



# Inverse method for development of an advanced vegetation index



Nadine Gobron

With collaborations of:

Bernard Pinty, Michel Verstraete & Jean-Luc Widlowski



## What is the main scientific issue in remote sensing?

- Remote sensing instruments on space platforms are but sophisticated detectors recording the occurrence of elementary events (the absorption of photons or electromagnetic waves) as variations in electrical currents or voltages.
- Users, on the other hand, require information on events and processes occurring in the geophysical environment (e.g., atmospheric pollution, oceanic currents, terrestrial net productivity, etc.)
- Extracting useful environmental information from data acquired in space is the main challenge facing remote sensing scientists.



## How is remote sensing exploited?

- The signals detected by the sensors are immediately converted into digital numbers and transmitted to dedicated receiving stations, where these data are heavily processed.
- The effective exploitation of remote sensing data to reliably generate useful, pertinent information hinges on the availability and performance of **specific tools and techniques of data analysis and interpretation**.
- A **variety of mathematical models** can be used for this purpose; they are implemented as computer codes that read the data or intermediary products and ultimately lead to the generation of products and services usable in specific applications.



## Why is remote sensing called an inverse problem?

- Since space borne instruments can only measure the properties of electromagnetic waves emitted or scattered by the Earth, scientists need first to understand where these waves originate from, **how they interact with the environment, and how they propagate towards the sensor.**
- To this effect, they develop **models of radiation transfer**, assuming that everything is known about the sources of radiation and the environment, and calculate the properties of the radiation field as the sensor should measure them. This is the so-called *direct problem*.



## Why is remote sensing called an inverse problem?

- In practice, one does acquire the measurements from the satellite, and would like to know what is going on in the environment:
  - This is the *inverse problem*, which is much more complicated: Knowing the value of the electromagnetic measurements gathered in space, how can we derive the properties of the environment that were responsible for the radiation to reach the sensor?



# Measurements of interpretation (1)

- A model representing a measurement expresses the dependency of this measurement with respect to the relevant variables and processes:

$$z = f(s_1, s_2, \dots, s_m)$$

where  $s_m$  are the state variables of the system

- Direct problem: if values  $s_m$  are known, such a model can accurately simulate the observation
- Inverse problem: Quantity  $\hat{z}$  is measured, and one seeks information on the state variables  $s_m$



## Measurements of interpretation (2)

- If a single state variable accounts for the physics of the measurement, the problem can be solved analytically or numerically, usually with great accuracy:

$$z = f(s_1) \Rightarrow \hat{s} = f^{-1}(\hat{z})$$

- In general, more than one state variable is required to describe the physics of the problem. The solution to the inverse problem then requires multiple equations and therefore multiple measurements.



## Role of independent variables (1)

- To acquire more information on an invariant system, it is necessary to gather **different measurements by changing a variable other than the state variables.**

$$z = f(x; s_1, s_2, \dots, s_m)$$

where the independent variable  $x$  describes the changing conditions of observations.

- If the system of interest can be observed in more than one way, multiple independent variables may be defined.
- In case of *RS from space*, the useful independent variables are *space, time, wavelength, illumination and observation geometry.*



## Role of independent variables (2)

- Multiple measurements may thus be acquired under different conditions of observation:

$$\hat{z}^1 = f(x_1^1, x_2^1, \dots, x_N^1; s_1, s_2, \dots, s_M) + \mathbf{e}_1$$

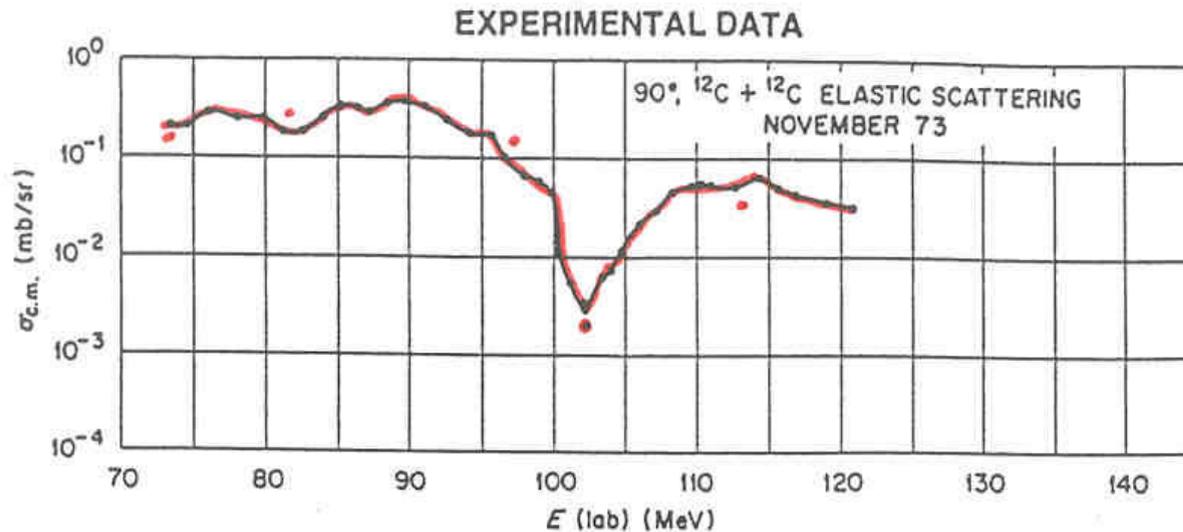
$$\hat{z}^2 = f(x_1^2, x_2^2, \dots, x_N^2; s_1, s_2, \dots, s_M) + \mathbf{e}_2$$

....

$$\hat{z}^K = f(x_1^K, x_2^K, \dots, x_N^K; s_1, s_2, \dots, s_M) + \mathbf{e}_K$$



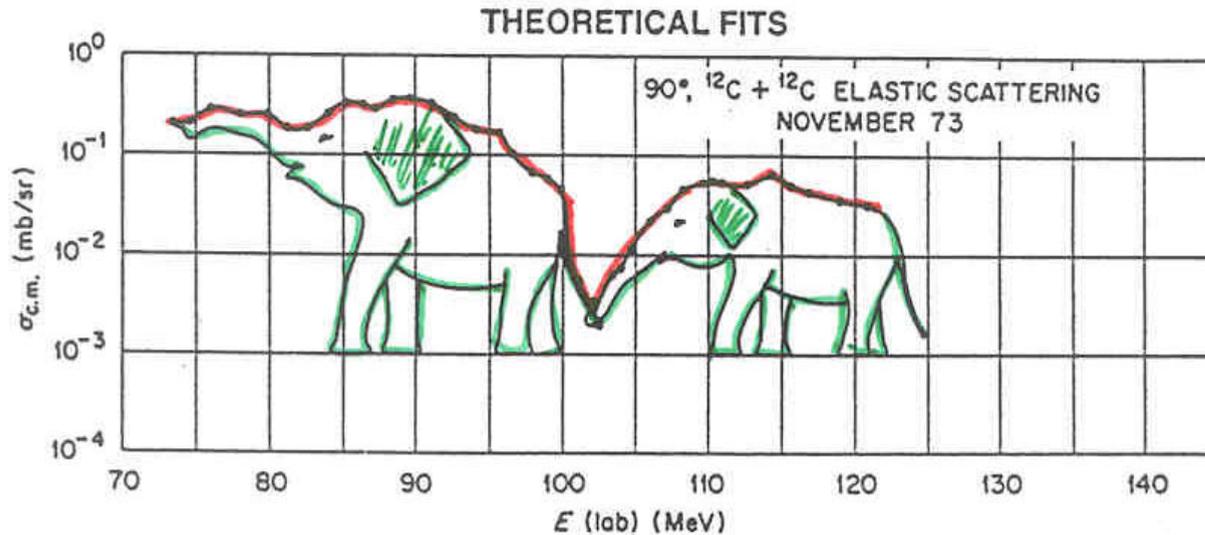
# Association of physical measurements, models & models representing the biosphere



The process of fitting data, as seen by Subramanian Raman in Science with a Smile.



