Wave spectra retrieval from complex ENVISAT Wave Mode data using a parametric inversion scheme

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Spaceborne synthetic aperture radar (SAR) is still the only instrument providing directional ocean wave information on a global and continuous basis. Different retrieval schemes for the derivation of two-dimensional ocean wave spectra $F_{\mathbf{k}}$ from SAR data have been developed (Krogstad, 1994; Hasselmann et al., 1996; Mastenbroek, 2000; Dowd, 2001; Johnson et al., 2003). The potential of the data for improving wave model forecasts has been demonstrated [Heimbach et. al, 2000].
More recent schemes are based on SAR cross spectra, which allow to resolve the directional wave propagation ambiguity present in conventional SAR image variance spectra (Engen, 1995). With the launch of the European satellite ENVISAT cross spectra have become available on anoperational basis. Actvities therefore exist at different weather centres to use the data for the assimilation of numerical ocean wave forecast models.
The cross spectra method is based on a special processing technique, where two SAR images (looks), which are separated by about half a second (Engen, 1995) are generated. By calculating the cross spectrum Φ_k of the two looks information can be obtained about the wave propagation direction. This information had to be taken from wave models in earlier retrieval schemes which were based on symmetric SAR image variance spectra.
A difficult problem in SAR wave retrieval schemes is the fact that SAR data contain information mainly about longer waves, whereas shorter waves in particular those propagating in the flight direction (azimuth) are strongly distorted or completely filtered out in many cases. In order to obtain a complete two-dimensional wave spectrum a retrieval scheme therefore has to blend SAR information and prior information in some consistent way. The **Par**tition **R**escaling and **S**hift **A**lgorithm (**PARSA**) scheme described in this paper is able to deal with this problem and has several additional features compared to the scheme presented in Hasselmann (1996):

- The scheme has the directional spreading of the different wave systems as an additional parameter.
- The algorithm is based on explicit models for the measurement error, errors in the forward model, and uncertainties in the prior wave spectrum.
- The scheme is based on a maximum a posteriori approach. The second iteration loop used in Hasselmann (1996), where the prior wave spectrum is adjusted and fed back into the optimal estimation problem is avoided. This approach has two advantages:

    - The sensitive cross assignment procedure used in Hasselmann (1996) is not required;
    - Based on the rigorous formulation as an optimal estimation problem it is possible to estimate the error covariance matrix of the retrieved parameters.

- The scheme makes use of the phase information contained in cross spectra to resolve ambiguities in wave propagation direction.
- The side condition $F_{\mathbf{k}} > 0$ is treated in a rigorous way.

The **PARSA** scheme is designed for the needs of global wave model assimilation, which is regarded as the primary application of the method. In this paper the basic concept of the method is explained. We will start with the basic retrieval approach and then describe the different statistical models for measurement errors and uncertainties in the wave model used in the inversion. Finally the performance of the method is illustrated based on examples and maps of retrieved parameters.

