = 20
Ensemble size = 50
Observations have random noise with variance 0.1
Background noise = 1

Constant Bias [+5, obs var 4]

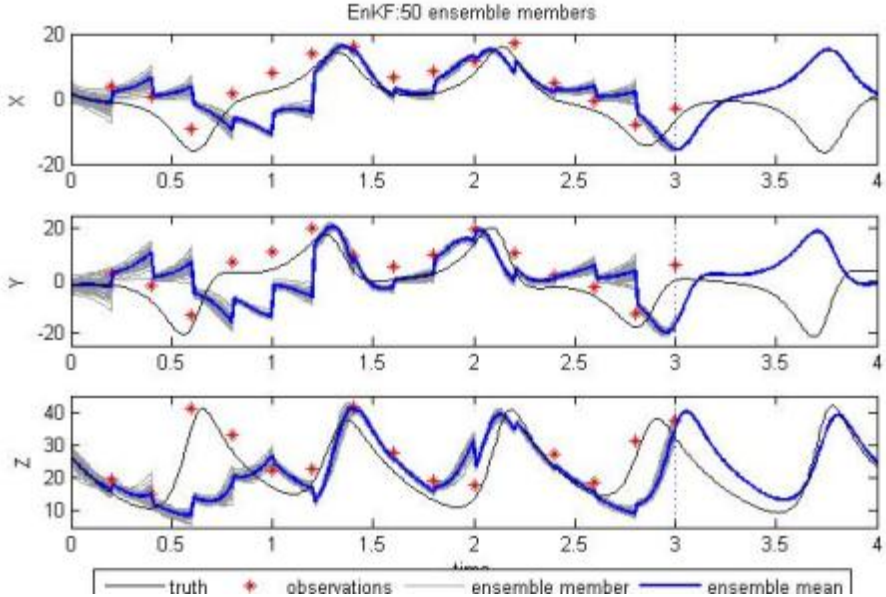


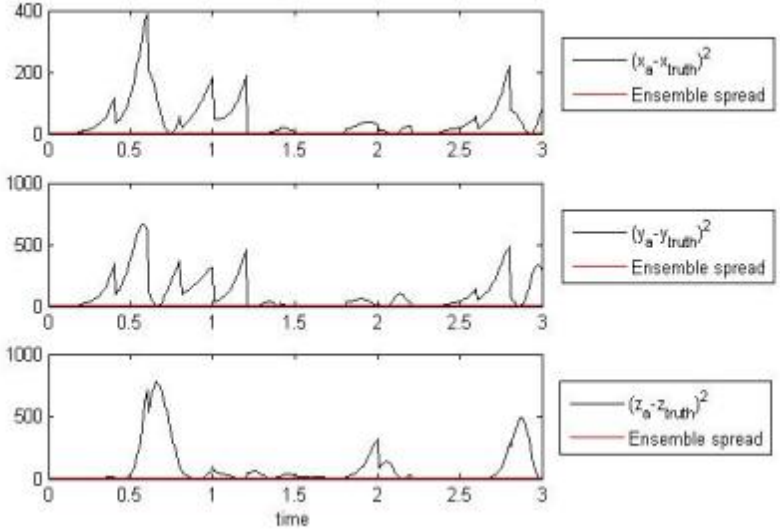


List of options chosen for the Ensemble Kalman filter

True (x,y,z) at t=0: (1.5,-1.5,25.5)
Length of assimilation window: 3
Length of subsequent forecast: 1
Time step: 0.01
Frequency of observations = 20
Ensemble size = 50
Observations have random noise with variance 4
Background noise = 1

Constant Bias [+5, bg var 4]

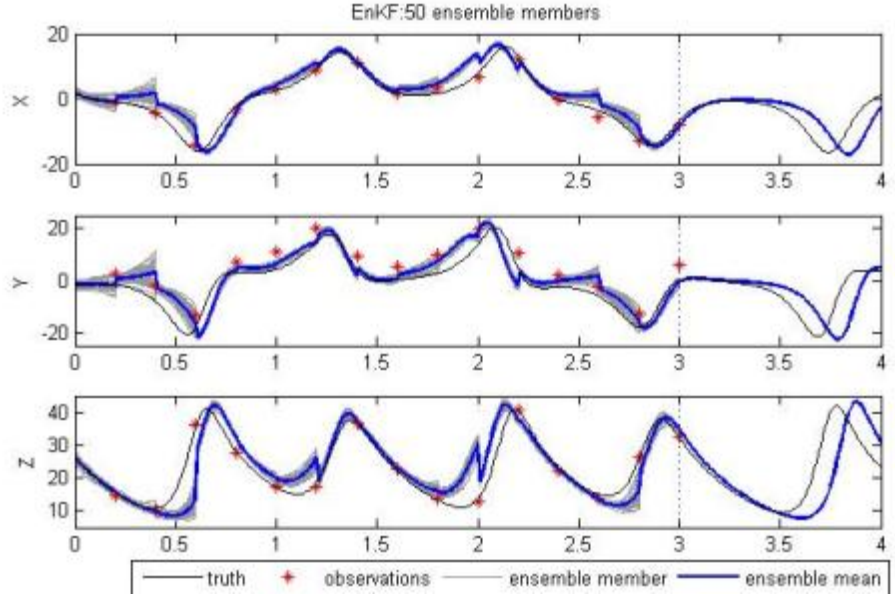


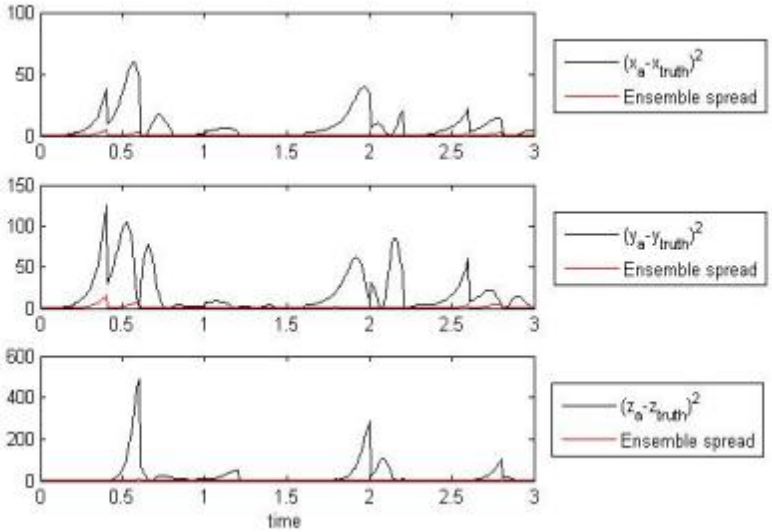


List of options chosen for the Ensemble Kalman filter

True (x,y,z) at t=0: (1.5,-1.5,25.5)
Length of assimilation window: 3
Length of subsequent forecast: 1
Time step: 0.01
Frequency of observations = 20
Ensemble size = 50
Observations have random noise with variance 1
Background noise = 4

Constant Bias in Y only [+5]

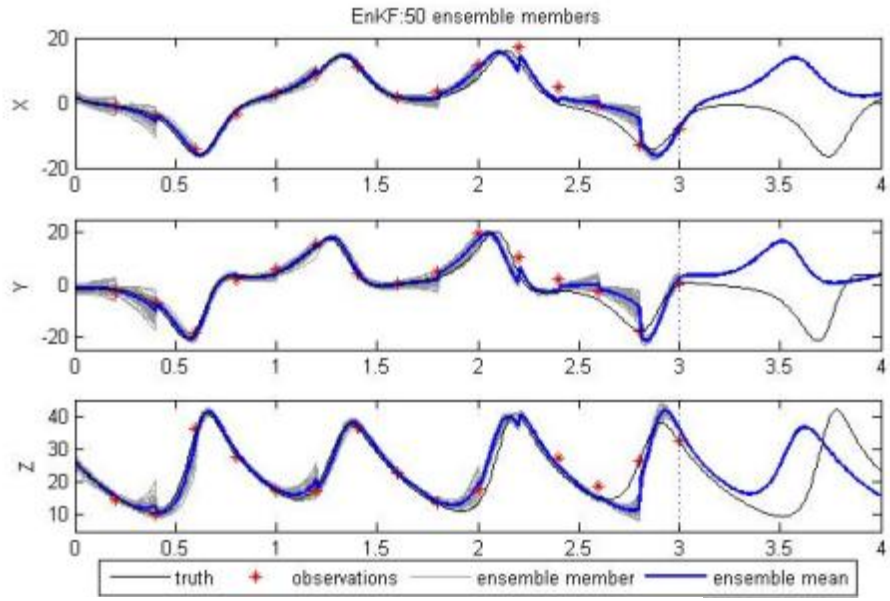


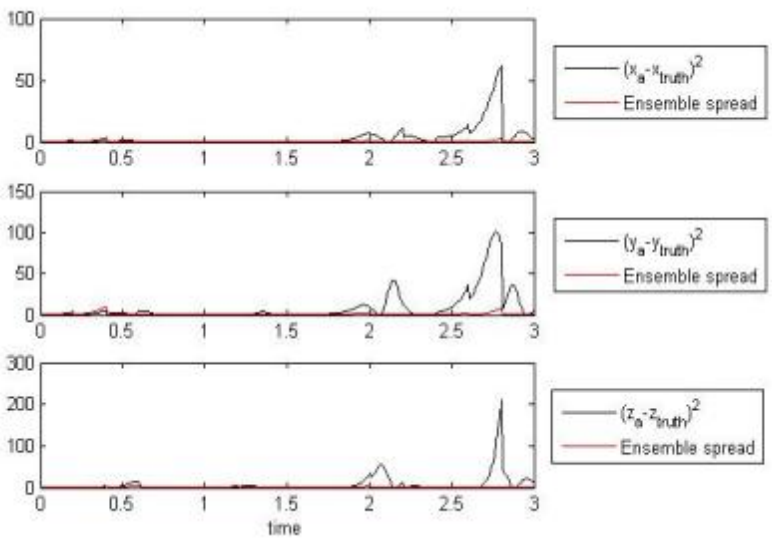


List of options chosen for the Ensemble Kalman filter

True (x,y,z) at t=0: (1.5,-1.5,25.5)
Length of assimilation window: 3
Length of subsequent forecast: 1
Time step: 0.01
Frequency of observations = 20
Ensemble size = 50
Observations have random noise with variance 1
Background noise = 1

Constant Bias on final obs [+5]

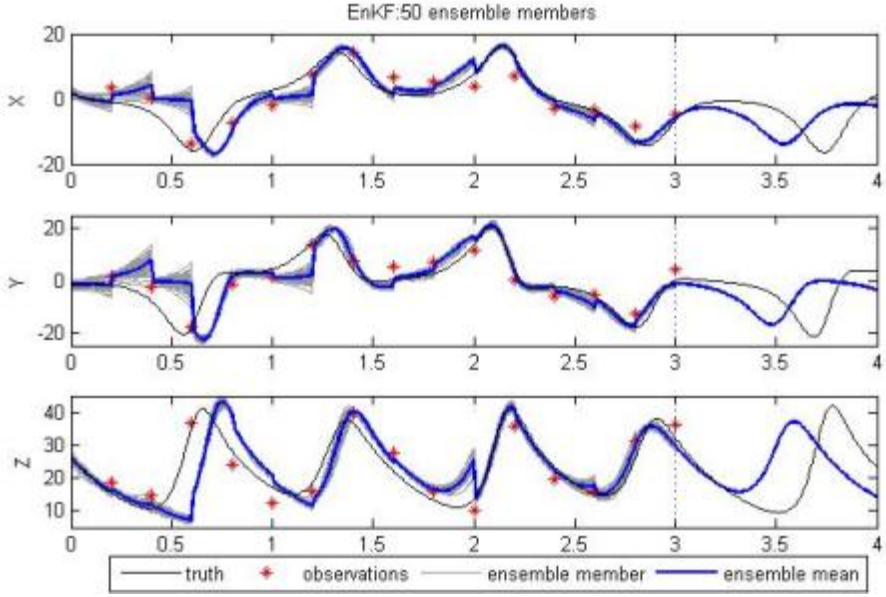


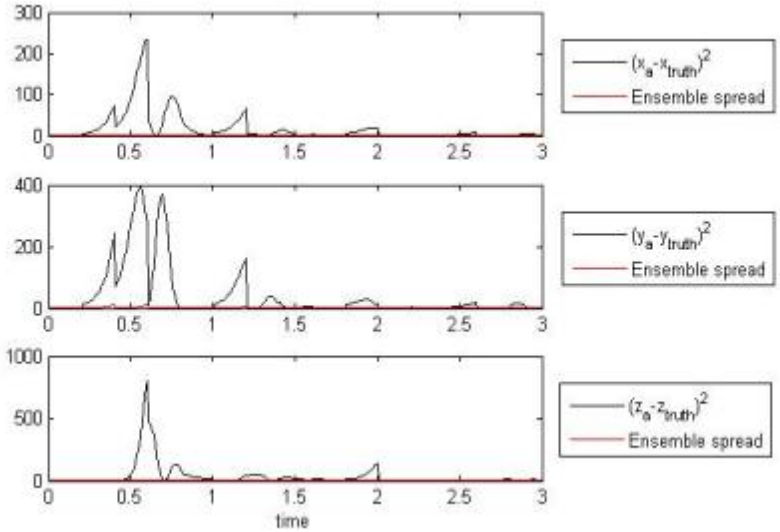


List of options chosen for the Ensemble Kalman filter

True (x,y,z) at t=0: (1.5,-1.5,25.5)
Length of assimilation window: 3
Length of subsequent forecast: 1
Time step: 0.01
Frequency of observations = 20
Ensemble size = 50
Observations have random noise with variance 1
Background noise = 1

Time-correlated Sinusoidal Bias





List of options chosen for the Ensemble Kalman filter

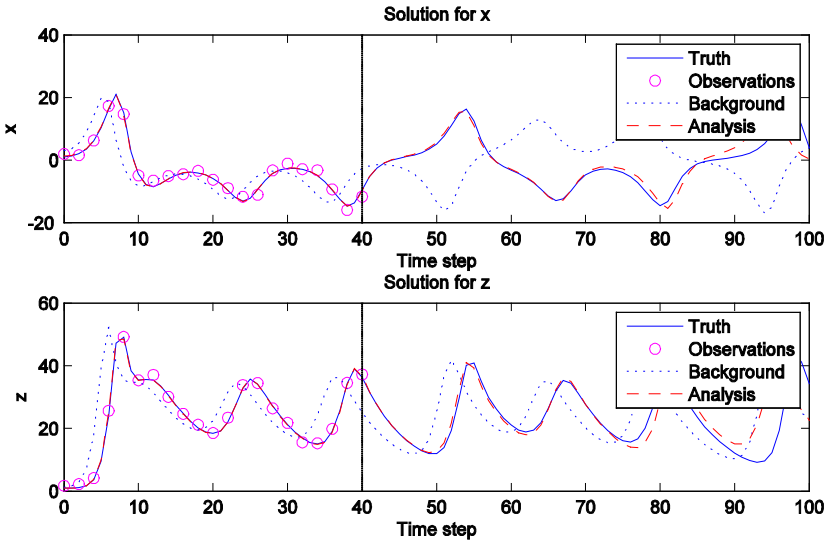
True (x,y,z) at t=0: (1.5,-1.5,25.5)
Length of assimilation window: 3
Length of subsequent forecast: 1
Time step: 0.01
Frequency of observations = 20
Ensemble size = 50
Observations have random noise with variance 1
Background noise = 1

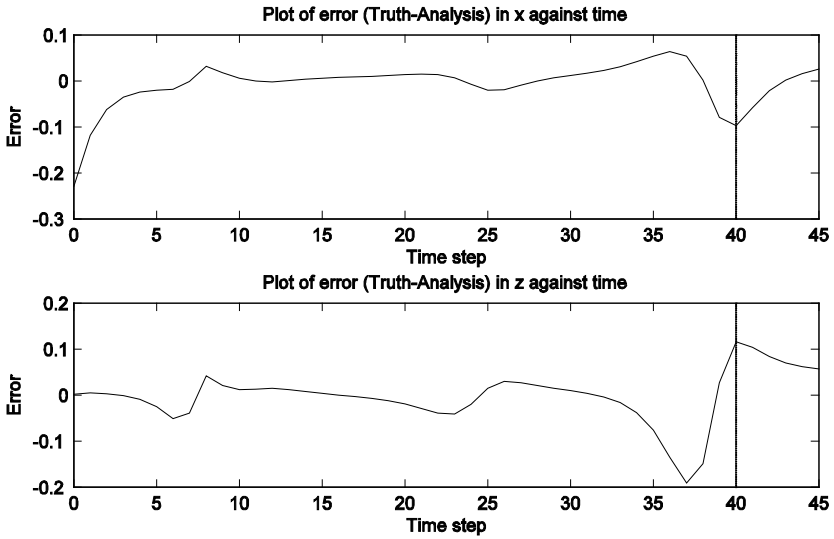
How do biased observations effect 4D VAR DA results using the Lorenz model?

Project 9

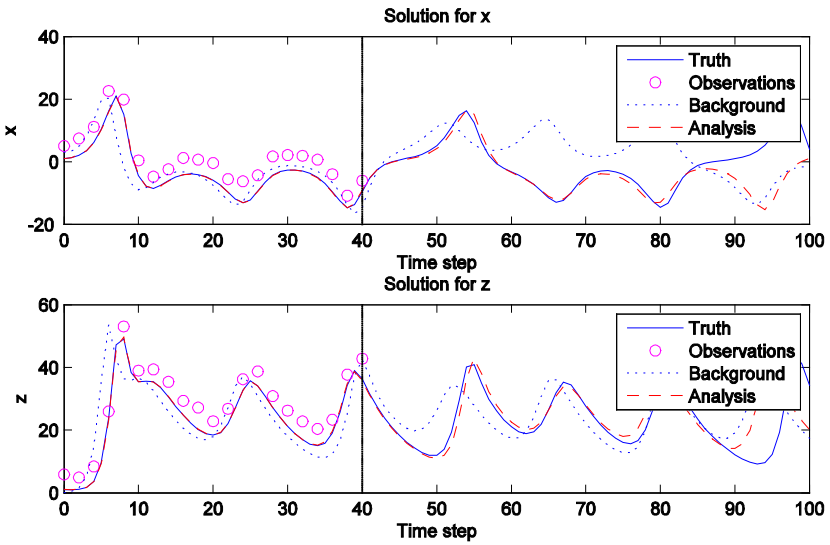
Richard Pope, UK
Yang Liu, China
Niko Wanders, NL
Hans Roelofsen, NL

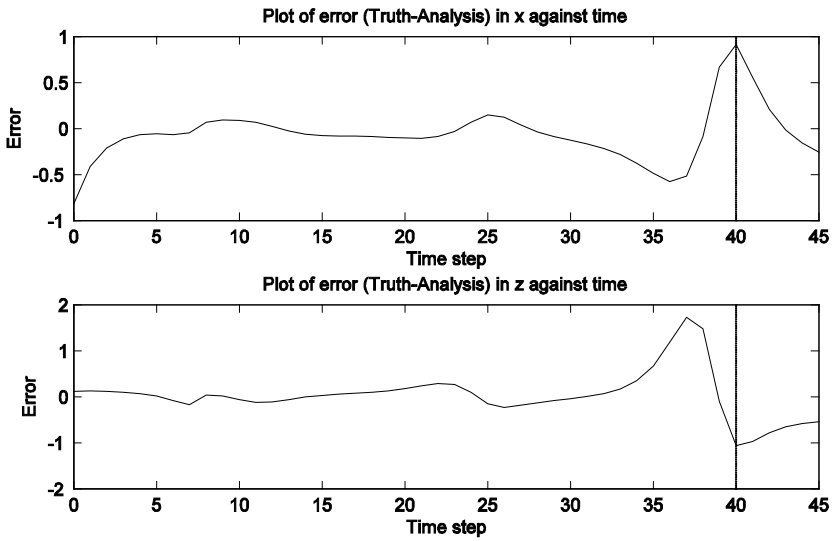
Control Analysis





Biased Observation Analysis (Obs + 5)



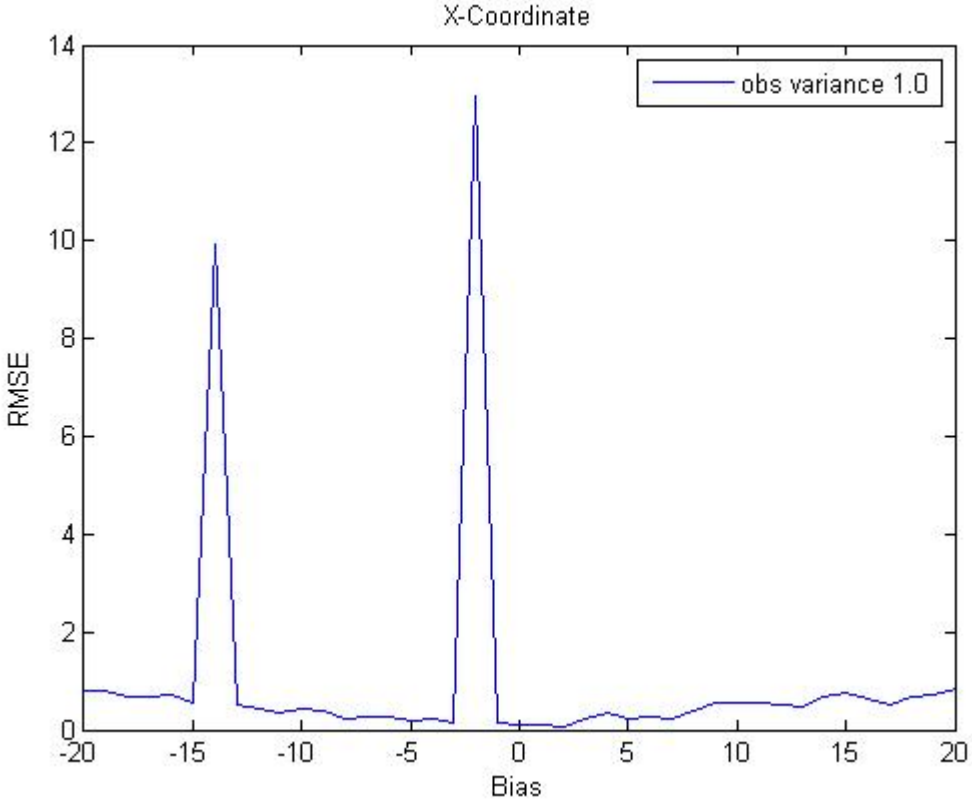


Errors as a function of bias

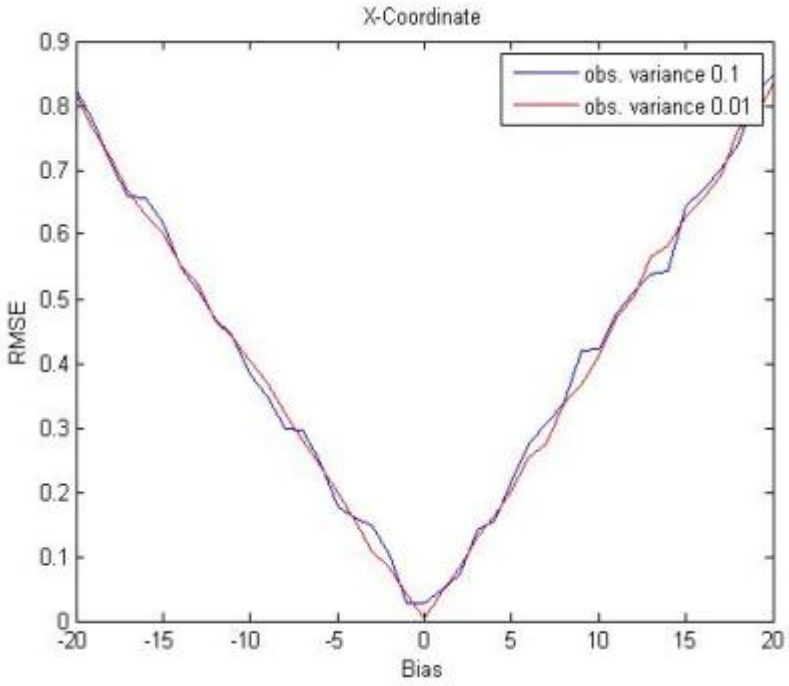
- Introduced bias in observation:
  - Observation = randn() + bias

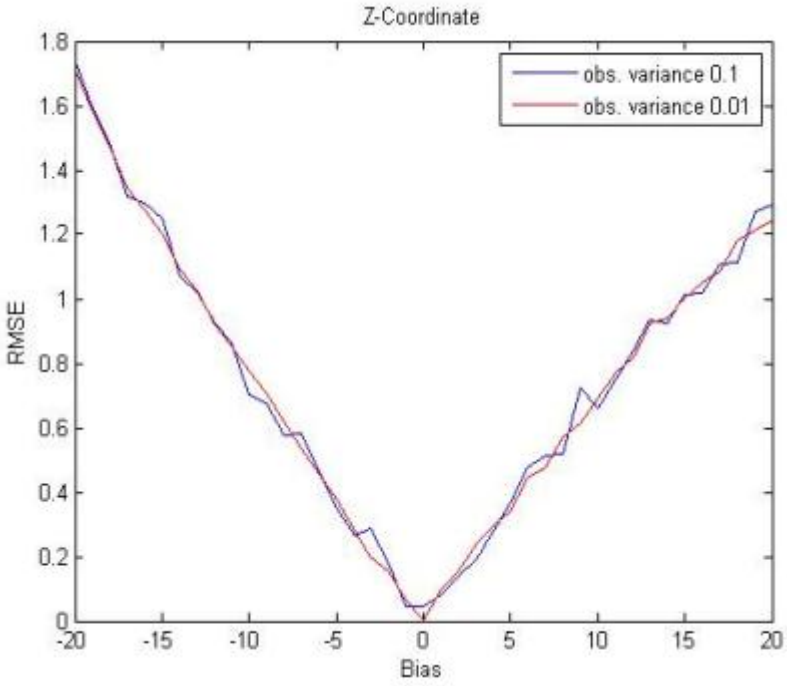
- Root Mean Square Error:
  - RMSE = f(bias)
  - RMSE = $(\Sigma(\text{analysis} - \text{truth})^2)/\text{n_tsteps})^{0.5}$

Dominant random errors



RMSE = f(bias)


X-Coordinate
obs. variance 0.1
obs. variance 0.01
RMSE
Bias
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-20
-15
-10
-5
0
5
10
15
20


Z-Coordinate
obs. variance 0.1
obs. variance 0.01
RMSE
Bias
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-20
-15
-10
-5
0
5
10
15
20

Operational 4D VAR DA example

- Hypothetical ESA spacecraft docking with the ISS using 4D VAR DA.
- Biased observations- GPS satellite instrument iced over so inaccurate positioning co-ordinates.
- So spacecraft analysis path misses ISS and ……..