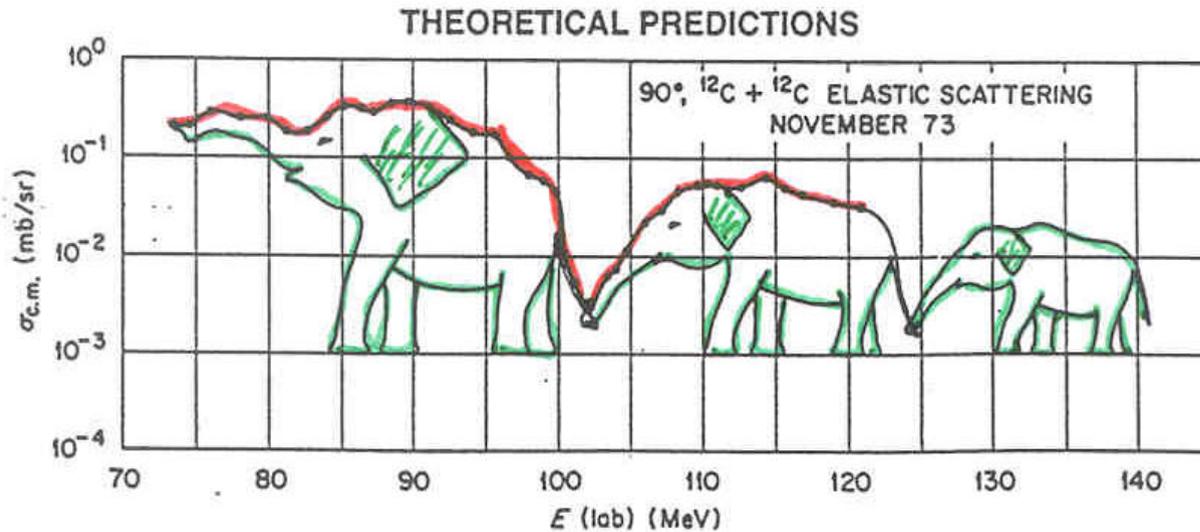
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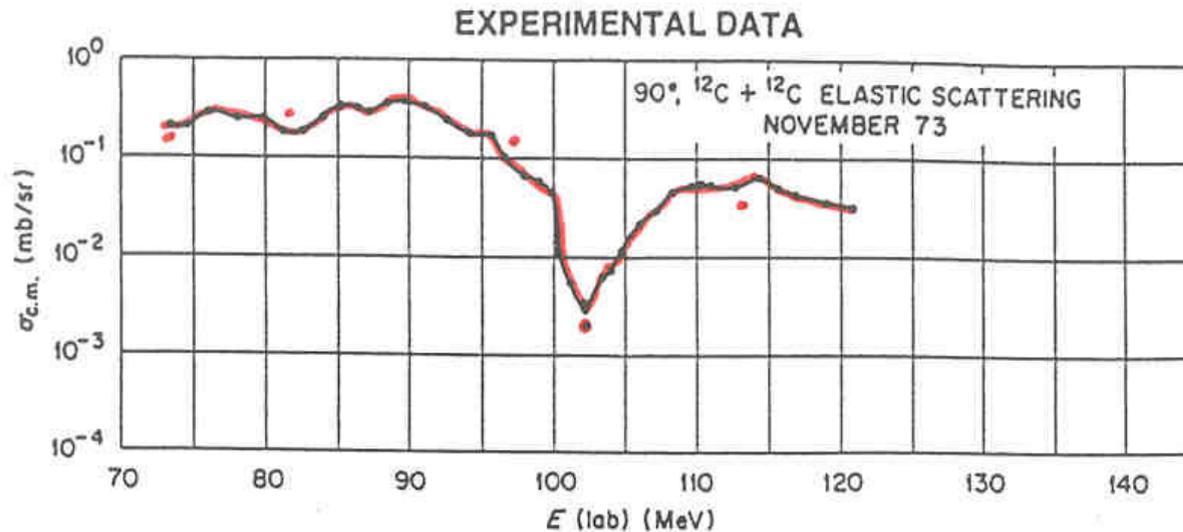
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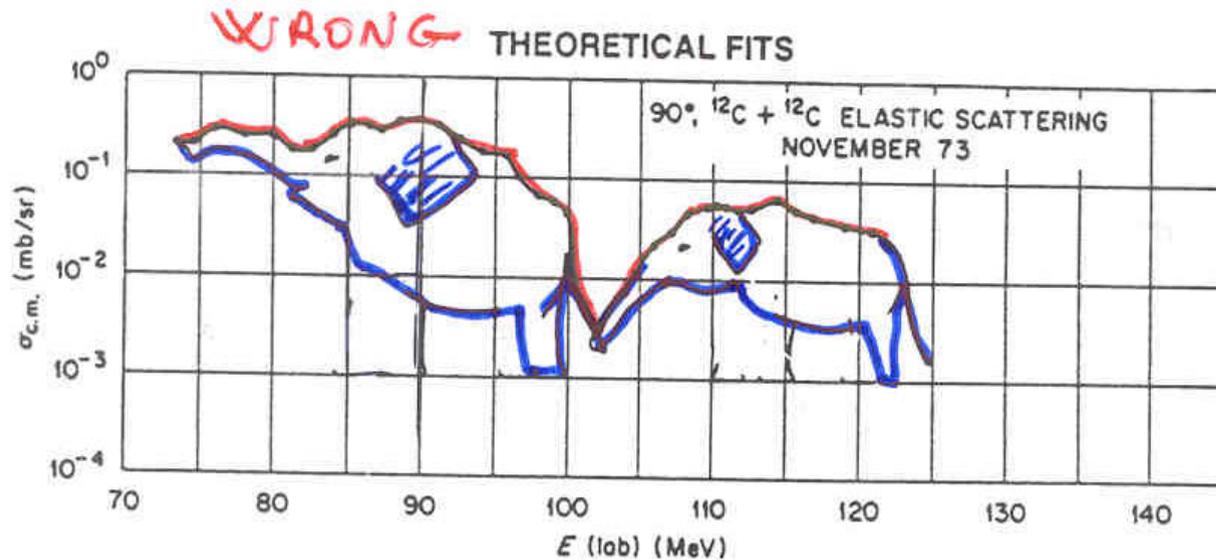
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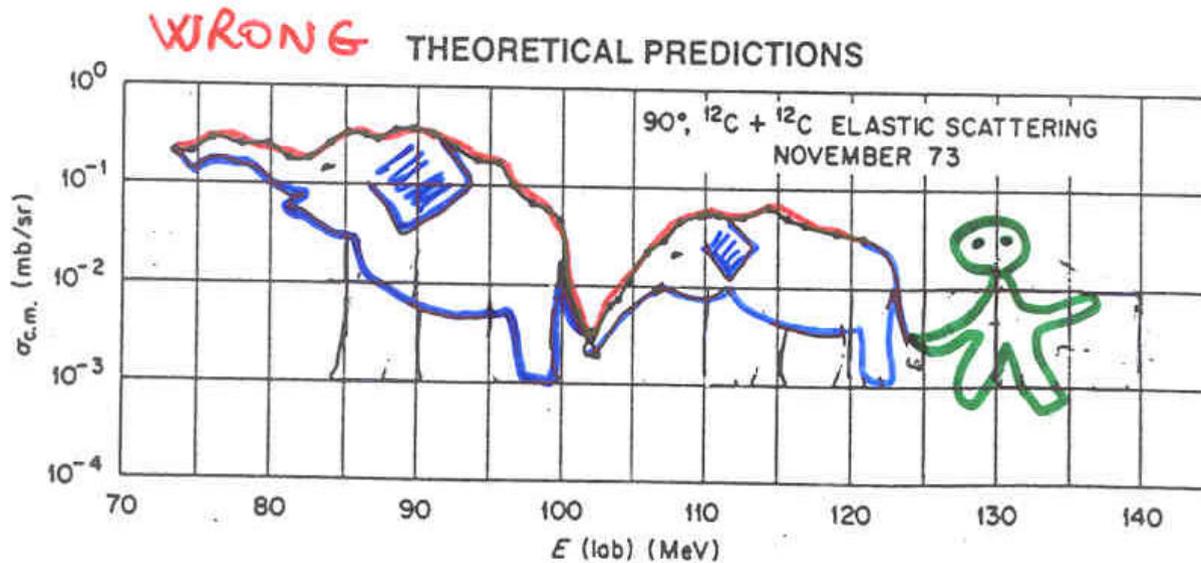
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The process of fitting data, as seen by Subramanian Raman in Science with a Smile.



# Model Inversion

- In general, the system of equations cannot be solved:
  - If  $K < M$ , the system is still underdetermined
  - If  $K=M$ , measurements errors may prevent the identification of the exact solution.
  - If  $K>M$ , the system is overdetermined.

**K depends on space observation strategy**

**M depend on the RT model**



# Space Observation Strategies

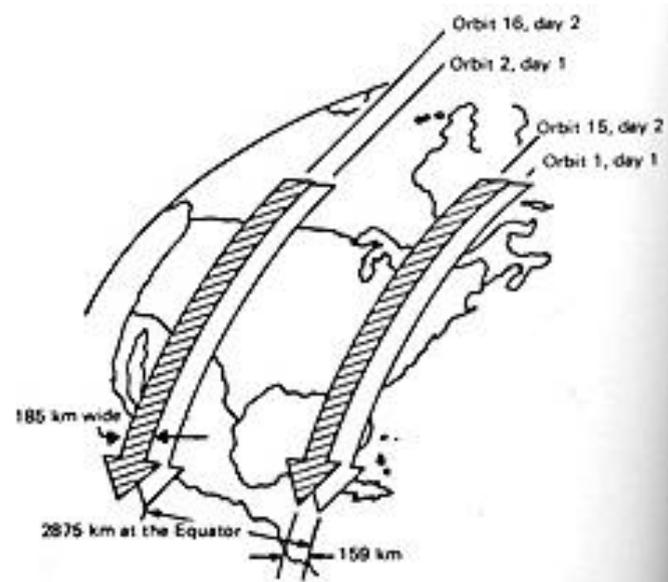
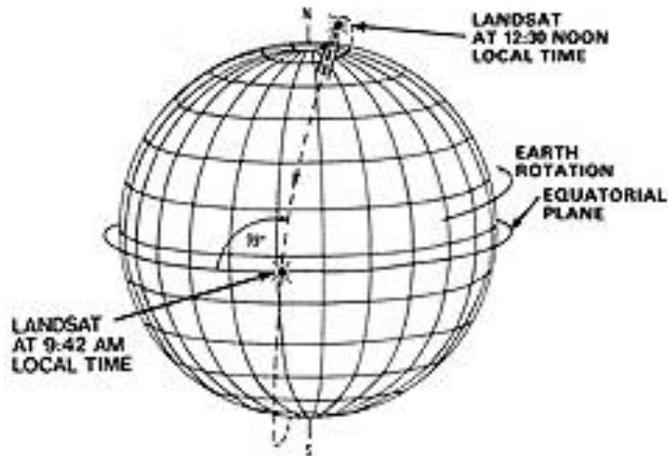
- Mono-directional multi-spectral scanning sensor on a Sun-synchronous orbit

→ single view of the same target during a given day (given Sun zenith angle)



