Satellite Gravimetry

Reiner Rummel
Institute of Advanced Study (IAS)
Technische Universität München

Lecture Three

5th ESA Earth Observation Summer School
2-13 August 2010, ESA-ESRIN, Frascati / Italy

plan for lecture three

Lecture Three

Measurement of the free fall of a test mass

Interpretation of the motion of a satellite in its orbit as test mass in free fall

From the orbit to the earth‘s gravitational field

The use of several test masses in free fall

Case One: the satellite mission GRACE

Measurement of temporal changes in gravitation

Case Two: the ESA Living Planet mission GOCE

The principle of gravitational gradiometry

Outlook

test mass in free fall

gravitation,

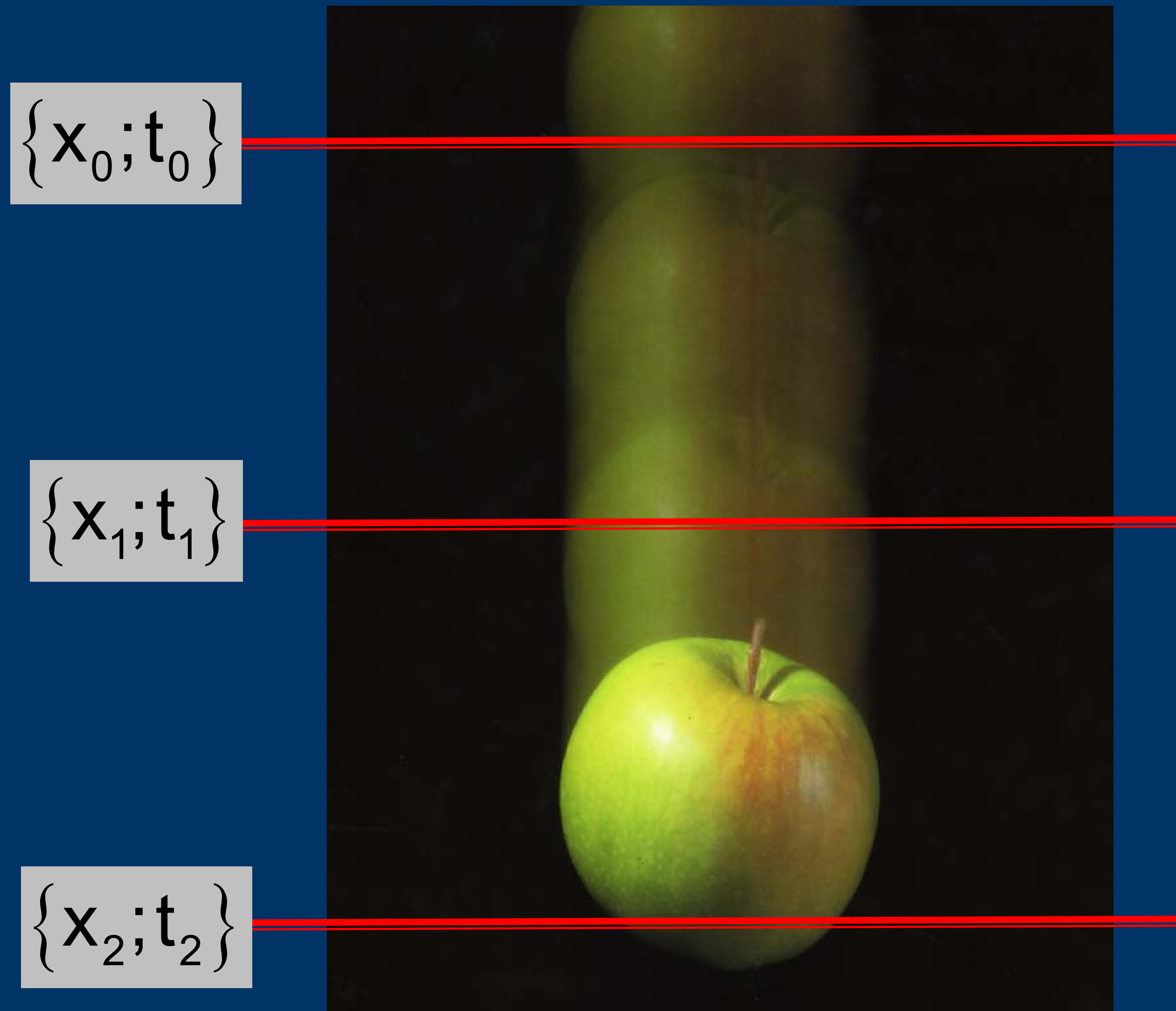
the story of the

falling apple

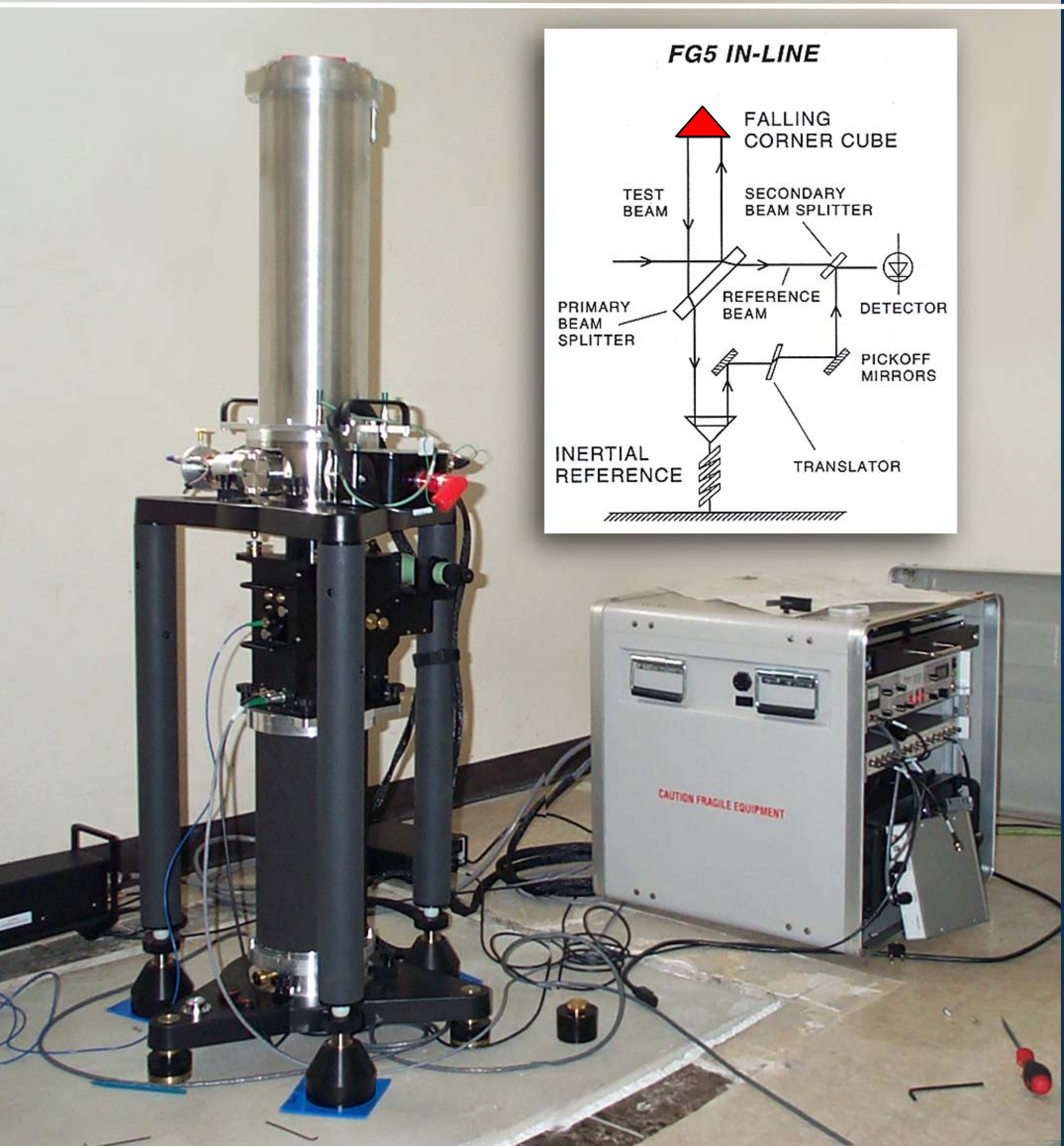
Reference:
R Westfall: The life of Isaac Newton,
Cambridge Univ Press, 1999
[Compare page 51 and 305]

test mass in free fall

the size of
gravity
from
measuring
position
and
time



test mass in free fall



Absolute-

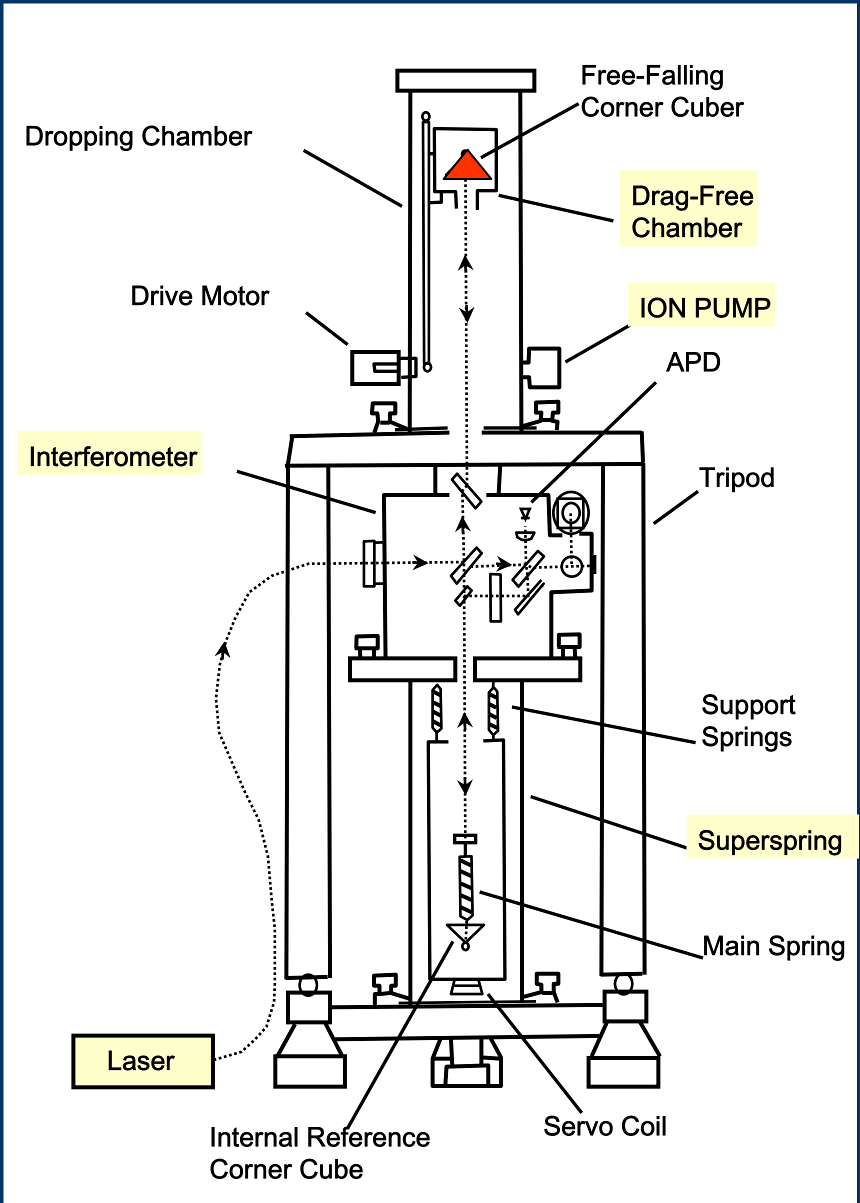
Gravimeter FG5

at

TU Hannover

test mass in free fall

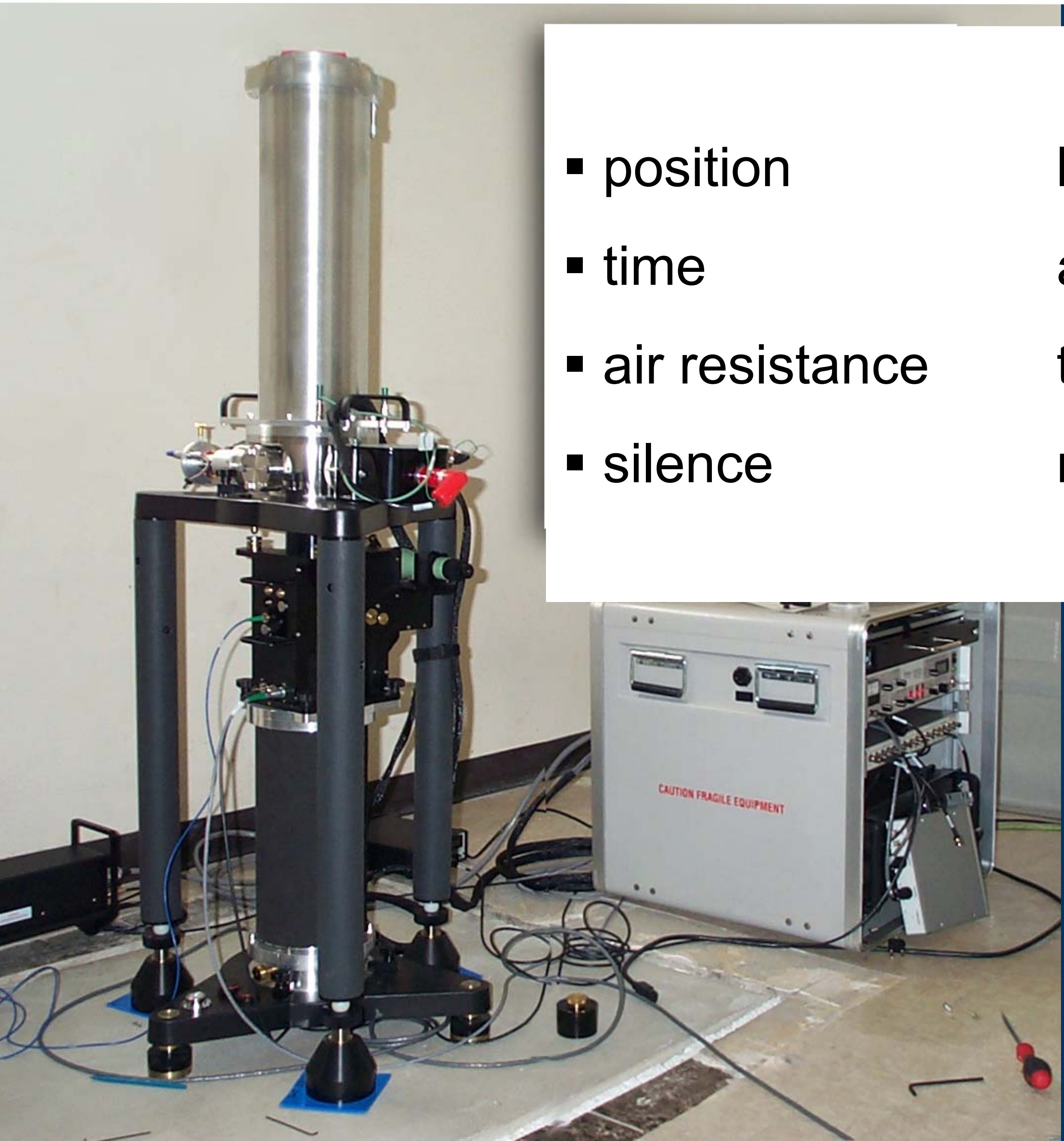
Absolut-Gravimeter FG5 (Micro-g Solutions, Inc.)



drop height: 0.2m / time: 0.2s
600 000 fringes per drop
100 drops per set; 25 sets
Mach-Zender laser interferometer
time: Rubidium atomic clock 10^{-10}
vacuum: 10^{-4} Pa
residual drag compensation
micro seismisity: super spring

test mass in free fall

- position — laser interferometry
- time — atomic clock
- air resistance — to be eliminated
- silence — no vibrations



Absolute-
Gravimeter FG5
at
TU Hannover

test mass in free fall

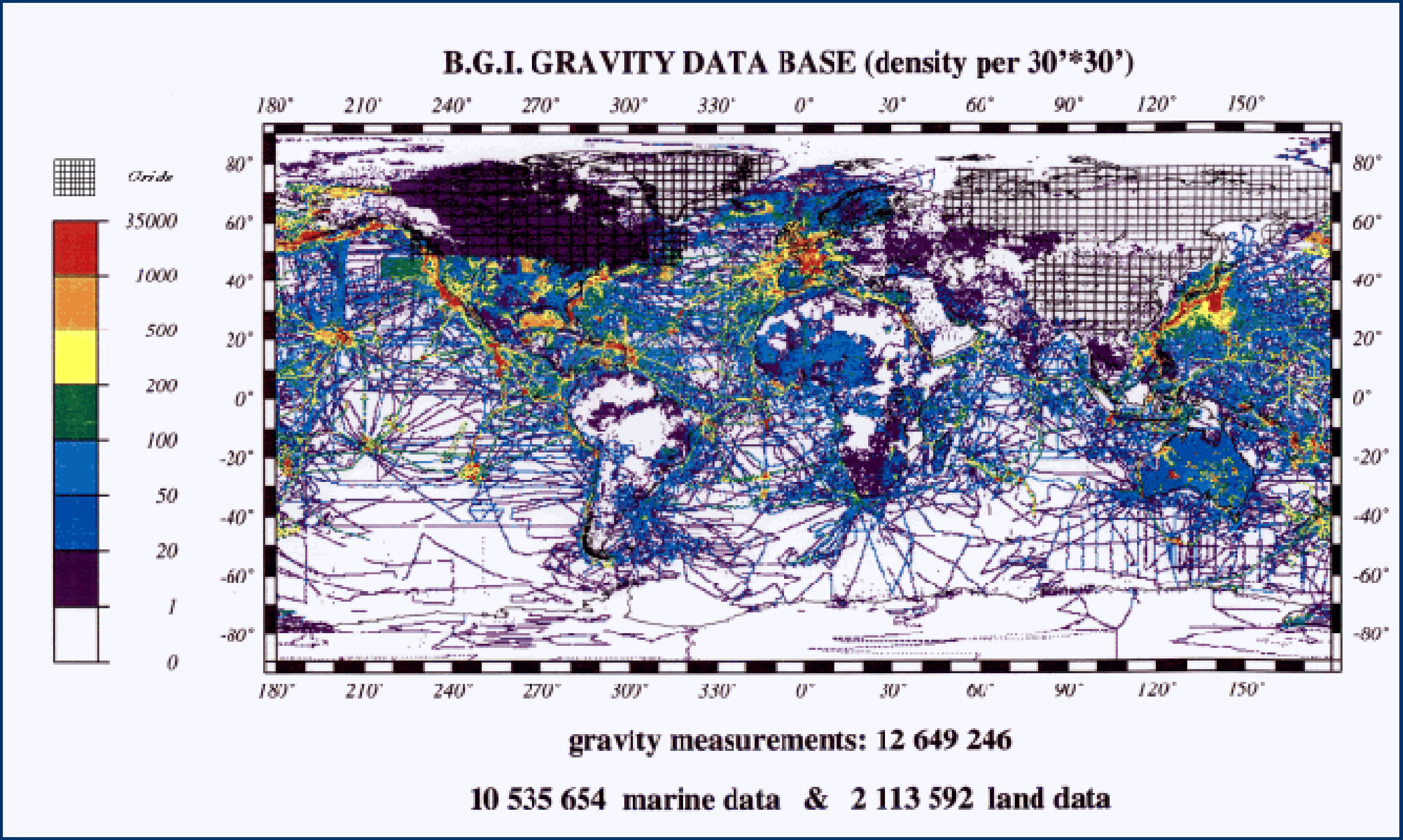
terrestrial absolute gravimetry in the mountains

test mass in free fall

gravity (in laboratory at TU München)
9.807 246 72 m/s^2

stationary	10^{0}	**spherical Earth**
	10^{-3}	**flattening & centrifugal acceleration**
	10^{-4}	**mountains, valleys, ocean ridges, subduction**
	10^{-5}	**density variations in crust and mantle**
	10^{-6}	**salt domes, sediment basins, ores**
variable	10^{-7}	**tides, atmospheric pressure**
	10^{-8}	**temporal variations: oceans, hydrology**
	10^{-9}	**ocean topography, polar motion**
	10^{-10}	**general relativity**