DA-Projects #10

Kikka and Shawn

Topic 10- Extended Kalman filter

- “ Investigate the performance of the extended Kalman filter when the model state is only partially observed.”
- à ‘change the code so that only two components of the state vector (x, y, z) are observed.’
- Thank you Alison Fowler!!!

Real World

- For example, 3-D winds where you have x, y, and z and your sensor breaks and only gives you 2 out of 3 directions.

- Or

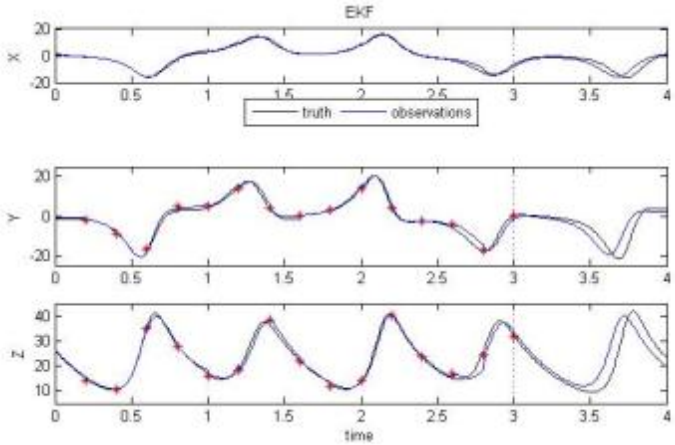
- For example, 3 separate sensors for chl a, chl b and xanthophylls, one stops working after sampling 60 of 100 trees.

Lorenz 63 Model

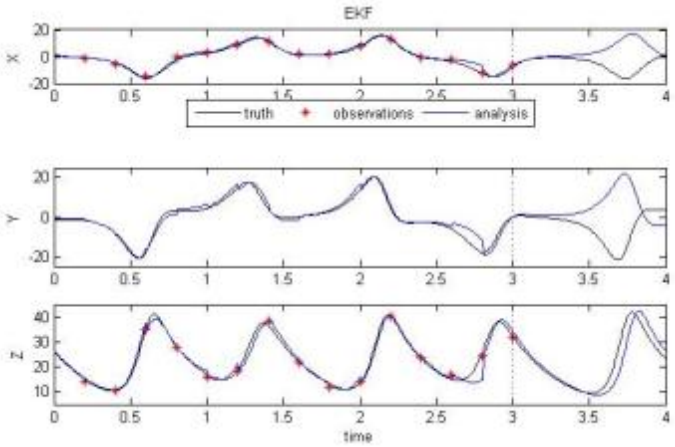
$$\frac{dx}{dt} = \sigma(y - x)$$
$$\frac{dy}{dt} = -xz + \rho x - y$$
$$\frac{dz}{dt} = xy - \beta z.$$

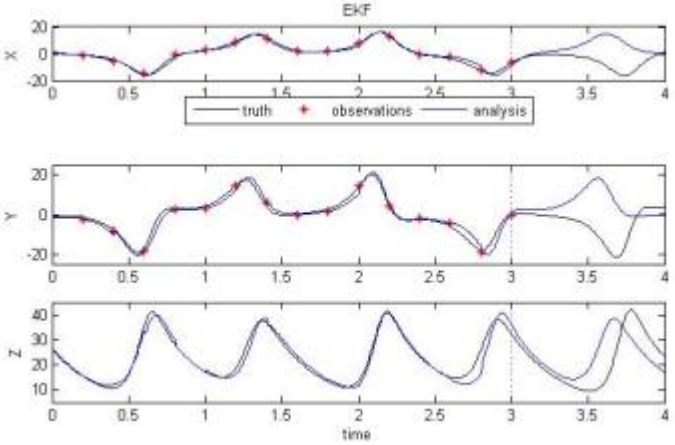
Figures-Models

Remove X

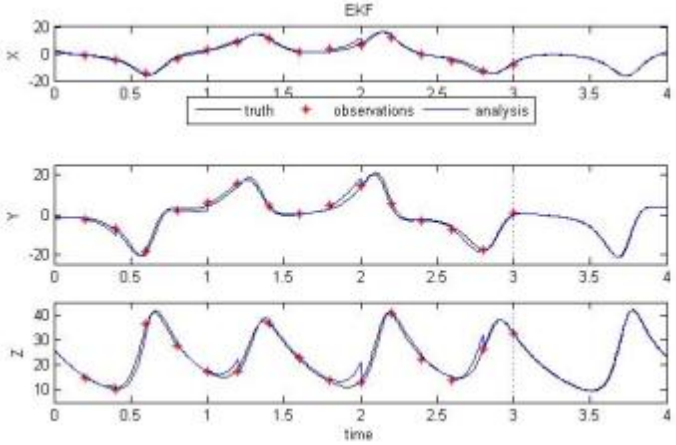


Remove Y





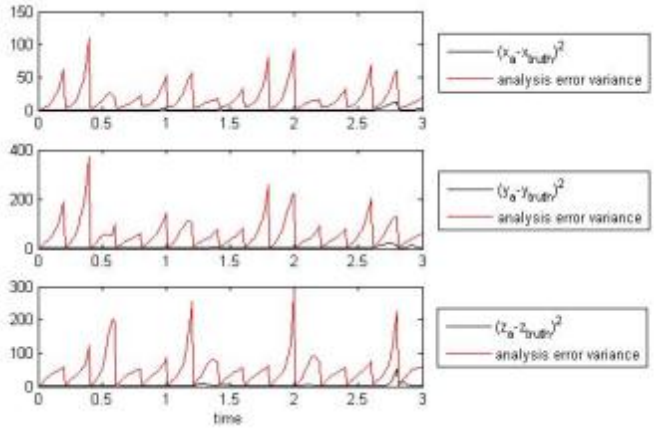
Remove Z



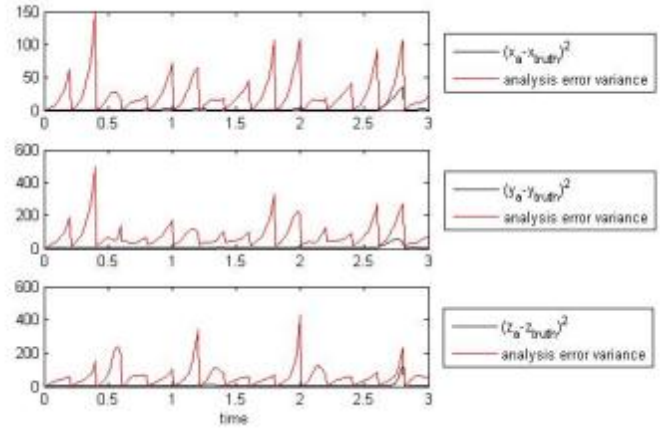
Normal

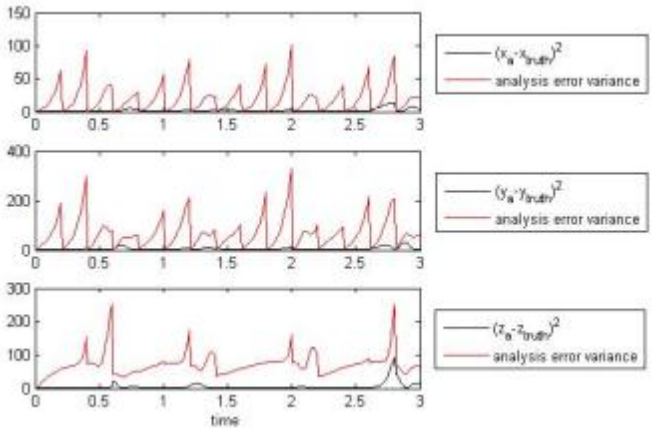
Figures-Errors

Remove X

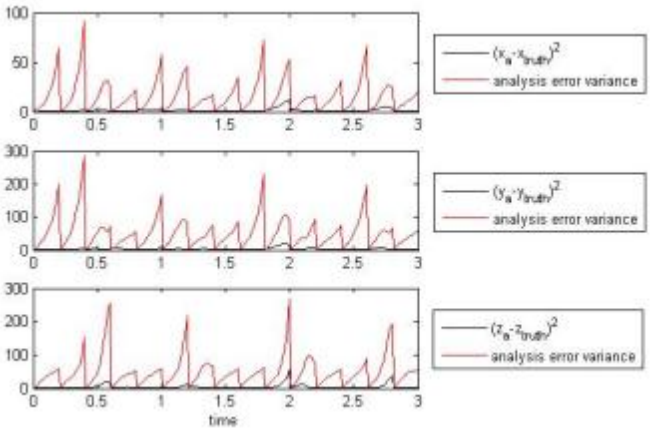


Remove Y





Remove Z



Normal

Figures – Parameters All

List of options chosen for the Extended Kalman filter

True (x,y,z) at t=0: (1.5,-1.5,25.5)
Length of assimilation window: 300
Length of subsequent forecast: 100
Time step: 0.01
Frequency of observations = 20
Observations have random noise with variance 1
Background noise = 1

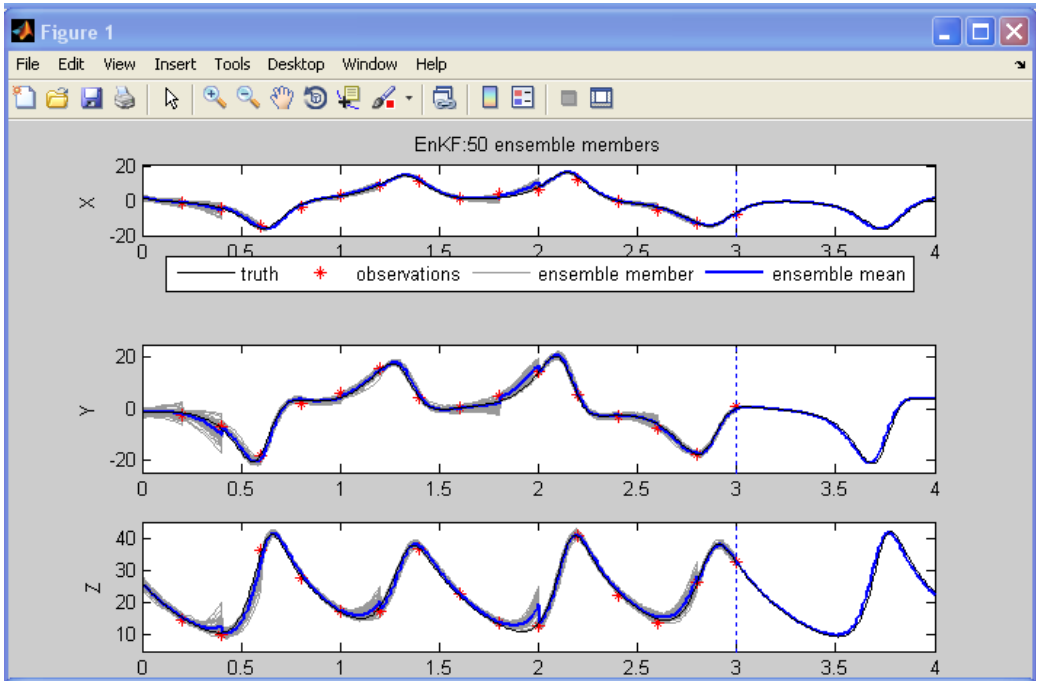
Conclusions

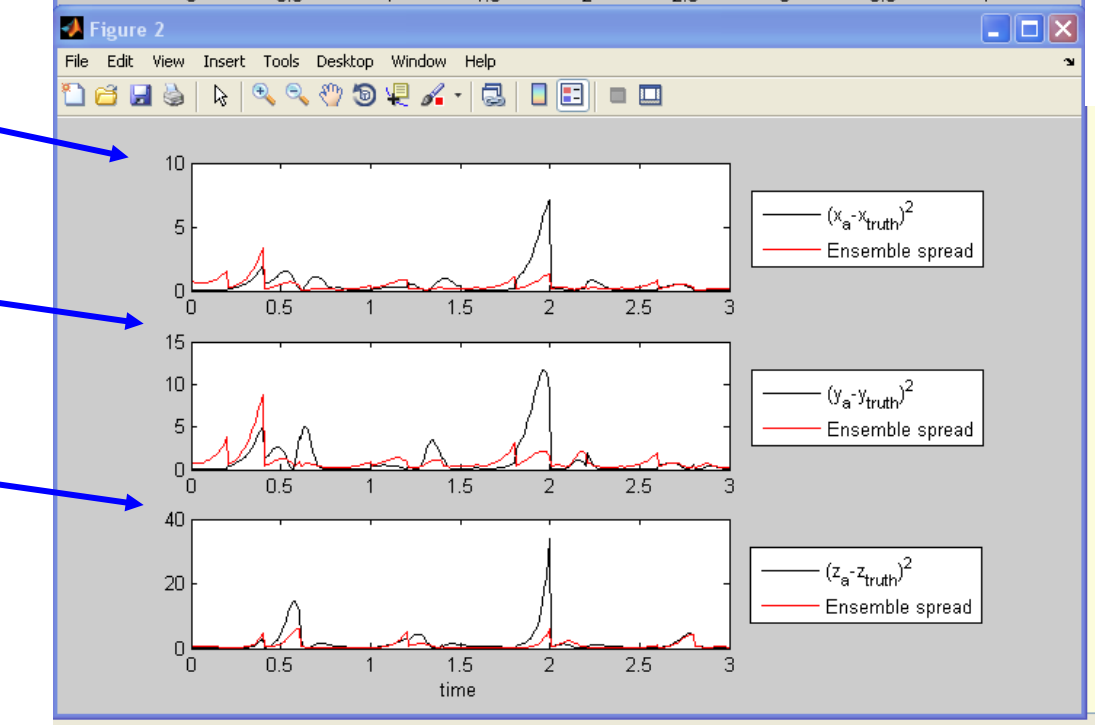
- Affect on total model determined by relationships of components
  - Removing X less impact than removing Y or Z
- With a priori knowledge, we can model the third component with only 2
- Expect some decreased model accuracy

11

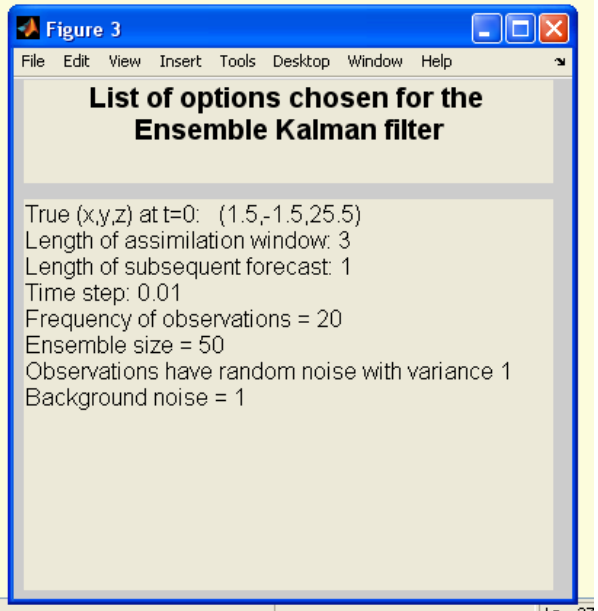
The ensemble Kalman filter: partially observation case

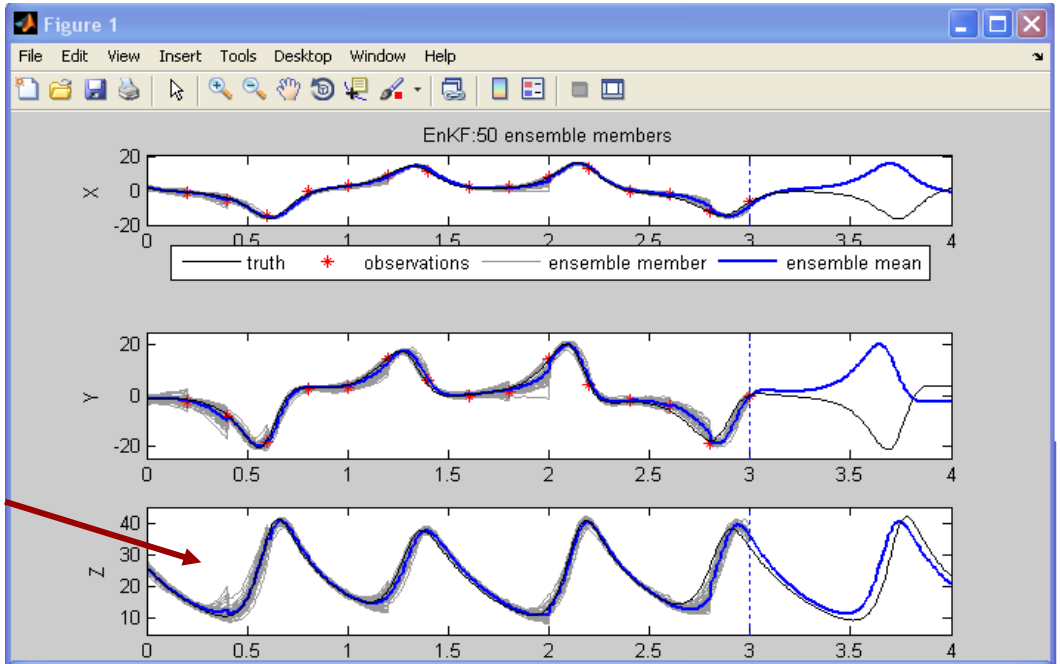
Ruggero Avezzano, Guido Ceccherini, Patrick Leinenkugel, Tuan Vo Quoc

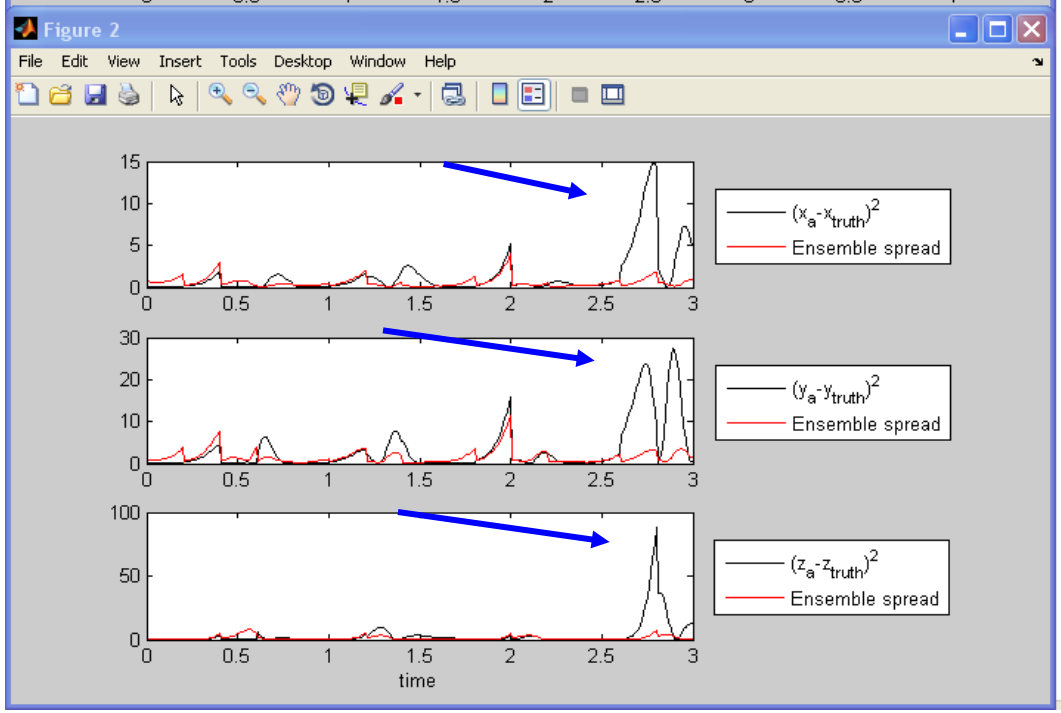




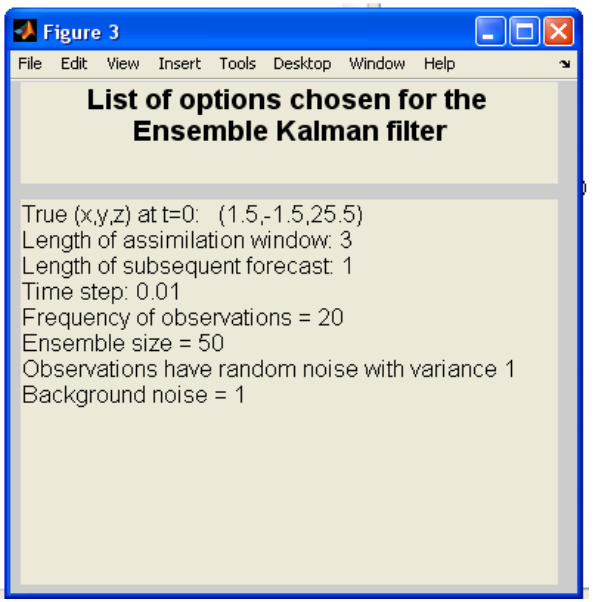
Fully observed i.e. observation on x,y,z