# Space Observation Strategies

**INCLINATION OF LANDSAT ORBIT TO MAINTAIN SYNCHRONOUS ORBIT**



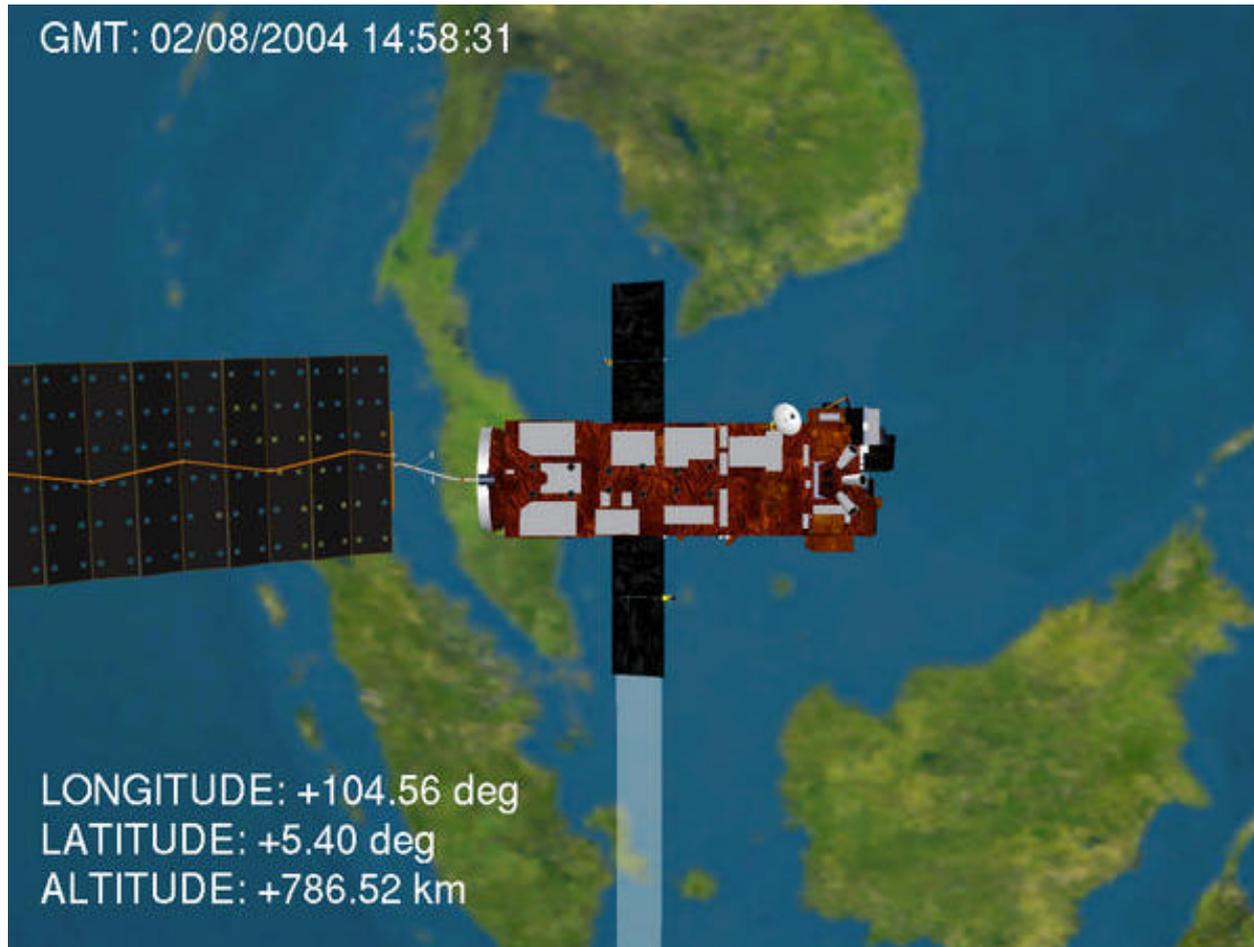


# Space Observation Strategies





# Space Observation Strategies





# Space Observation Strategies

- Mono-directional multi-spectral scanning sensor on a Sun-synchronous orbit

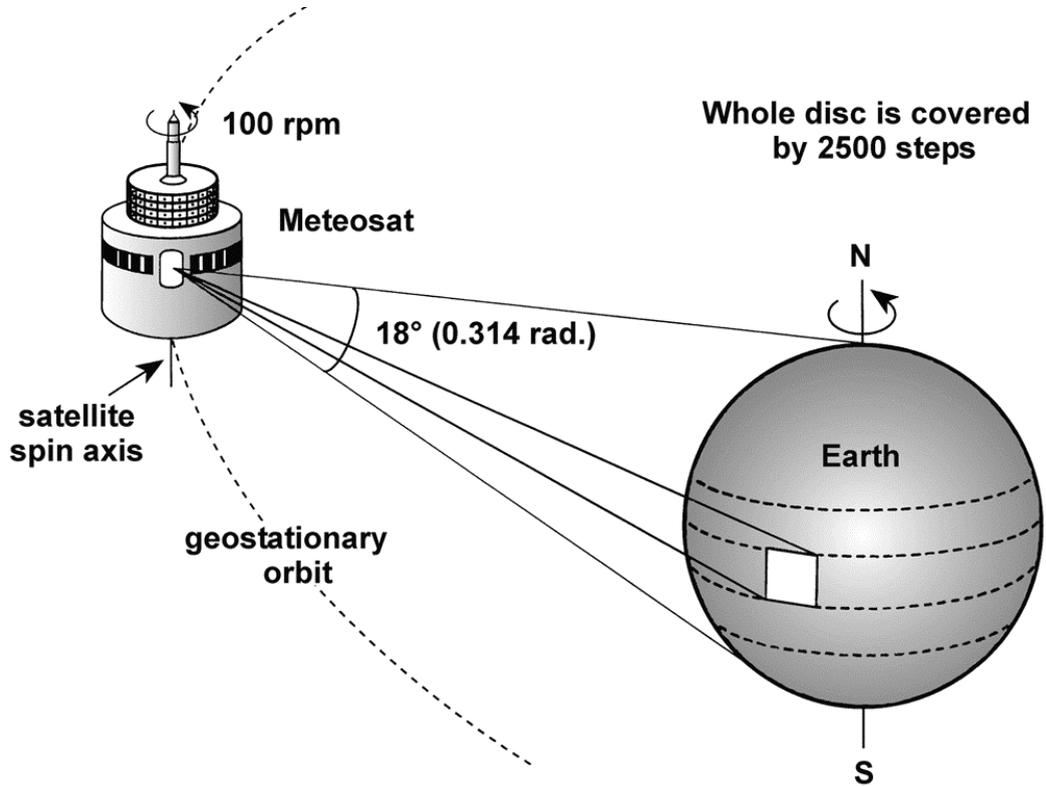


single view of the same target during a given day (given Sun zenith angle)

- Spinning sensor on a geostationary orbit



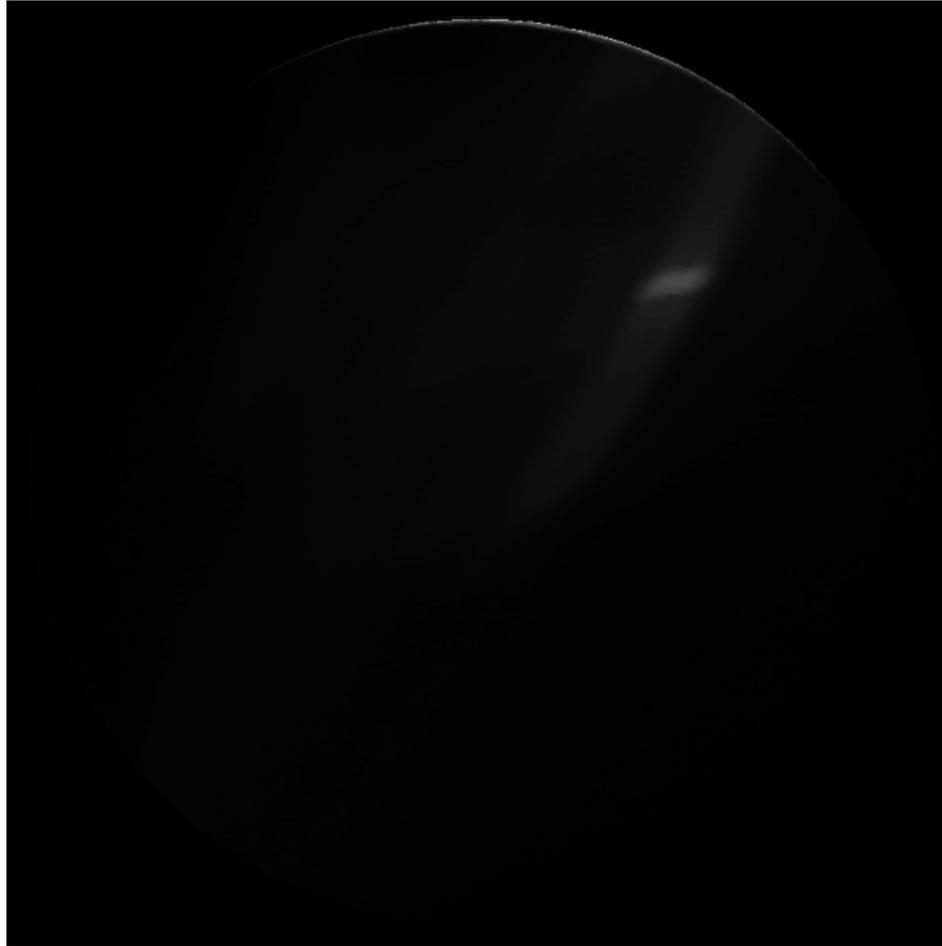
# Space Observation Strategies





# Space Observation Strategies

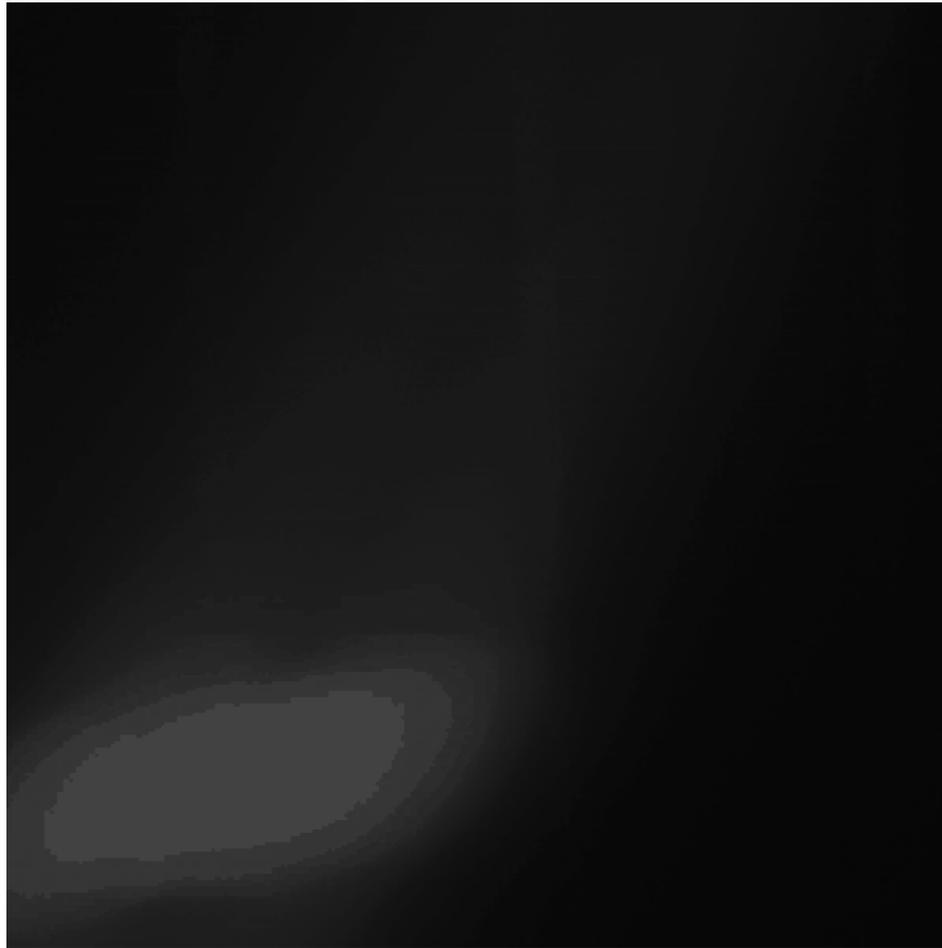
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# Space Observation Strategies

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# Space Observation Strategies

- Mono-directional multi-spectral scanning sensor on a Sun-synchronous orbit

→ single view of the same target during a given day (given Sun zenith angle)

- Spinning sensor on a geostationary orbit

→ multiple views of the same target during a given day (various Sun zenith angles)



# Space Observation Strategies

- Mono-directional multi-spectral scanning sensor on a Sun-synchronous orbit



single view of the same target during a given day (given Sun zenith angle)

- Spinning sensor on a geostationary orbit

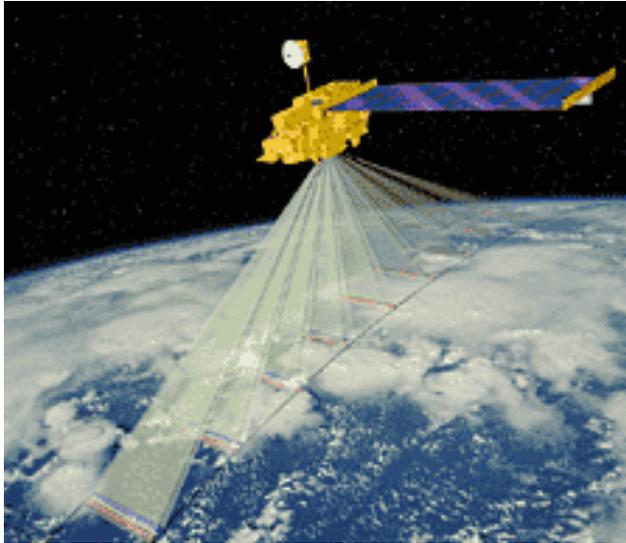


multiple views of the same target during a given day (various Sun zenith angles)

- Multi-directional multi-spectral sensor on a Sun-synchronous orbit



## Overview of MISR



- 9 cameras at  $\pm 70.5$ ,  $\pm 60$ ,  $\pm 45.6$ ,  $\pm 26.1$ ,  $0^\circ$
- Each camera at 446, 558, 672, and 866 nm
- Spatial resolution: 275 m (250 m nadir)
- Global mode: Full res. nadir and red, 1.1 km otherwise
- Local mode: Full resolution all cameras and all bands
- Swath: 360 km
- Coverage: global (9 days)



79 km

Multiangle animation

Emigrant Gap Fire,  
California

13 August 2001





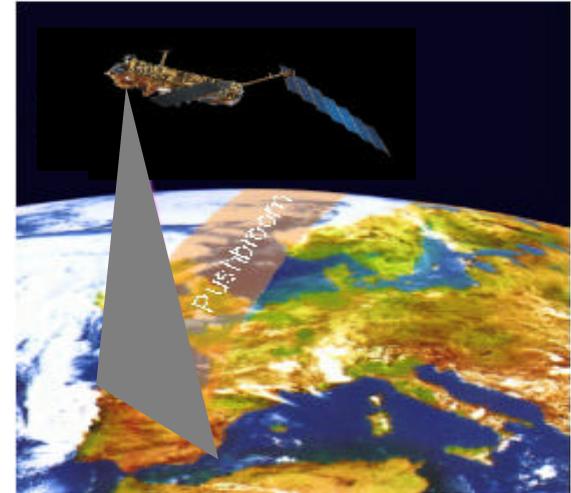
# **Design of the MERIS Global Vegetation Index**