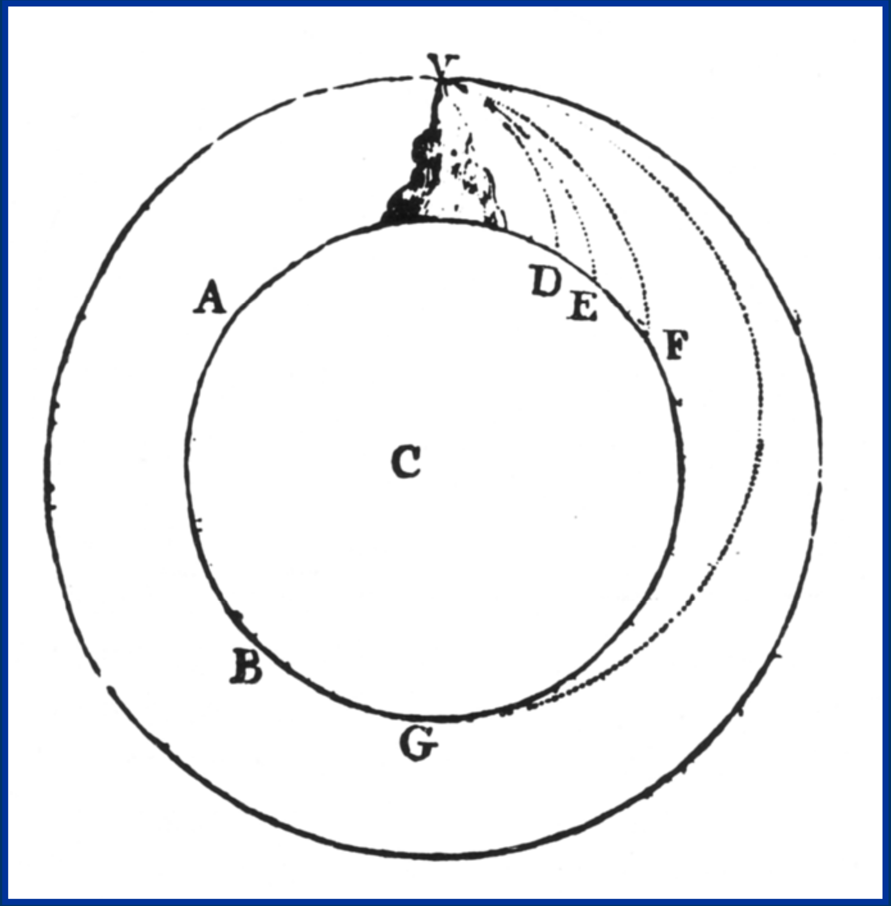
test mass in free fall



map with global distribution of in-situ measurements

satellites - test masses in free fall

Newton‘s brilliant conclusion:
Orbit motion of planets (Kepler) obeys the same law as a free falling „apple“ (Galileo)

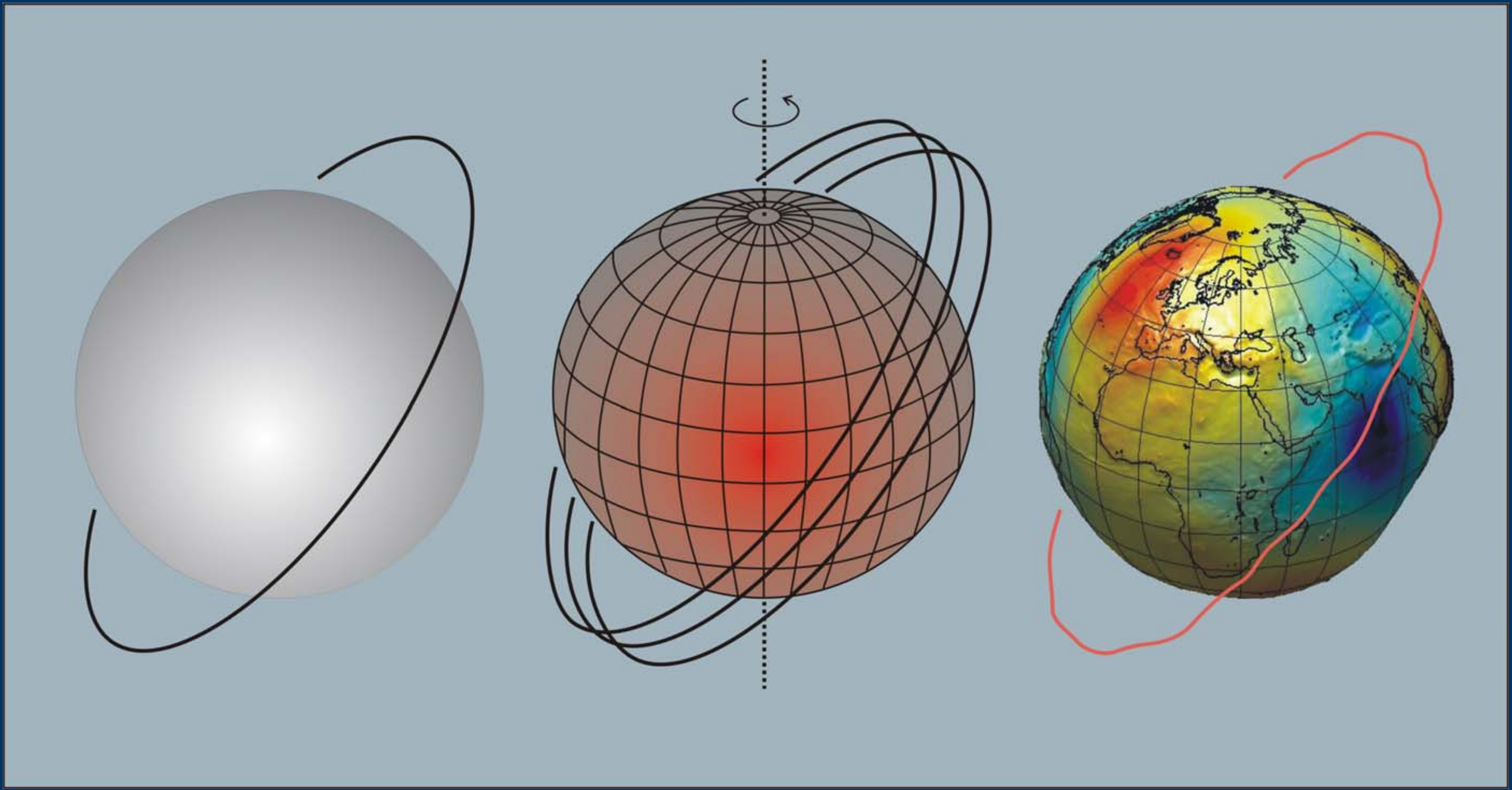


I. Newton “De mundi systemate“ 1715

satellites - test masses in free fall

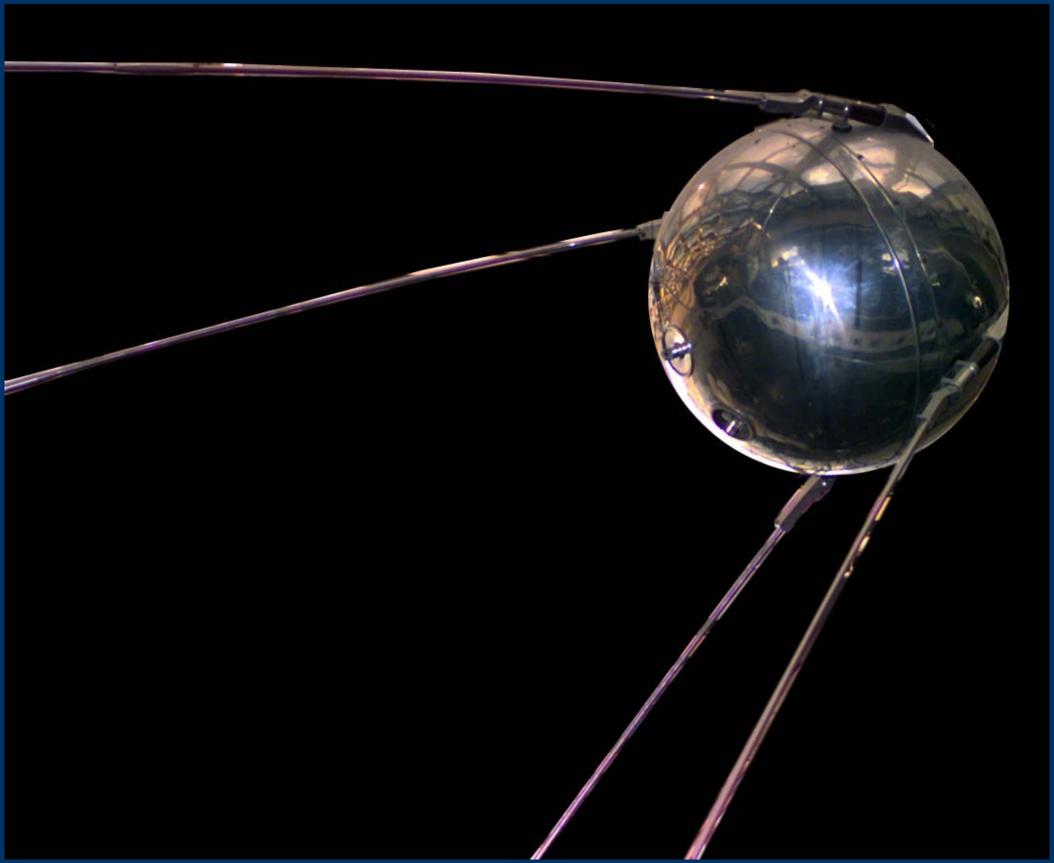
satellite orbit

Case 1 homogeneous sphere:	space fixed (Kepler) ellipse
Case 2 oblate sphere:	precessing ellipse (spiral)
Case 3 actual Earth:	modulation from gravitation



satellites - test masses in free fall

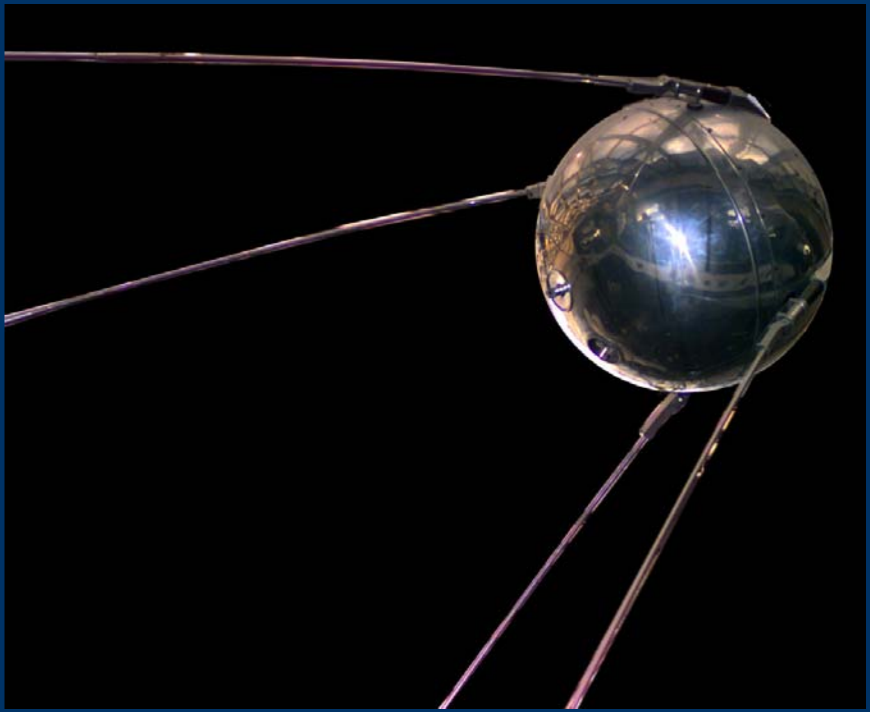
earth oblateness deduced from orbit plane precession



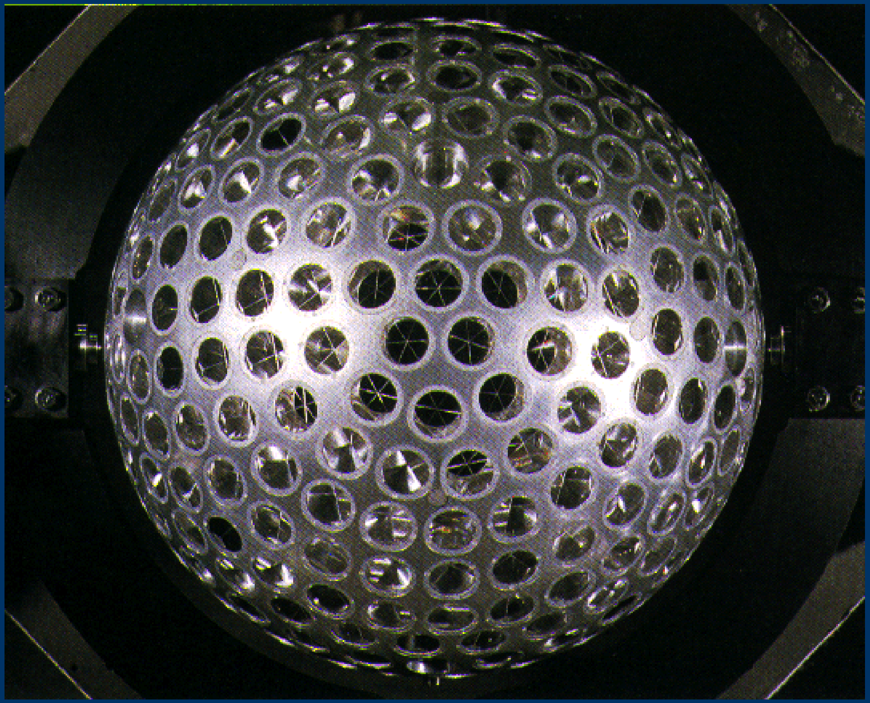
1957: Sputnik 1

earth flattening $J_2 = 1084 \cdot 10^{-6}$

test mass in free fall



1957: Sputnik 1

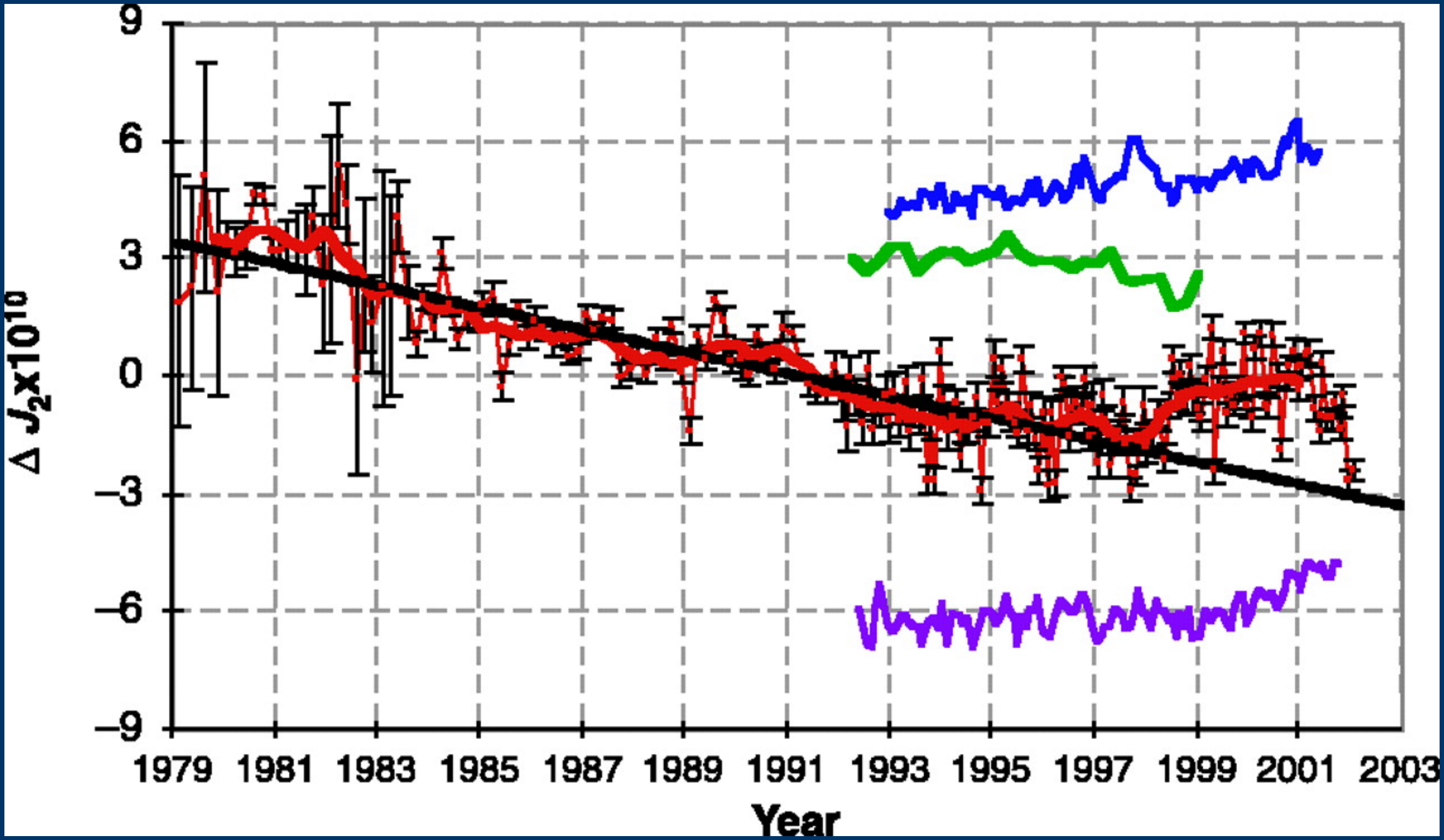


today: LAGEOS I and II

satellites - test masses in free fall

steady decrease of earth flattening

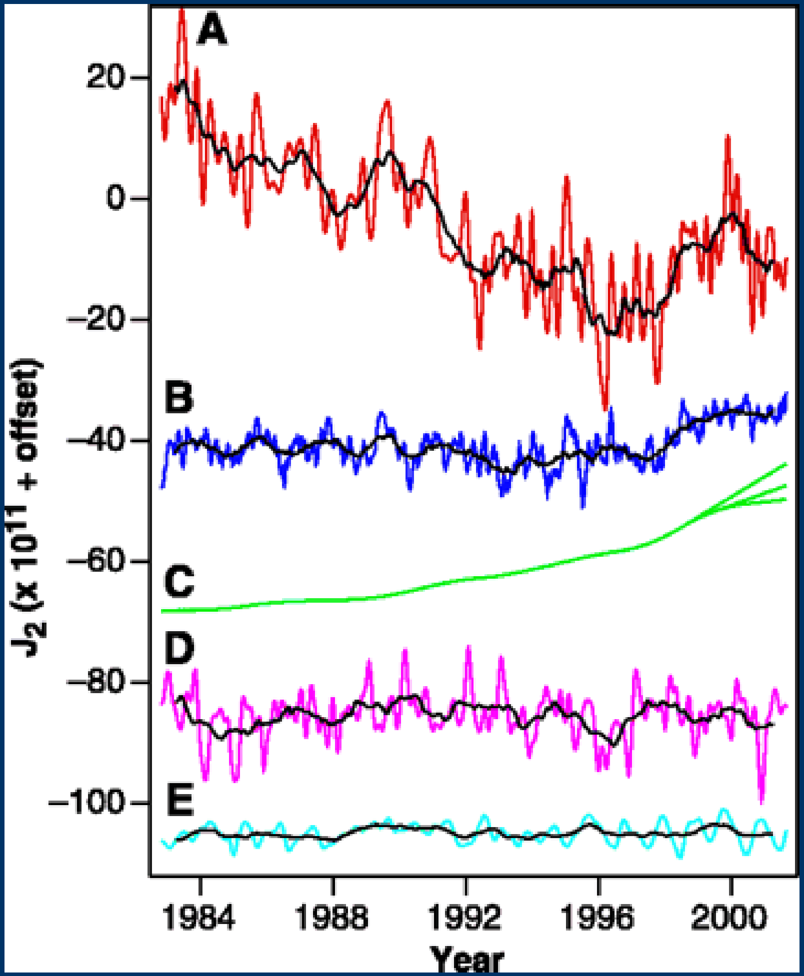
change of trend in recent years



Cox & Chao, Science, 297, 2002

satellites - test masses in free fall

Temporal changes of the Earth‘s flattening: What are the causes?