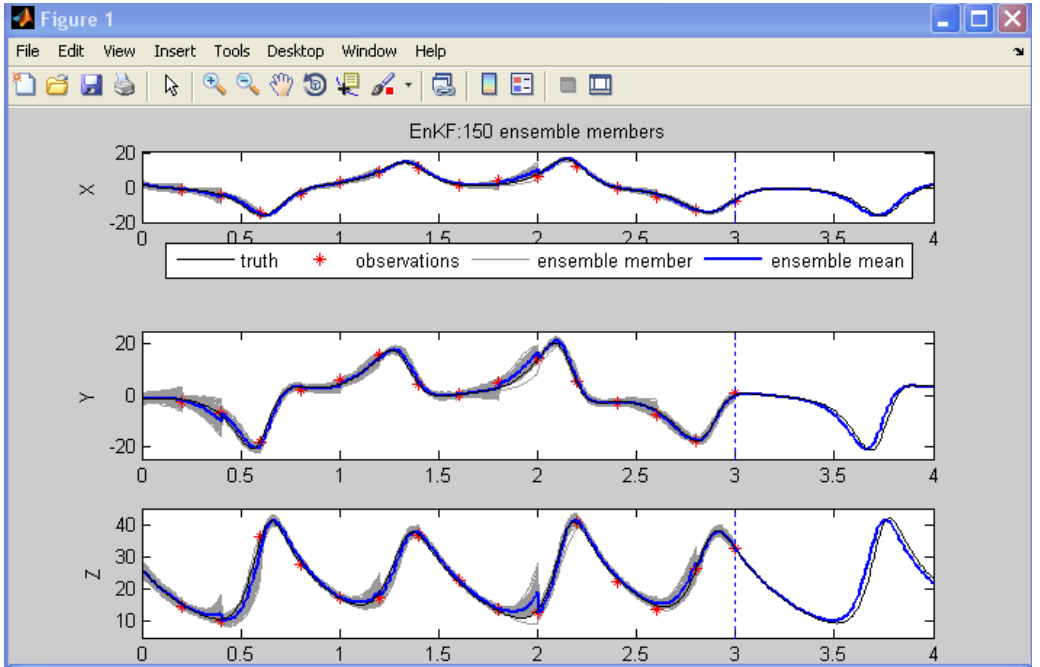


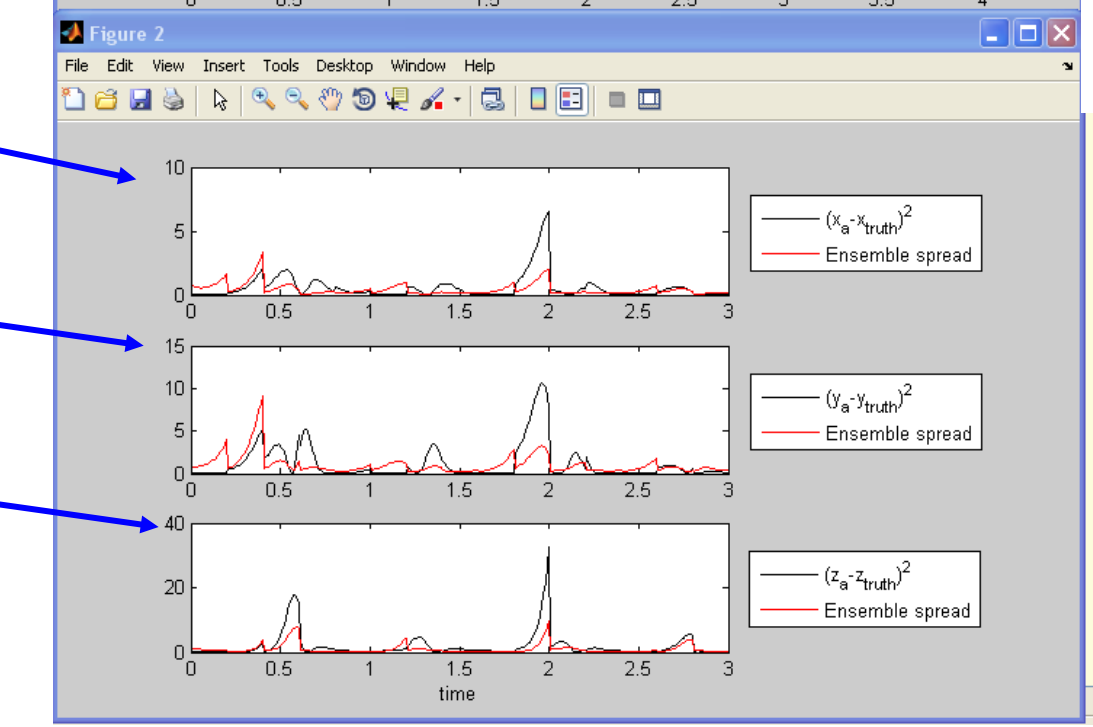


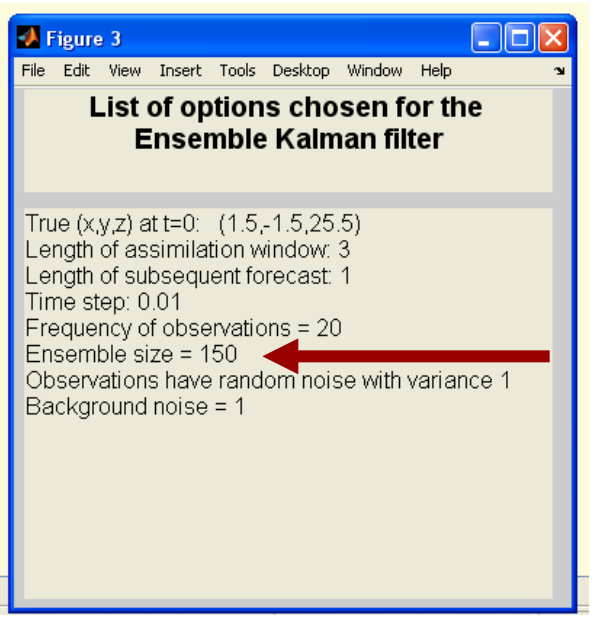
Partially observed i.e. observation on x,y,~~z~~



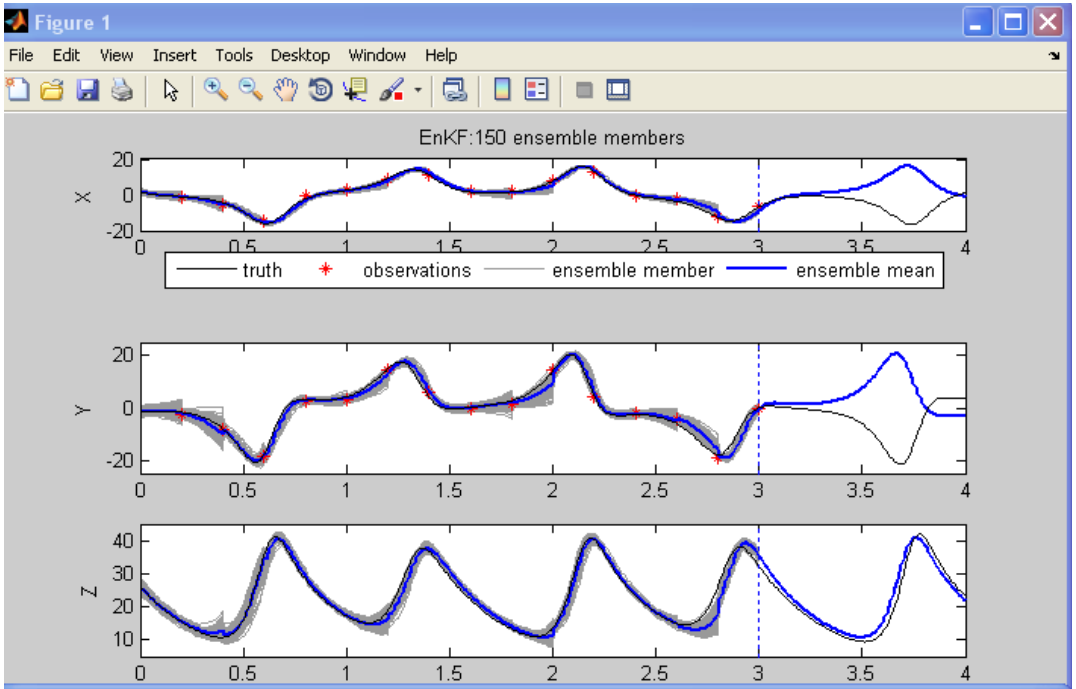
Fully observed increasing the number of ensambles

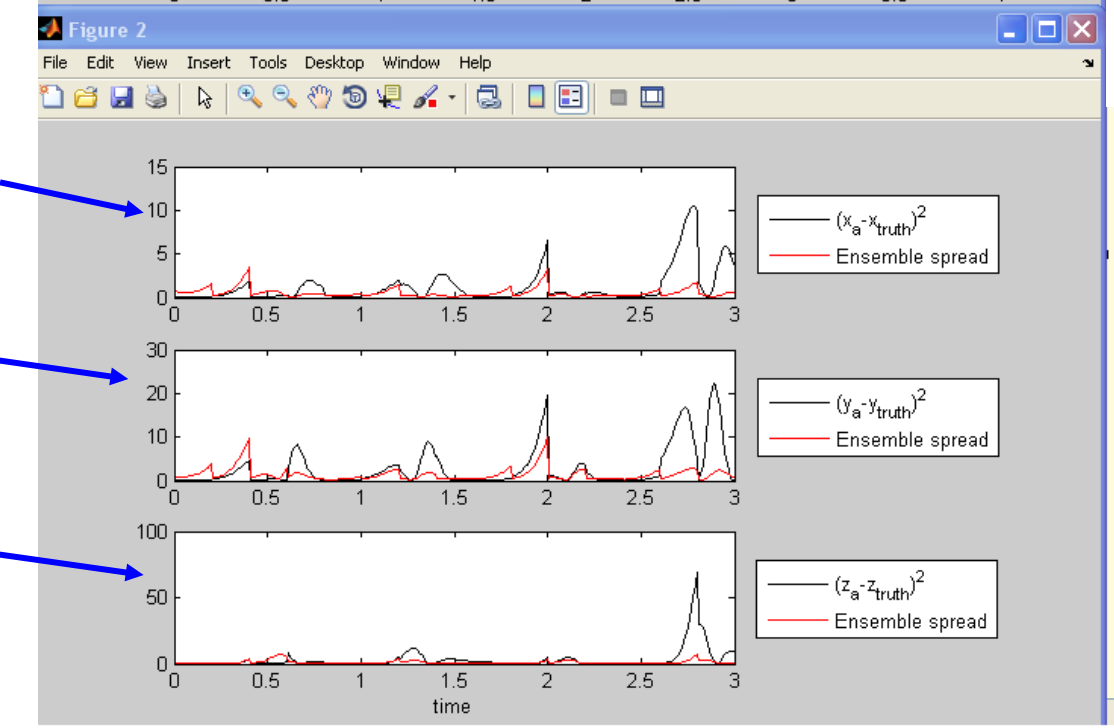


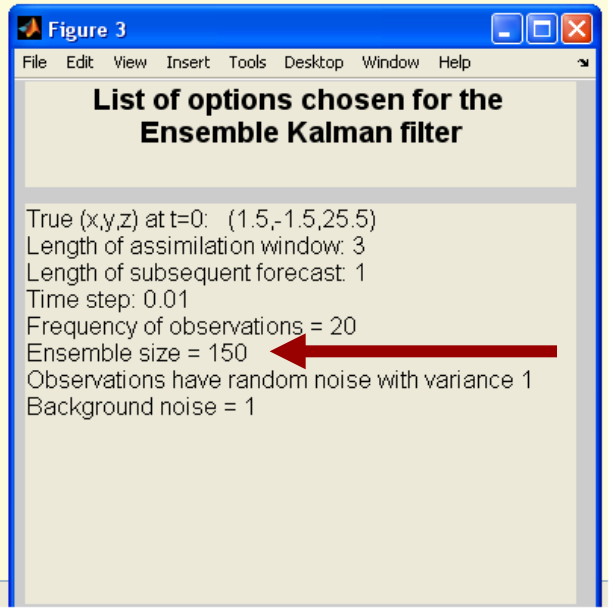


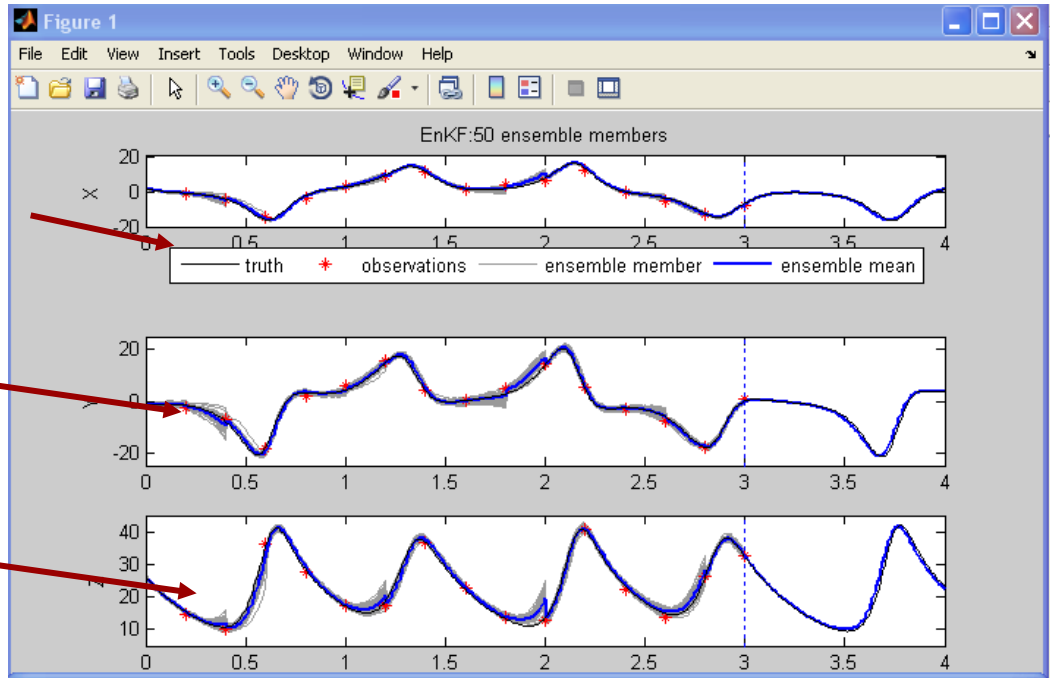


Partially observed increasing the number of ensambles

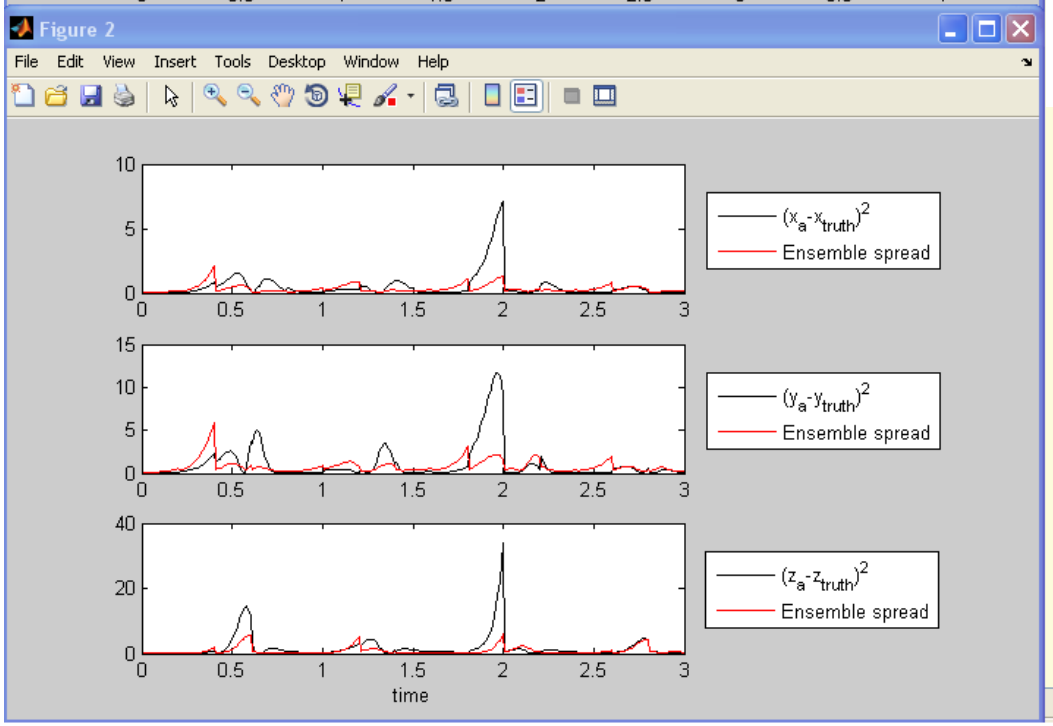


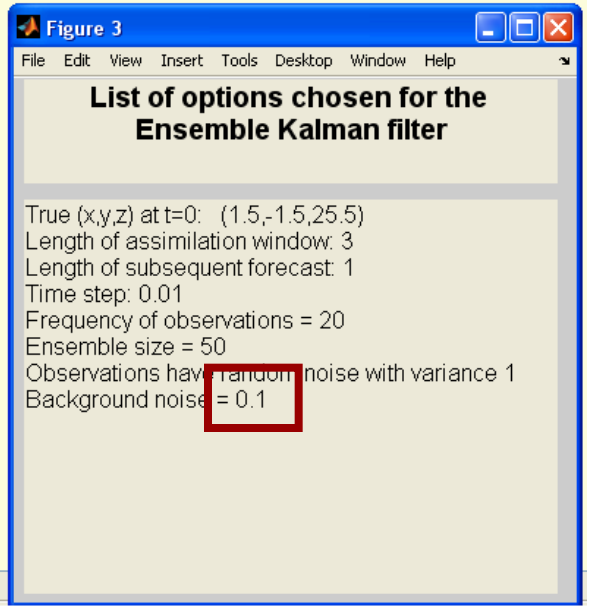


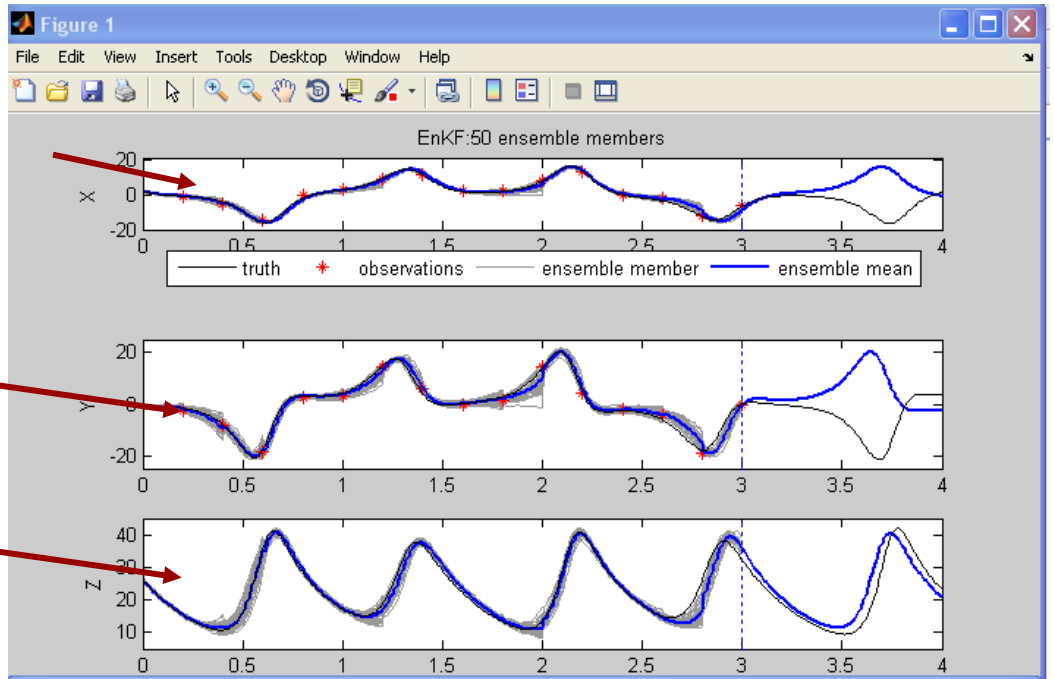




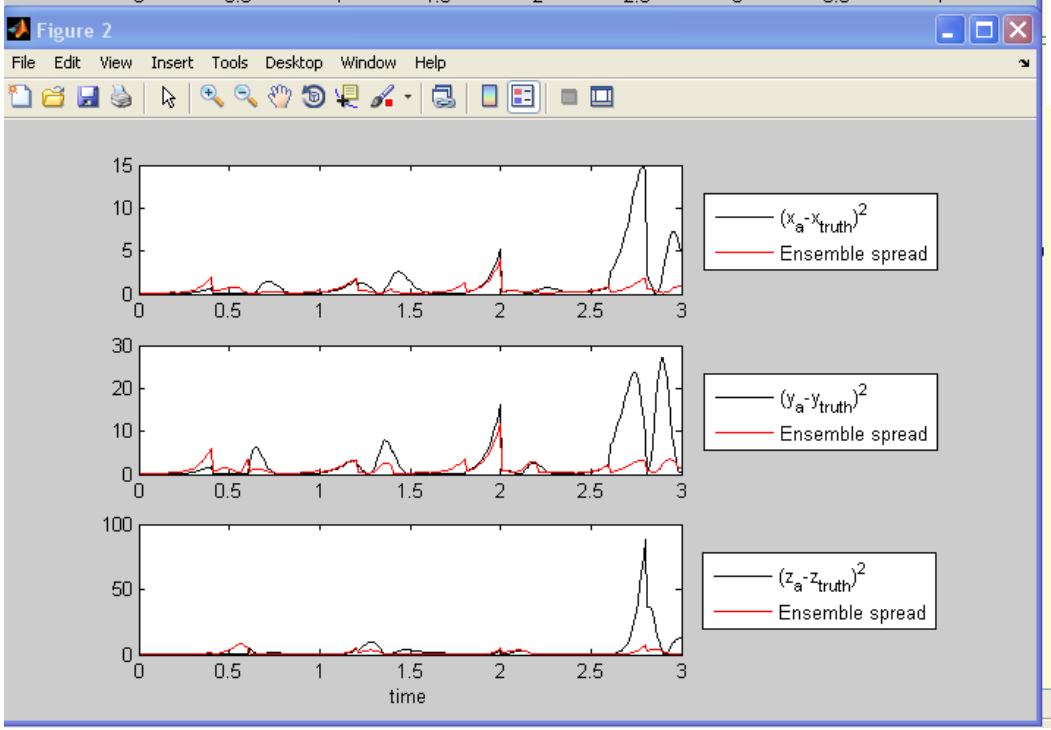
Fully observed with low background error covariance

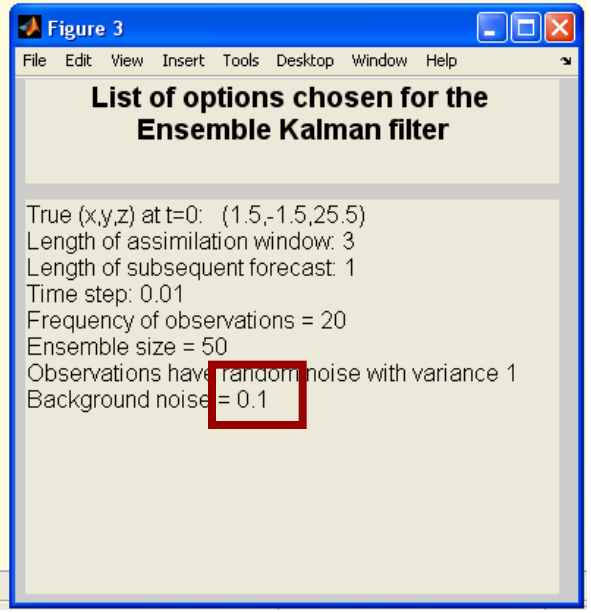






Partially observed with low background error covariance





Real world

- Meteorological model with 3d wind field where wind measurements are collected only for one elevation layer. Estimate wind speed also at other elevations.

Project 12

Investigate the performance of 4D-Var when the model state is only partially observed

Group members : Alijafar, Javier, Simon, Alireza

How to investigate ?