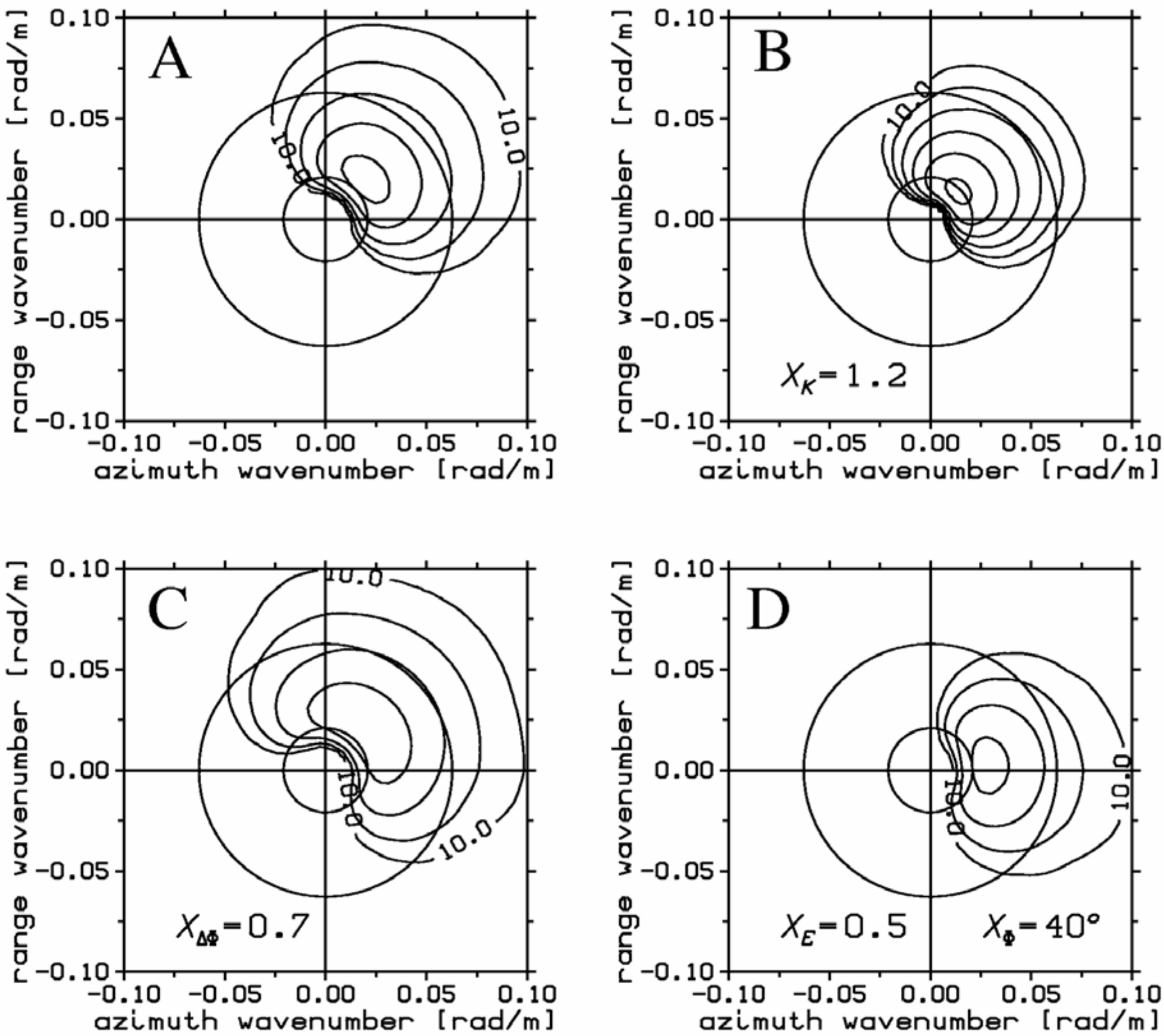

Figure 1: Transformations of wave systems used in the PARSA retrieval scheme. (A) Prior wave system with 250 m peak wavelength. (B-D) Transformed wave spectra with wavenumber rescaled (B), directional spreading changed (C) and simultaneous rotation and energy rescaling (D).

1. RETRIEVAL APPROACH

The **PARSA** scheme is based on a maximum a posteriori approach where the conditional probability of the retrieved wave spectrum given the SAR measurement and the prior information is maximised.

Using the Bayes theorem this probability can be written as

$$pdf(F_k,\alpha|\Phi_k)=\frac{pdf(\Phi_k\,|\,F_k,\alpha)\,pdf(F_k)\,pdf(\alpha)}{pdf(\Phi_k)} \qquad (1)$$

where the different factors have the following meanings:

$\sigma_{\alpha 1}$	$\sigma_{\alpha 2}$	$\sigma_{\nu RF}$	$\sigma_{\nu IF}$
0.2	250 m^2	0.1	0.1

Table 1: Parameters describing uncertainties in the SAR imaging model

α_E	α_k	σ_Φ	$\sigma_{\Delta\Phi}$
0.1	0.1	20°	0.1

Table 2: Parameters describing uncertainties in the prior wave

***pdf*(Φ_k|F_k, α)** is the conditional distribution of the measured cross spectrum Φ_k given an ocean wave spectrum F_k and a forward model, which contains a stochastic parameter vector α.
pdf(α) is the prior distribution of parameters in the forward model;
***pdf*(F_k)** is the prior distribution of the ocean wave spectrum F_k;
***pdf*(Φ_k)** is the (irrelevant) prior distribution of the cross spectrum.