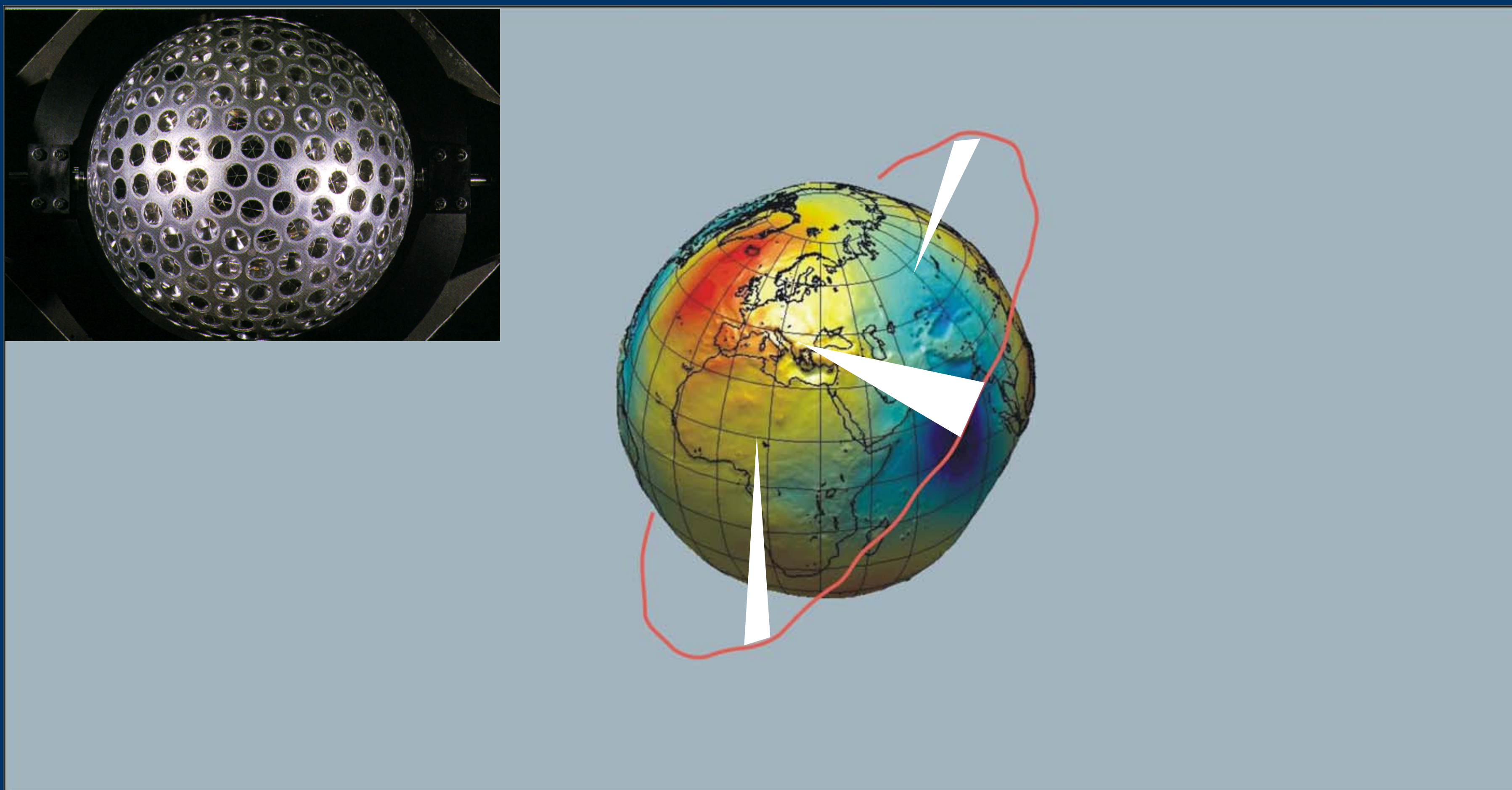
candidates:

- ocean masses
- melting ice caps
- atmospheric masses
- hydrology

Dickey, Marcus, de Viron, Fukumori, 2002

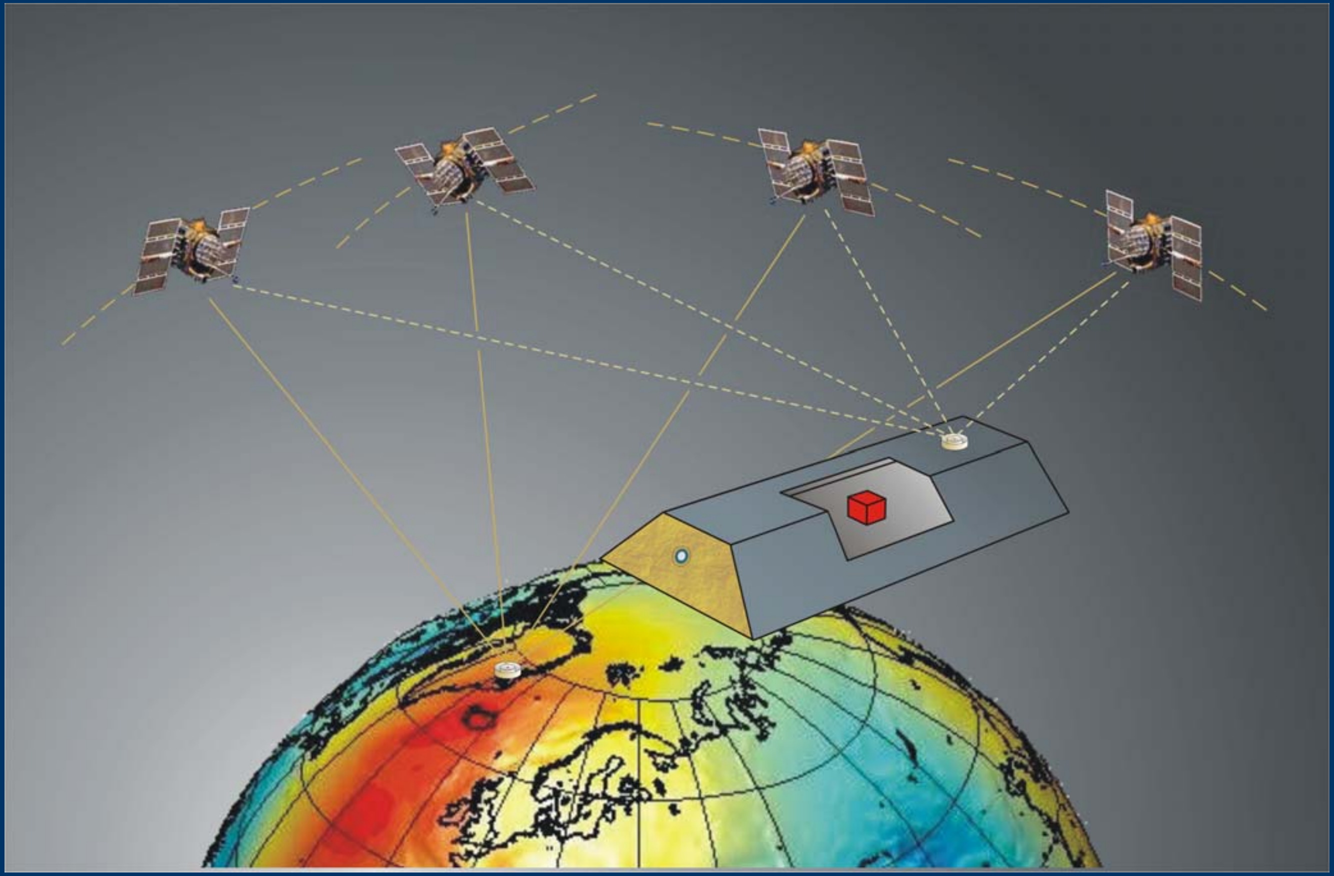
satellites - test masses in free fall

observatories “see” only
short arc segments



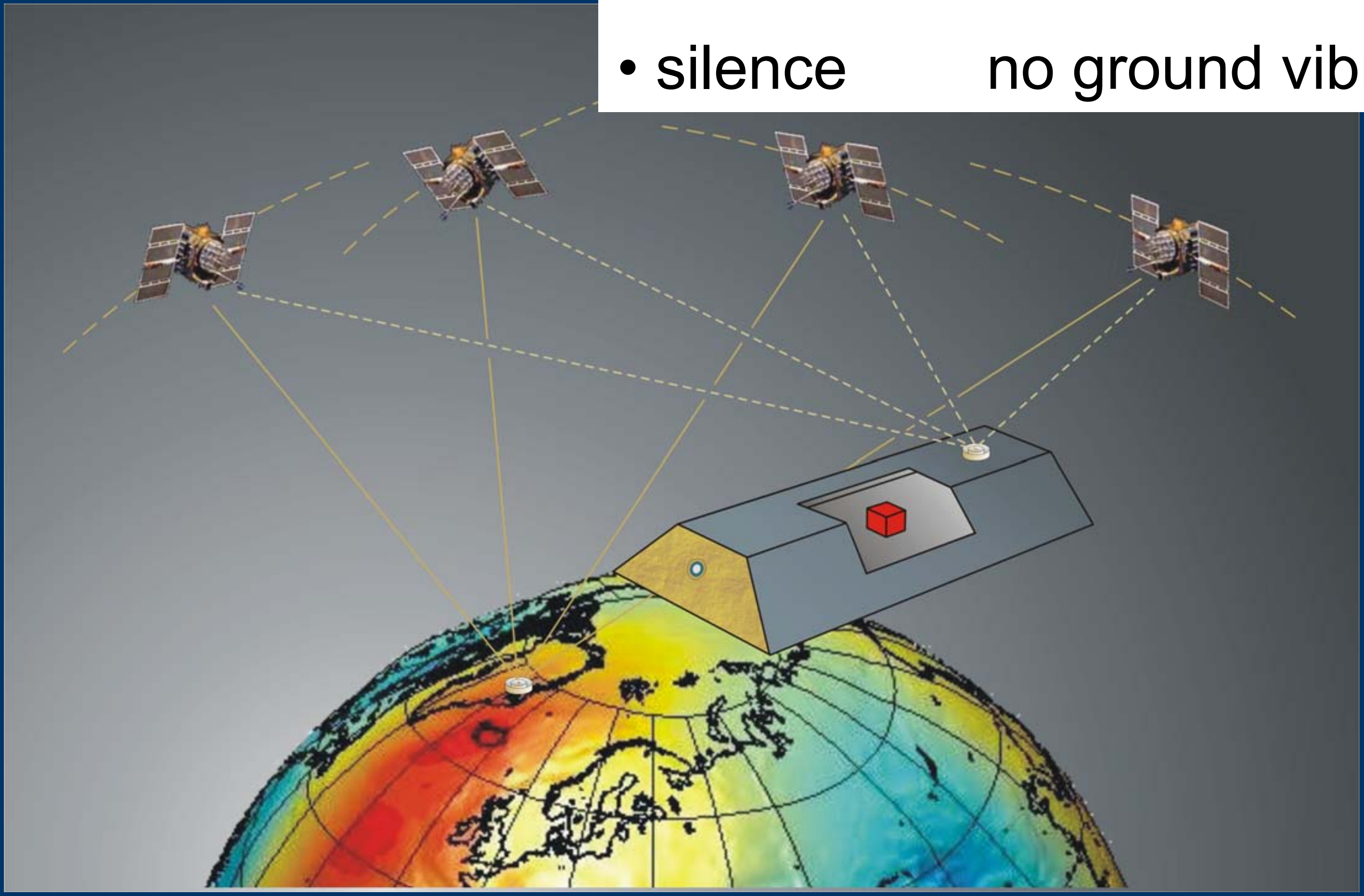
satellites - test masses in free fall

A new era: continuous tracking in 3D
of low earth orbiters (LEOs) by navigation satellites
(GPS, GALILEO, GLONASS…)



satellites - test masses in free fall

• location	GPS
• time	synchronized atomic clocks
• air resistance	measured
• silence	no ground vibrations



satellites - test masses in free fall

atmospheric pressure at satellite altitude

Table 13.5. Physical properties of the atmosphere[a]

Height (km)	Pressure (mb)	Temperature (K)	Density (kg/m³)	Mean molecular mass (u)
0	$1.01 \cdot 10^{3}$	288	$1.23 \cdot 10^{0}$	28.96
5	$5.40 \cdot 10^{2}$	256	$7.36 \cdot 10^{-1}$	28.96
10	$2.65 \cdot 10^{2}$	223	$4.14 \cdot 10^{-1}$	28.96
20	$5.53 \cdot 10^{1}$	217	$8.89 \cdot 10^{-2}$	28.96
40	$2.87 \cdot 10^{0}$	250	$4.00 \cdot 10^{-3}$	28.96
60	$2.20 \cdot 10^{-1}$	247	$3.10 \cdot 10^{-4}$	28.96
80	$1.05 \cdot 10^{-2}$	199	$1.85 \cdot 10^{-5}$	28.96
100	$3.20 \cdot 10^{-4}$	195	$5.60 \cdot 10^{-7}$	28.40
150	$4.54 \cdot 10^{-6}$	634	$2.08 \cdot 10^{-9}$	24.10
200	$8.47 \cdot 10^{-7}$	855	$2.54 \cdot 10^{-10}$	21.30
300	$8.77 \cdot 10^{-8}$	976	$1.92 \cdot 10^{-11}$	17.73
400	$1.45 \cdot 10^{-8}$	996	$2.80 \cdot 10^{-12}$	15.98
500	$3.02 \cdot 10^{-9}$	999	$5.21 \cdot 10^{-13}$	14.33
600	$8.21 \cdot 10^{-10}$	1,000	$1.14 \cdot 10^{-13}$	11.51

[a] Above 120 km of altitude, the atmospheric parameters given here depend on the phase of the sunspot cycle. The values given refer to the year 1976 (sunspot minimum).

laboratory:

10^{-4}Pa = 10^{-6}mbar

300 km altitude:

$8.77 \cdot 10^{-8}$mbar

Emiliani C,1992, p.272

Table 10.1 Units of Pressure

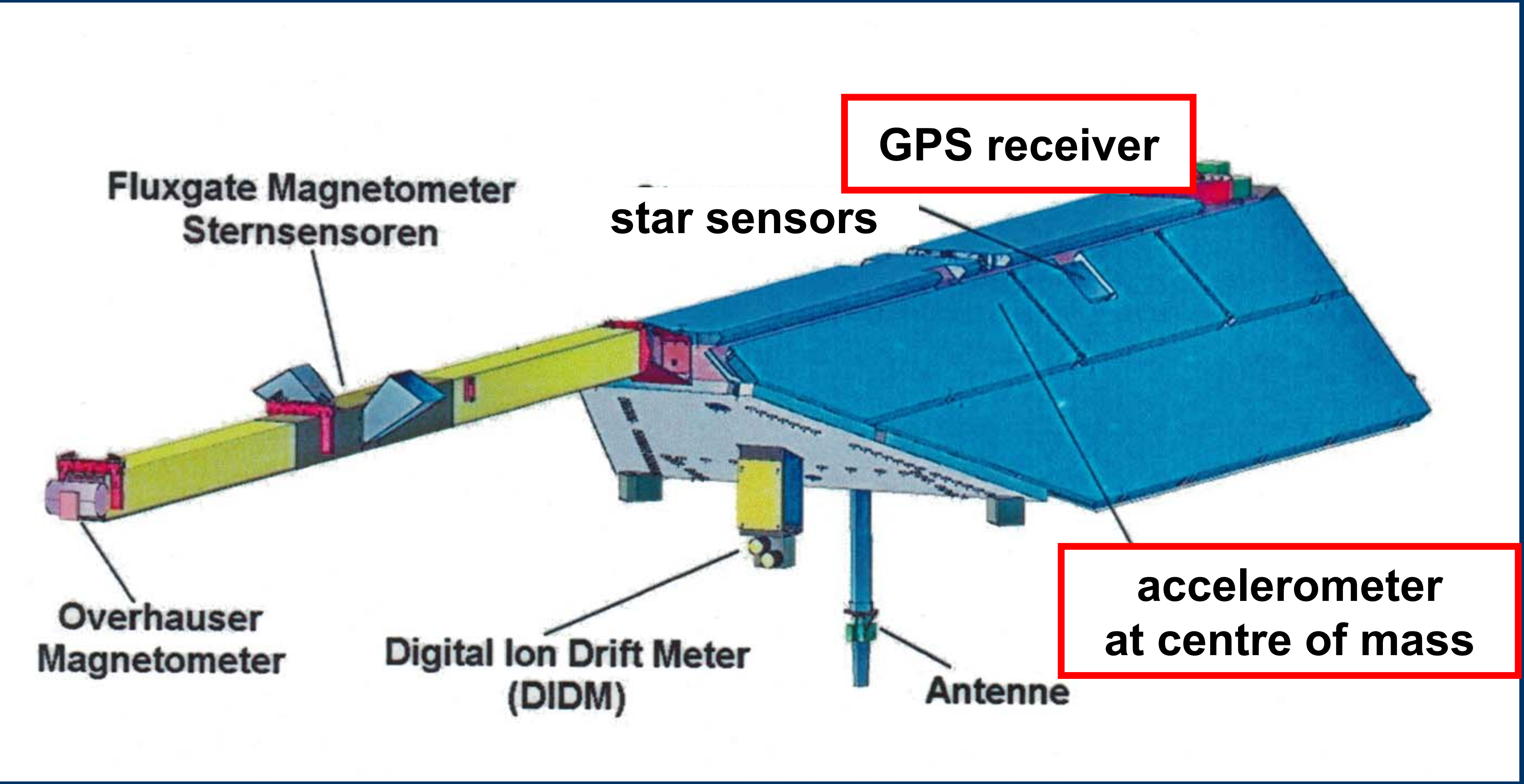
1 pascal (Pa)	=	$1\ \mathrm{N/m^2} = 1\ \mathrm{kg \cdot s^{-2} \cdot m^{-1}}$
1 bar	=	10^5 Pa
1 decibar	=	10^4 Pa
1 millibar	=	100 Pa