1. Run using all variables
2. Ignoring one variables
3. comparing the results

List of options chosen

True (x,y,z) at t=0: (1,1,1)
Length of assimilation window: 2
Length of subsequent forecast: 3
Time step: .05
Frequency of observations = 2
Maximum iterations: 30
Tolerance: 1d-3
Observations have random noise with variance 2
Background noise = 20

Background (x,y,z) at t=0: (3,3,3)
Analysis (x,y,z) at t=0: (1.5855,0.26078,0.97614)

All variables

List of options chosen

True (x,y,z) at t=0: (1,1,1)
Length of assimilation window: 2
Length of subsequent forecast: 3
Time step: 0.05
Frequency of observations = 2
Maximum iterations: 30
Tolerance: 1d-3
Observations have random noise with variance 2
Background noise = 20

Background (x,y,z) at t=0: (3,3,3)
Analysis (x,y,z) at t=0: (1.2115,0.69093,0.94045)

No X

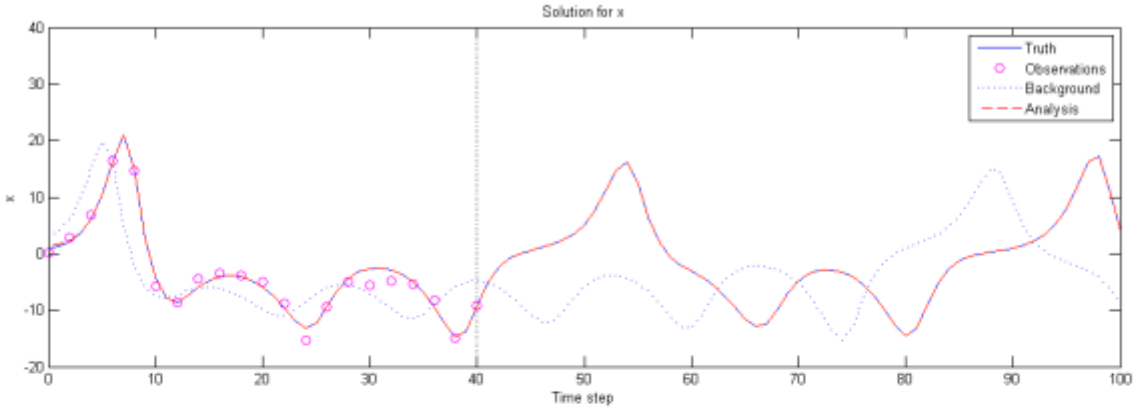
List of options chosen

True (x,y,z) at t=0: (1,1,1)
Length of assimilation window: 2
Length of subsequent forecast: 3
Time step: 0.05
Frequency of observations = 2
Maximum iterations: 30
Tolerance: 1d-3
Observations have random noise with variance 2
Background noise = 20

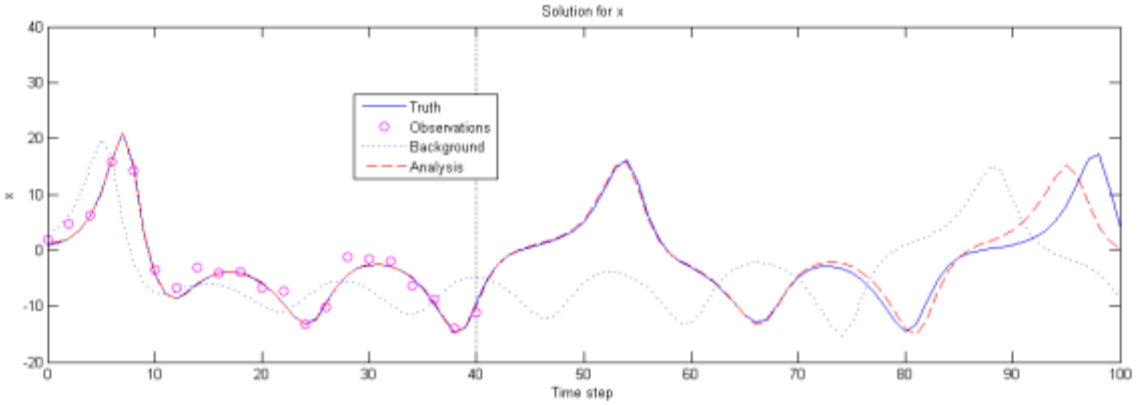
Background (x,y,z) at t=0: (3,3,3)
Analysis (x,y,z) at t=0: (0.63879,1.4637,1.1014)

No Z

All variables



No X



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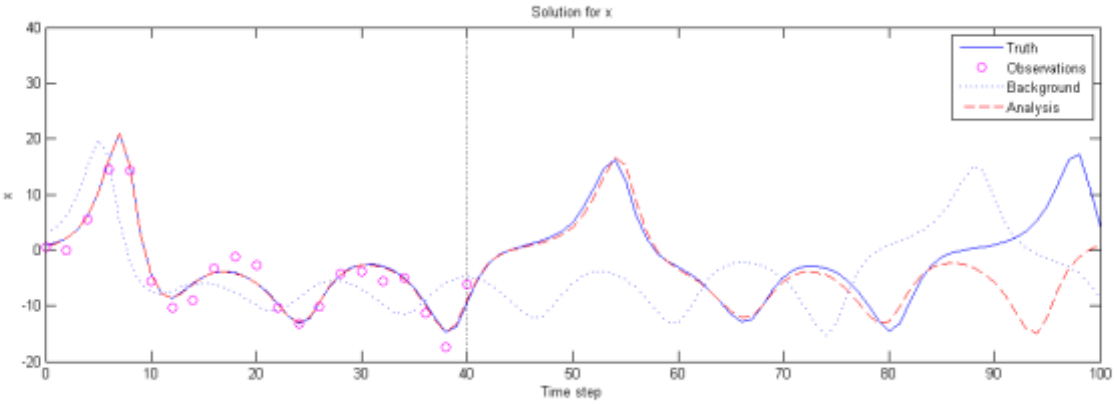
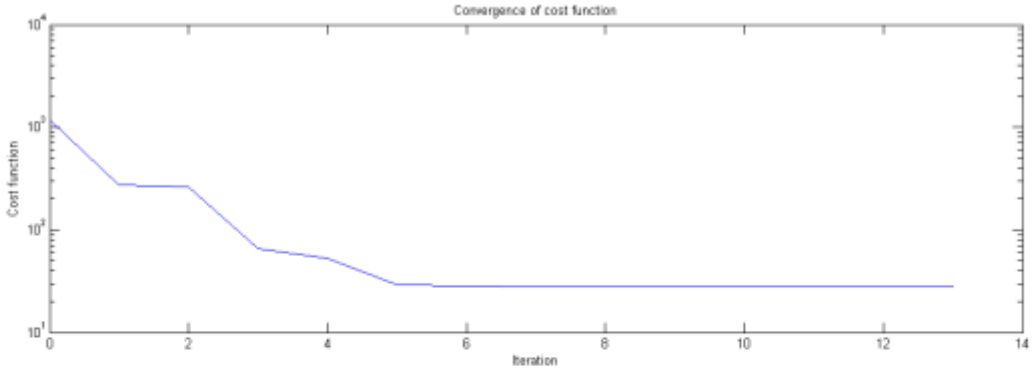
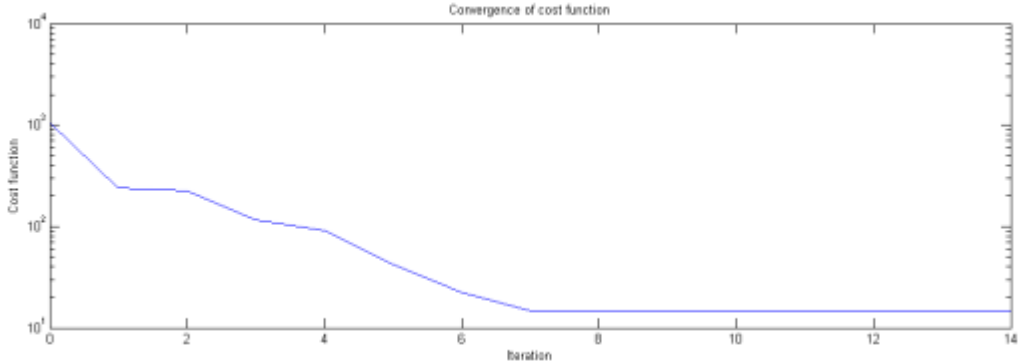




All variables



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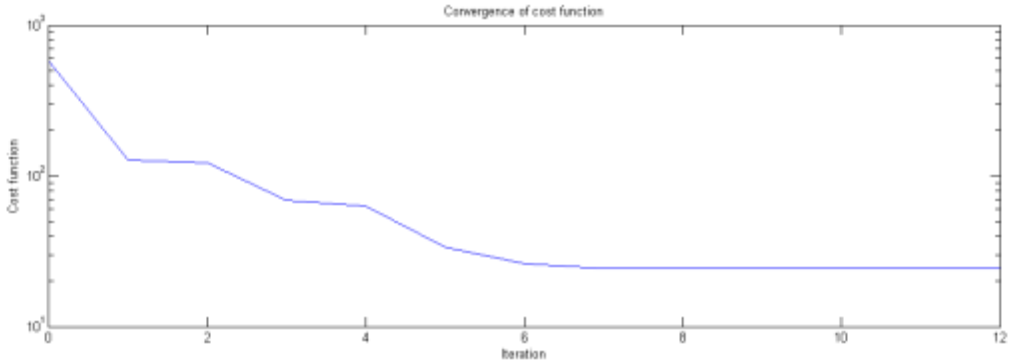
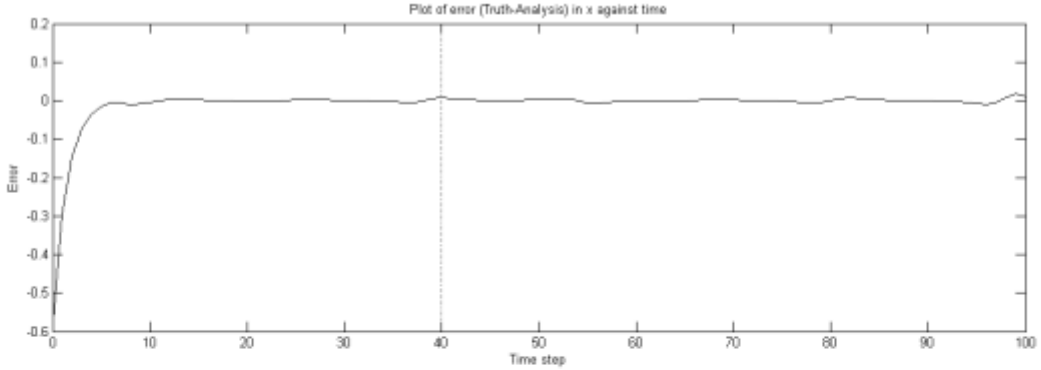
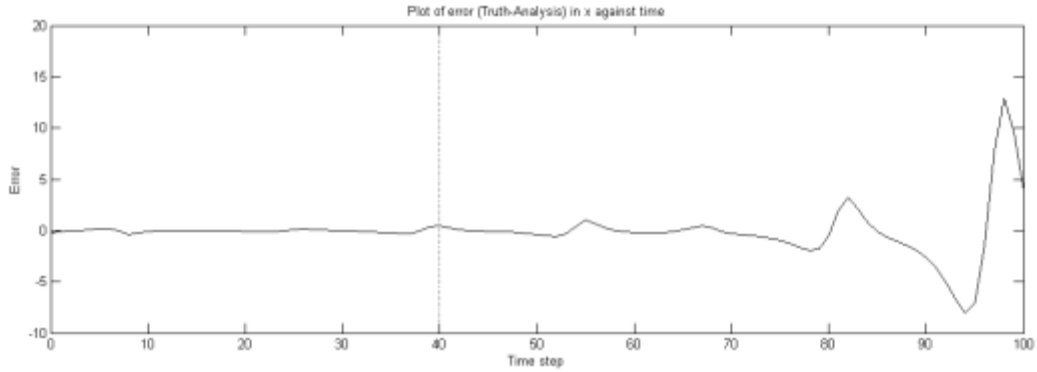


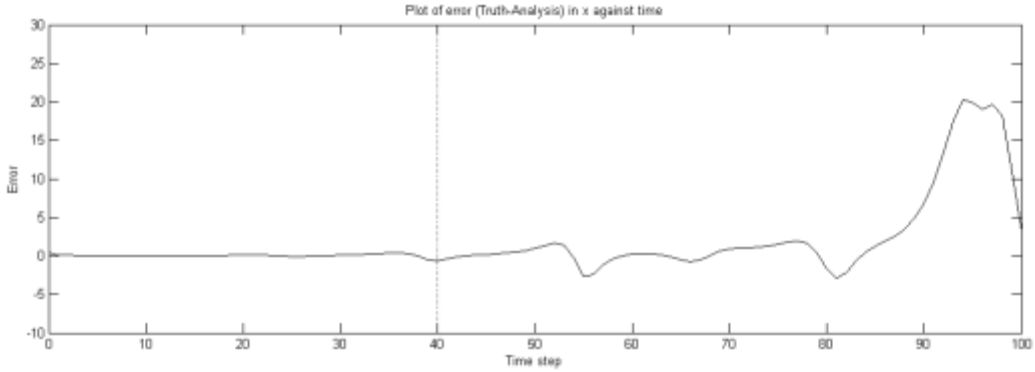
No Z



All variables



No X



No Z

Project 13

Investigate different forms of the model error covariance matrix, **Q**, in the extended Kalman filter

Elizabeth Wong, Melina Nikolaidou, Zina Mitraka, Rado Guzinski

Introduction

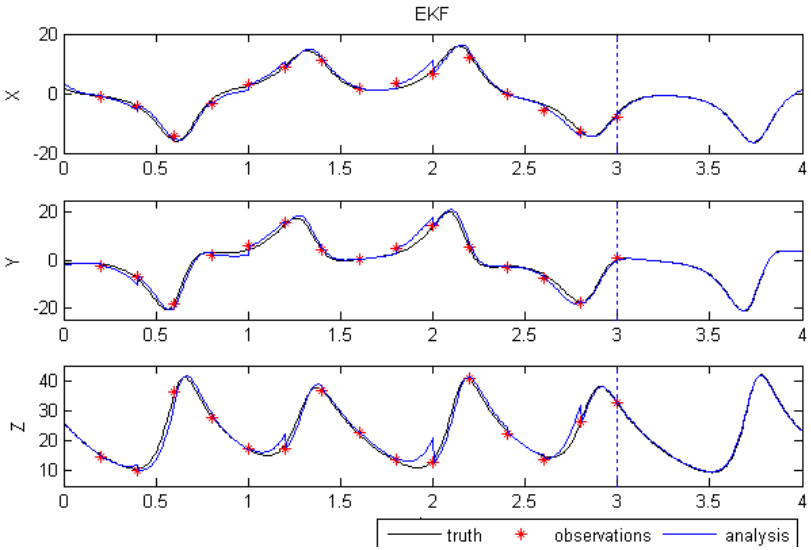
- The covariance matrix Q is the perception of the accuracy of the model M.
- The greater the values in Q, the lower confidence in the model.

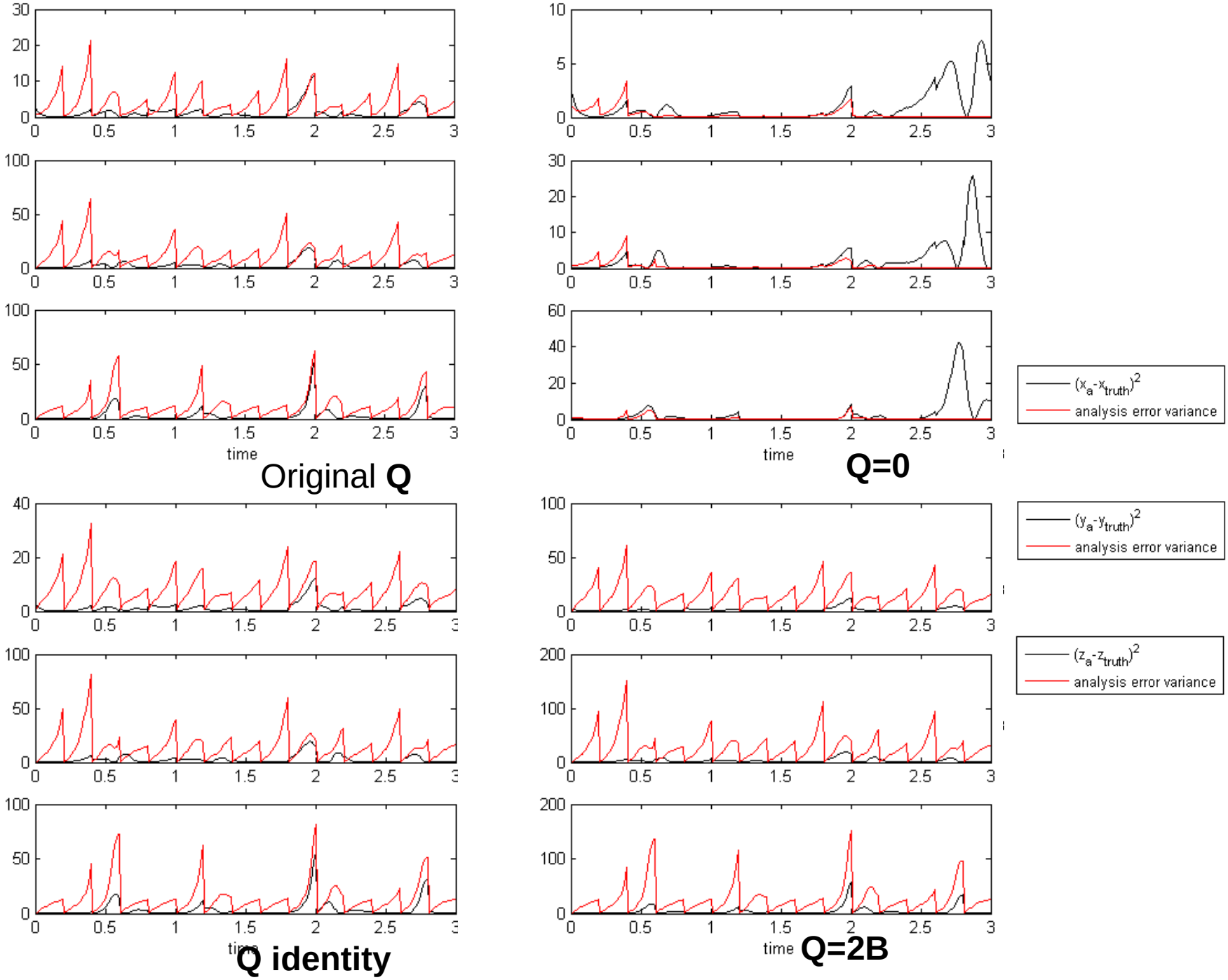
a) if Q=0 then we fully trust our model.
b) if Q is identity then we assume that the errors in different dimension don’t correlate with each other
c) we can use Q=a*B when we don’t have a good estimation of Q and we can assume that the error in the model is directly proportional to the error of the first guess.

Original **Q**

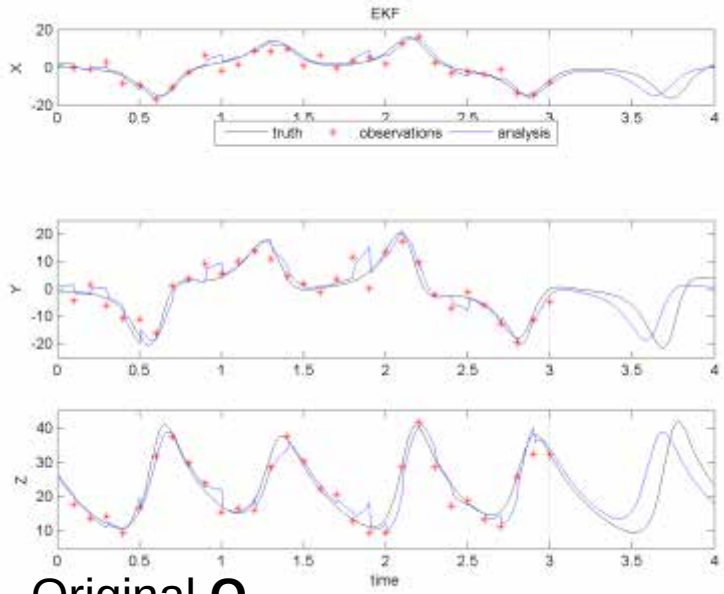
$$Q = \begin{matrix} 0.1491 & 0.1505 & 0.0007 \\ 0.1505 & 0.9048 & 0.0014 \\ 0.0007 & 0.0014 & 0.9180 \end{matrix}$$

For Lorenz 63 model

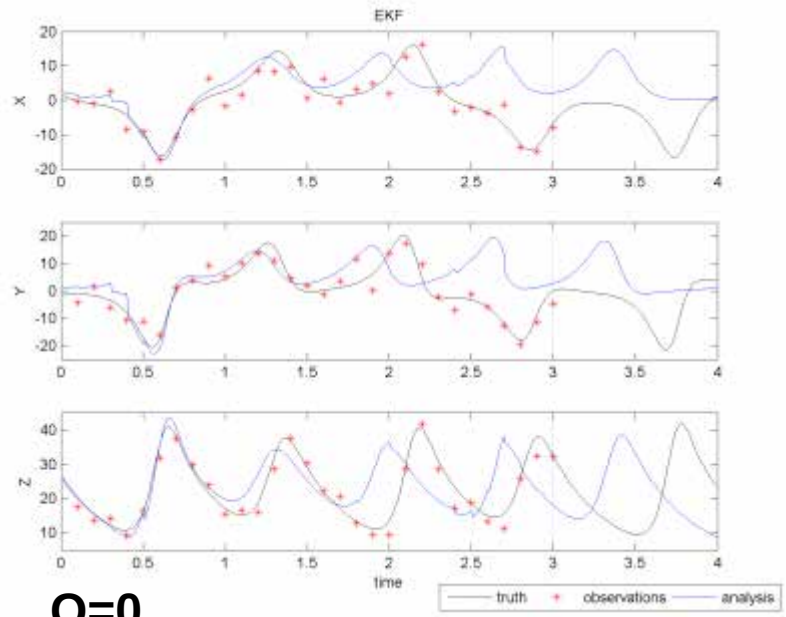



time
Original Q
Q=0
Q identity
Q=2B
$(x_a-x_{truth})^2$
analysis error variance
$(y_a-y_{truth})^2$
analysis error variance
$(z_a-z_{truth})^2$
analysis error variance