## **decontaminated from atmospheric and geometrical effects**



# MERIS/ENVISAT

- Passive optical instrument of Earth Observation
- Primary mission: Ocean productivity
- Secondary missions: Atmosphere and **land surface characterization**
- Ground segment support (up to L2)
  
- Global coverage: = 3 days (depends on latitude)
- Swath: 1150 km
- Spatial resolution:  $\pm 300$  m (FR) &  $\pm 1200$  m (RR)
  
- Spectral band positions, widths and gains are programmable
- Radiometric and spectral calibration on-board mechanisms (white & pink Spectralon, Fraunhofer lines)



Source: <http://envisat.esa.int/instruments/meris/>



# Nominal MERIS spectral bands

Band	Location [nm]	Width [nm]	Applications
1	412.5	10	Yellow substance and detrital pigments
2	442.5	10	Chlorophyll absorption maximum
3	490	10	Chlorophyll and other pigments
4	510	10	Suspended sediment, red tides
5	560	10	Chlorophyll absorption minimum
6	620	10	Suspended sediment
7	665	10	Chlorophyll absorption and fluorescence
8	681.25	7.5	Chlorophyll fluorescence peak
9	708.75	10	Fluorescence, atmospheric corrections
10	737.75	7.5	Vegetation, clouds
11	760.625	3.75	Oxygen absorption R-branch
12	778.75	15	Atmospheric corrections
13	865	20	Vegetation, water vapour reference
14	885	10	Atmospheric corrections
15	900	10	Water vapour, land

Ref: <http://envisat.esa.int/instruments/meris/descr/concept.html>



## Objective

- To provide useful, quantitative, reliable, accurate and low cost information on the state of terrestrial vegetation using remote sensing data

## Approach

- Development of Optimized Vegetation Indices using a physically-based approach



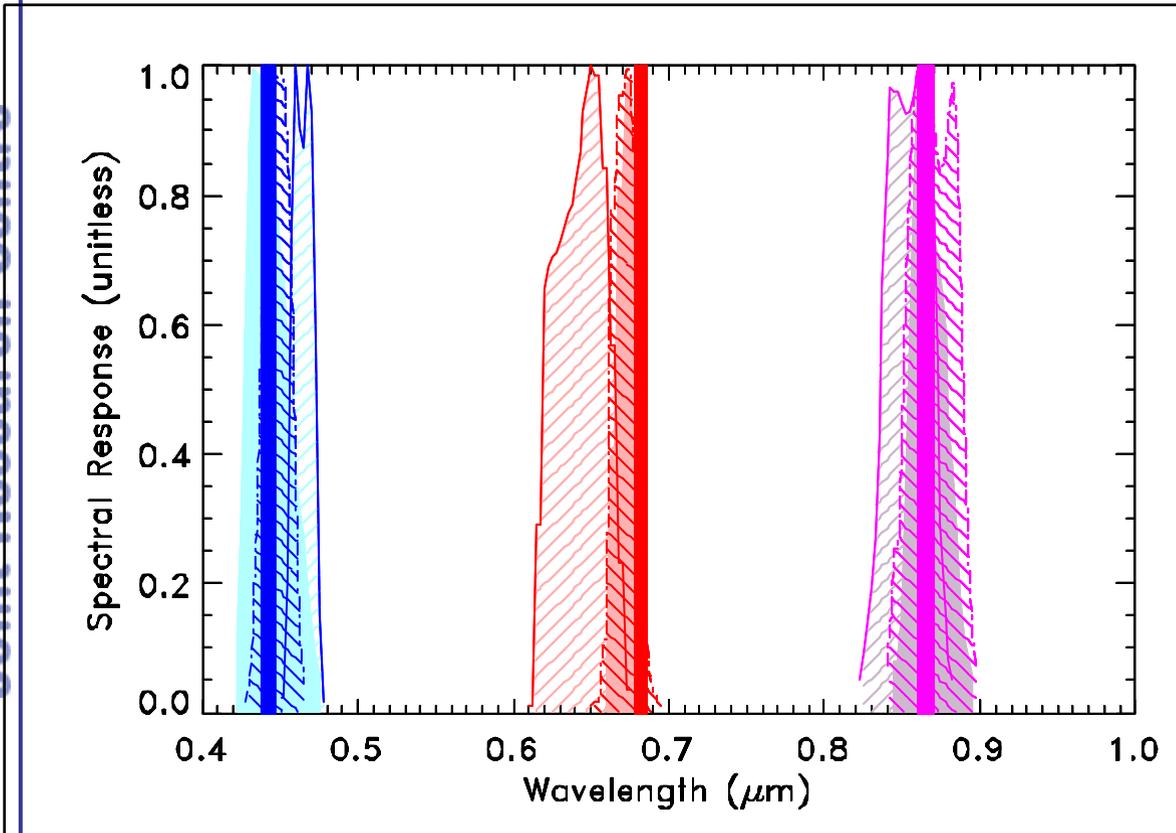
# Scientific Constraints

- Maximal sensitivity to the Fraction of Absorbed Photosynthetically Active Radiation (FAPAR)
- Minimal sensitivity to atmospheric perturbations
- Minimal sensitivity to soil colour and brightness changes
- Minimal sensitivity to angular effects



# Spectral Sensor Properties

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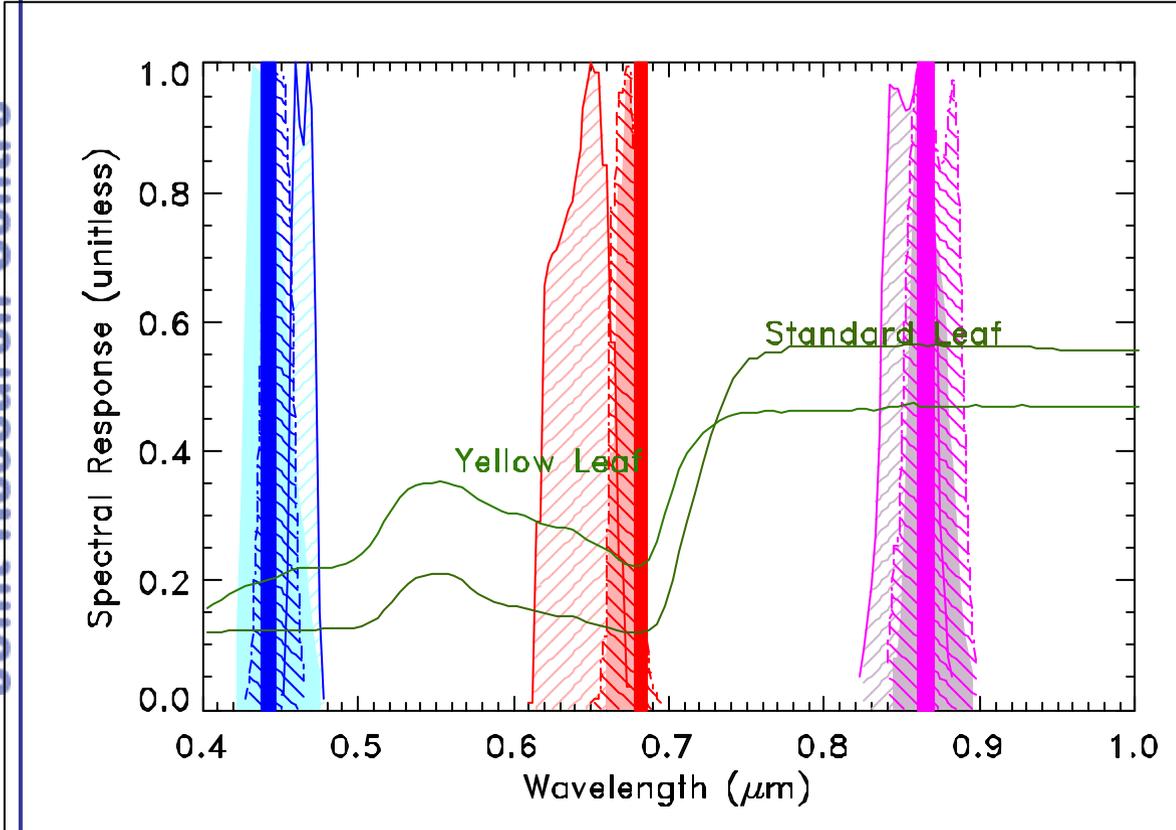