satellites - test masses in free fall

CHAMP satellite

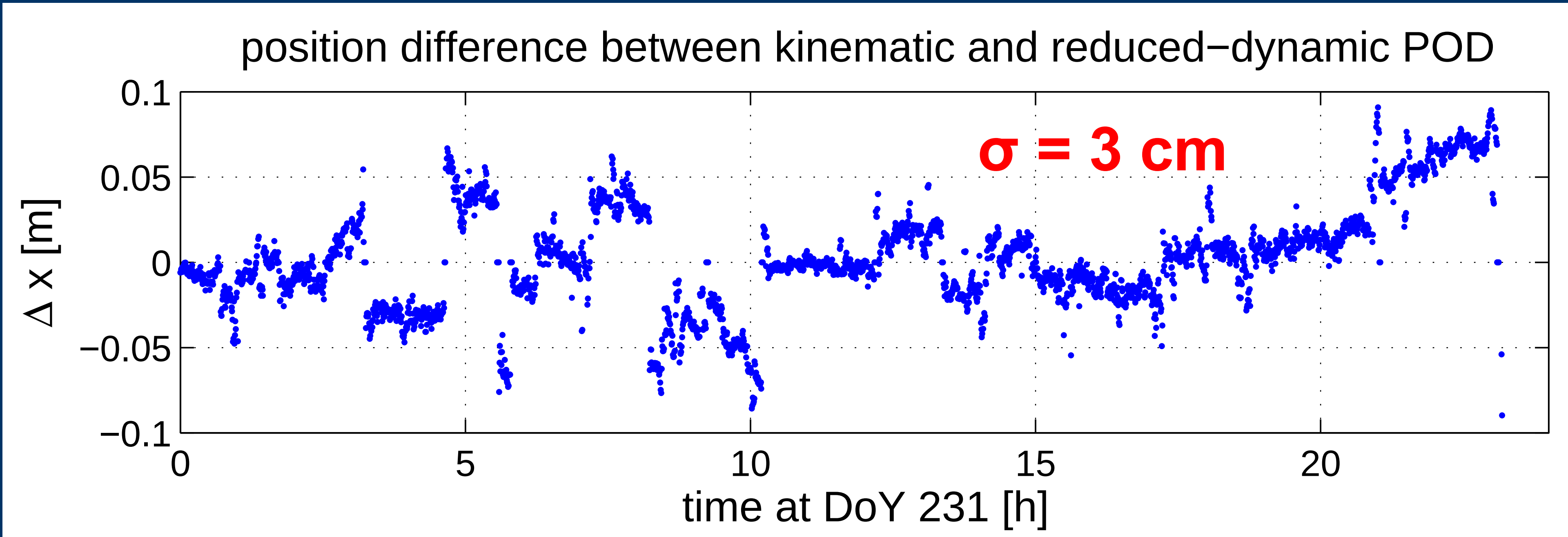
GeoForschungsZentrum Potsdam

mission life time: 2000 – 2010



from the orbit to gravitation

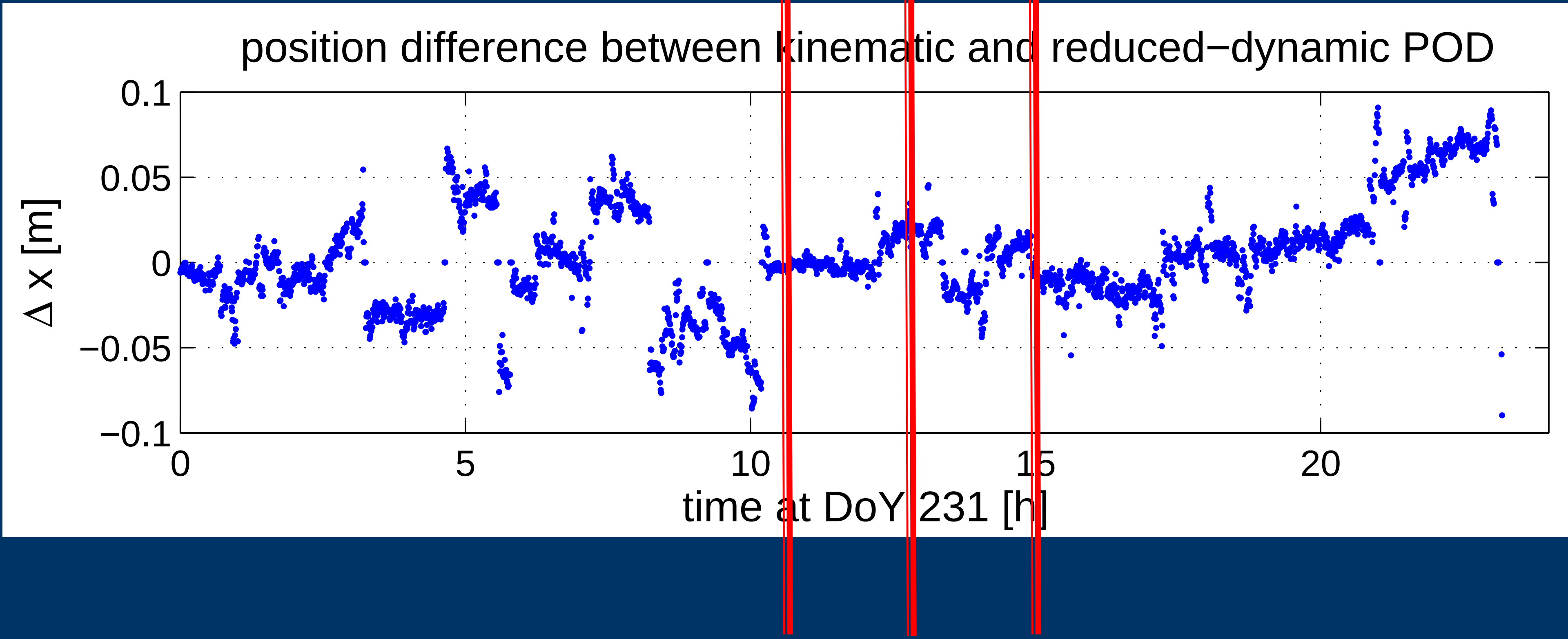
positioning (orbit determination)
of CHAMP by GPS



(Rothacher & Svehla, 2003)

from the orbit to gravitation

gravitation from $\{\vec{x}_0;t_0\}$ $\{\vec{x}_1;t_1\}$ $\{\vec{x}_2;t_2\}$



from the orbit to gravitation

energy conservation

kinetic energy = potential energy

$$\frac{1}{2}mv^2 = \Phi$$

however:

non conservative contributions

residual air resistance and

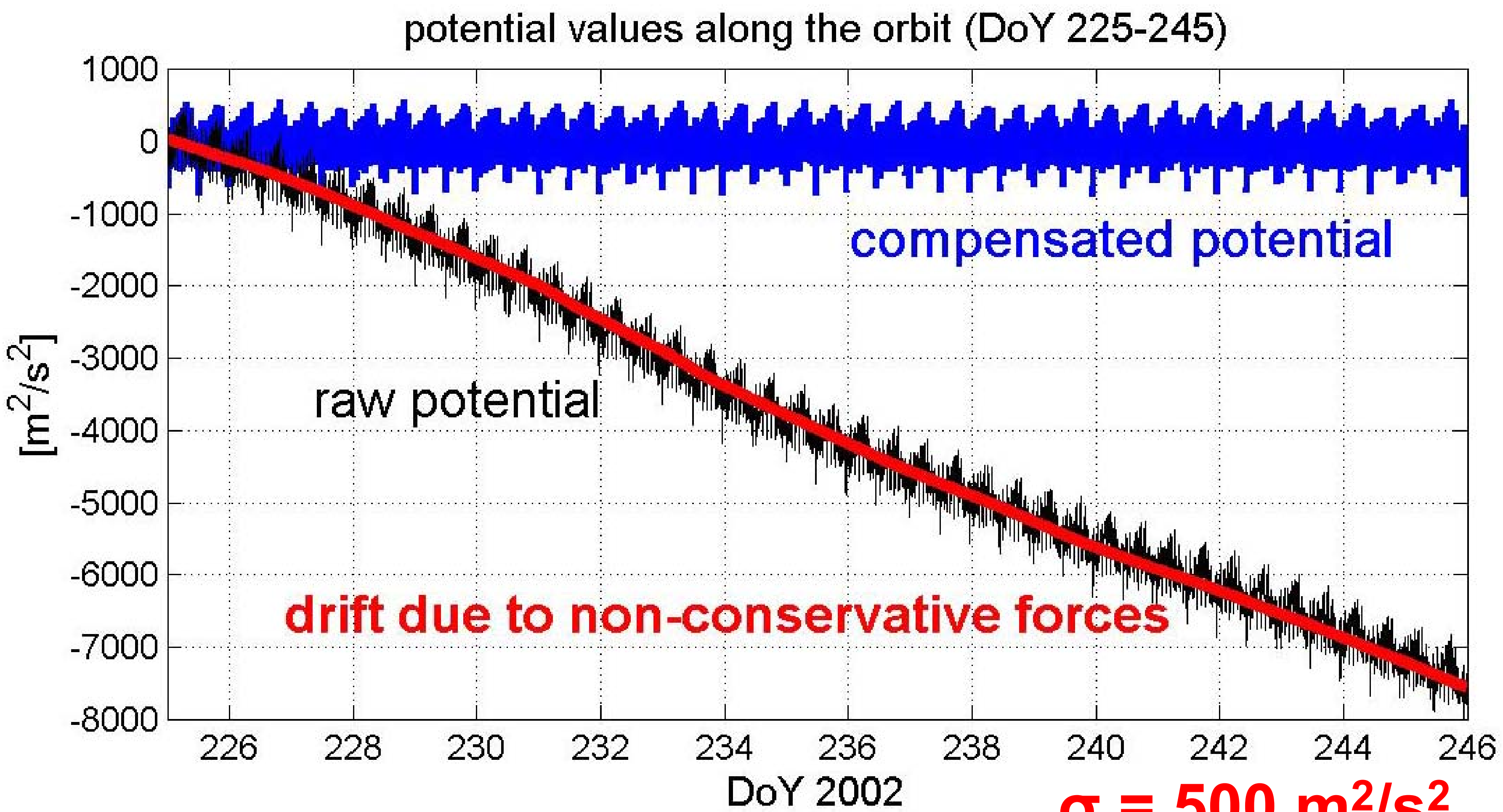
gravitation changes due to

direct, solid earth & ocean tides,

atmosphere oceans…

from the orbit to gravitation

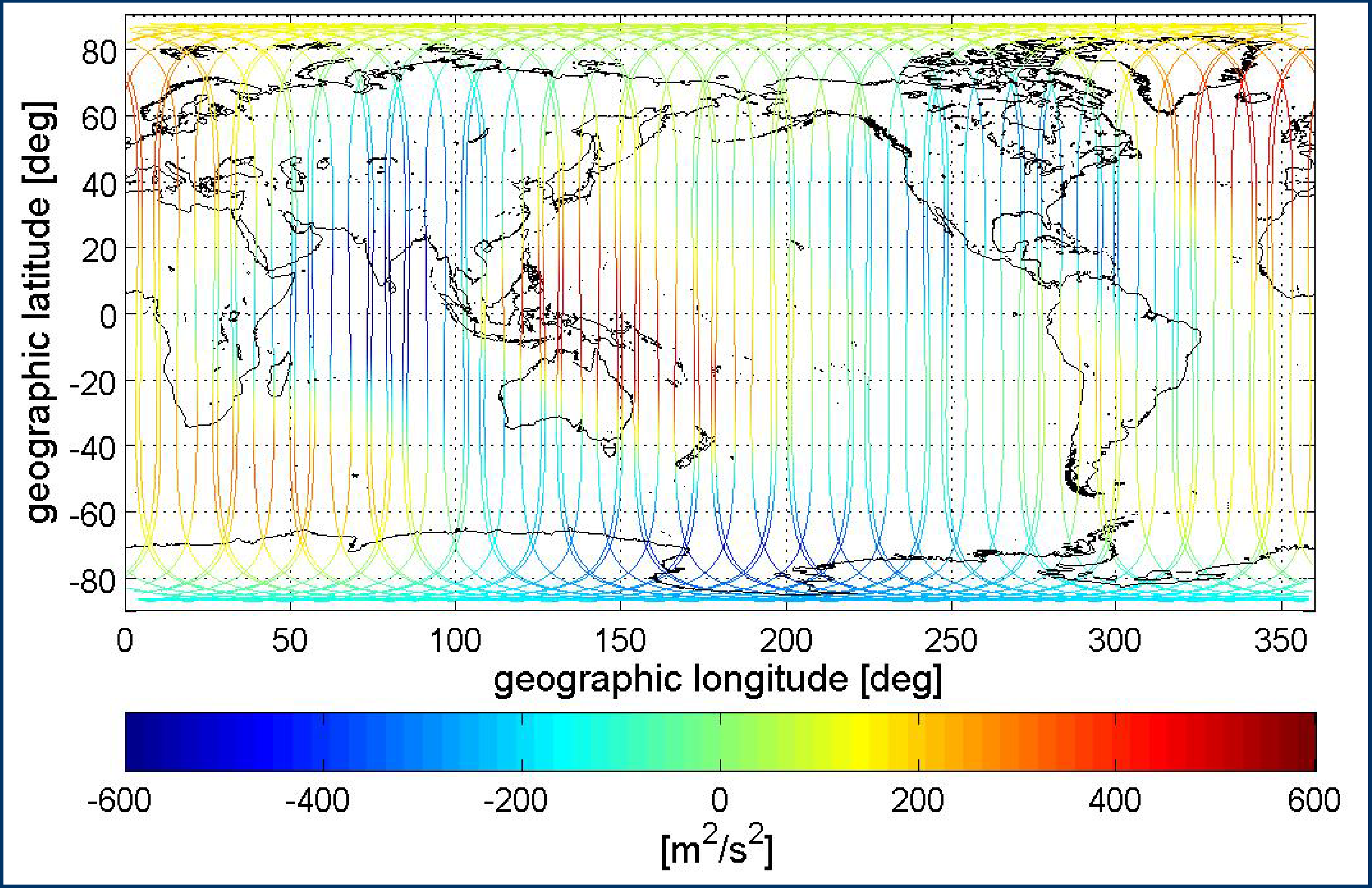
gravitational potential along the orbit trajectory

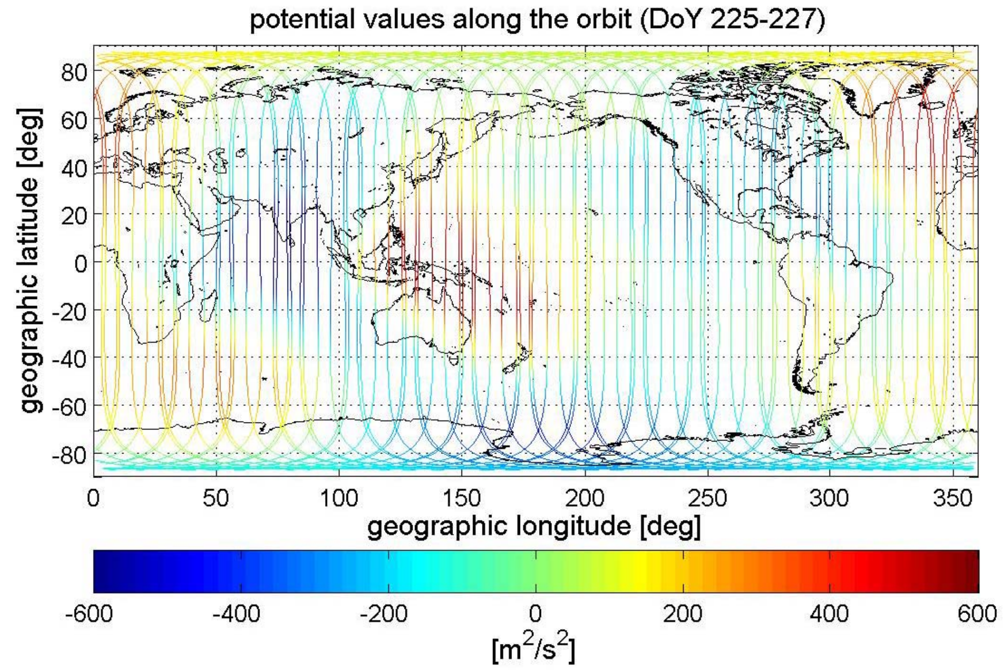


$\sigma = 500\ m^2/s^2$

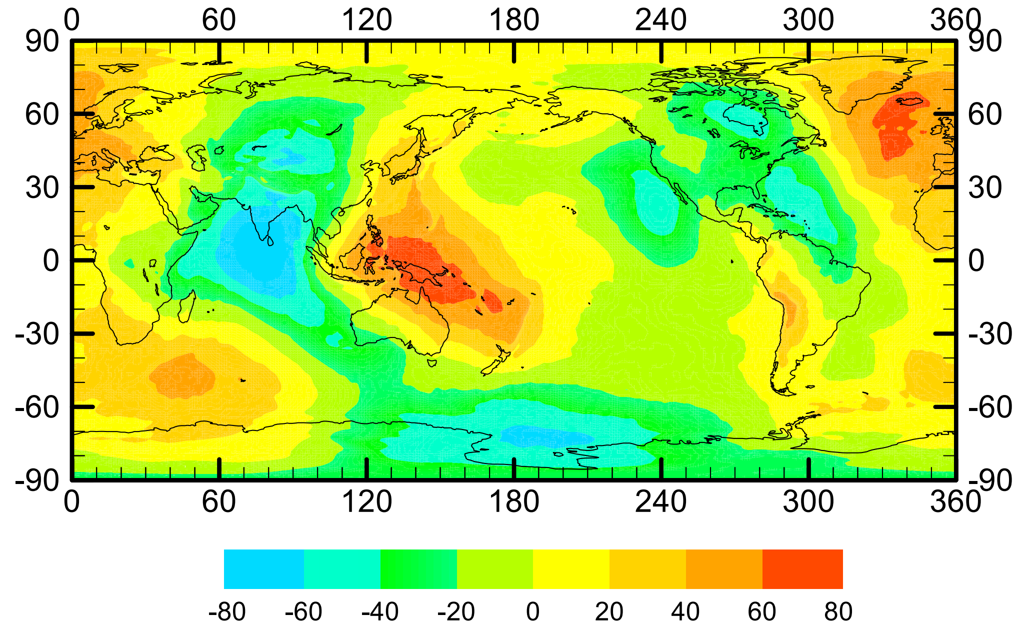
from the orbit to gravitation

gravitational potential along the orbit mapped onto the globe





potential along the orbit mapped onto the globe [m/s²]