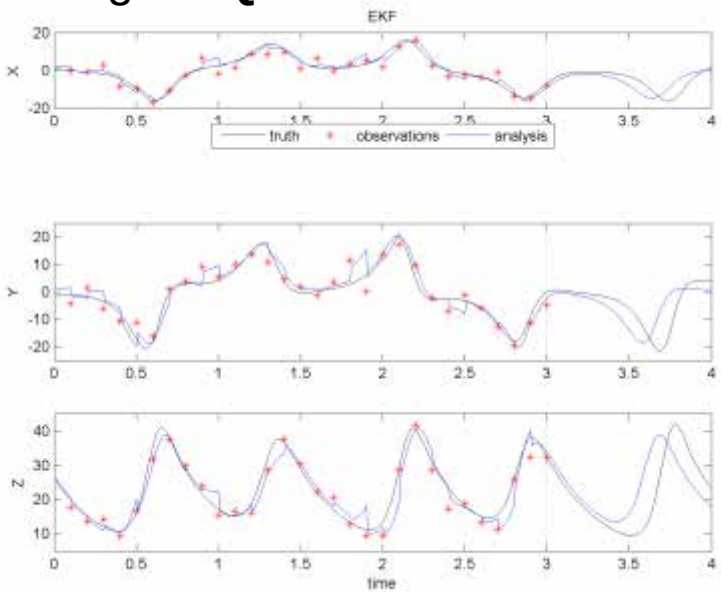
Frequent, inaccurate observation



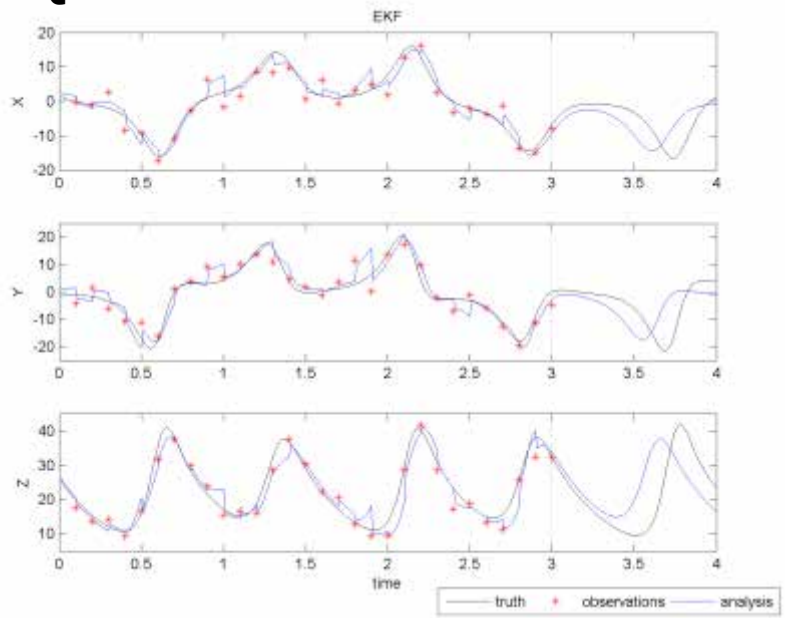
Original **Q**



Q=0

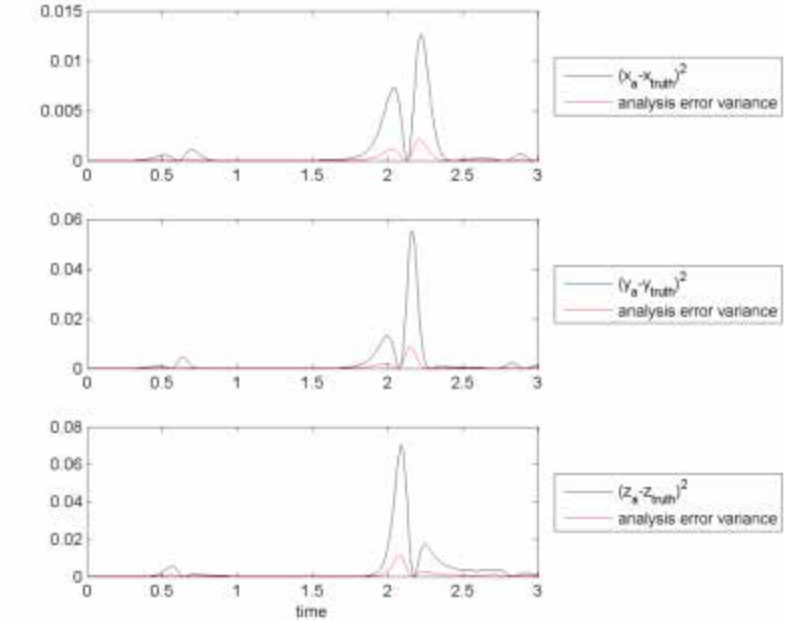


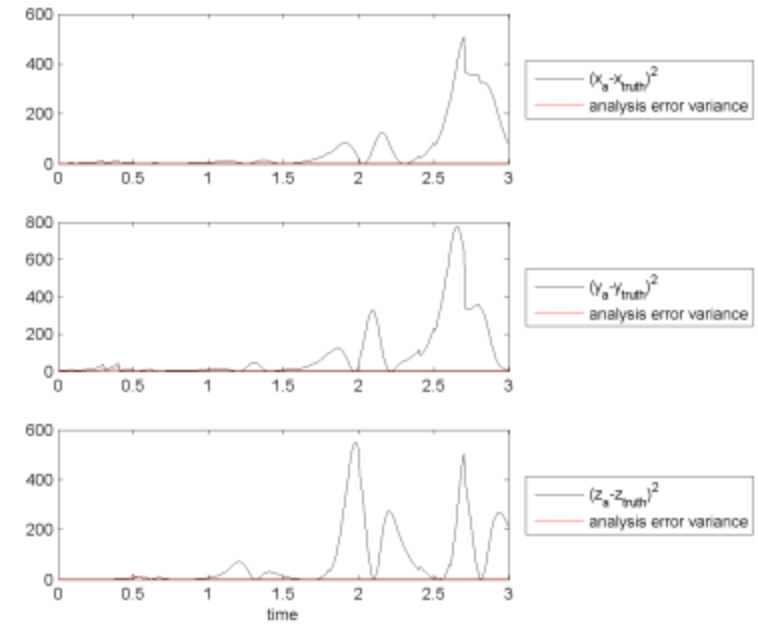
Q identity

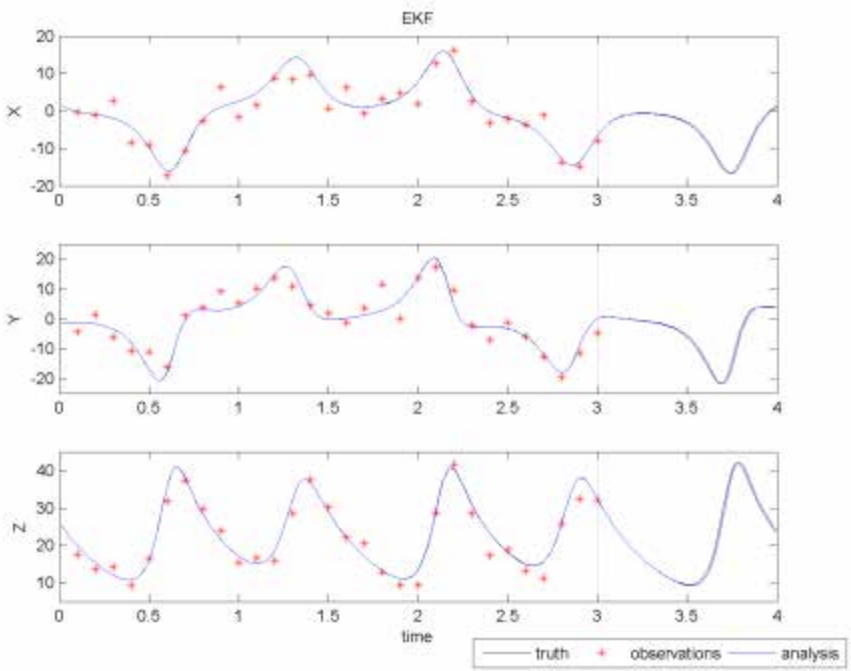


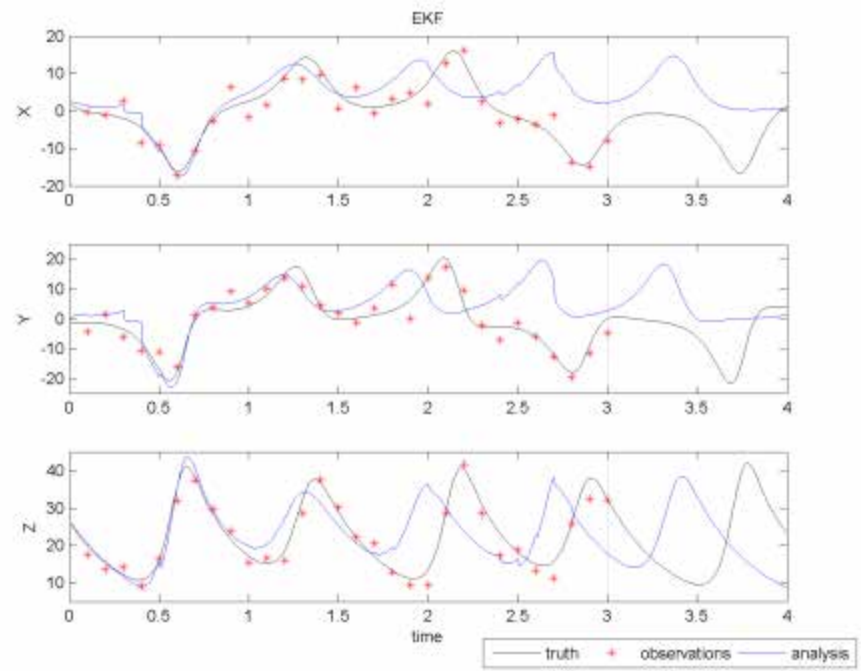
Q=2B

Importance of first guess when the model is fully trusted (**Q** = 0)







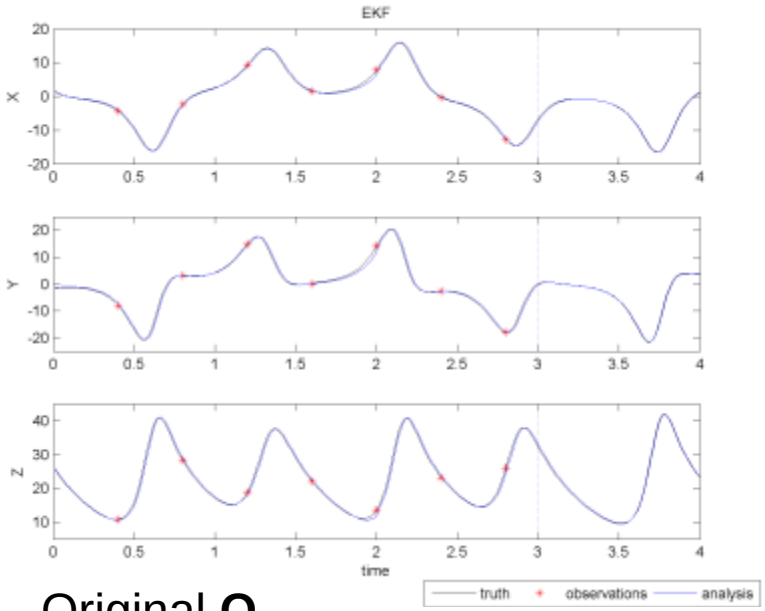


Conclusions

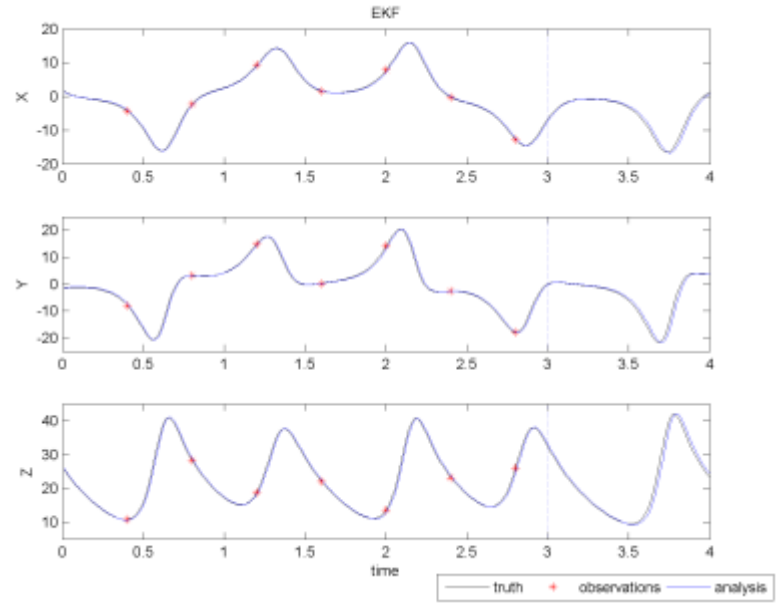
- If no model error covariance matrix Q is assumed then the initial background has to be very accurate

- In Lorenz model the error in different dimensions is weakly correlated. Therefore Q=I works fine

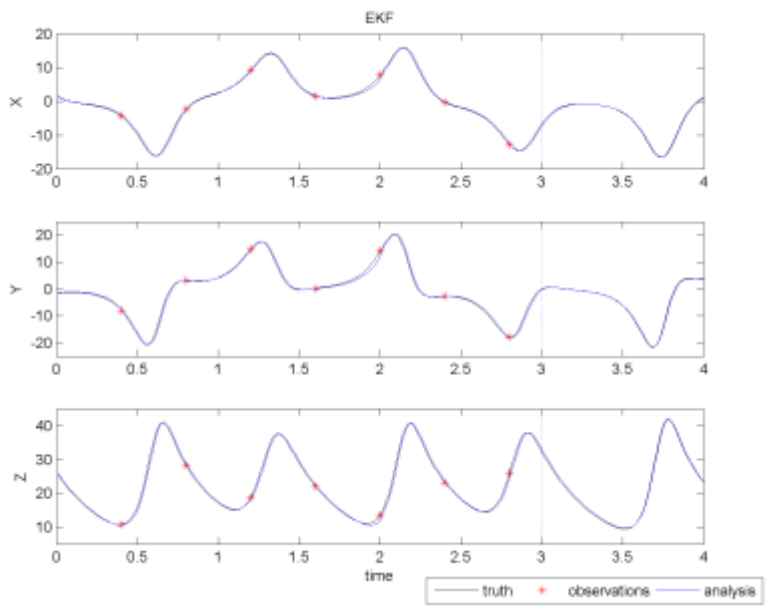
Infrequent, accurate observations



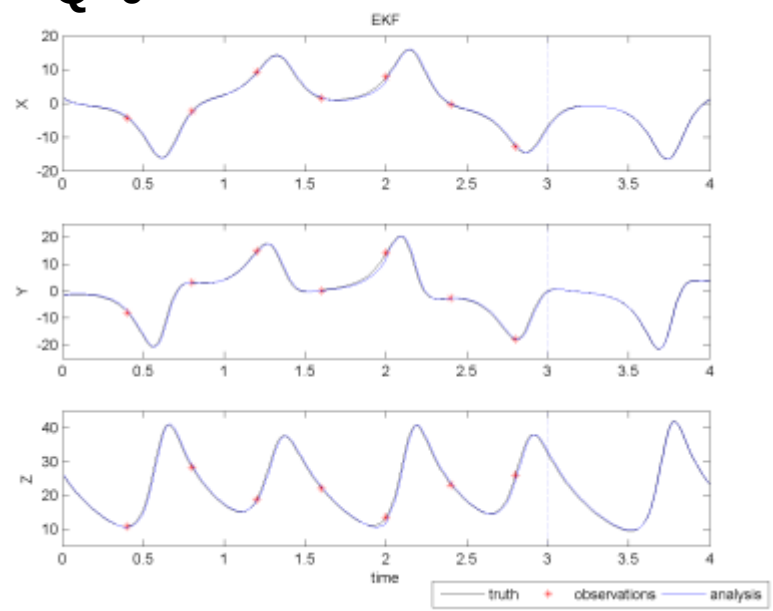
Original **Q**



Q=0



Q identity



Q=2B

Project 14

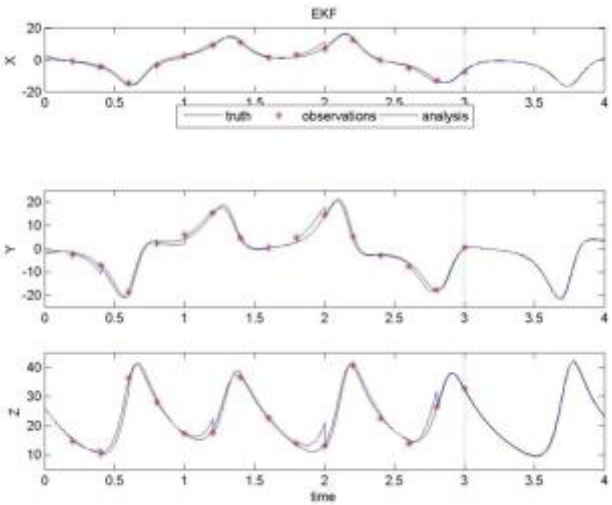
Investigate the effect of model error in the extended Kalman Filter.

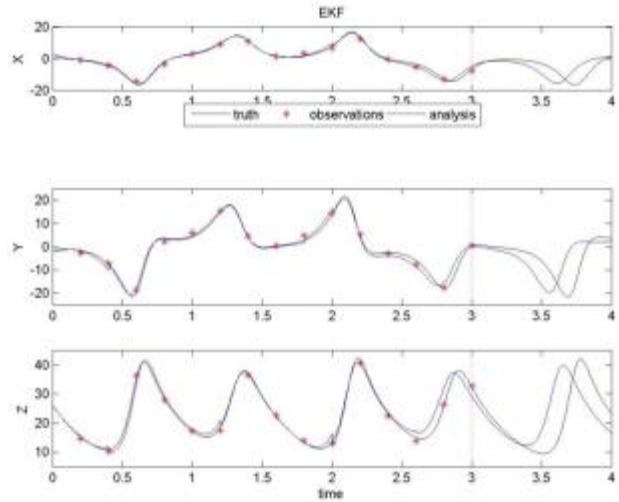
a) random stochastic error
b) error in the parameters
c) bias error