SeaWiFS  
MISR  
MODIS  
MERIS



# Spectral Leaves Profile

Joint Research Centre



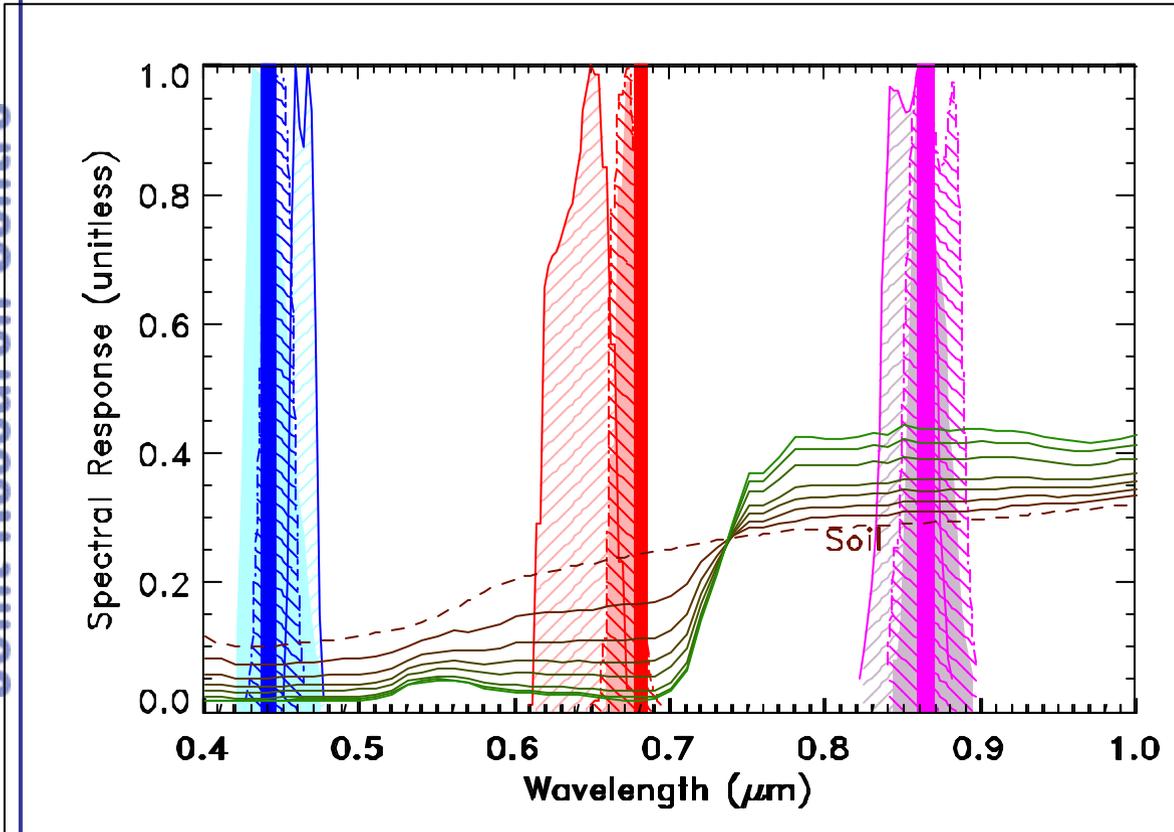
SeaWiFS  
MISR  
MODIS  
MERIS

Prospect Model



# Bidirectional Reflectance Factor at Top Of Canopy : Nadir View

Joint Research Centre



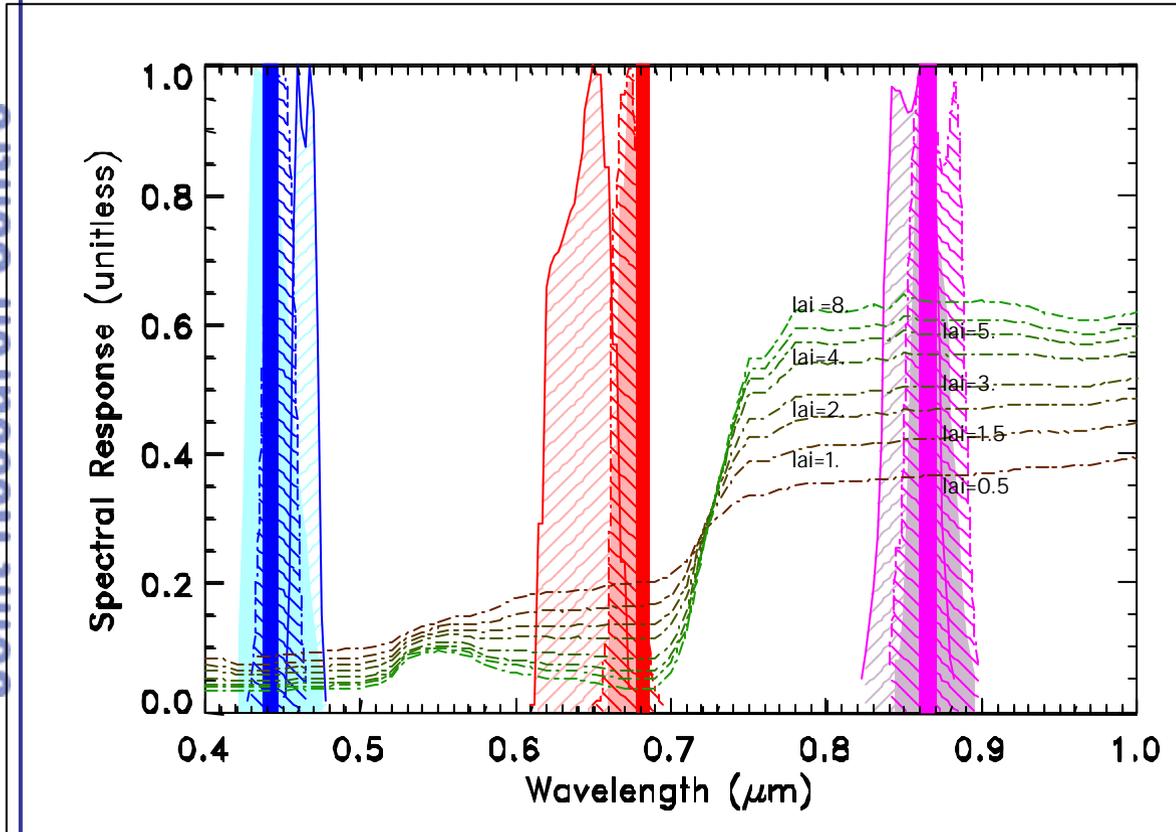
SeaWiFS  
MISR  
MODIS  
MERIS

Semi-discret Model



# Bidirectional Reflectance Factor at Top Of Canopy

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SeaWiFS  
MISR  
MODIS  
MERIS

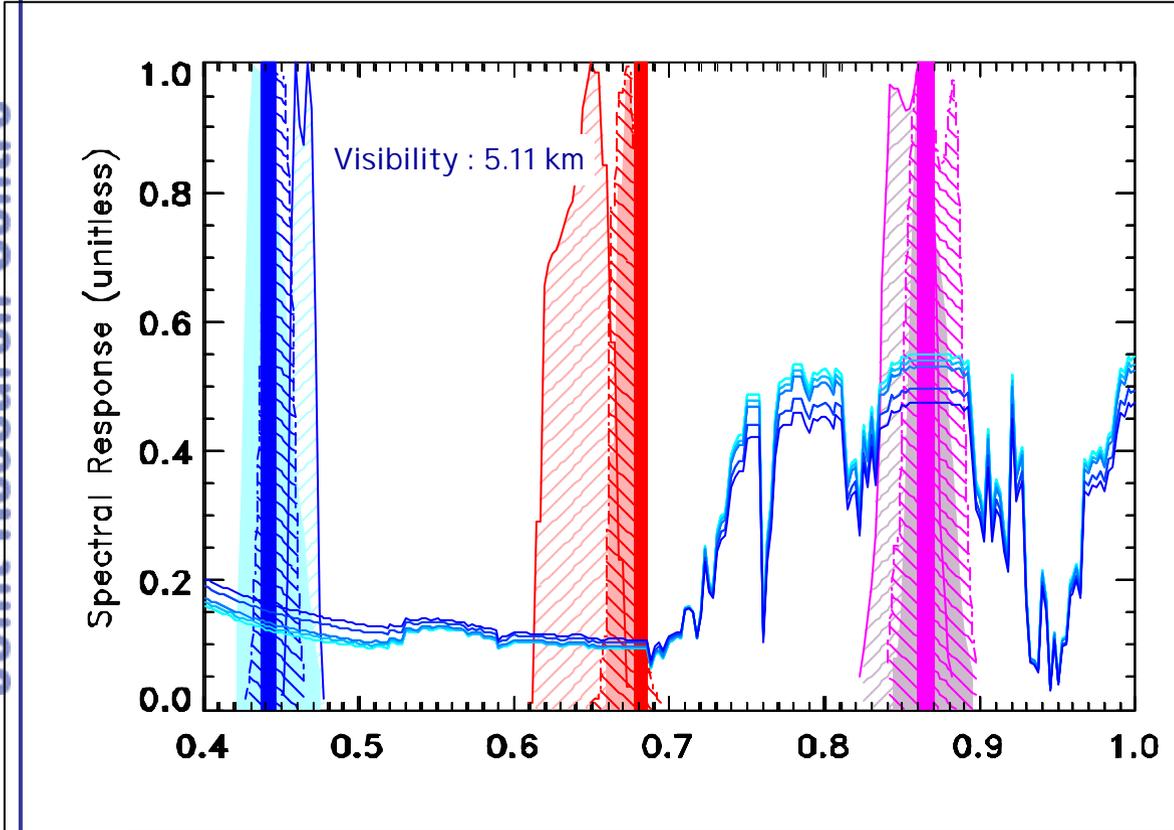
Semi-discret Model





# Bidirectional Reflectance Factor Top Of Atmosphere: Aerosols Optical Depth

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SeaWiFS  
MISR  
MODIS  
MERIS

6S Model





# Operational Constraints

- Exploit only the target's spectral variations as measured by the satellite instruments
- Take the actual spectral response of the sensor into account
- Minimise the computational load
- Require data from a single orbit only



# Scheme for the algorithm optimization (1)





# Mathematical Implementation (1)

Models:

- 6S atmospheric radiation transfer model (Vermote *et al.*, 1997)
- Semi-discrete vegetation radiation transfer model (Gobron *et al.*, 1997)

## Geophysical properties

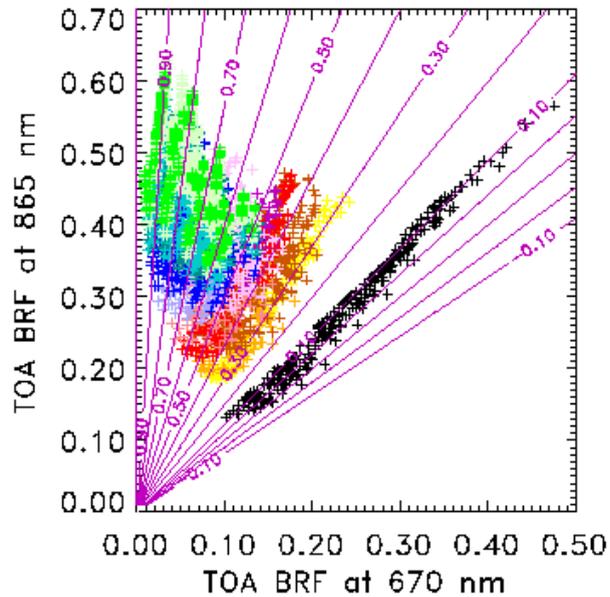
Medium	Variables	Range of values
Atmosphere	$\tau_s$	0.05, 0.3 and 0.8
Vegetation	LAI $H_C$ $D_1$ LAD	0, 1, 2, 3, and 5 0.5 and 2 m 0.01 and 0.05 m Erectophile, Planophile
Soil	$r_s$	5 Soil spectra (Price, 1995)

## Illumination and observation geometries

Angle	Values
Solar zenith ( $\theta_0$ )	20 and 50 degrees
Satellite zenith ( $\theta_v$ )	0, 25 and 40 degrees
Sun-Satellite relative azimuth ( $\Delta\phi$ )	0, 90 and 180 degrees

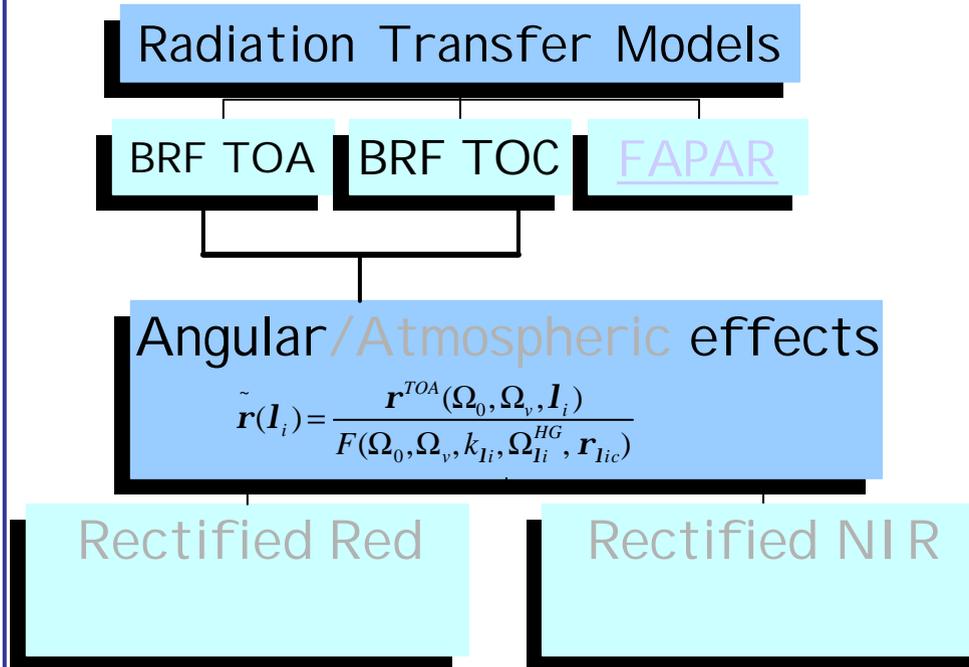


## RT model driven improvement for FAPAR





## Scheme for the algorithm optimization (2)





# The RPV parametric model

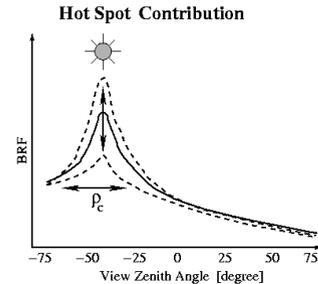
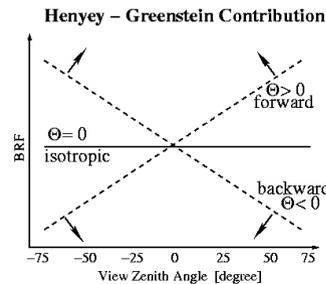
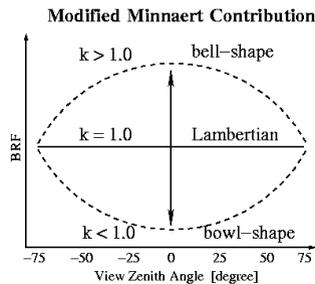
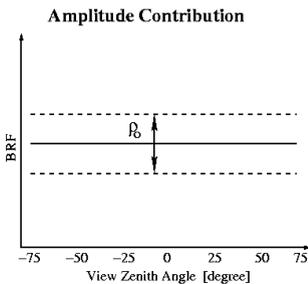
$$\text{BRF}(z, \Theta_0 - \Theta) = \rho_0 \text{MI}(k) \text{FHG}(T) \text{H}(\rho_c)$$