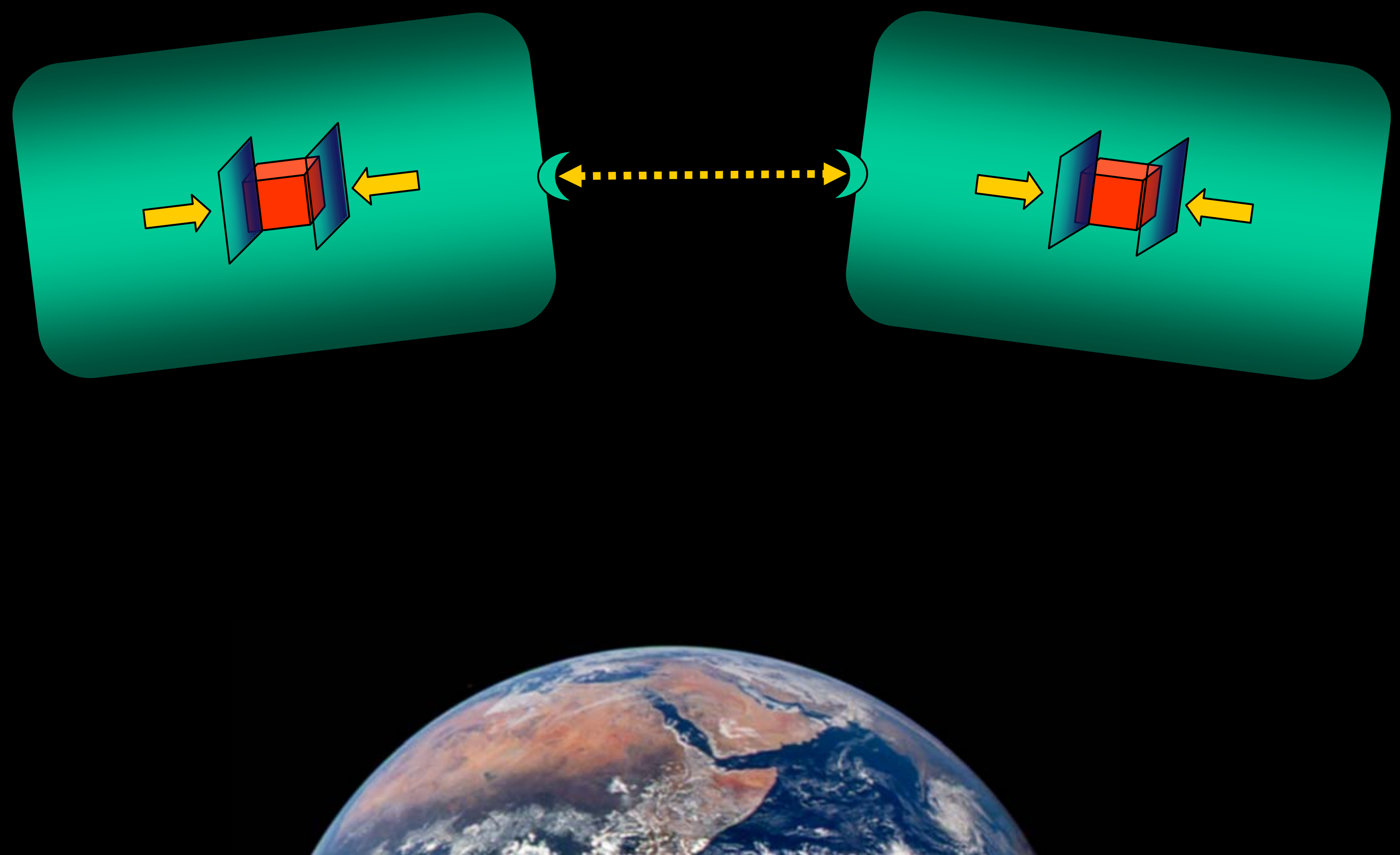


geoid [m]

the use of several free falling test masses

the use of several free falling test masses

from absolute to differential measurement

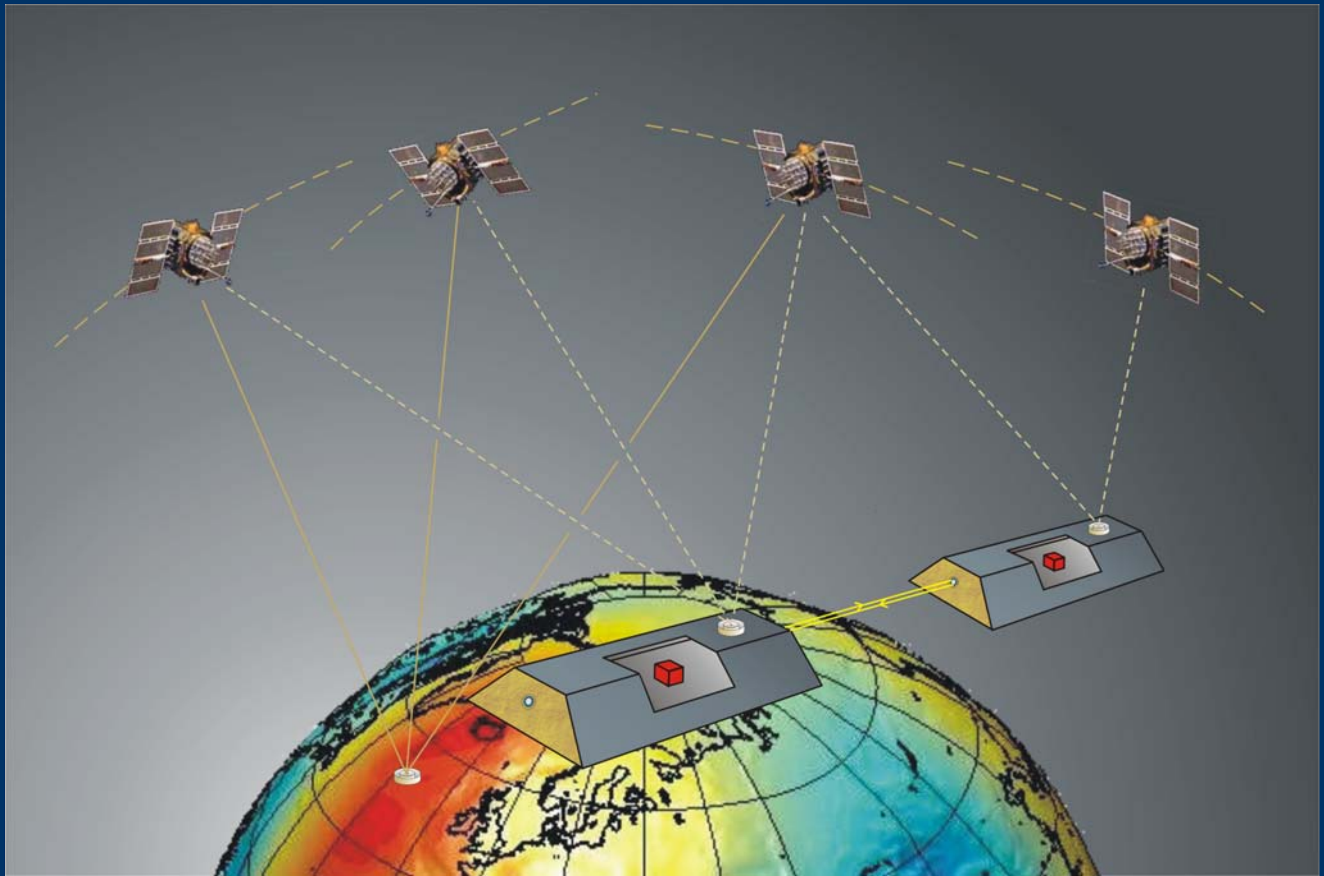


the use of several free falling test masses

GRACE

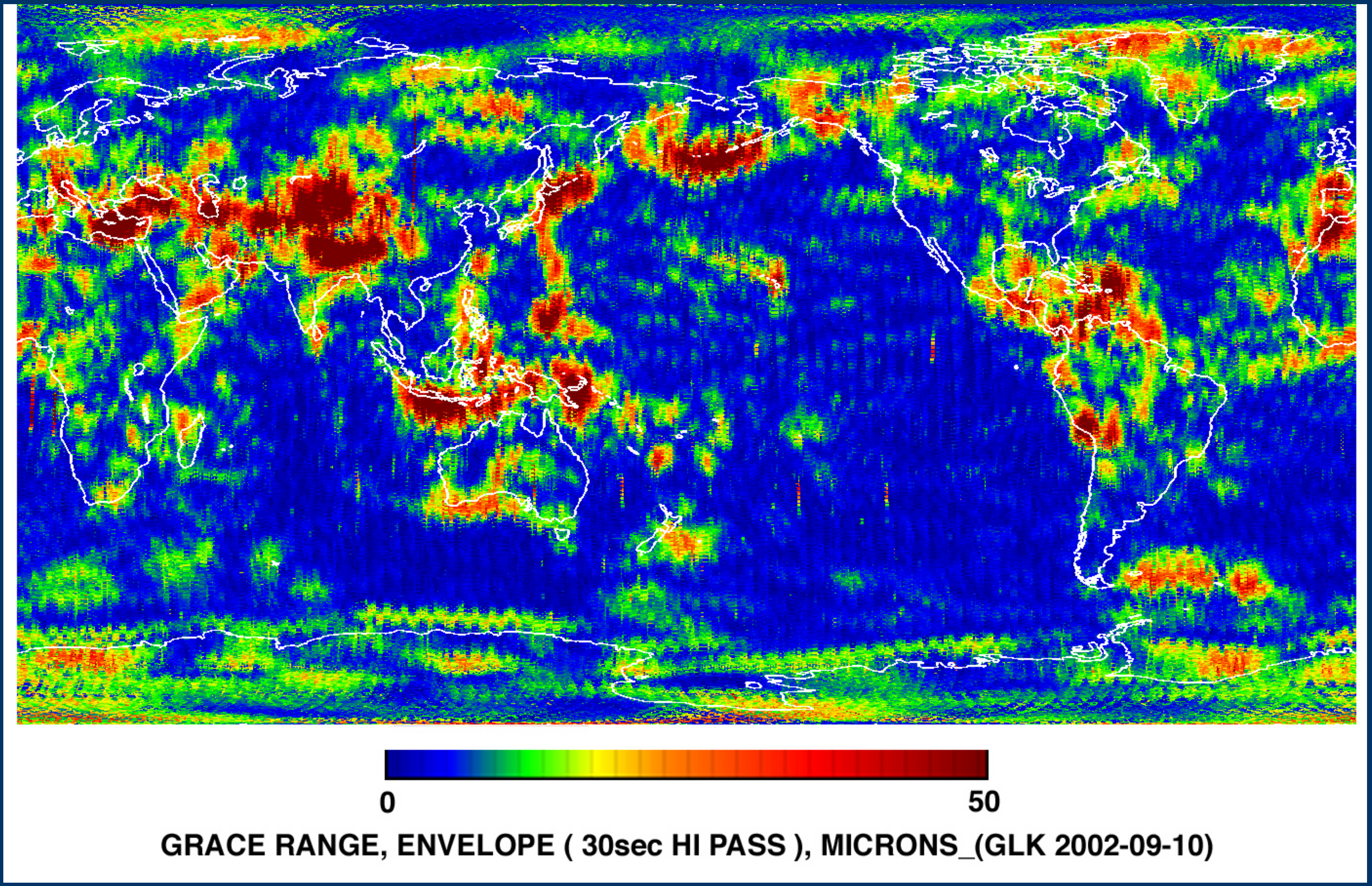
NASA + DLR mission (in orbit since 2002)

Gravitation from very precise measurement (1µm) of
changes of inter satellite distance
of two satellites following each other in the same orbit (200 km)



the use of several free falling test masses

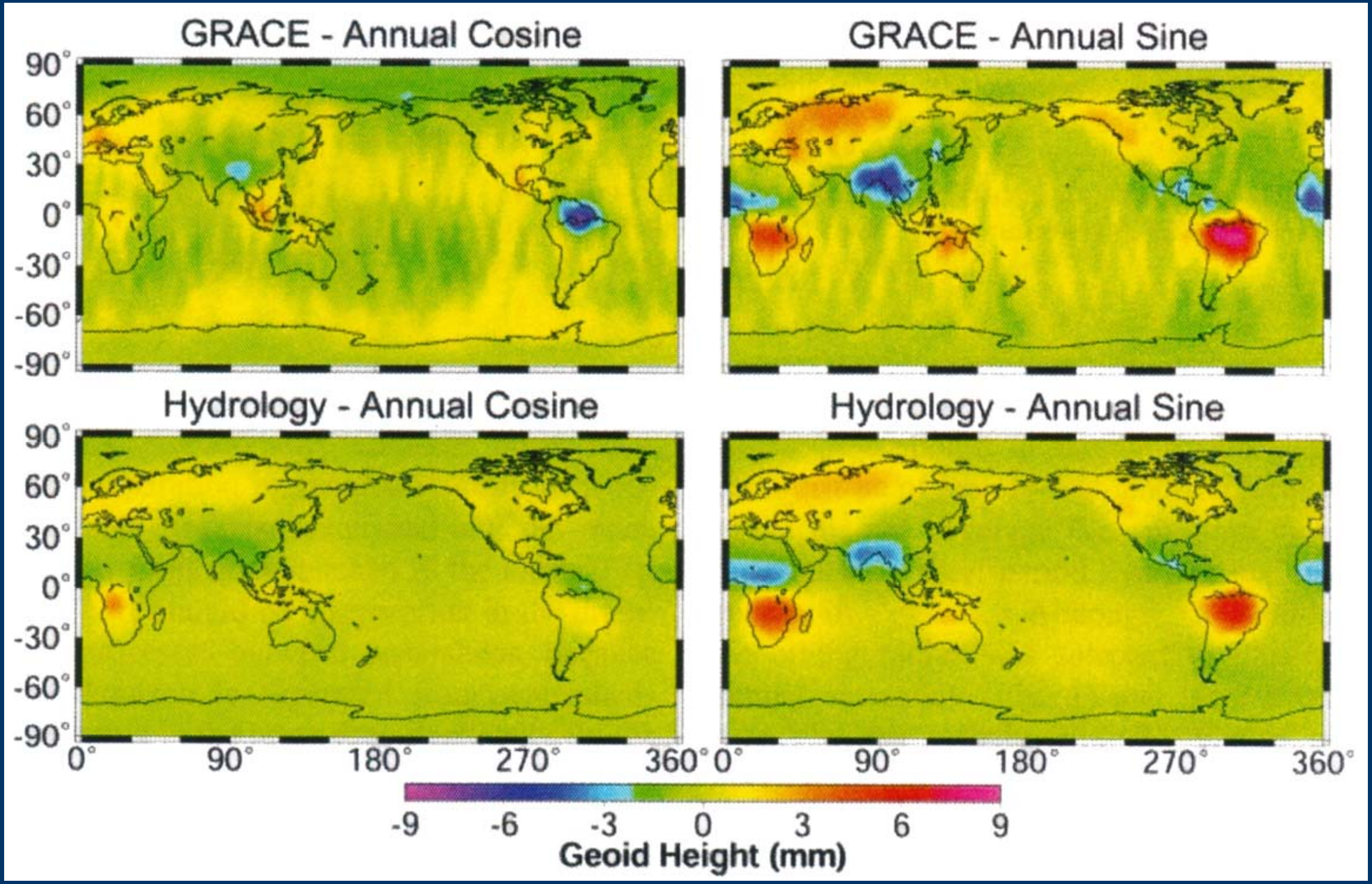
GRACE: measures tiny changes of gravitational acceleration



Gerard Kruizinga, JPL, 2002

the use of several free falling test masses

GRACE measures temporal gravitational changes
example: seasonal changes of continental hydrology

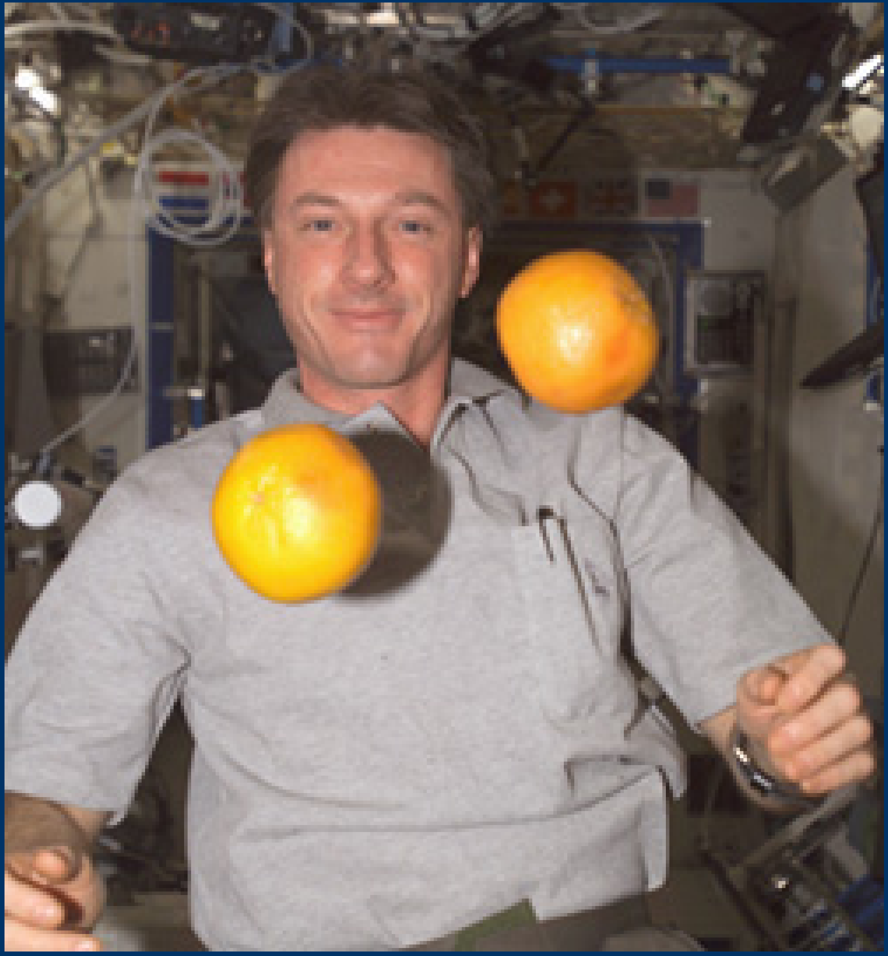


Tapley et al., Science, 2004

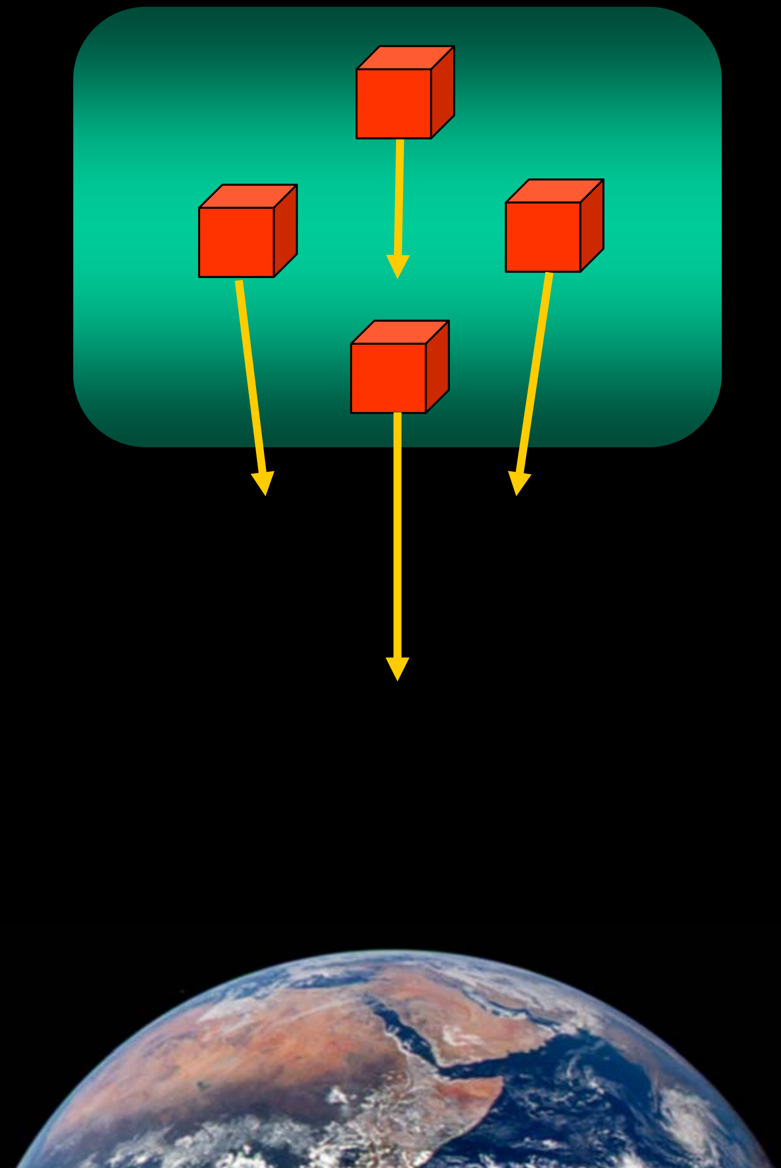
the use of several free falling test masses