Olga Zawadzka, Benedita Santos, Antonio Neves, Tomohiro Sato

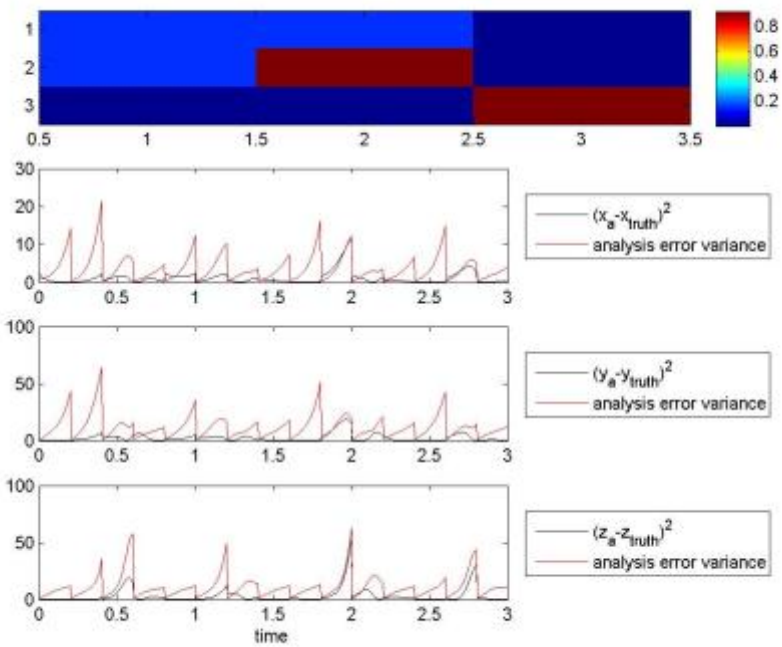
Control

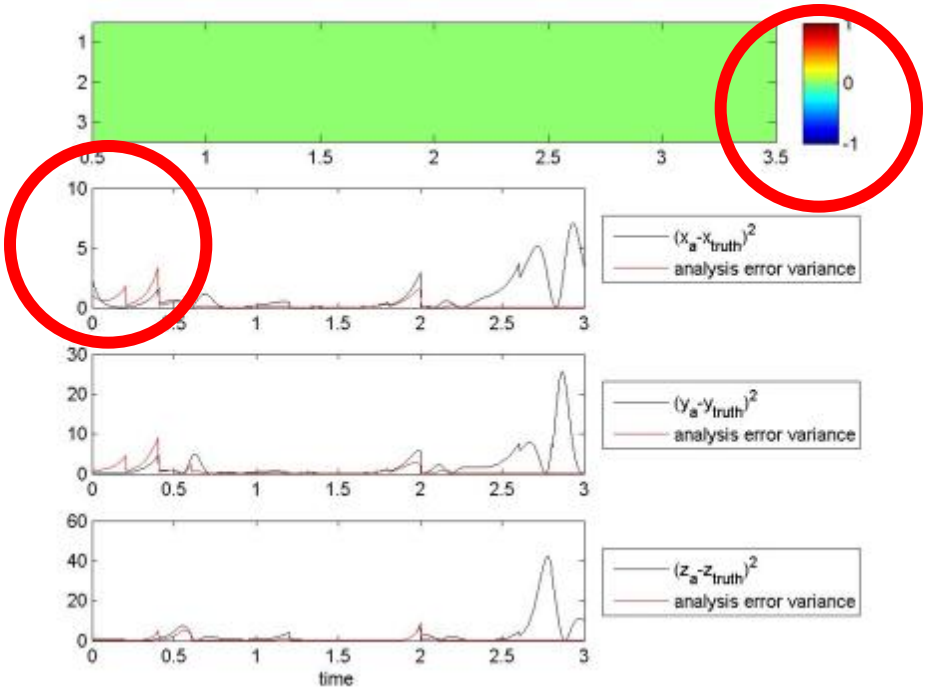
Without model error



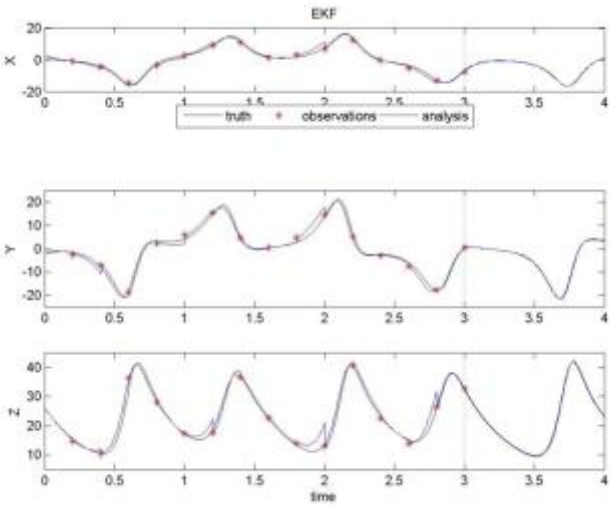


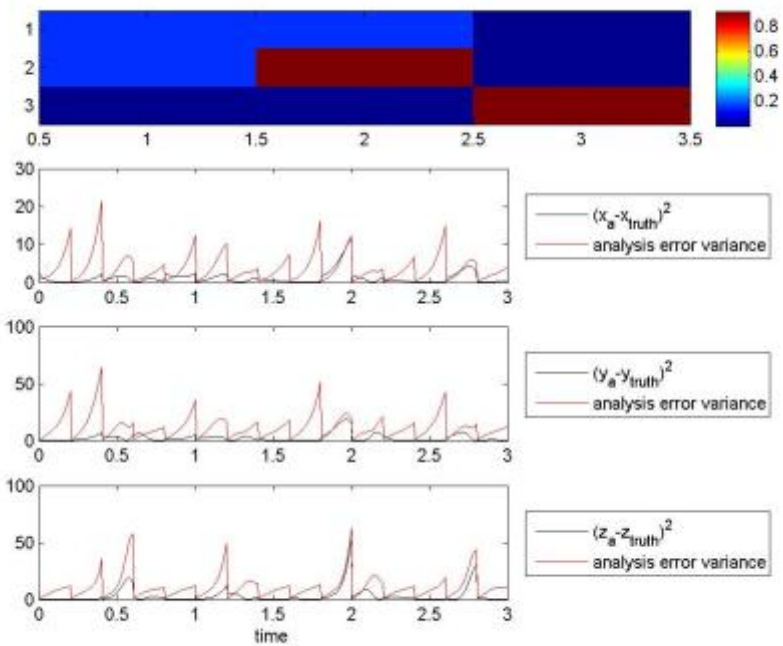
Q=0



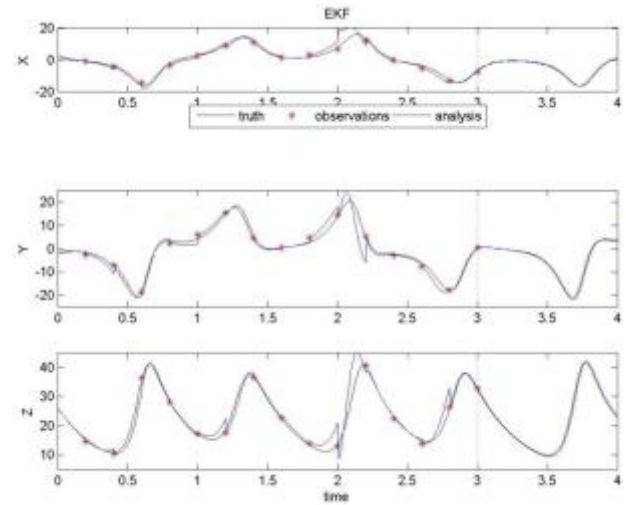


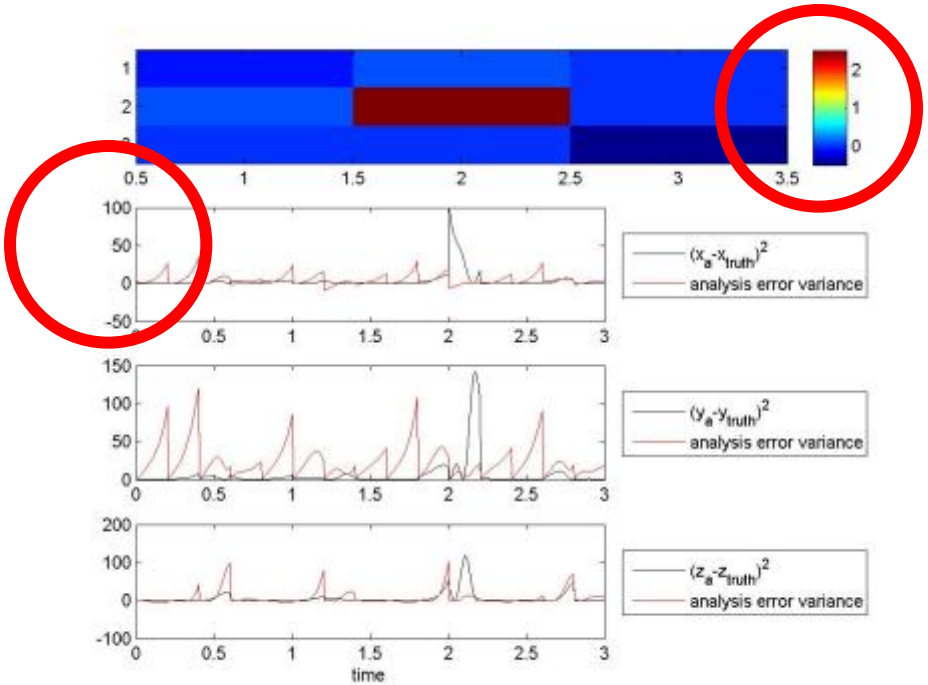
Control





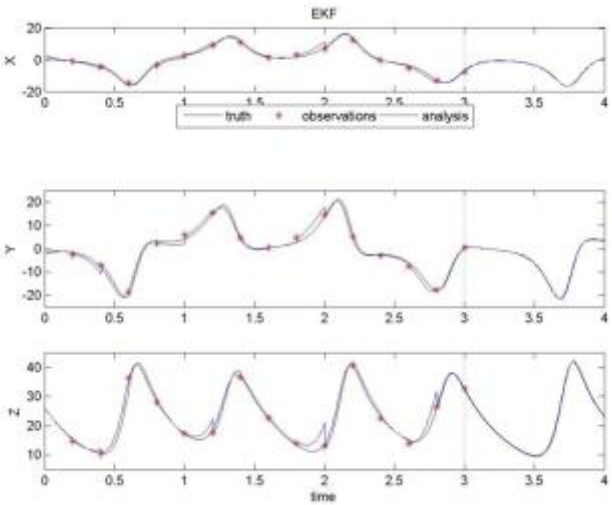
Random error added

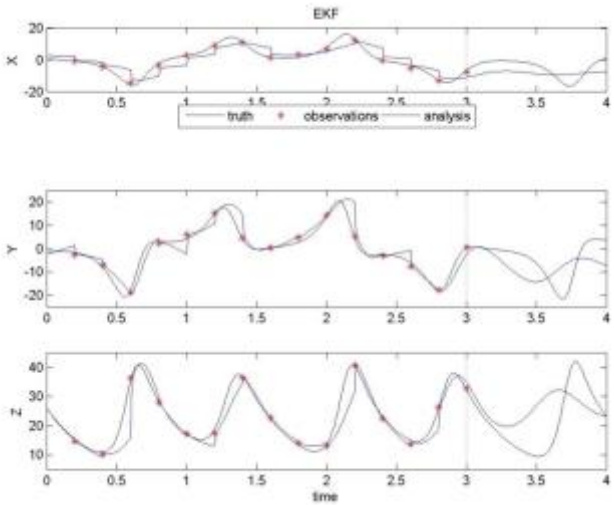




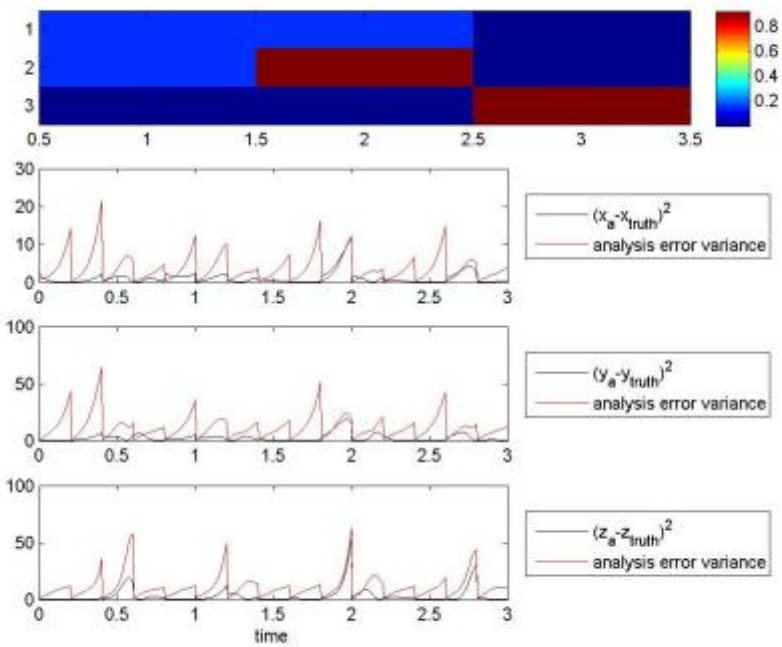
Control

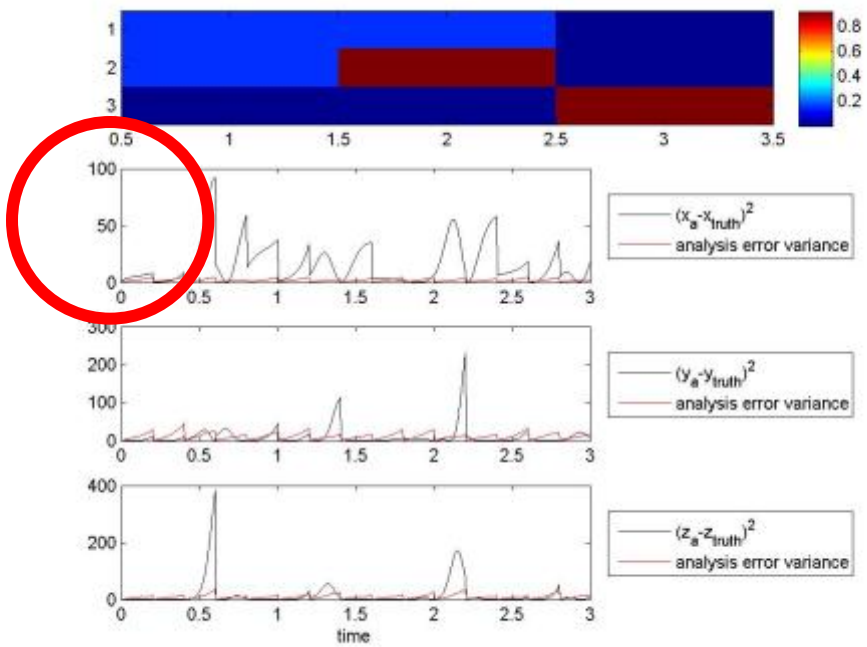
Error in model parameters





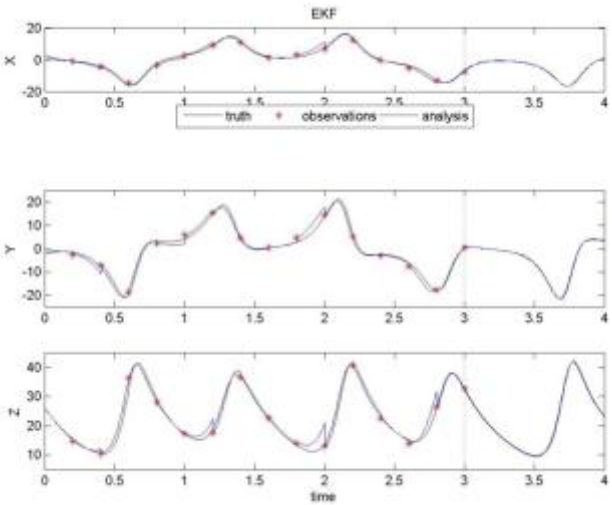
Sigma=10 → Sigma=2

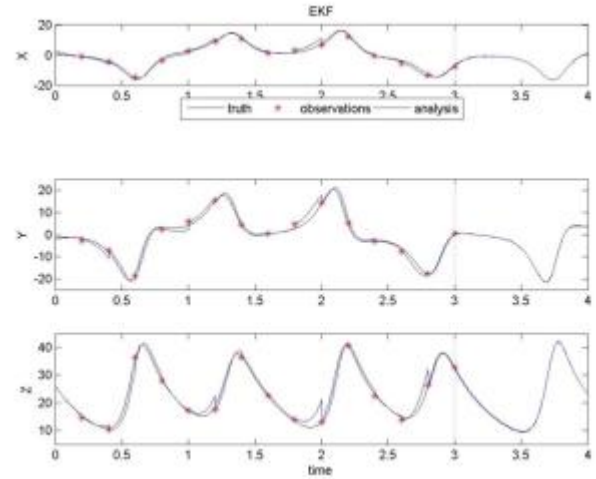


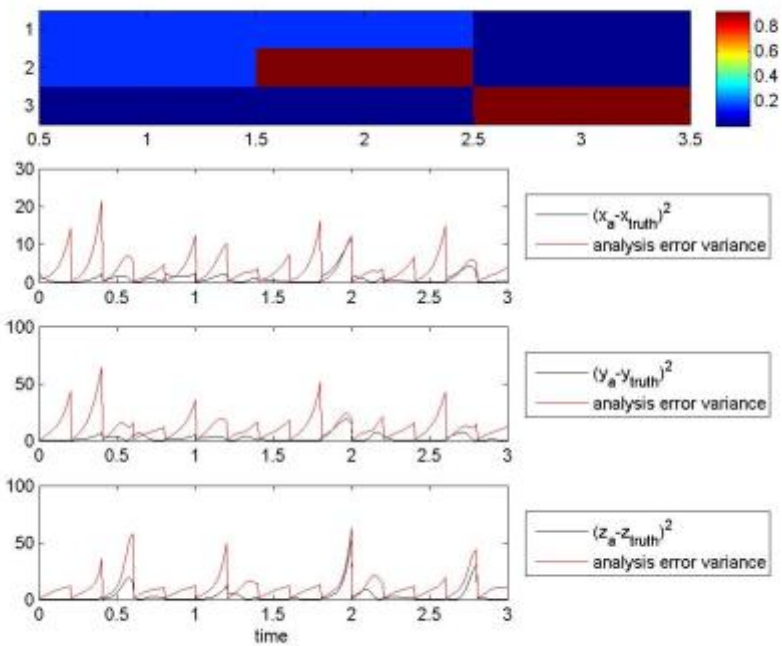


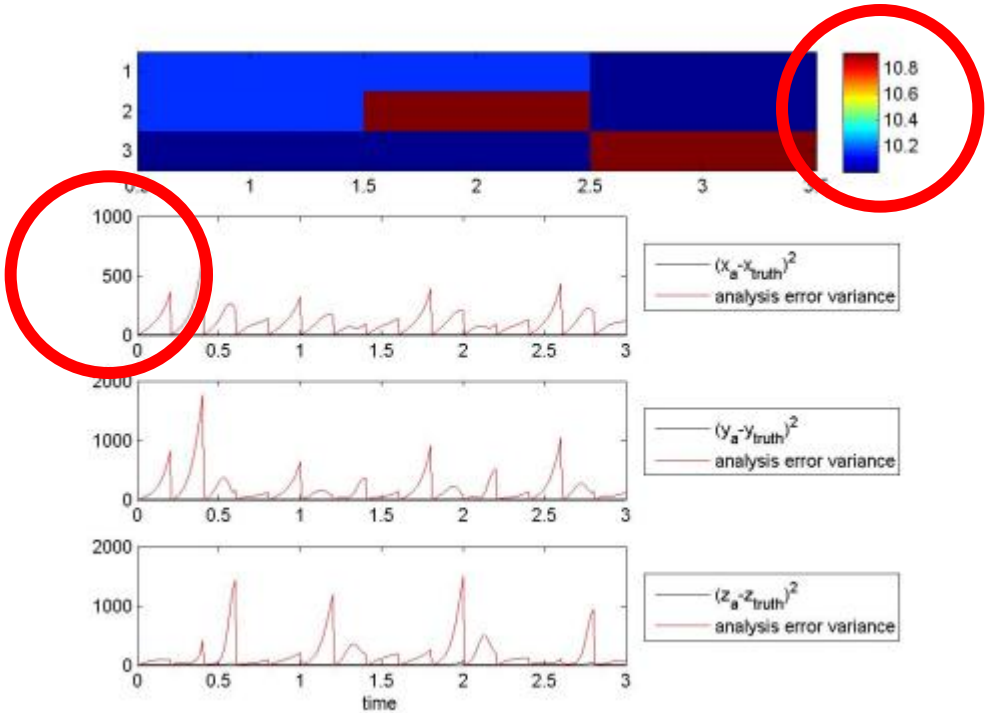
Control

Bias = 10





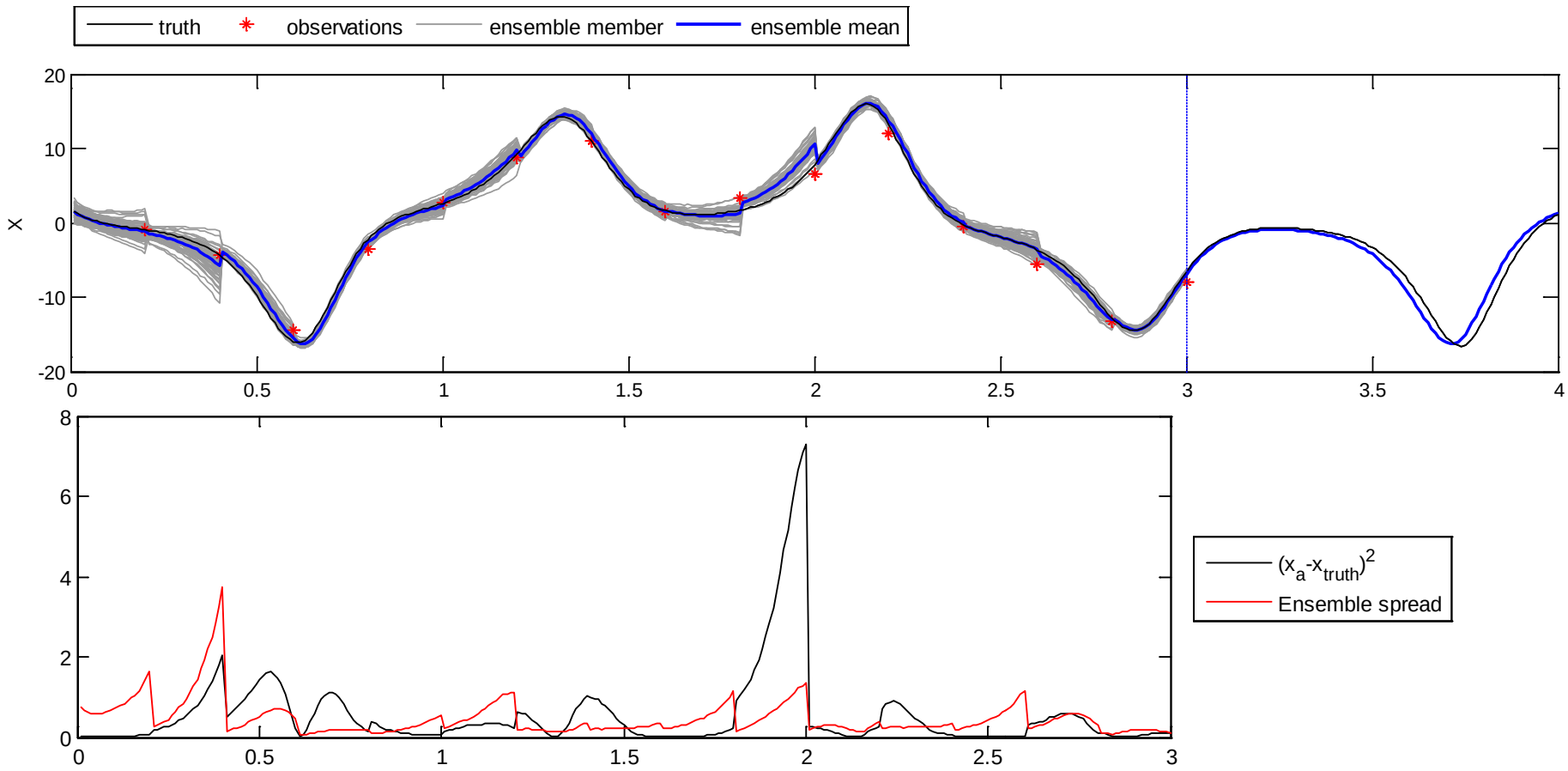




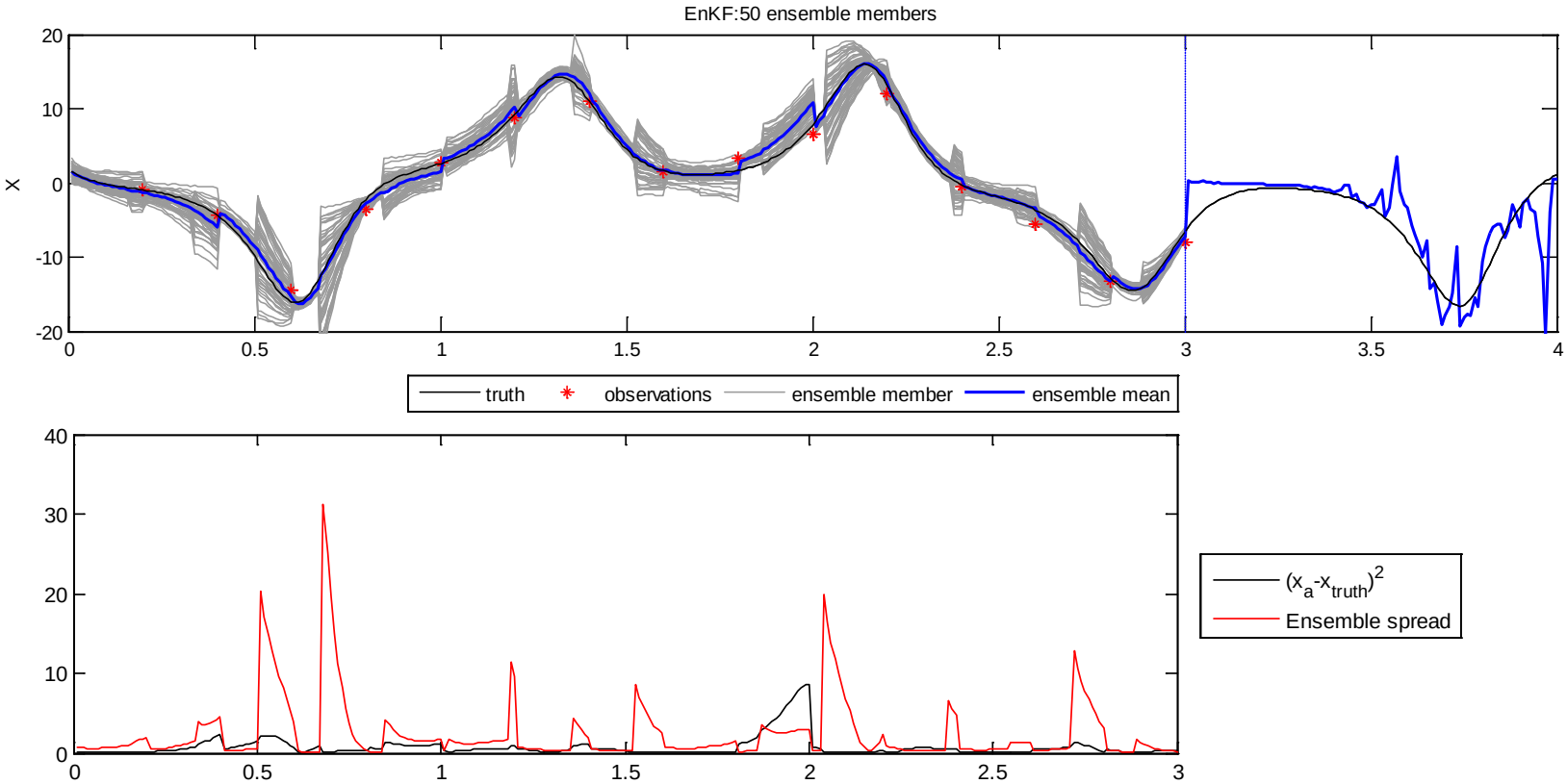
Project 15

Investigation of the effect of model error in the ensemble Kalman Filter

Original results



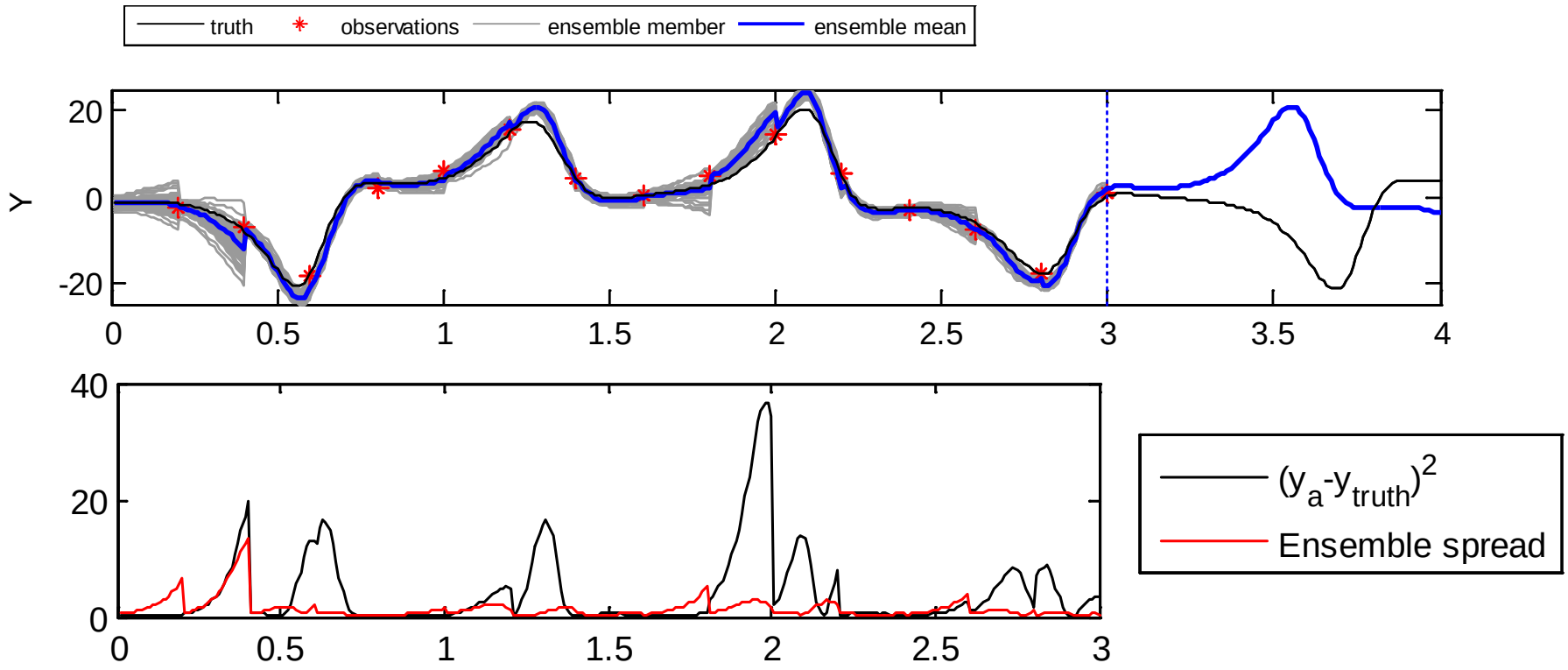
Random stochastic errors



X = x + 0.5d0 * (kx1+kx2) *** (-10+22.0*rand(1))**;

- Random error introduced in x every 17th timestep.
- Error appears as increases in ensemble spread.
- Because we are dealing with the Lorenz model, the modelled values converge towards the mean. (spread decreases)

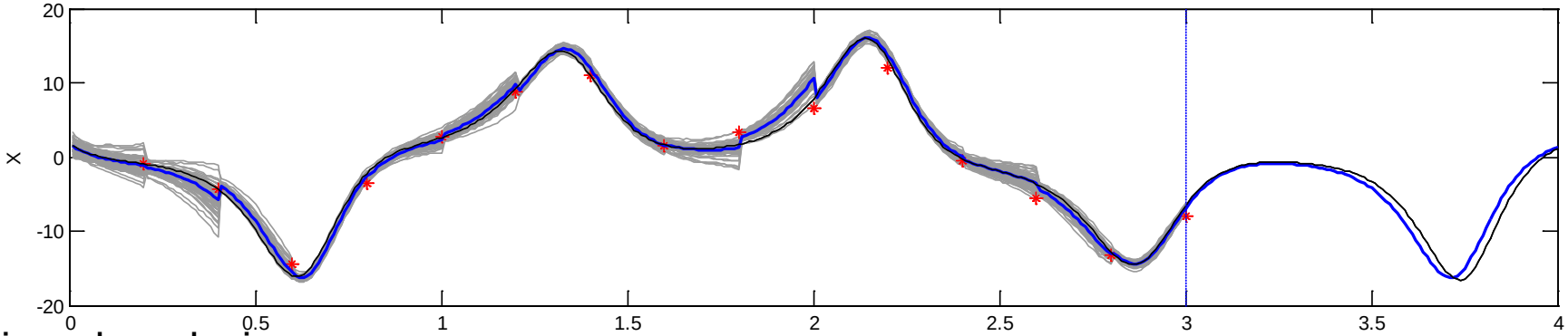
Parameter error



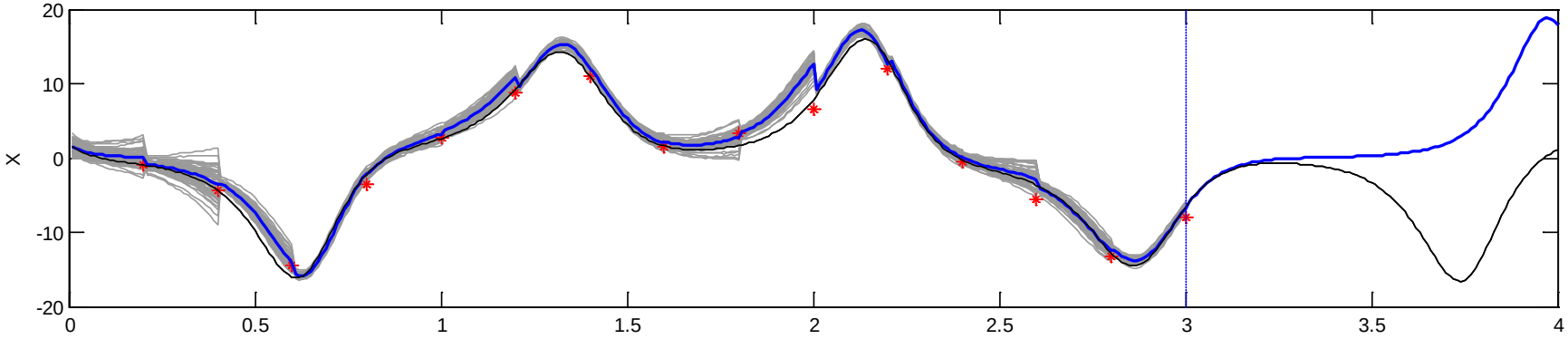
- Parameter error introduced : 10% increase in Rho.
- Difference between analysis and truth increases significantly.
    - greater effect than the (larger!) stochastic error.
- Ensemble spread has similar variations than before error introduction.