Taking the logarithm of eq. 1 leads to a cost function minimisation problem, the exact form of which is determined by the error models described in the next section.

2. ERROR MODELS

2.1 Model for prior knowledge

The error model for the prior wave spectrum F_p^k is based on a partitioning scheme. For each subsystem B^i, i=1,...,n_p of F_p^k the confidence in the mean direction, the mean wavelength, the energy, and the directional spreading is quantified by defining respective stochastic models. With the partitions given on a polar grid (k, Φ), the corresponding processes B^i can be written as (compare Fig. 2):

$$\begin{aligned}\widetilde{B}^i(\phi,k) = X_E^i X_{\Delta\phi} X_k^i \cdot \\ B^i\left(\Phi_0^i + (\Phi - X_\Phi^i - \Phi_0^i) X_{\Delta\Phi}, X_k^i k\right)\end{aligned} \quad (2)$$

for i=1,...,n_p. The stochastic vector (X_E^i, X_k^i, X_Φ^i, $X_{\Delta\Phi}$) is assumed to be Gaussian with mean (1,1,0,1) and standard deviations as given in the lower right table 2.

3.1 Model Measurement Errors

The following model is used for deviations between the simulated and the observed cross spectrum Φ^{sim}_k due to errors in the SAR imaging model:

$$\overline{\Phi}_k = \alpha_1 \exp[-k_x^2 \alpha_2] \Phi_k^{sim} + \varepsilon_k^F \quad (3)$$

Here, k_x is the azimuth component of the wavenumber vector and α_1, α_2 and ε_k^F have the following meanings:
meanings:

- α_1 is a Gaussian distributed variable with unit mean and standard deviation $\sigma_{\alpha 1}$, which describes errors in the overall energy level of the spectrum;
- α_2 is a Gaussian distributed variable with zero mean and standard deviation $\sigma_{\alpha 2}$, which describes uncertainties in the cut-off wavelength of the forward model;
- ε^F is additive white Gaussian noise with independent real and imaginary part and zero mean. It is supposed to take into account errors in the fine scale structure of the spectrum.
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The standard deviations used here are given in the upper right table in Fig. 2, where $\sigma_{\mu RF}$, $\sigma_{\mu IF}$ denote the relative errors of the real and imaginary part of ε^F.