gravitational measurement

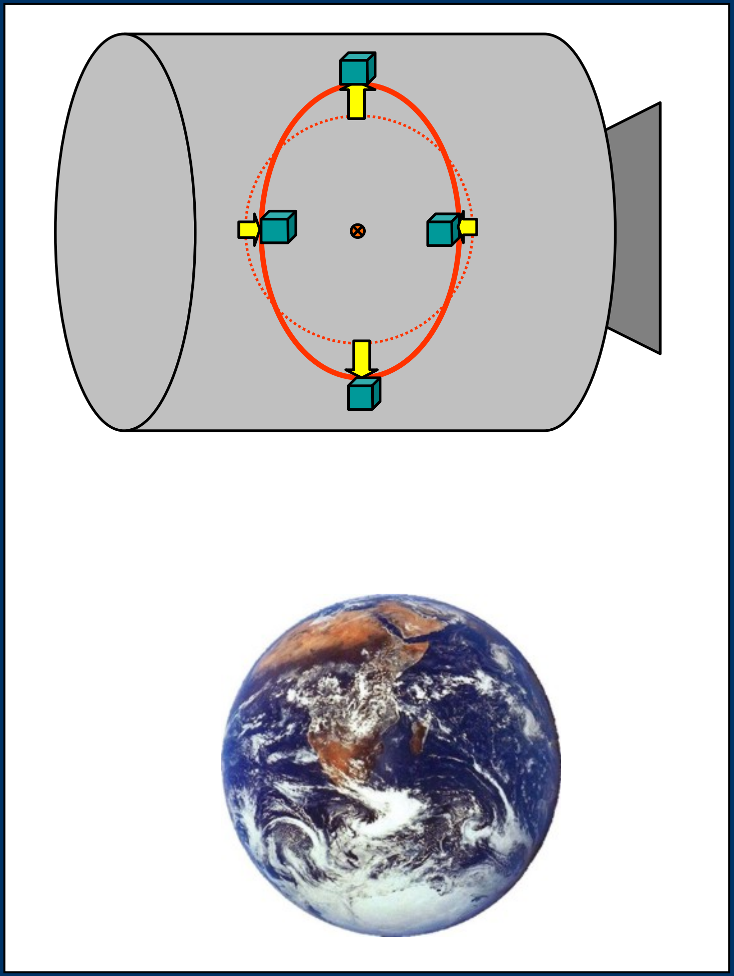
in a micro-g environment



the use of several free falling test masses



tidal attraction of earth acting on a satellite



satellite

260 km

earth surface

mass ration 1: $6 \cdot 10^{21}$

gravitational gradiometry – principle

$$\vec{g} = \begin{pmatrix} g_x \\ g_y \\ g_z \end{pmatrix} = \begin{pmatrix} \partial V / \partial x \\ \partial V / \partial y \\ \partial V / \partial z \end{pmatrix} = \nabla V = \begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix}$$

$$\begin{pmatrix} \partial g_x / \partial x & \partial g_x / \partial y & \partial g_x / \partial z \\ \partial g_y / \partial x & \partial g_y / \partial y & \partial g_y / \partial z \\ \partial g_z / \partial x & \partial g_z / \partial y & \partial g_z / \partial z \end{pmatrix} = \begin{pmatrix} V_{xx} & V_{xy} & V_{xz} \\ V_{yx} & V_{yy} & V_{yz} \\ V_{zx} & V_{zy} & V_{zz} \end{pmatrix}$$

gravity tensor

gravitational gradiometry – principle

gravity tensor

=

tidal tensor

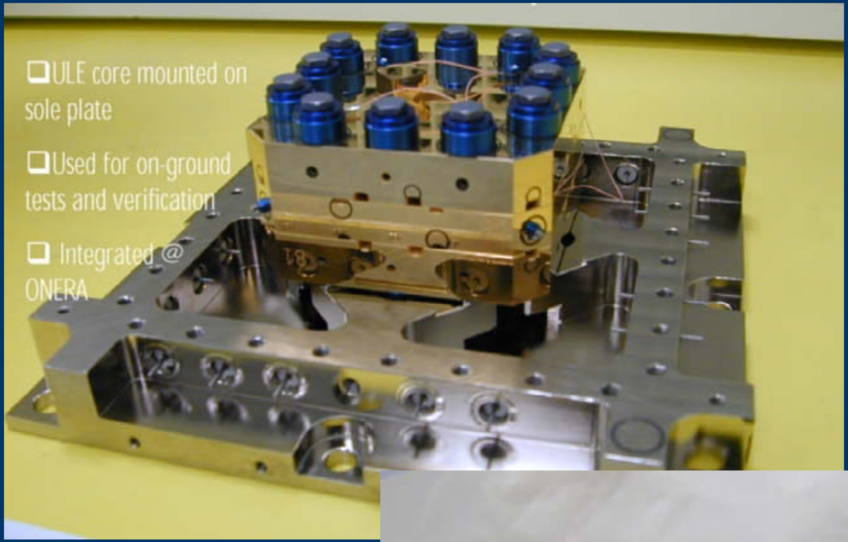
=

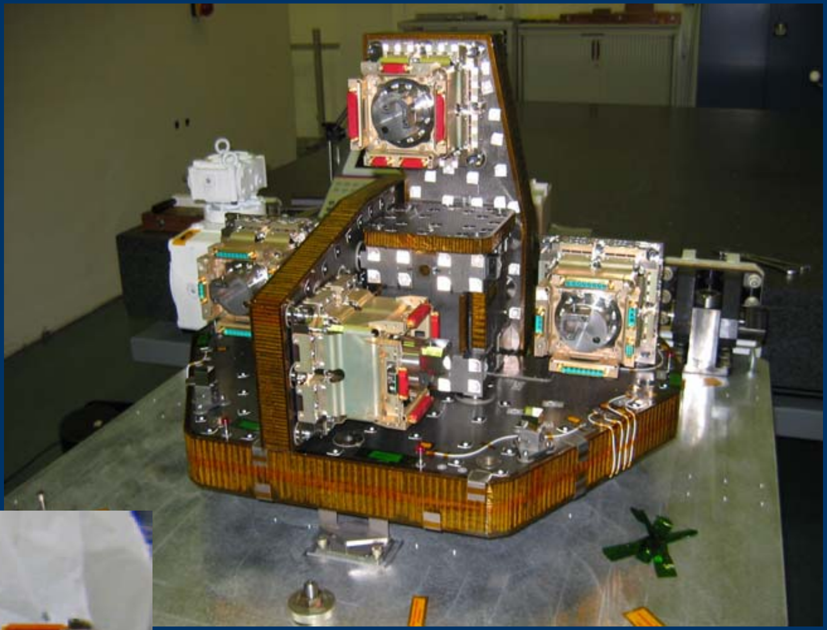
curvature tensor

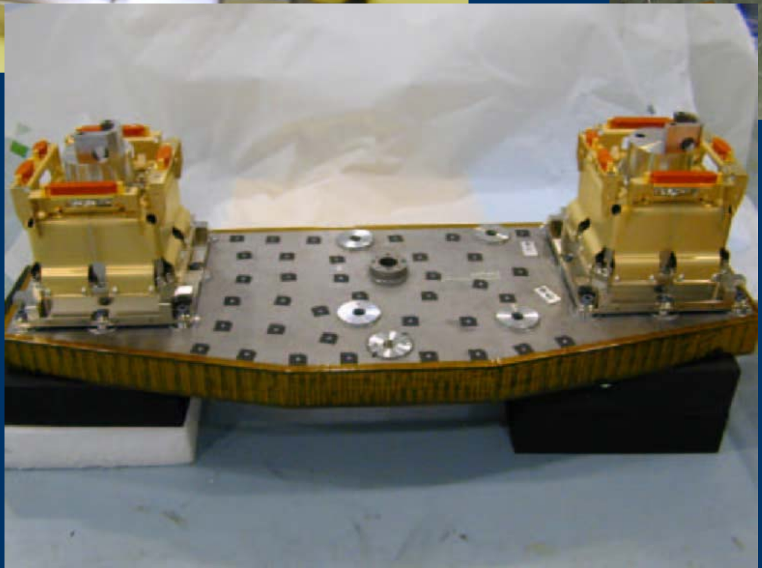
$$R^{i}_{0j0} = V_{ij} = -g\begin{pmatrix} k_{NS} & t & f_{NS} \\ t & k_{OW} & f_{OW} \\ f_{NS} & f_{OW} & \partial g \big/ g\,\partial z \end{pmatrix}$$

GOCE and gravitational gradiometry

single accelerometer



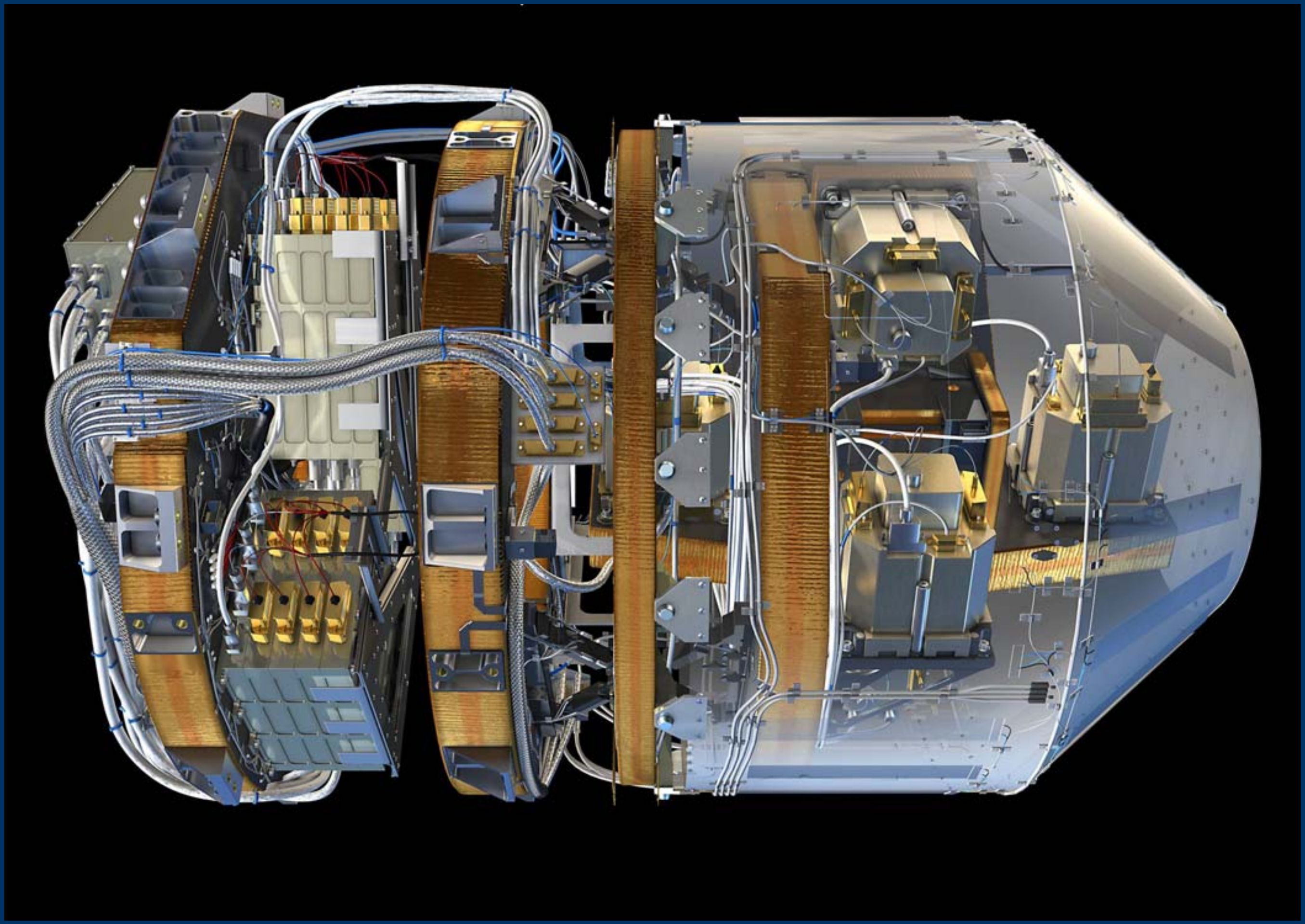




one axis gradiometer

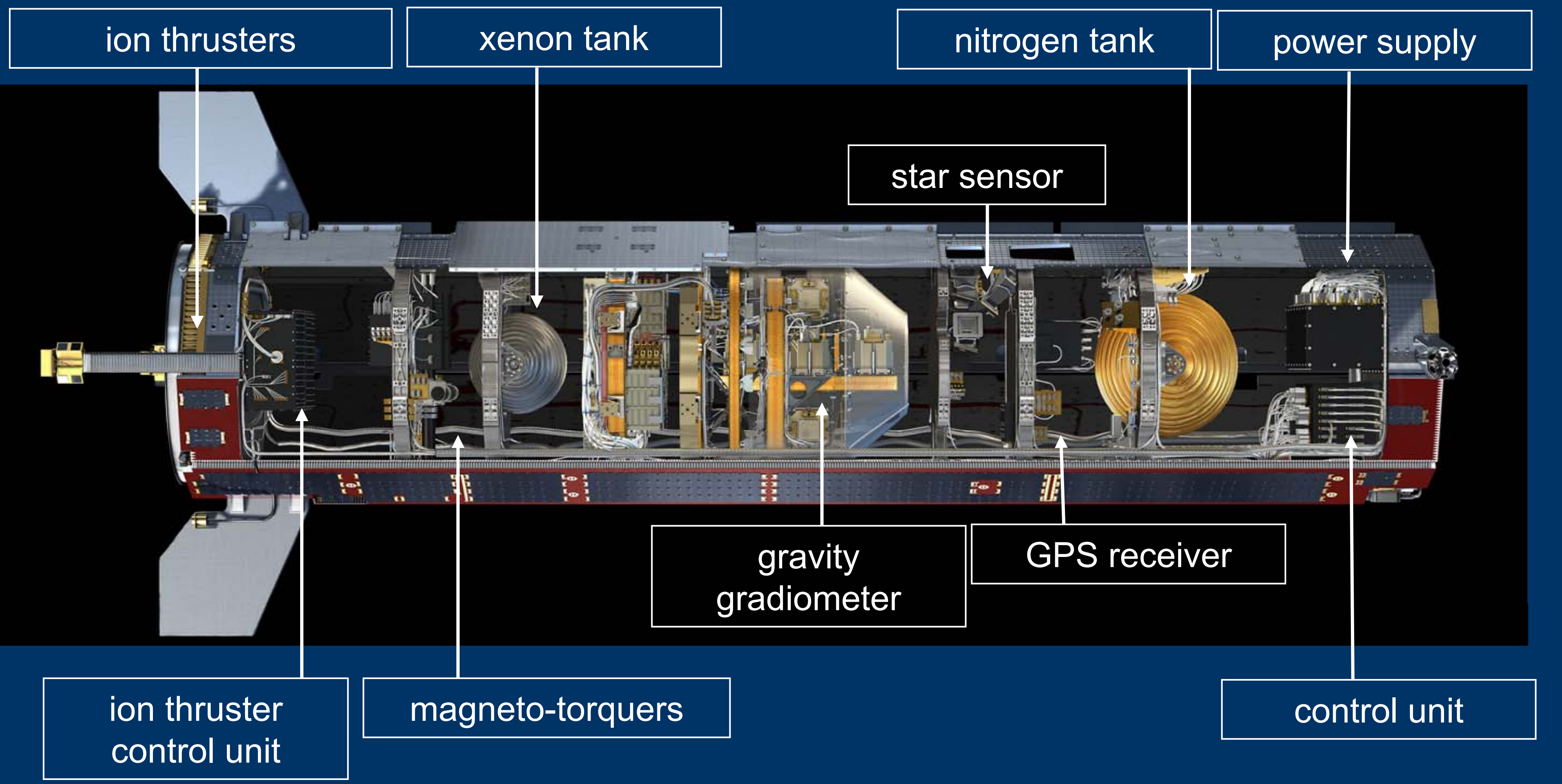
three axes gradiometer consisting of 6 accelerometers