$\rho_0$  - controls amplitude level

$k$  - controls bowl/bell shape

$T$  - controls forward/backward scattering

$\rho_c$  - controls hot spot peak





# The RPV parametric model

- **Direct Mode**

$$r(z_0, \Omega_s \rightarrow \Omega; r_0, r_c, \Theta, k) = r_0 \tilde{r}(z_0, \Omega_s \rightarrow \Omega; r_c, \Theta, k)$$

$$\tilde{r}(z_0, \Omega_s \rightarrow \Omega; r_c, \Theta, k) = M_I(\mathbf{q}_s, \mathbf{q}; k) F_{HG}(g, \Theta) H(\mathbf{r}_c; G)$$

$$M_I(\mathbf{q}_s, \mathbf{q}; k) = \frac{\cos^{k-1} \mathbf{q}_s \cos^{k-1} \mathbf{q}}{(\cos \mathbf{q}_s + \cos \mathbf{q})^{1-k}} \quad F_{HG}(g, \Theta) = \frac{1 - \Theta^2}{[1 + 2\Theta \cos g + \Theta^2]^{3/2}}$$

$$H(\mathbf{r}_c; G) = 1 + \frac{1 - r_c}{1 + G}$$

$$\cos g = \cos \mathbf{q} \cos \mathbf{q}_s + \sin \mathbf{q} \sin \mathbf{q}_s \cos f$$

$$G = [\tan^2 \mathbf{q}_s + \tan^2 \mathbf{q} - 2 \tan \mathbf{q}_s \tan \mathbf{q} \cos f]^{1/2}$$

- **Inverse Method when mono-angle data (MERIS)**

➤  $k, \Theta, \rho_c$  parameters are optimized with simulated-data for each wavelength at TOA & TOC: representative values of anisotropic at the global scale.



# The RPV parametric model

- **Direct Mode**

$$r(z_0, \Omega_s \rightarrow \Omega; r_0, r_c, \Theta, k) = r_0 \tilde{r}(z_0, \Omega_s \rightarrow \Omega; r_c, \Theta, k)$$

$$\tilde{r}(z_0, \Omega_s \rightarrow \Omega; r_c, \Theta, k) = M_I(\mathbf{q}_s, \mathbf{q}; k) F_{HG}(g, \Theta) H(\mathbf{r}_c; G)$$

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- **Inverse Method when multi-angles data (MISR, SPECTRA)**

- Resolution of second order equations (Gobron and Lajas, 2002)
- Users' controlled accuracy of the fit (model/data)
- Retrieval of a set of acceptable solutions
- Selection on the most representative solutions



# Parametric Model: MRPV

- **Direct Mode**

$$\mathbf{r}(z_0, \Omega_s \rightarrow \Omega; \mathbf{r}_0, b_m, k) = \mathbf{r}_0 \tilde{\mathbf{r}}(z_0, \Omega_s \rightarrow \Omega; b_m, k)$$

$$\tilde{\mathbf{r}}(z_0, \Omega_s \rightarrow \Omega; \bar{\mathbf{r}}, b_m, k) = M_I(\mathbf{q}_s, \mathbf{q}; k) F(g; b_m) H(\bar{\mathbf{r}}; G)$$

$$M_I(\mathbf{q}_s, \mathbf{q}; k) = \frac{\cos^{k-1} \mathbf{q}_s \cos^{k-1} \mathbf{q}}{(\cos \mathbf{q}_s + \cos \mathbf{q})^{1-k}}$$

$$F(g; b_m) = \exp(-b_m \cos g)$$

$$H(\bar{\mathbf{r}}; G) = 1 + \frac{1 - \bar{\mathbf{r}}}{1 + G}$$

$$\cos g = \cos \mathbf{q} \cos \mathbf{q}_s + \sin \mathbf{q} \sin \mathbf{q}_s \cos f$$

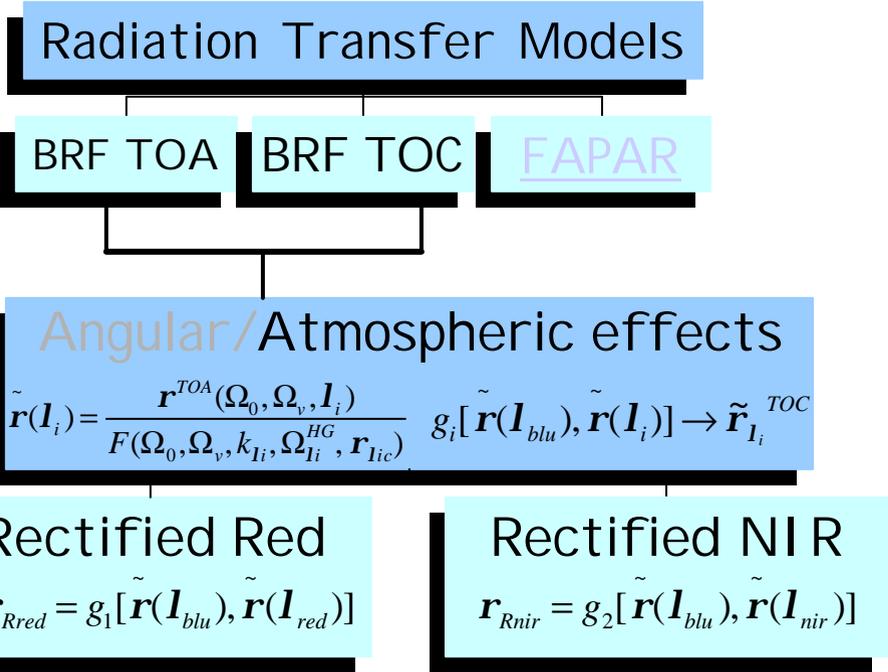
$$G = [\tan^2 \mathbf{q}_s + \tan^2 \mathbf{q} - 2 \tan \mathbf{q}_s \tan \mathbf{q} \cos f]^{1/2}$$

- **Inverse Method**

- Resolution of a linear system (Engelsen et al, 1996)
- No predefined accuracy
- Only one solution
- Very fast method



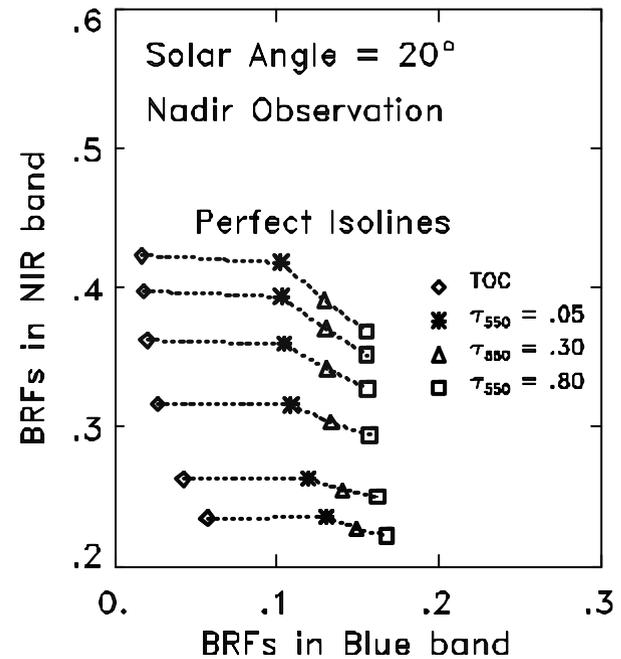
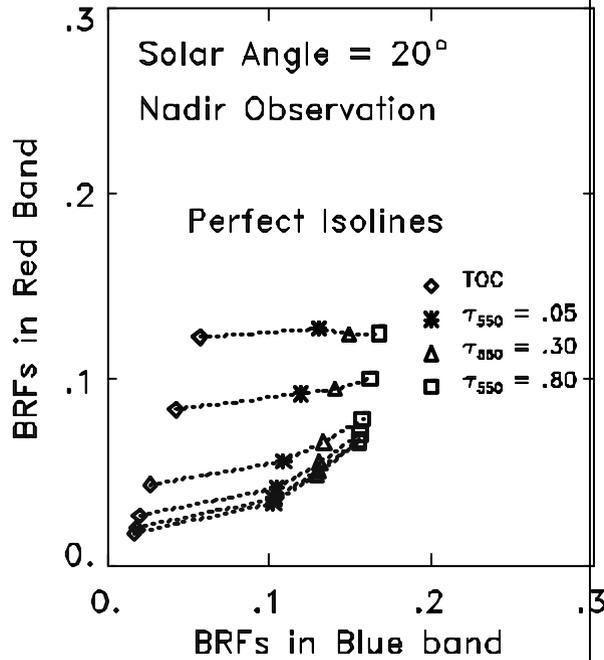
## Scheme for the algorithm optimization (2)





# Scheme for the algorithm optimization (3)

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## Mathematical Implementation (2)

- Ratios of polynomials are assumed to be appropriate to generate the “rectified channels” :

$$g_n(x, y) = f(x, y, l_{n, m})$$

Where  $l_{n,m}$  are the  $m$  coefficients of the polynomials, and  $x$  and  $y$  are either the measured or rectified spectral reflectances, depending on the step



# Cost Functions (1)

- The optimisation procedure is applied to the RED and NIR bands to find the coefficients of  $g_1$  and  $g_2$ :

$$d_{g_i}^2 = \sum_V [g_i(\tau_{blu}(\Omega), \tau_j(\Omega)) - \tau_j^{TOC}] \rightarrow 0$$

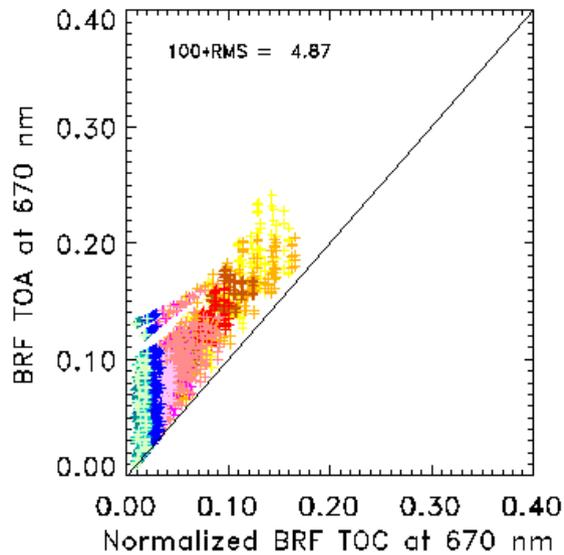
$$g_1(x, y) = l_{1,1}(x + l_{1,2})^2 + l_{1,3}(y + l_{1,4})^2 + l_{1,5}xy$$

$$g_2(x, y) = \frac{l_{2,1}(x + l_{2,2})^2 + l_{2,3}(y + l_{2,4})^2 + l_{2,5}xy}{l_{2,6}(x + l_{2,7})^2 + l_{2,8}(y + l_{2,9})^2 + l_{2,10}xy}$$