Bias error

Unbiased analysis



Biased analysis



- the assimilation ‘corrects’ the bias (as long as the observations are correct).
- however, the forecast quickly shows large differences with the truth

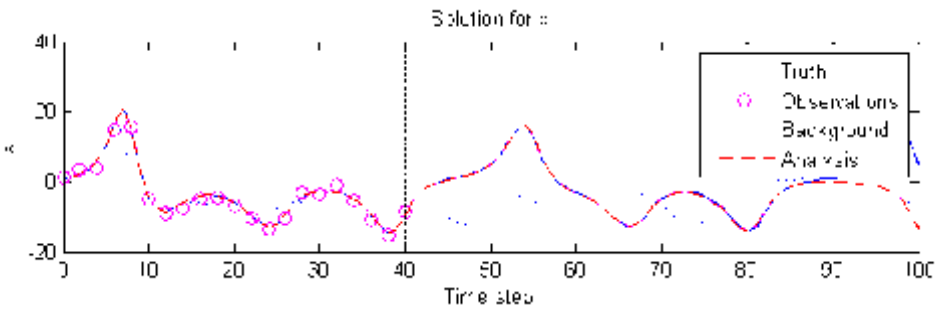
Implications: Model errors in EnKF

- Even slight errors in parameters or bias can lead to significant errors in the analysis.
- Stochastic errors have less influence on the overall results
  - … in the case of the Lorenz model.

Investigating the effect of model error in 4D -Var

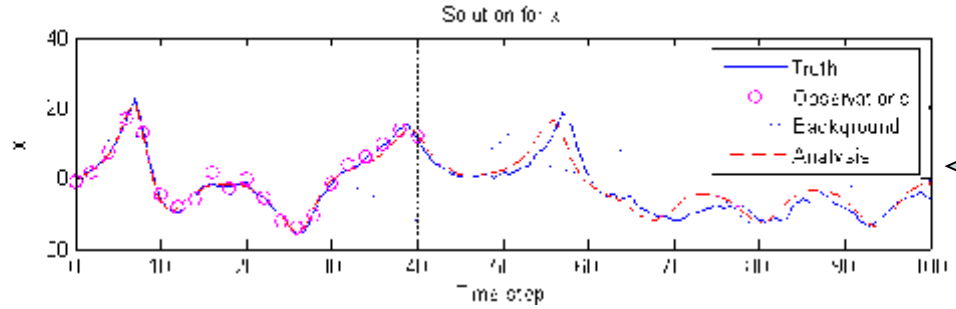
Group 16: Ujjwal Kumar, Chandrakanta Ojha, Xi Xi, Iacob I. Ciprian

- **ESA SUMMER SCHOOL 2012** -

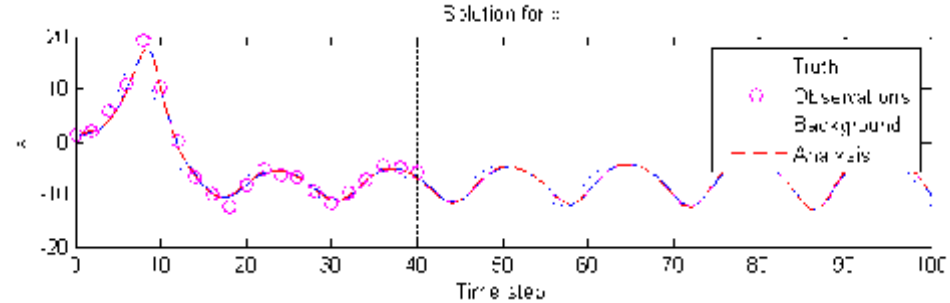


True (x,y,z) at t=0: (1,1,1)
Length of assimilation window: 2
Length of subsequent forecast: 3
Time step: 0.05
Frequency of observations = 2
Maximum iterations: 30
Tolerance: 1d-3
Observations have random noise with variance 1
Background noise = 2

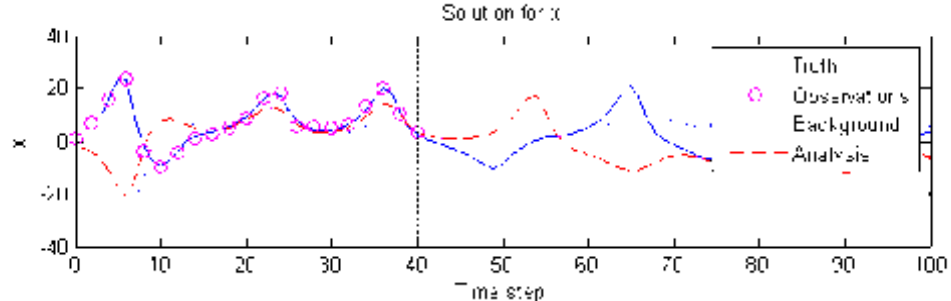
Background (x,y,z) at t=0: (2.9735,1.4539,3.2958)
Analysis (x,y,z) at t=0: (0.947,1.1008,1.0239)



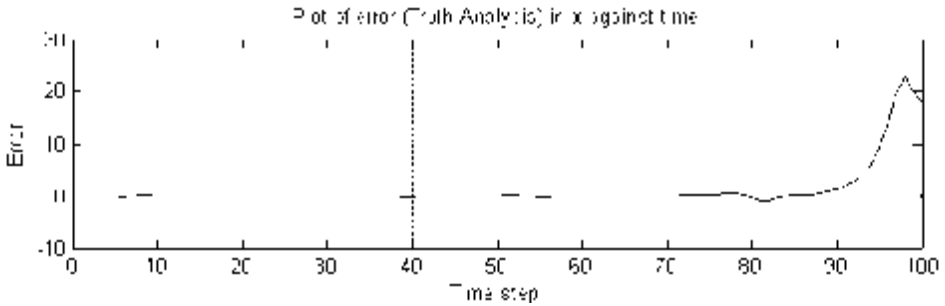
Random stochastic error



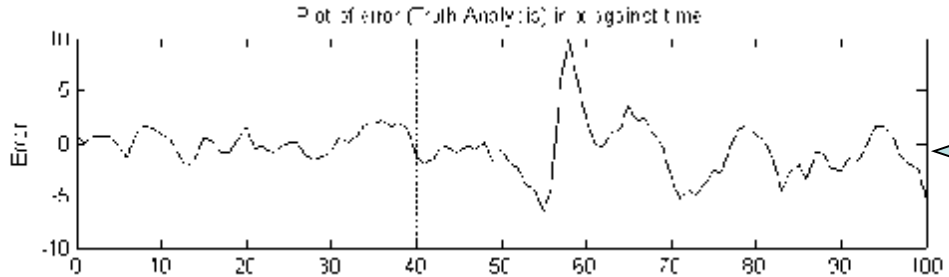
Change in the parameter σ = 8



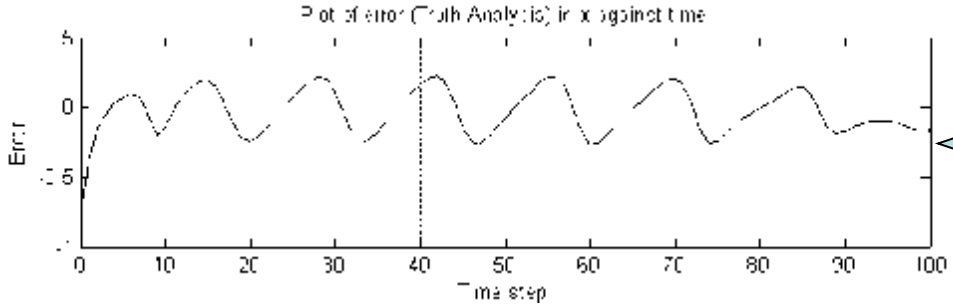
An additive bias error in x



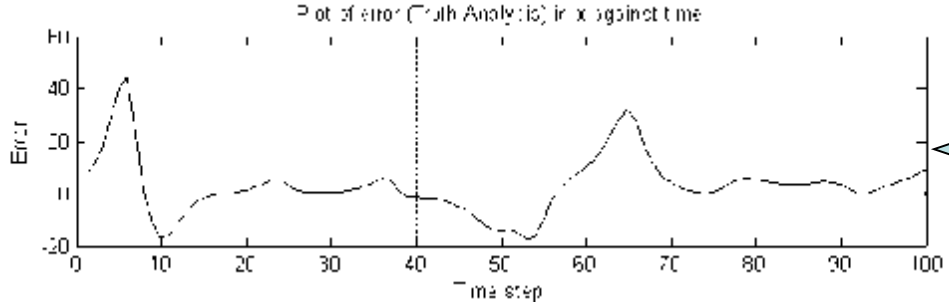
Default Parameters



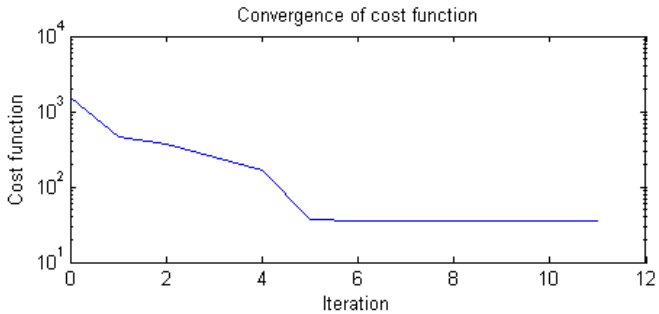
Random stochastic error



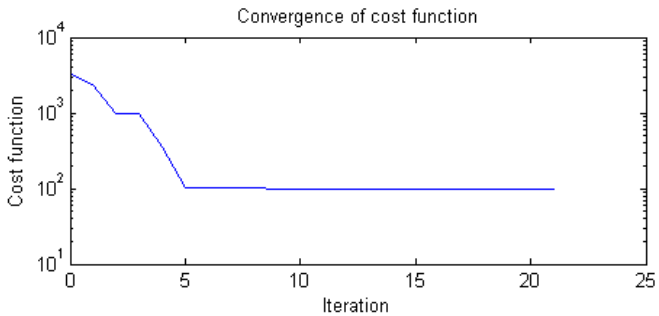
Change in the parameter σ = 8



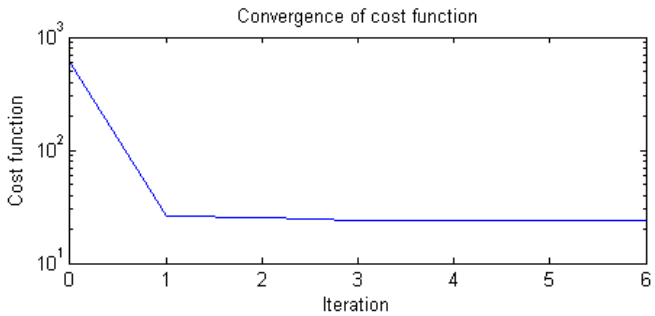
An additive bias error in x



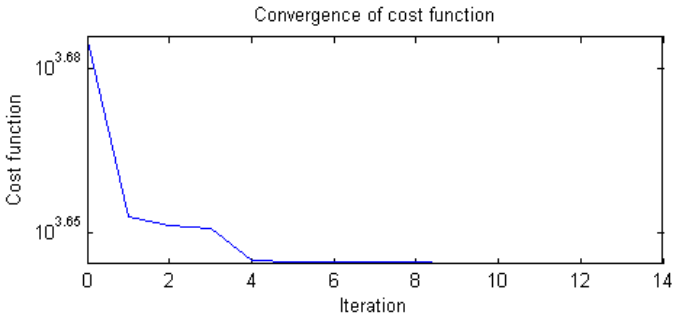
Default Parameters



Random stochastic error



Change in the parameter σ = 8



An additive bias error in x

Conclusion & Discussions

- In our experiment, 4D-var analysis results seem to be the most sensitive to the model bias error.
- Cost function appears to converge faster when errors are introduced.

Data assimilation projects : Project 17

What is an optimal ensemble size ?

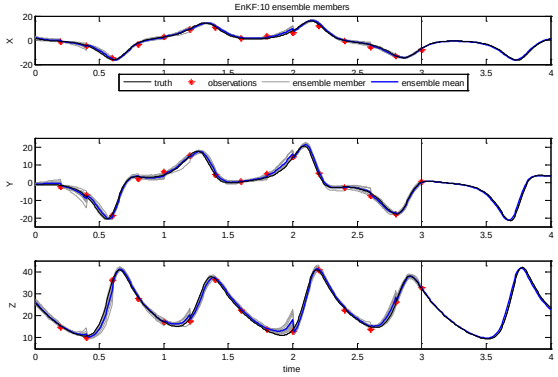
Ensemble Kalman Filter

Flore Tocquer
Elke Ludewig
Alex Geddes
Arjen Terwisscha van Scheltinga

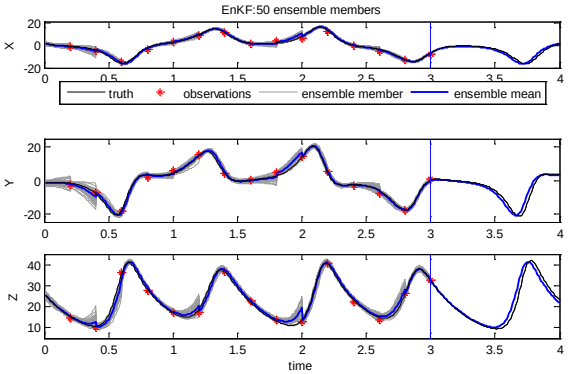
Does the solution converge?

Does the error reduce?

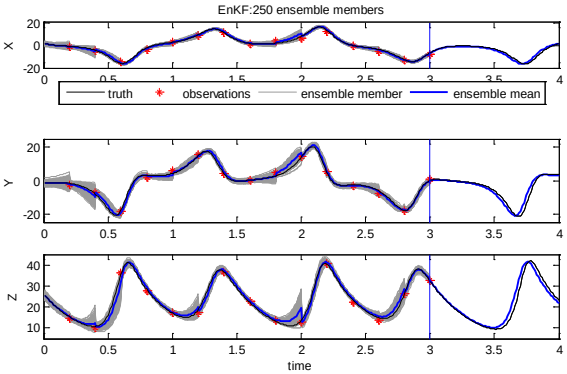
10 members

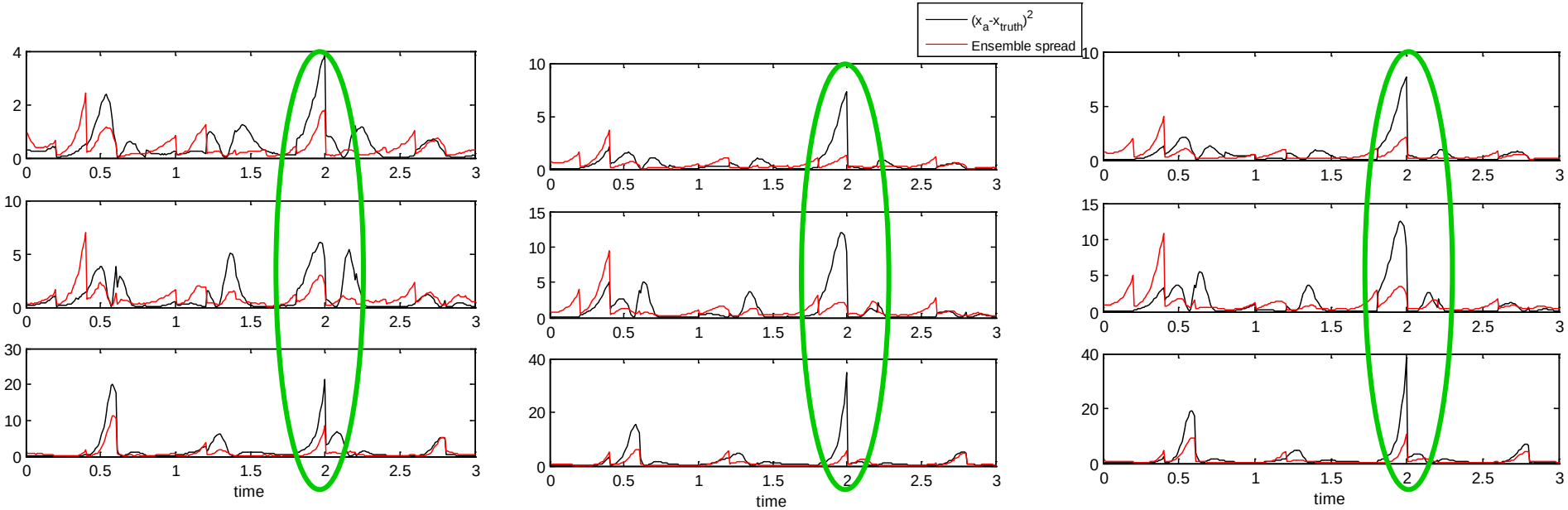


50 members



250 members





True (x,y,z) at t=0: (1.5,-1.5,25.5)
Length of assimilation window: 3
Length of subsequent forecast: 1
Time step: 0.01
Frequency of observations = 20
Ensemble size = 10
Observations have random noise with variance 1
Background noise = 1

True (x,y,z) at t=0: (1.5,-1.5,25.5)
Length of assimilation window: 3
Length of subsequent forecast: 1
Time step: 0.01
Frequency of observations = 20
Ensemble size = 50
Observations have random noise with variance 1
Background noise = 1

True (x,y,z) at t=0: (1.5,-1.5,25.5)
Length of assimilation window: 3
Length of subsequent forecast: 1
Time step: 0.01
Frequency of observations = 20
Ensemble size = 250
Observations have random noise with variance 1
Background noise = 1

Does the solution converge?

As you increase the ensemble size the solution converges, a notable example is the peak at time 2

Running increasing numbers of members does not further improve the solution and increases computation time

Does the error reduce?

A reduction in error was not observed with increasing ensemble size, this could partly be due to the lack of accurate observations and background

In The Real World

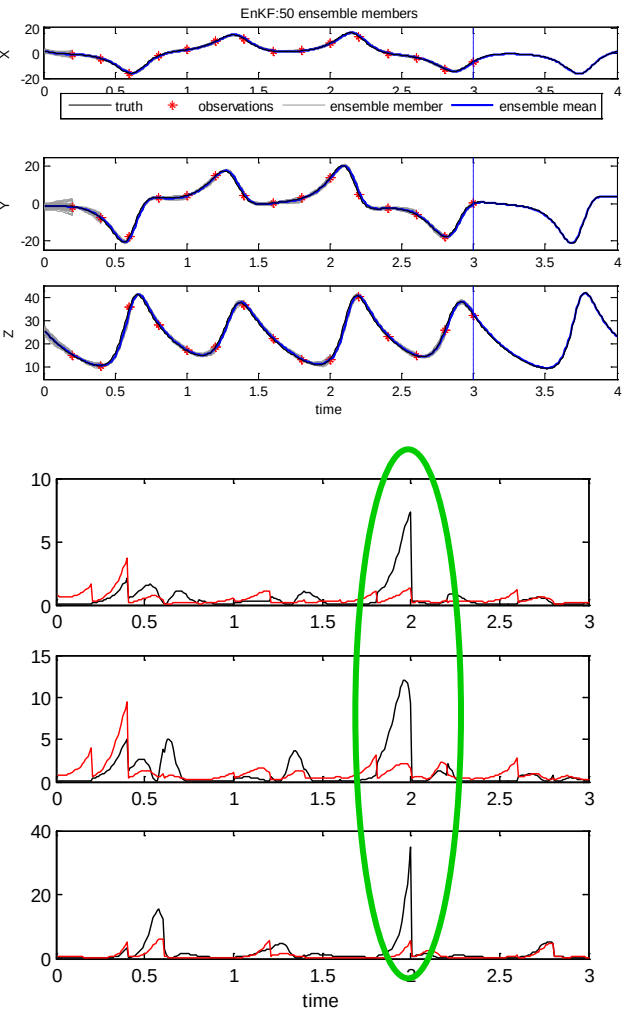
Increasing the ensemble size does not necessarily improving your solution, for instance a weather prediction would not be more accurate by merely adding more members

However this must be balanced by having sufficient members in order for the solution to converge

How is the convergence affected by the frequency and accuracy of the observations?