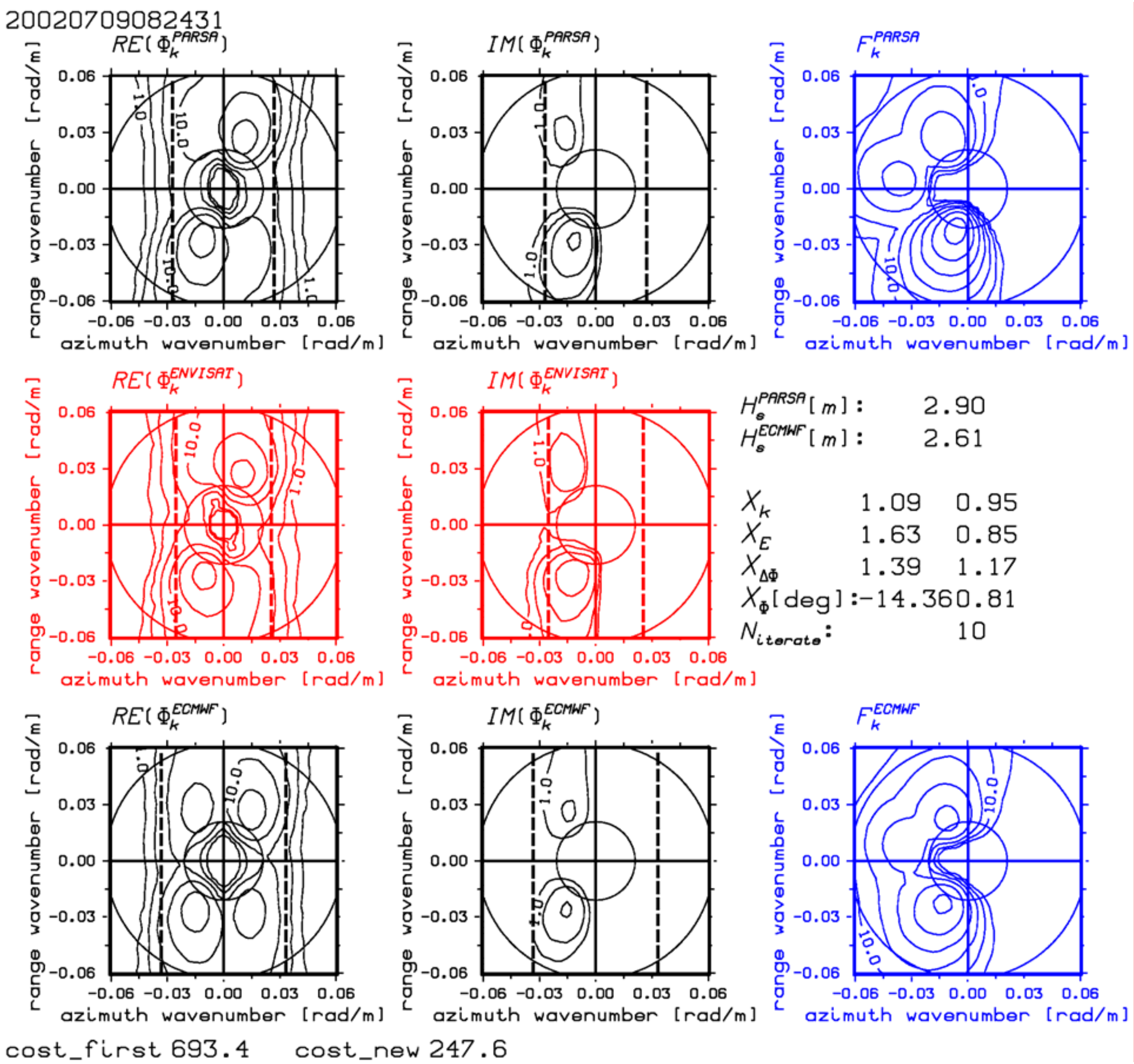

FIG. 3: PARSA **TRIEVAL FOR AN ENVISAT CROSS SPECTRUM ACQUIRED IN THE INDIAN OCEAN AT -64.81°N 130.7° W ON JUL 9, 2002, 8:24 UTC. THE SATELLITE HEADING IS 337.2°. THE DASHED VERTICAL LINES INDICATE THE AZIMUTHAL CUT-OFF WAVELENGTH (SEE TEXT FOR DETAILS).**

2.2 Numerical Inversion Procedure

From the mathematical point of view the **PARSA** method solves a minimisation problem with 4 n_p unknown parameters. The optimisation problem is solved on a polar grid using a Levenberg-Marquard (LM) (Rodgers, 1998} method, with an iteration scheme of the following form:

$$X^{n+1} = X^{a} + \left(C_X + \lambda^n I_N\right)^{-1} D_n^T S_\varepsilon^{-1} \left(\phi^{obs} - \phi^{sim} + D_n (X^n - X^a) + \lambda^n (X^n - X^a)\right) \quad (4)$$

Here, X^a, X^n are the prior and nth iterate parameter vectors, C_X is the covariance matrix