GOCE and gravitational gradiometry



measurement of „micro-g“ with micro-precision

GOCE and gravitational gradiometry

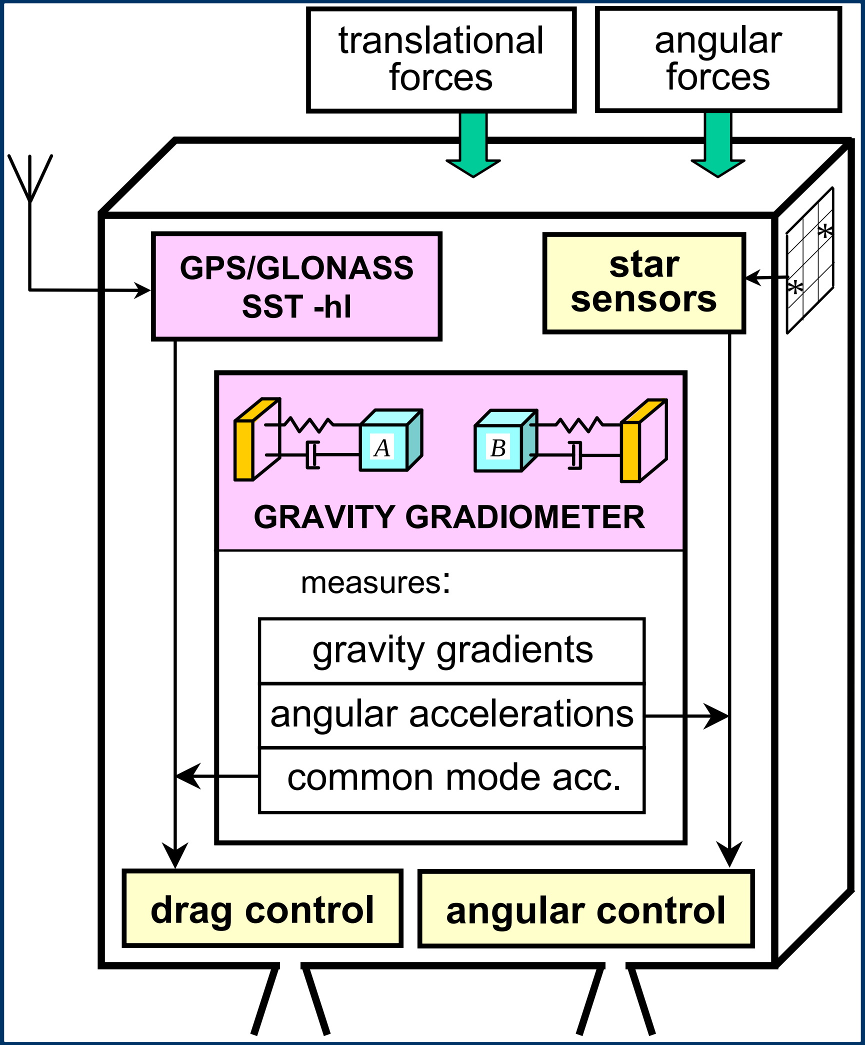


a perfect laboratory in space

source: ESA

GOCE and gravitational gradiometry

instrument and control concept



GOCE and gravitational gradiometry

1. The first gravitational gradiometer in space
2. European geodetic GPS-receiver on board
3. Extremly low orbit altitude (260 km)
4. Free fall (air drag is compensated along track)
5. Very „soft“ angular control by magnetic torquing
6. Absolutely quiet and stiff satellite materials and environment

GOCE and gravitational gradiometry

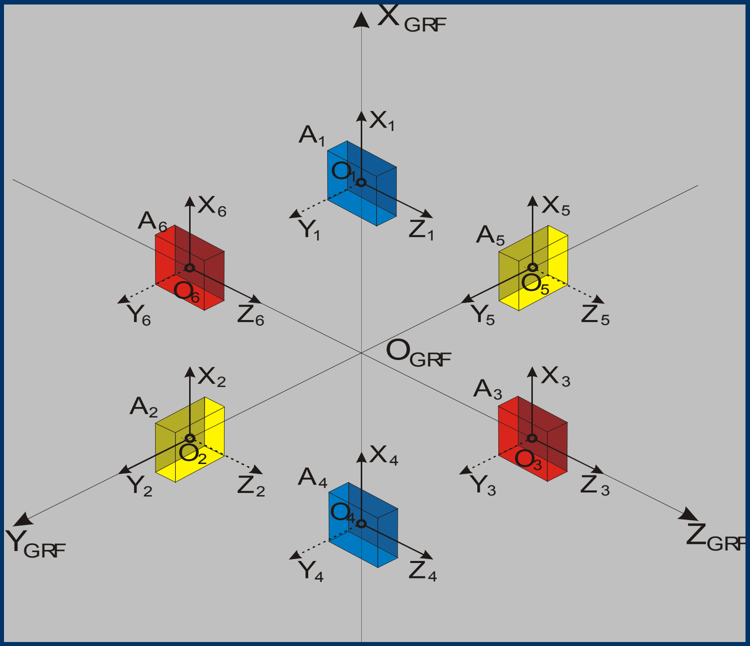
measurement in a rotating frame

gravitation tensor | centrifugal part (angular velocities) | angular accelerations

$$
\begin{pmatrix} V_{xx} & V_{xy} & V_{xz} \\ V_{yx} & V_{yy} & V_{yz} \\ V_{zx} & V_{zy} & V_{zz} \end{pmatrix}
+
\begin{pmatrix} -\omega_y^2 - \omega_z^2 & \omega_x\omega_y & \omega_x\omega_z \\ \omega_y\omega_x & -\omega_z^2 - \omega_x^2 & \omega_y\omega_z \\ \omega_z\omega_x & \omega_z\omega_y & -\omega_x^2 - \omega_y^2 \end{pmatrix}
+
\begin{pmatrix} 0 & \dot{\omega}_z & -\dot{\omega}_y \\ -\dot{\omega}_z & 0 & \dot{\omega}_x \\ \dot{\omega}_y & -\dot{\omega}_x & 0 \end{pmatrix}
$$

symmetric | symmetric | skew symmetric

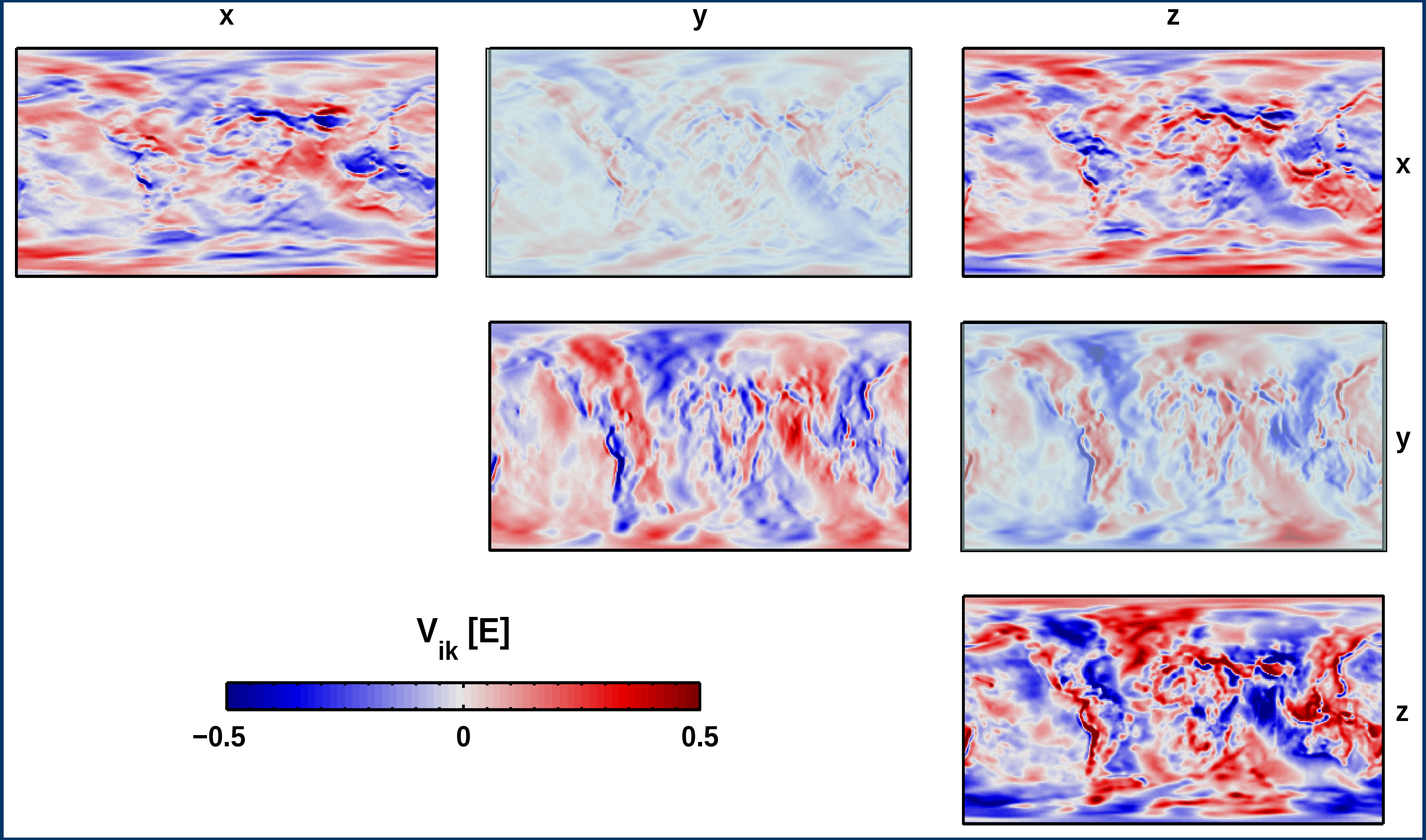
GOCE and gravitational gradiometry



two sensitive and one less sensitive direction

$$\begin{pmatrix} V_{xx} & V_{xy} & V_{xz} \\ V_{yx} & V_{yy} & V_{yz} \\ V_{zx} & V_{zy} & V_{zz} \end{pmatrix} + \begin{pmatrix} -\omega_y^2 - \omega_z^2 & \omega_x\omega_y & \omega_x\omega_z \\ \omega_y\omega_x & -\omega_z^2 - \omega_x^2 & \omega_y\omega_z \\ \omega_z\omega_x & \omega_z\omega_y & -\omega_x^2 - \omega_y^2 \end{pmatrix} + \begin{pmatrix} 0 & \dot{\omega}_z & -\dot{\omega}_y \\ -\dot{\omega}_z & 0 & \dot{\omega}_x \\ \dot{\omega}_y & -\dot{\omega}_x & 0 \end{pmatrix}$$

GOCE and gravitational gradiometry

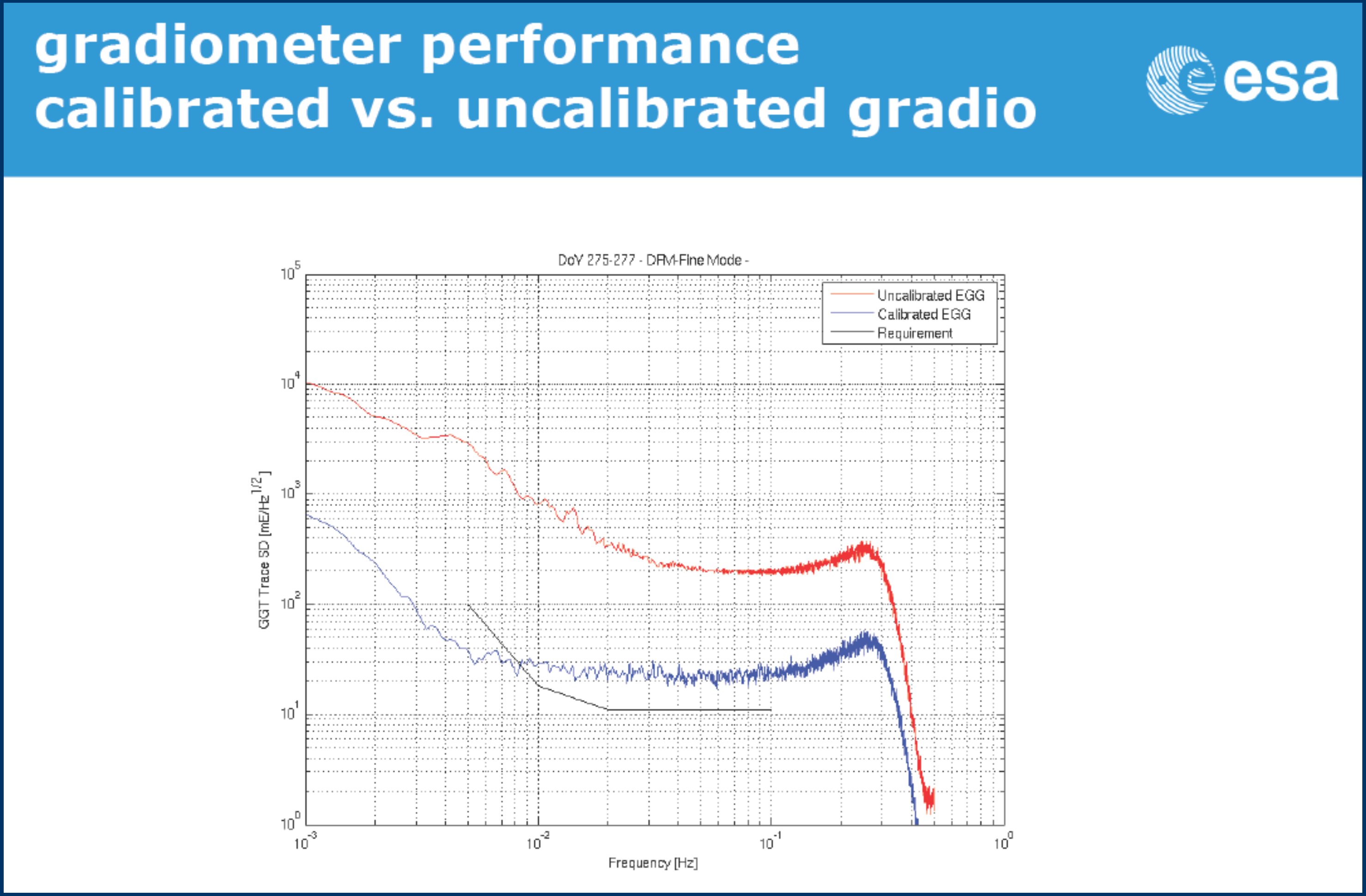


4 components measured with high precision and two less precise

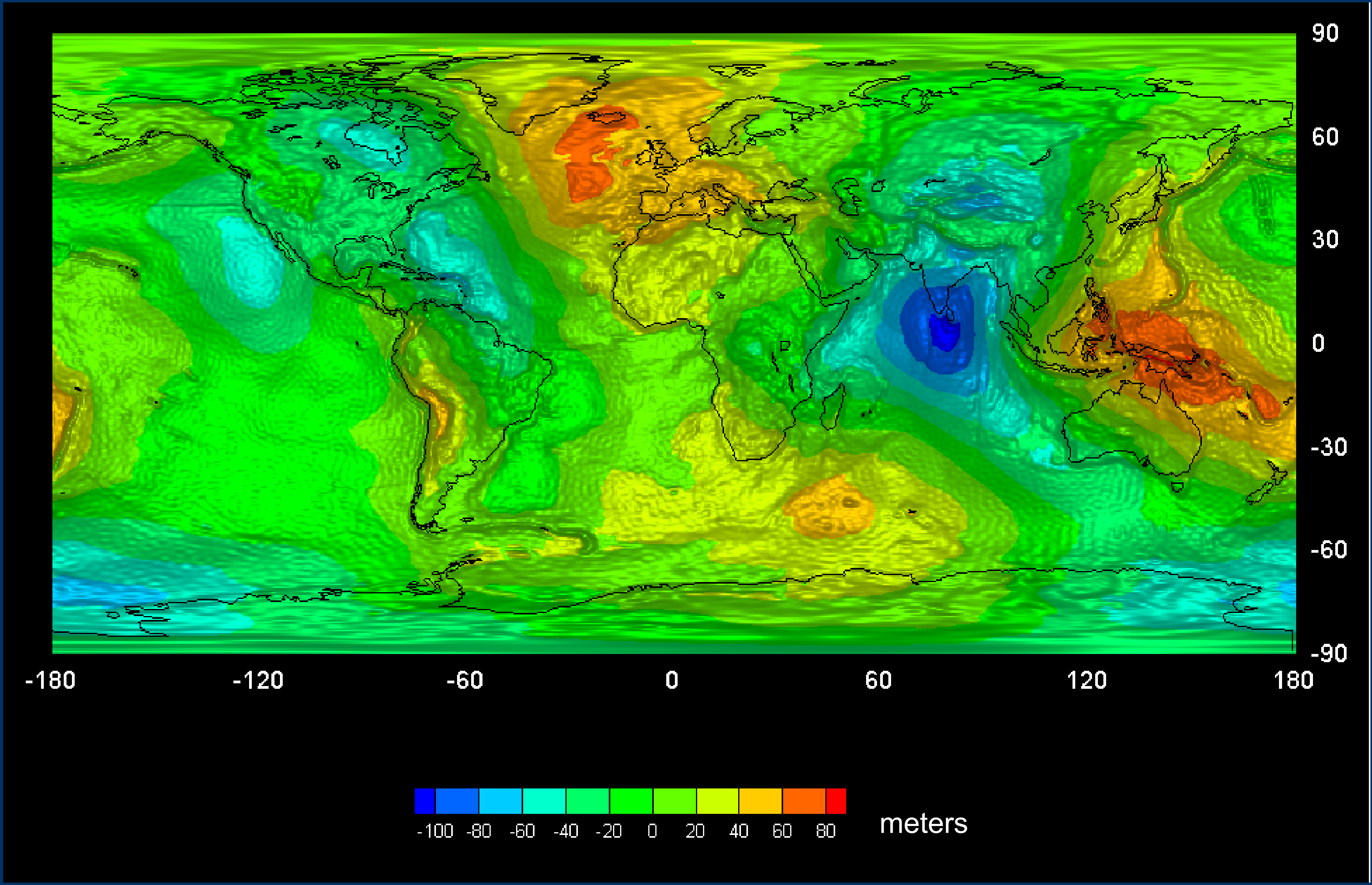
GOCE and gravitational gradiometry

trace condition (Laplace condition):