Observation errors: 1.0

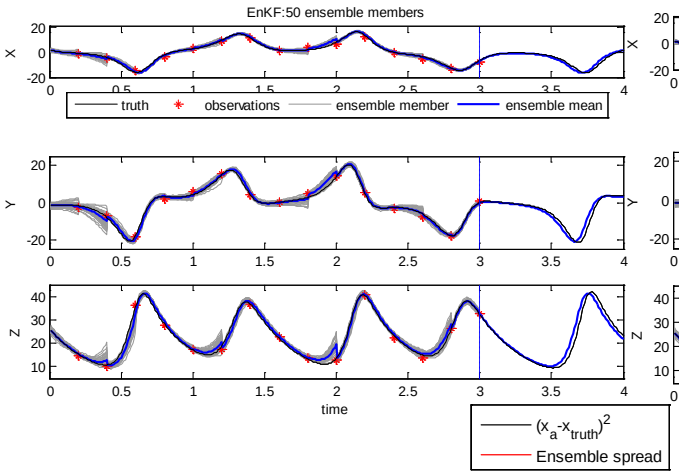
Frequency observation : 20

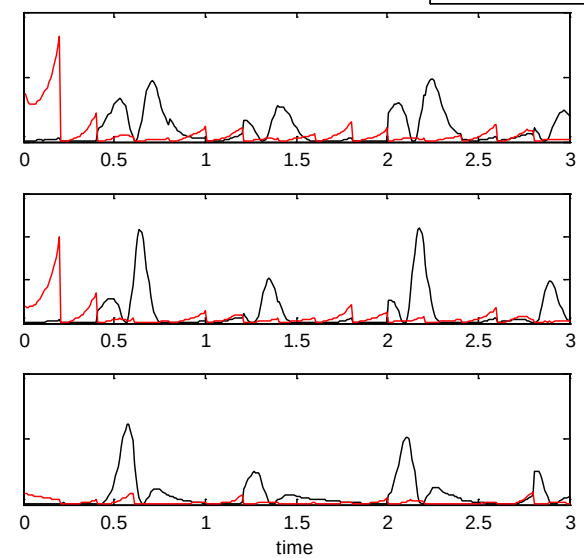


True (x,y,z) at t=0: (1.5,-1.5,25.5)
Length of assimilation window: 3
Length of subsequent forecast: 1
Time step: 0.01
Frequency of observations = 20
Ensemble size = 50
Observations have random noise with variance 1
Background noise = 1

Observation errors : 0.01

Frequency observation : 20

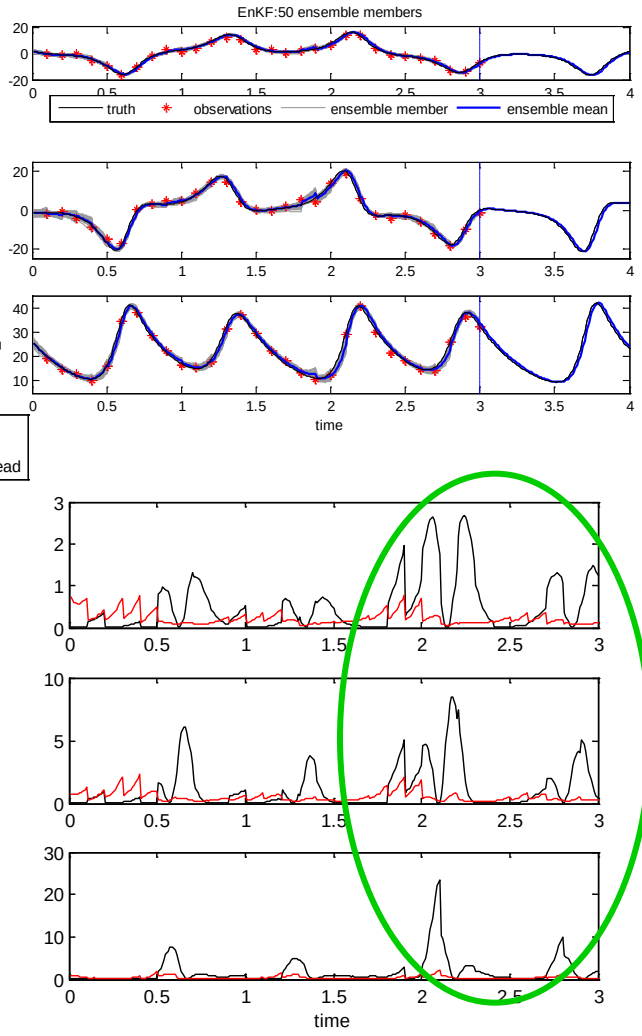




True (x,y,z) at t=0: (1.5,-1.5,25.5)
Length of assimilation window: 3
Length of subsequent forecast: 1
Time step: 0.01
Frequency of observations = 20
Ensemble size = 50
Observations have random noise with variance 0.01
Background noise = 1

Observation errors : 0.01

Frequency observation : 10



True (x,y,z) at t=0: (1.5,-1.5,25.5)
Length of assimilation window: 3
Length of subsequent forecast: 1
Time step: 0.01
Frequency of observations = 10
Ensemble size = 50
Observations have random noise with variance 1
Background noise = 1

How is the convergence affected by the frequency and accuracy of the observations?

The error reduces with increasing **accuracy** of the observations

Having **more** observations makes the forecast more accurate.

This only in the case where the background error is significantly larger than the observation error

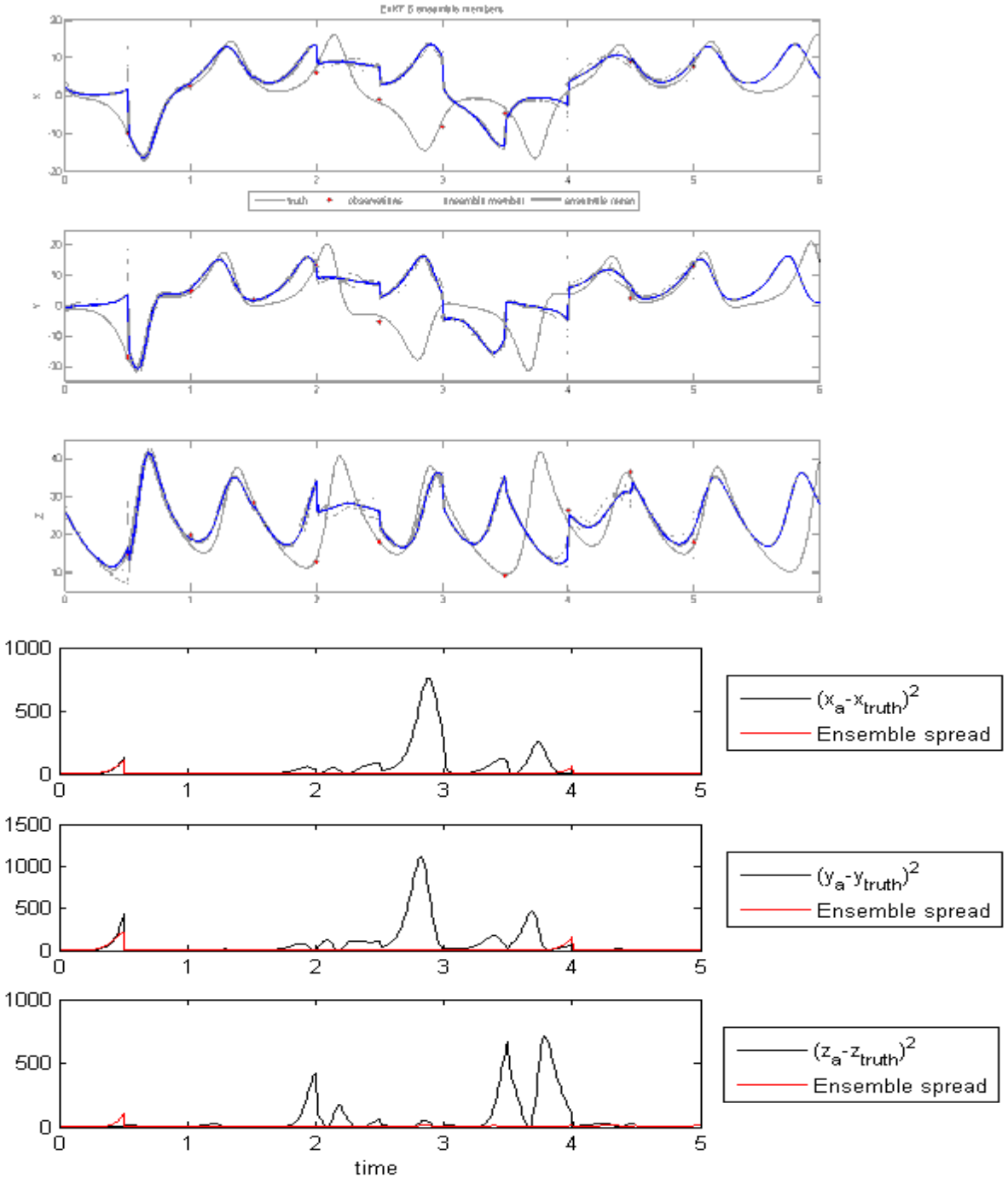
Ensemble Kalman Filter

Thomas Webb
Sarah Kemp
Robin Locatelli
Anne-Lise Dhomps

Project 18

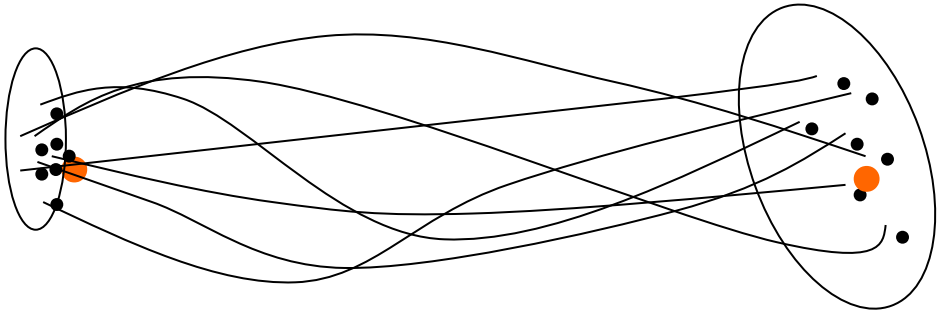
What options are there
to increase the ensemble spread
without increasing the ensemble size ?

Constraint: 5 ensemble members


EnKF 5 ensemble members
truth
observations
ensemble member
ensemble mean
$(x_a-x_{truth})^2$
Ensemble spread
$(y_a-y_{truth})^2$
Ensemble spread
$(z_a-z_{truth})^2$
Ensemble spread
time

Issue

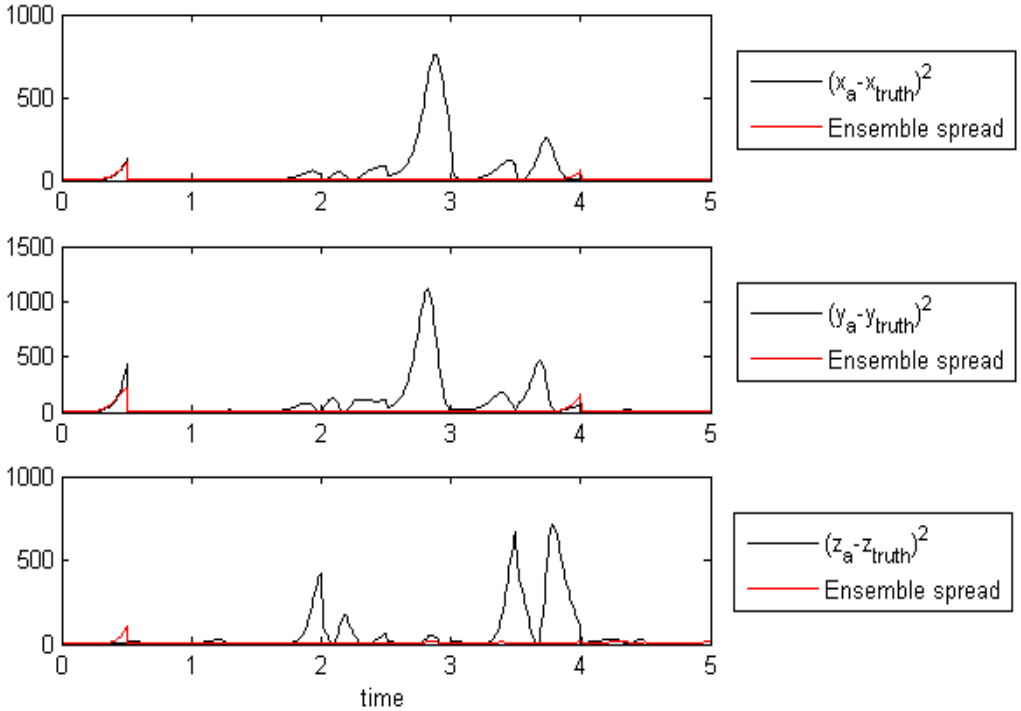
In many applications the ensemble size is limited. When the ensemble size is small it can underestimate the uncertainties in the model prediction and not give the observation enough weight in the update step.



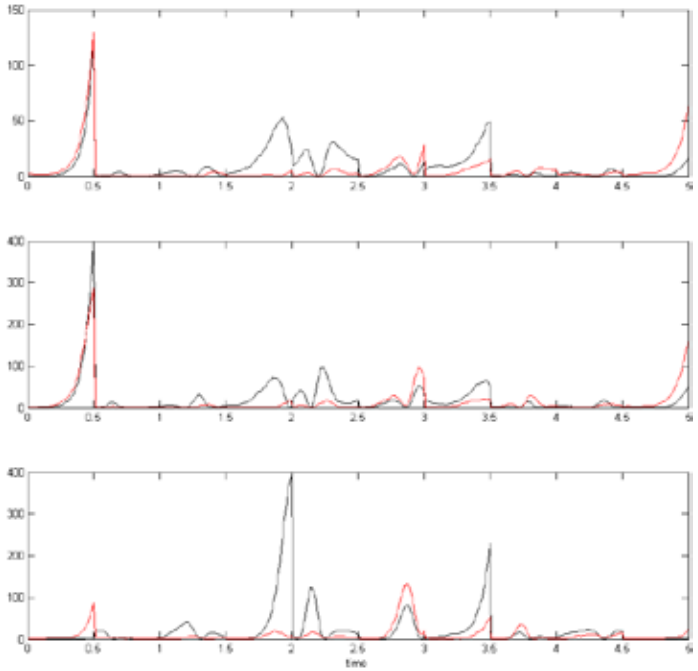
1. Modifying the Q matrix

$$\mathbf{P}^f(t)=\mathbf{M}\mathbf{P}^f(t\text{-}1)\mathbf{M}^T+\mathbf{Q}$$

Initial Q

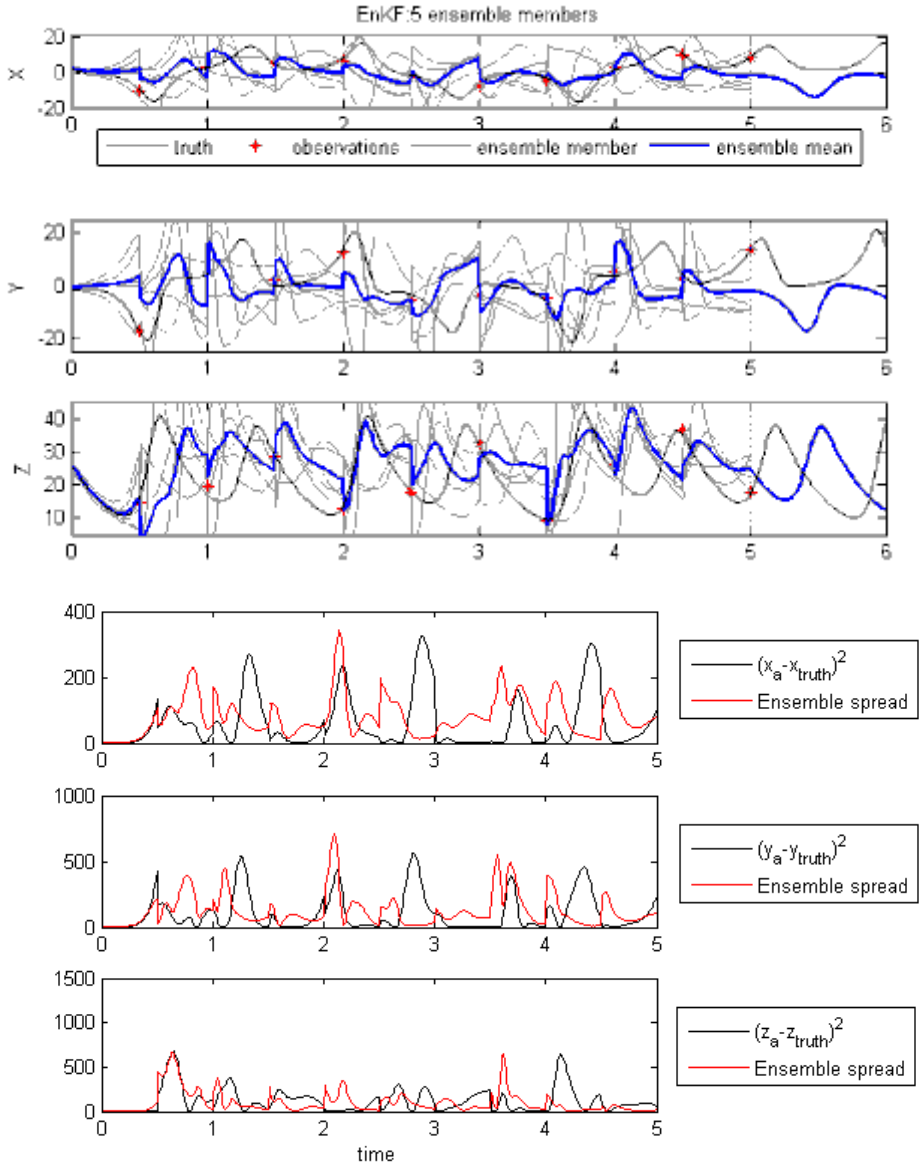


Initial Q x 5



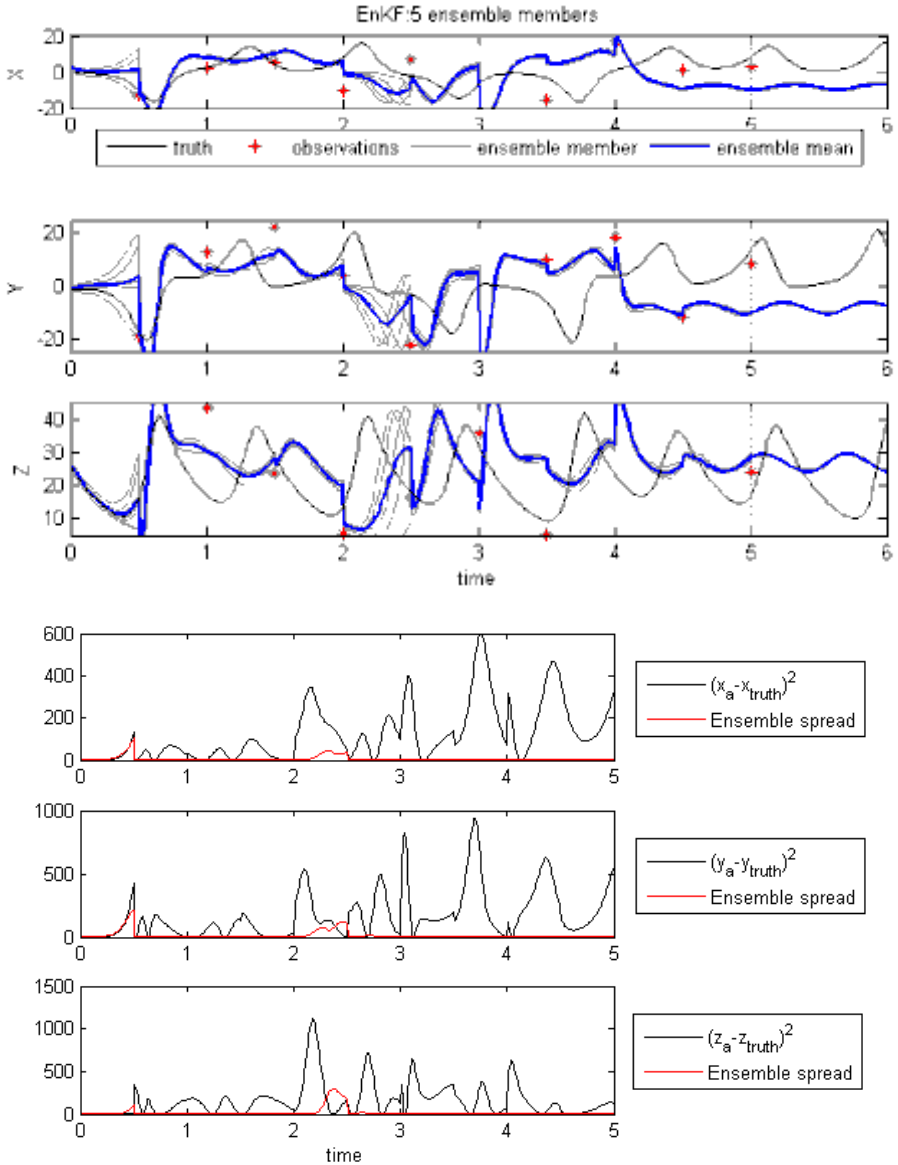
2. Modifying the size of obs perturbations

Xa = Xb + K*(y – H[Xb])

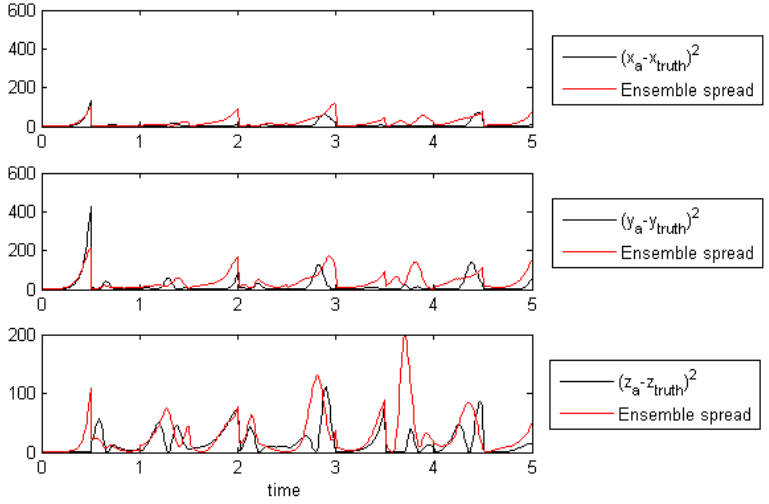


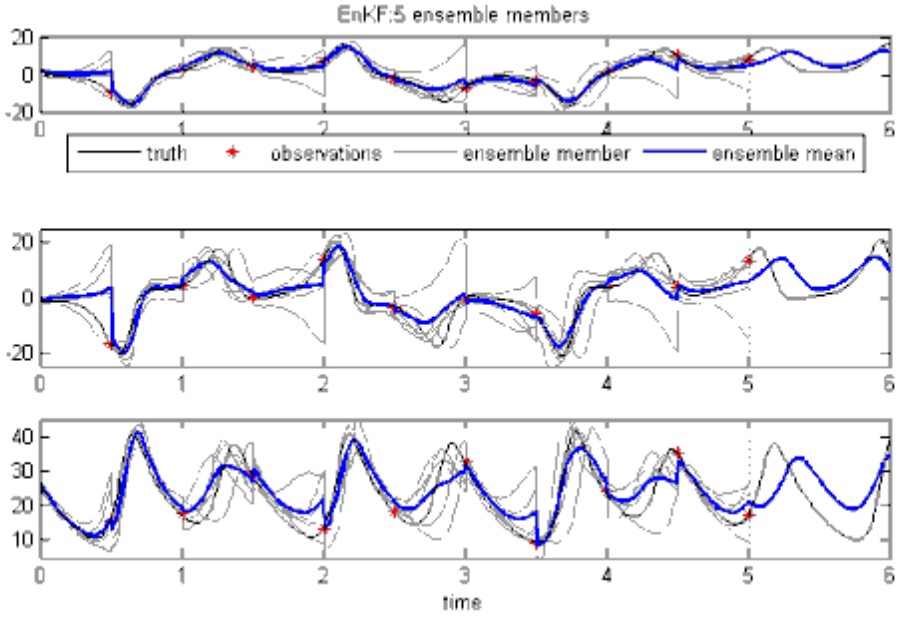
2. Modifying the size of obs perturbations

$$Xa = Xb + K*(y - H[Xb])$$



3. Modifying obs error and the size of perturbation





Conclusion

To increase the ensemble spread without increasing the ensemble size, we can “increase” the Q matrix or the model observation perturbations.
In reality, this would save CPU time but introduce error.

Questions ?