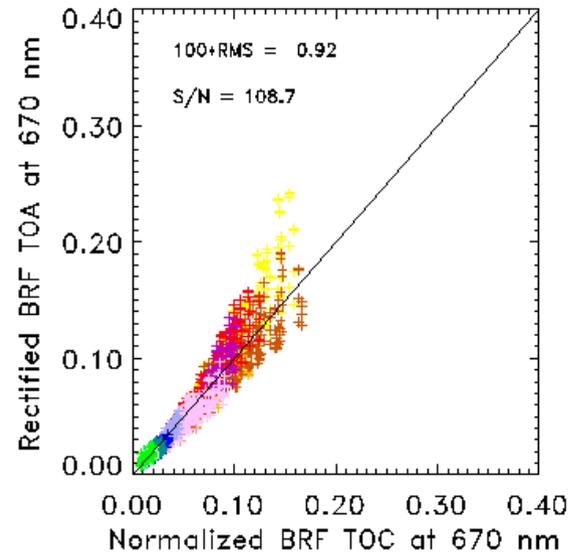


# Rectification Results in the Red Band

Before



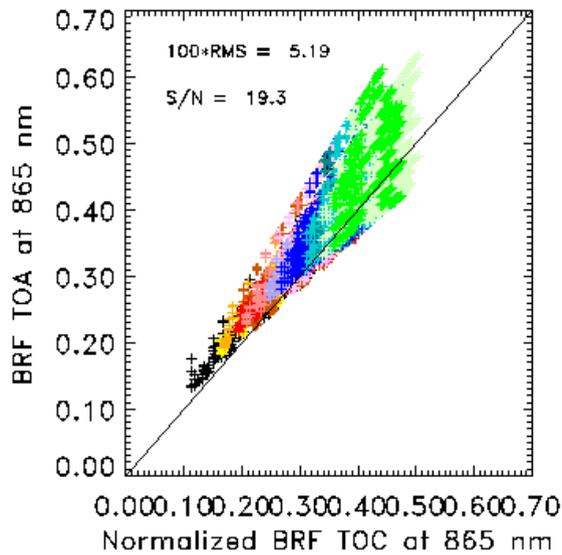
After



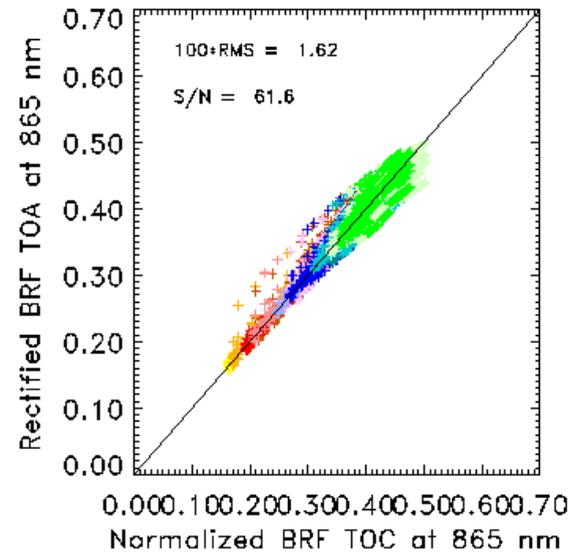


# Rectification Results in the NIR Band

Before

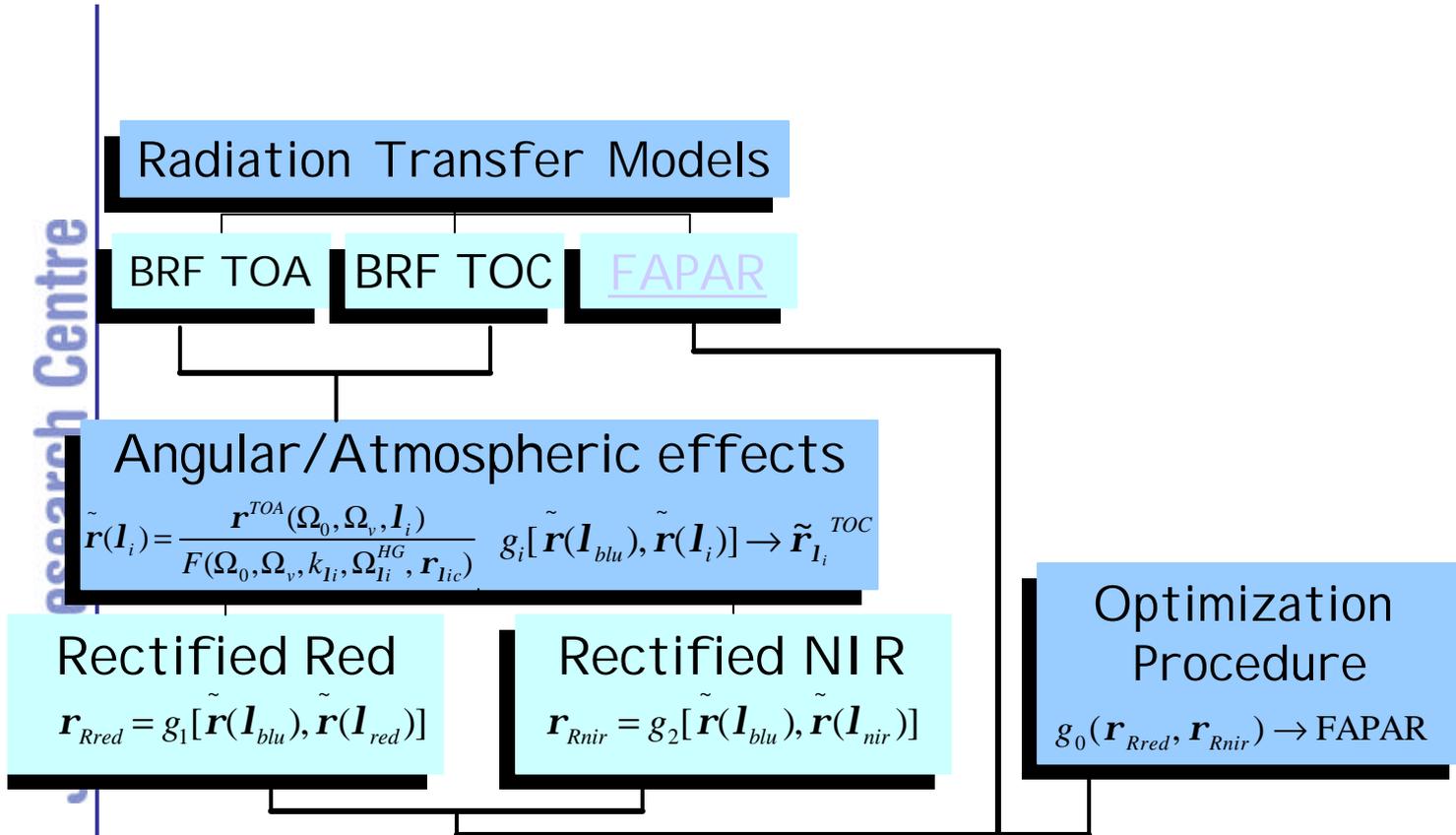


After





# Scheme for the algorithm optimization





## Mathematical Implementation (3)

- The index formula is generated on the basis of the rectified channel values,  $\rho_{Rred}$  and  $\rho_{Rnir}$ , estimated as follow:

$$\rho_{Rred} = g_1(\rho_{blu}, \rho_{red}) \quad \rho_{Rnir} = g_2(\rho_{blu}, \rho_{nir})$$

$$MGVI = g_0(\mathbf{r}_{Rred}, \mathbf{r}_{Rnir})$$



## Cost Functions (2)

- The coefficients of  $g_0$  are optimised so that:

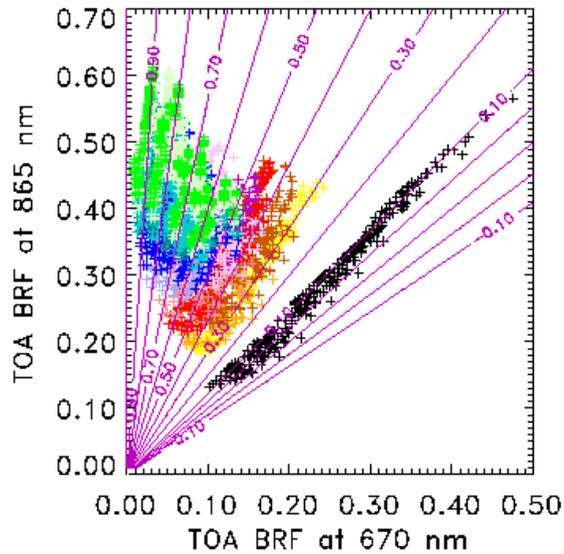
$$d_{g_0}^2 = \sum_V [g_0(\tau_{Rred}, \tau_{Rnir}) - FAPAR] \rightarrow 0$$

$$g_0(x, y) = \frac{l_{0,1}y - l_{0,2}x - l_{0,3}}{(l_{0,4} - x)^2 + (l_{0,5} - y)^2 + l_{0,6}}$$