$$C_X = D_n^T S_\varepsilon^{-1} D_n + S_a^{-1} \quad (5)$$

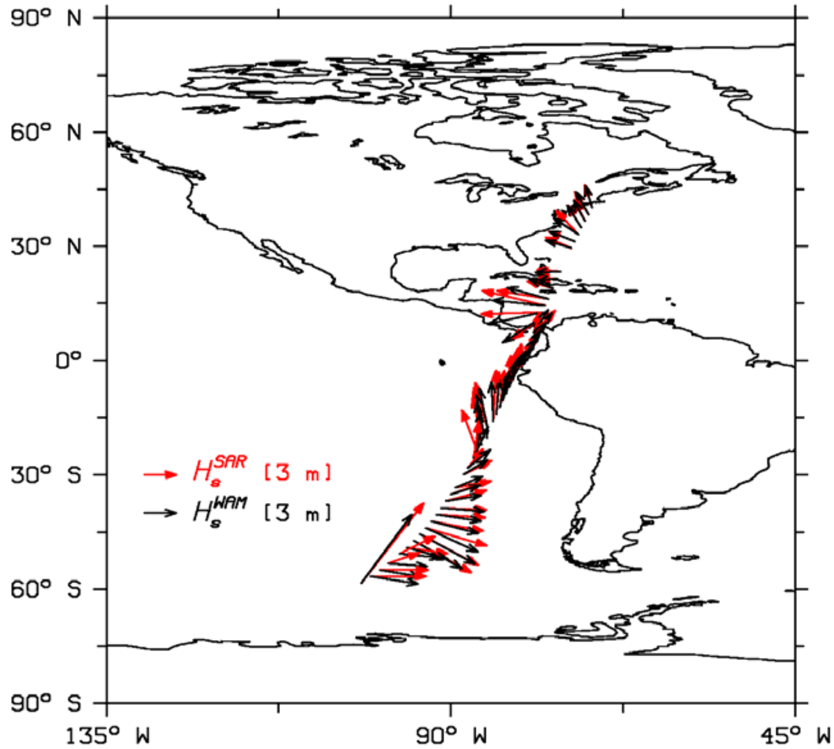

Figure 4: Mean wave direction and significant waveheight derived from ENVISAT ASAR wave mode data using the PARSA scheme (read arrows) and respective colocated ECMWF model spectra (black arrows) for one (descending) track acquired on July 9, 2002.

of X^a, D_n is the Jacobian matrix of the SAR imaging model, S_ε is the errors covariance matrix of the measured cross spectrum, I_N is the identity matrix, and λ^n is the relaxation parameter of the LM method.

3. PARSA inversion of ENVISAT ASAR data

Fig. 3 shows a **PARSA** inversion of an ENVISAT wave mode cross spectrum. The prior wave spectrum provided by the European centre for Medium-Range Weather Forecast (ECMWF) on the lower right has two wave systems propagating in opposite directions. The observed cross spectrum in the centre (red) only shows the shorter wave system propagating to upper left as indicated by the imaginary part (right). As can be seen in the inverted spectrum on the upper right the PARSA scheme amplifies the energy of this wave system, at the same time scaling down the opposite system. The cross spectra simulated from the prior and the inverted wave spectrum on the lower left and upper left (black) show that the consistency with the SAR observation is in fact improved by the **PARSA** scheme. The example is interesting as it demonstrates that the cross spectrum really contains information essential to resolve ambiguities, which can occur even if the overall shape of the spectrum is taken from a prior wave spectrum.

Fig. 4 shows a map with mean wave directions and significant wave heights derived from ENVISAT ASAR wave mode data (read arrows) and colocated ECMWF model spectra (black arrows) for one track in the Pacific acquired on July 9, 2002. One can see that the corrections applied by the PARSA scheme are smooth and consistent. The deviations in mean propagation directions are in most cases due to redistribution of energy among different subsystems (up to three occurred for this track), rather than rotations of single wave systems.

4. Conclusion and Outlook

An inversion scheme was presented which estimates two dimensional wave spectra from ENVISAT look cross spectra using prior information. The scheme is able to blend SAR information and wave model output in a consistent way based on rigorous errors models for the SAR measurement and the prior information, and is therefore ideally suited for wave model assimilation. The approach in particular allows to estimate the covariance matrix of the retrieved parameters. It was furthermore shown that the method makes use of the complex information contained in cross spectra although the overall shape of the spectrum is taken from a wave model.

The scheme is currently validated in close cooperation with UK Met office and the French Met office. The validation is performed in the framework of the ENVISAT Cal/Val activities in which a large data set of buoy co-locations is generated.

It is furthermore planned to extend the scheme for use with ENVISAT dual polarisation data, which will help to reduce a couple of uncertainties in the current SAR imaging model.

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