the sum of the measured diagonal components should be zero

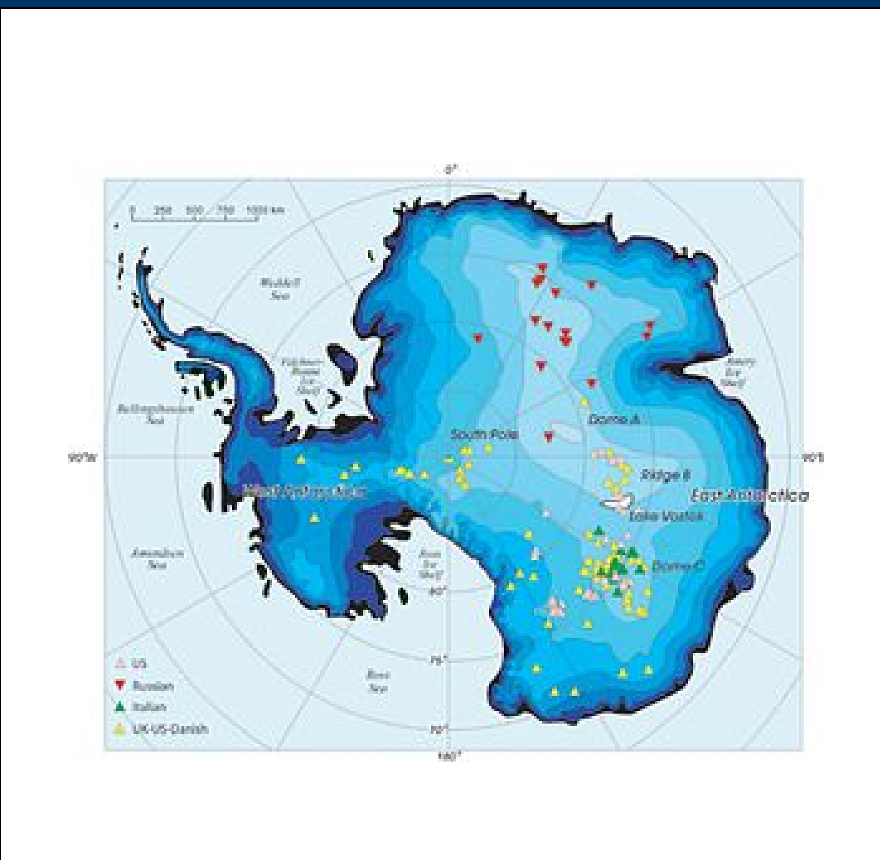


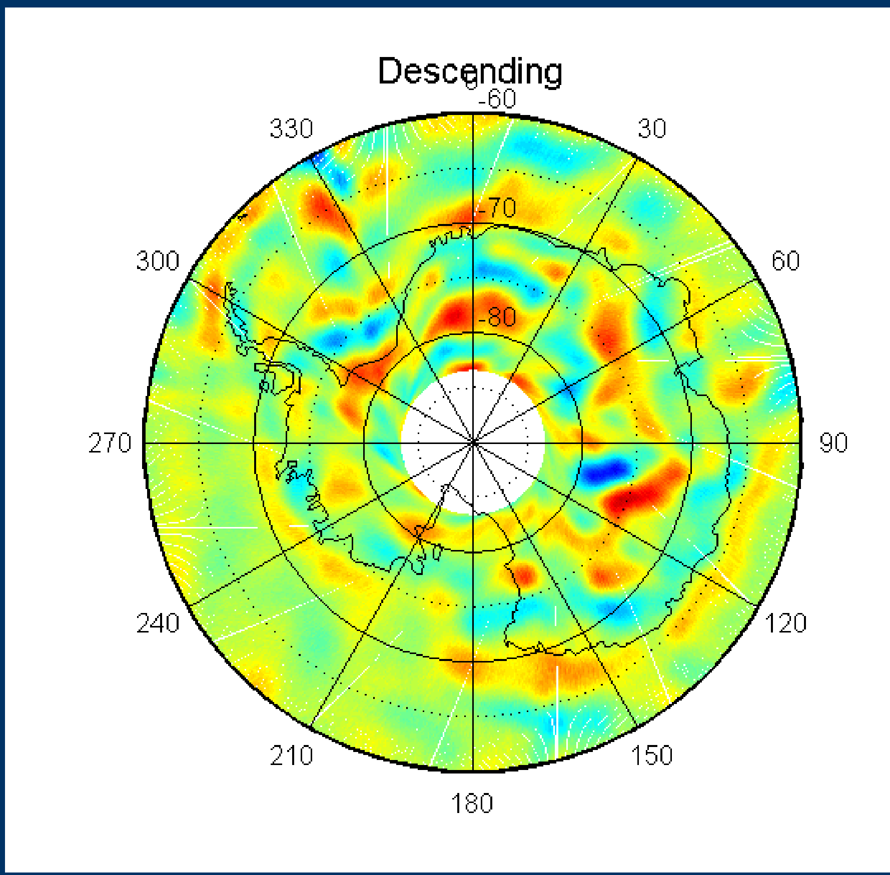
GOCE gravity field



a global geoid map based on two months of data

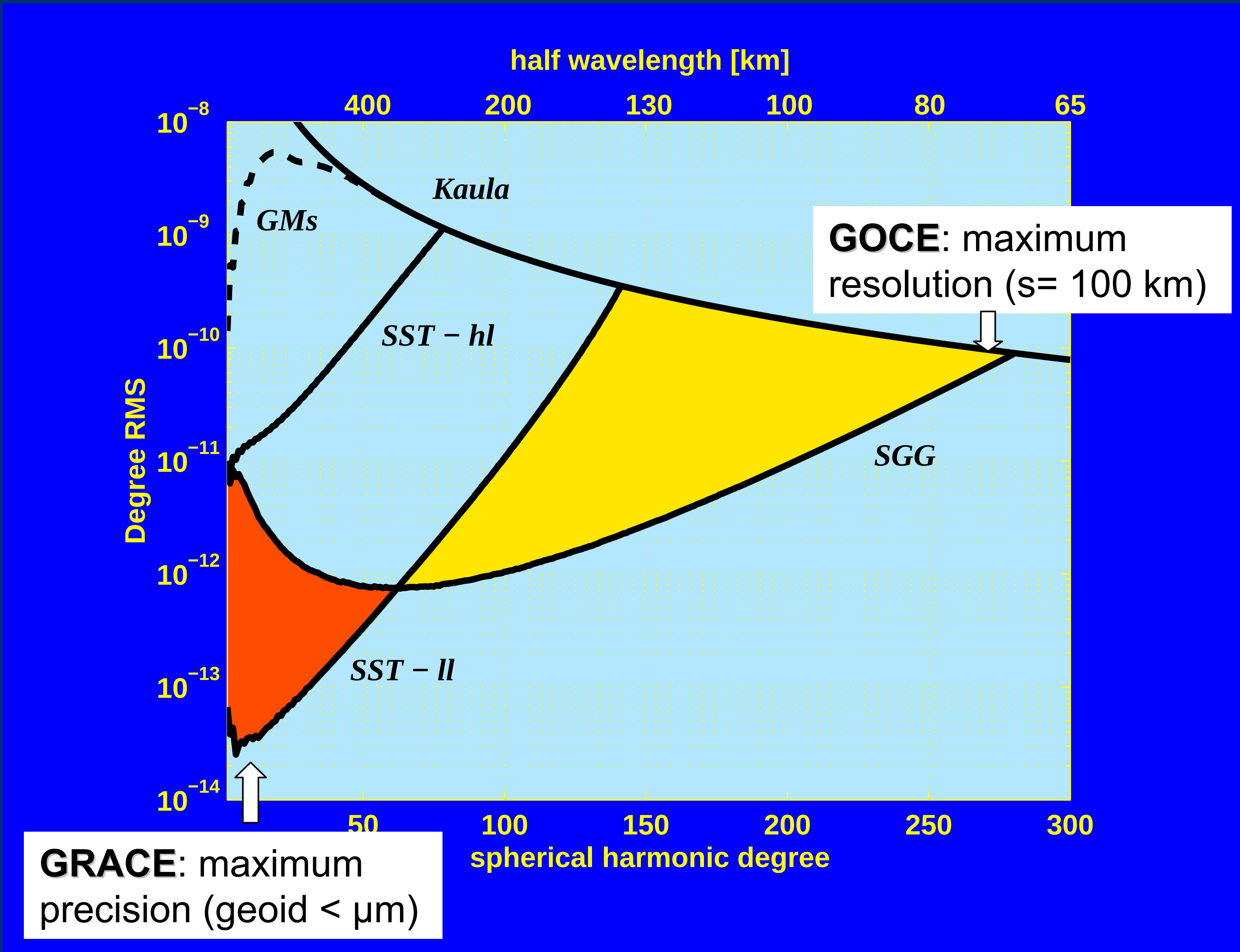
GOCE gravity field



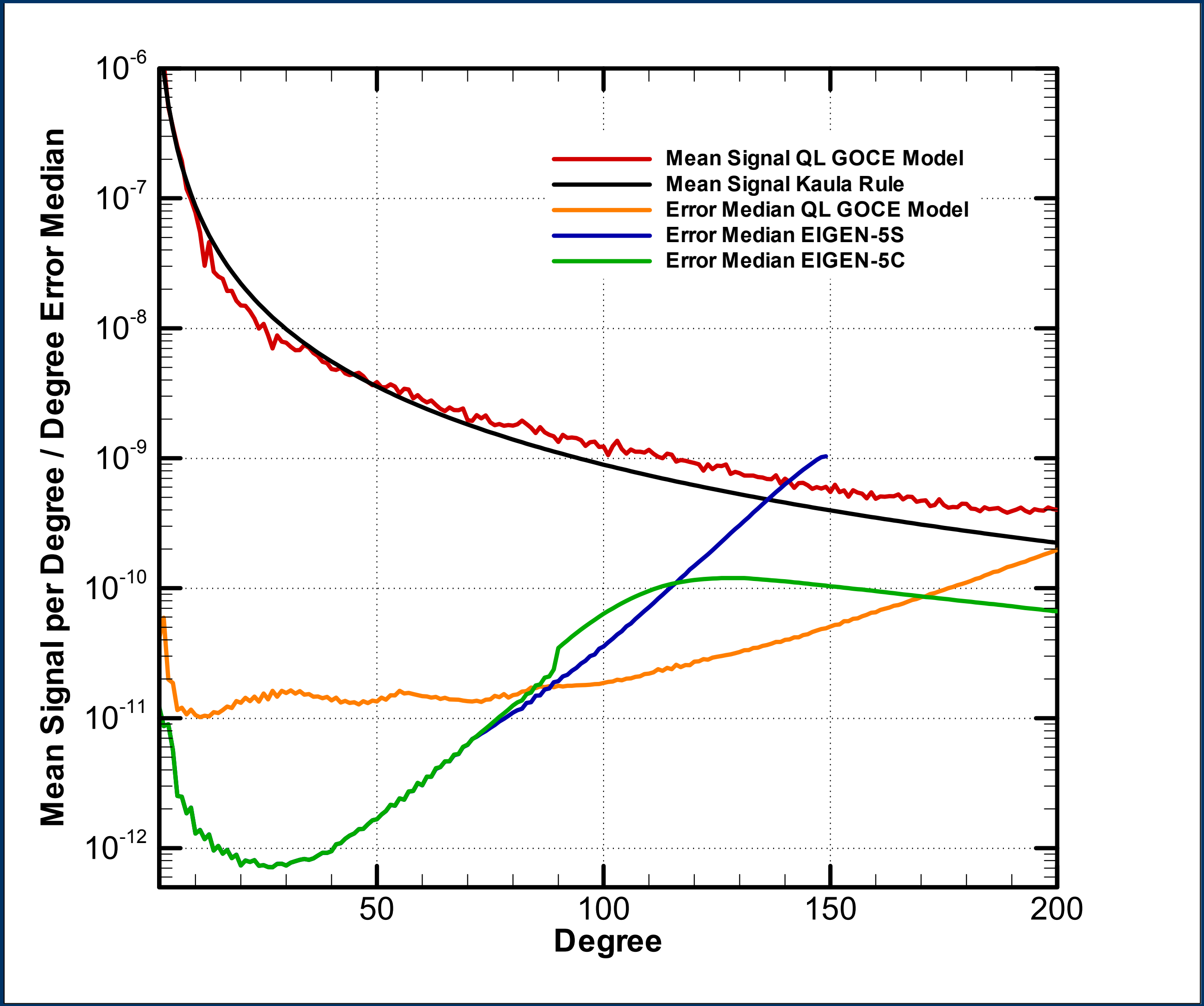


Map compiled by M.Studinger, LDEO,
Using data from Siegert et al. (2005) and NSIDC

GOCE versus GRACE



GOCE versus GRACE



degree variances (median) of signal and noise

GOCE versus GRACE

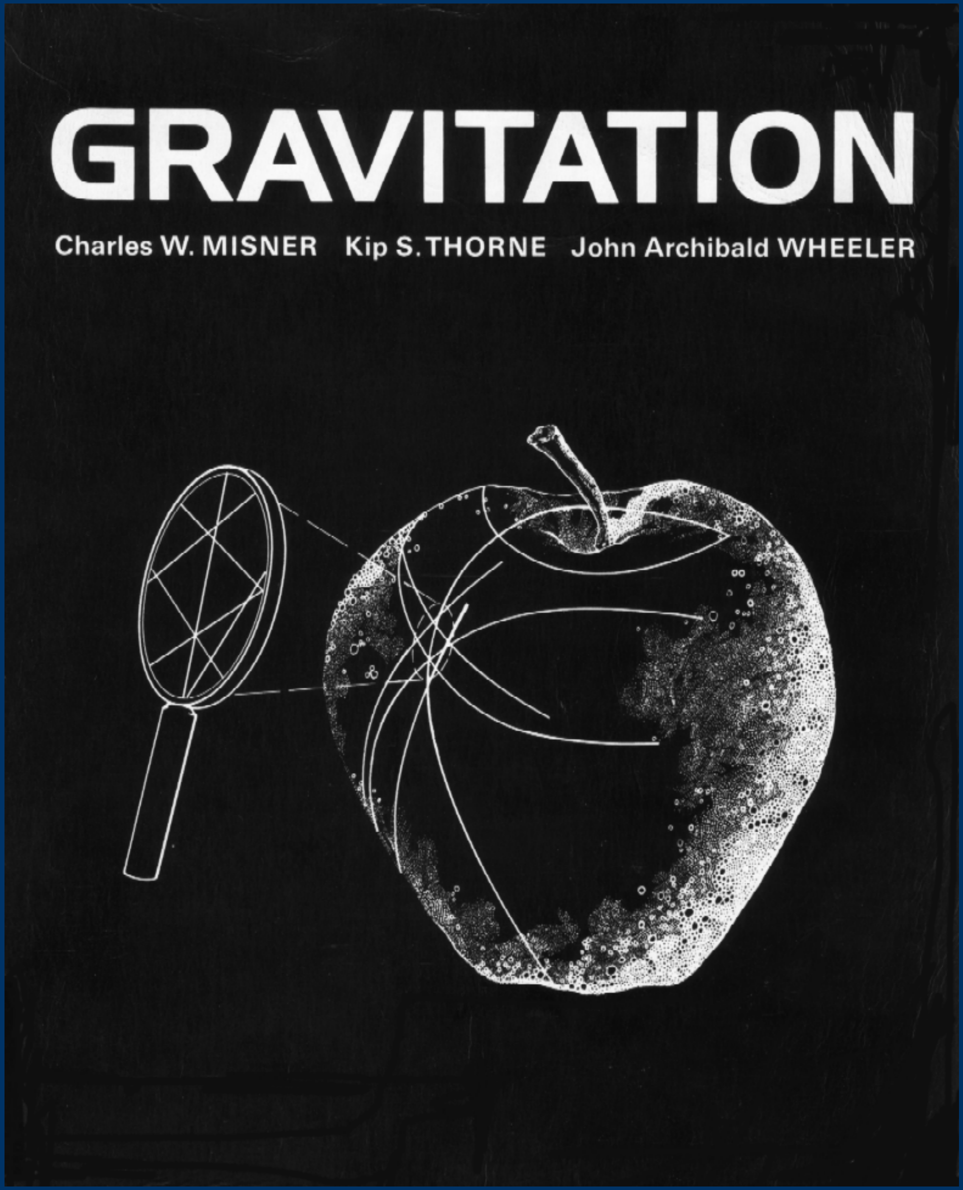
GRACE measures the long wavelength structure of gravity and geoid with extremely high precision

GRACE can therefore even detect temporal changes in the earth system due to mass redistribibition (ice, sea level, continetal hydrology)

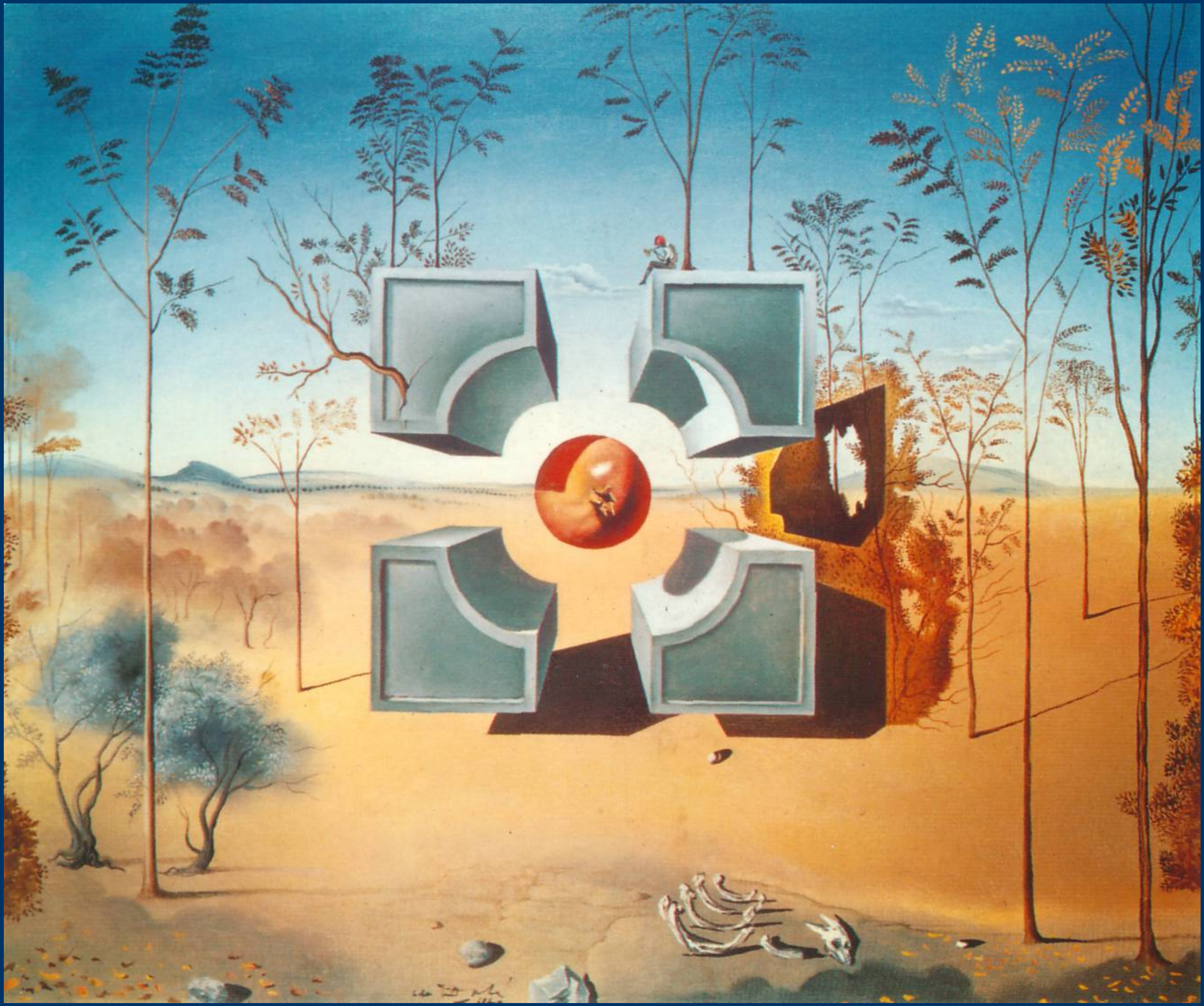
GOCE gives much higher spatial resolution

This resolution is needed when using the geoid e.g. as refence level surface for studies of ocean circulation or for geodynamics

gravitation and the story pf the apple



The tale of two ants walking on the surface of an apple: “They start at A and A’ and walk on two adjacent paths along shortest distance (geodesics) on the curved apple to B and B’. We measure the changing distance between the two ants. From these measured distances we deduce the local curvature of the apple.” (analogy to the satellite mission GRACE and GOCE)



S. Dali: Sans titre, 1948

summary of lecture three

- uninterrupted tracking of a low orbiting satellite (LEO) by GPS in combination with measurement of non-gravitational forces by accelerometry corresponds to free fall absolute gravimetry in an laboratory on earth
- CHAMP (2000) was the first mission of this kind
- differential measurement of the relative motion of two satellites increases the sensitivity, (GRACE, 2002)
- gravitational gradiometry is differential accelerometry between several test masses inside one satellite
- GOCE is the first satellite with a gravitational gradiometer
- GRACE can be regarded as one-arm gradiometer with an arm length of 200 km