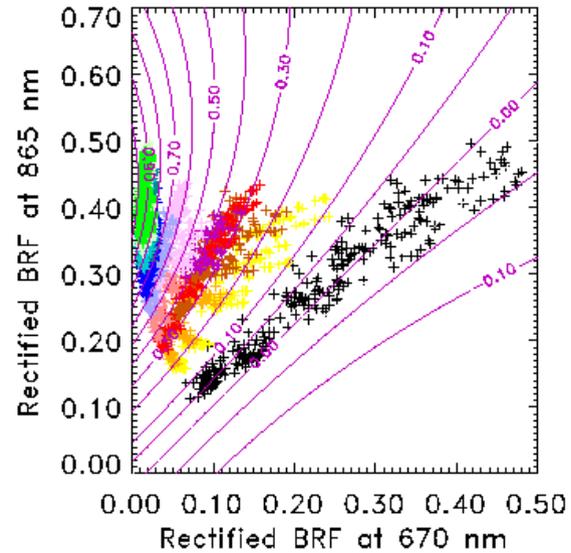


# MGVI Optimization

Before

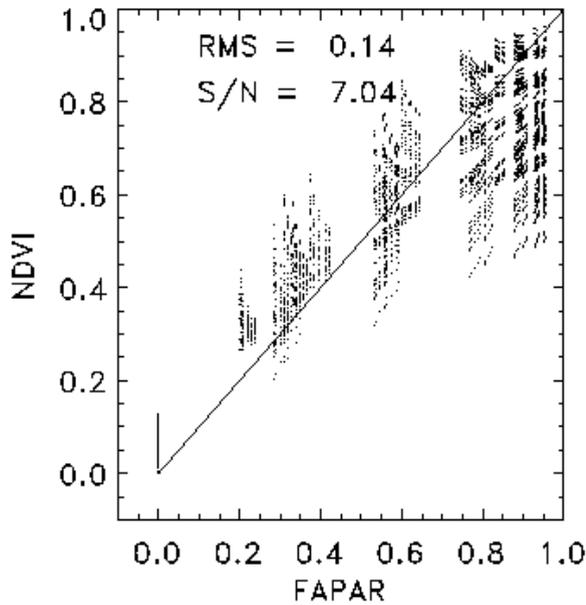


After

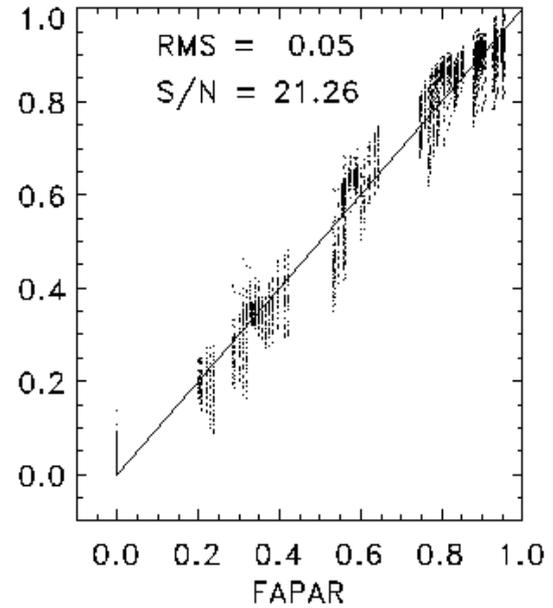




# RT model driven improvement for FAPAR

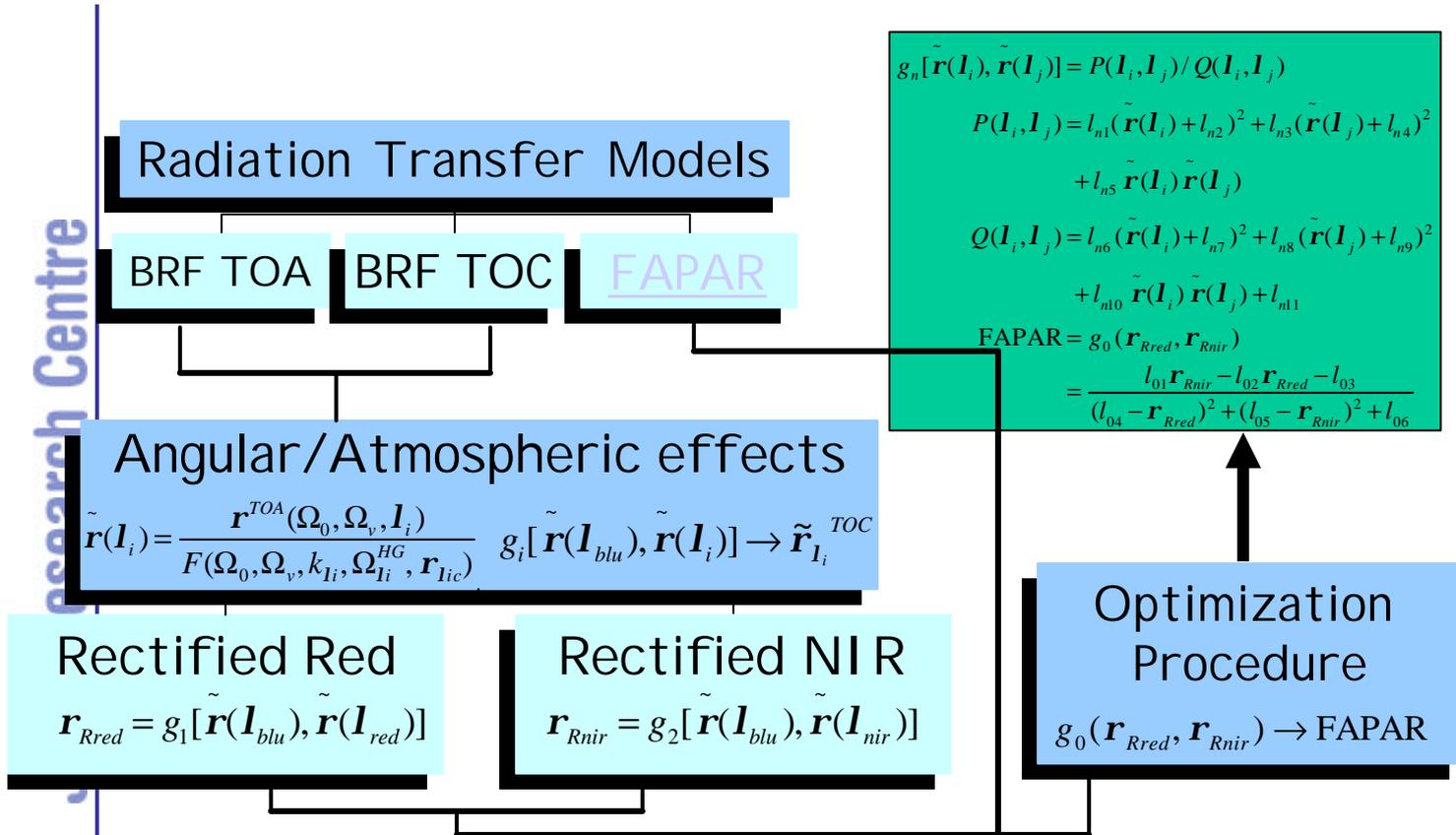


## MGVI





# Scheme for the algorithm optimization



$$g_n[\tilde{\mathbf{r}}(\mathbf{l}_i), \tilde{\mathbf{r}}(\mathbf{l}_j)] = P(\mathbf{l}_i, \mathbf{l}_j) / Q(\mathbf{l}_i, \mathbf{l}_j)$$

$$P(\mathbf{l}_i, \mathbf{l}_j) = l_{n1}(\tilde{\mathbf{r}}(\mathbf{l}_i) + l_{n2})^2 + l_{n3}(\tilde{\mathbf{r}}(\mathbf{l}_j) + l_{n4})^2 + l_{n5} \tilde{\mathbf{r}}(\mathbf{l}_i) \tilde{\mathbf{r}}(\mathbf{l}_j)$$

$$Q(\mathbf{l}_i, \mathbf{l}_j) = l_{n6}(\tilde{\mathbf{r}}(\mathbf{l}_i) + l_{n7})^2 + l_{n8}(\tilde{\mathbf{r}}(\mathbf{l}_j) + l_{n9})^2 + l_{n10} \tilde{\mathbf{r}}(\mathbf{l}_i) \tilde{\mathbf{r}}(\mathbf{l}_j) + l_{n11}$$

$$\text{FAPAR} = g_0(\mathbf{r}_{Rred}, \mathbf{r}_{Rnir}) = \frac{l_{01} \mathbf{r}_{Rnir} - l_{02} \mathbf{r}_{Rred} - l_{03}}{(l_{04} - \mathbf{r}_{Rred})^2 + (l_{05} - \mathbf{r}_{Rnir})^2 + l_{06}}$$

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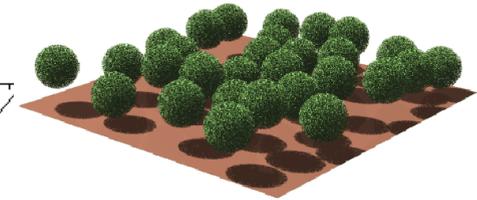


# Sensitivity to scenes heterogeneity

Case a



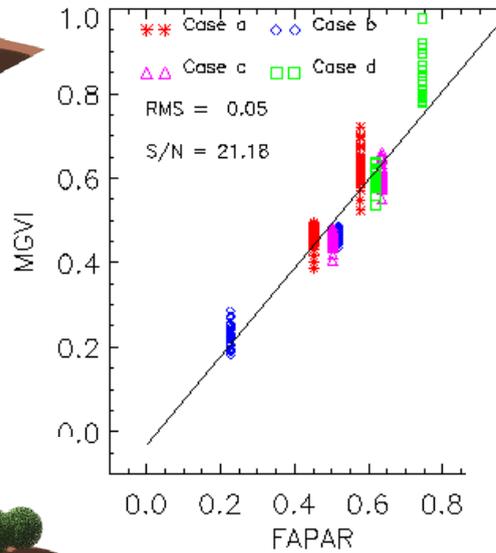
Case b



Case c



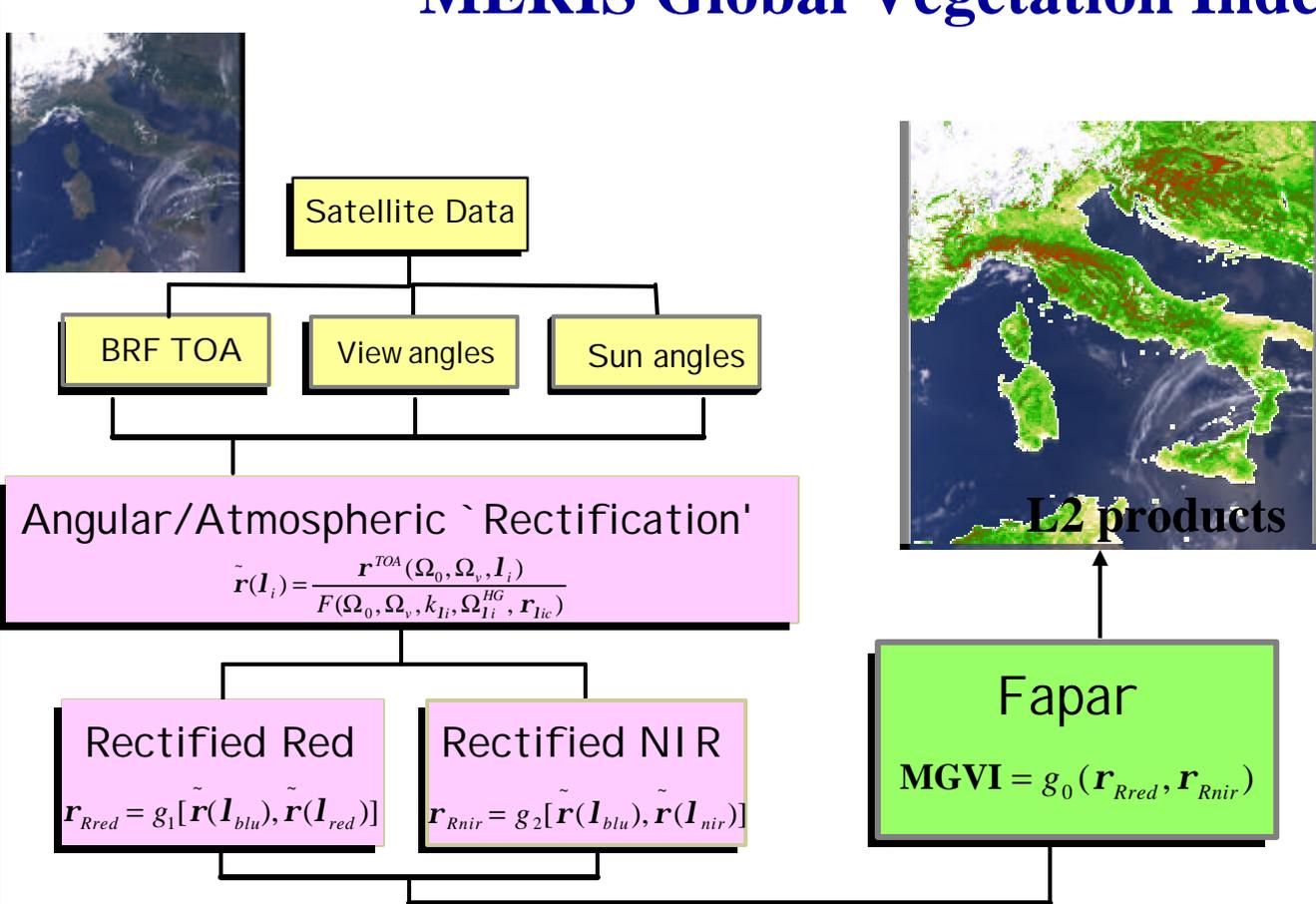
Case d





# MERIS Global Vegetation Index

Joint Research Centre





# Conclusions (1)

- Retrieval geophysical parameters is a coupled atmosphere-vegetation problem.
- Inverse methods depend on the space remote sensing data types (multi/mono spectral & multi/mono angular).
- Retrieval of geophysical parameters values are always associated to accuracies.
- MGVI (instantaneous) can be read in the LEVEL 2 land product (TOAVI) and represents FAPAR.



## Conclusions (2)

- Remote sensing products require a validation exercise through:
  - Ground measurements
  - Inter-comparison with other sensor derived products
- Temporal & spatial scale production also depend on specific applications (global or regional).