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The multiresolution approach in SAR interferometry

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Abstract

The high interest to derive digital elevation models from interferometric SAR data stimulated the research to optimally and numerically efficient solve the phase unwrapping problem. Several solutions have been proposed. We address the solution of the phase unwrapping stated as a least squares problem and its multiresolution solutions. The multiresolution algorithms are computationaly efficient implementations of the phase unwrapping solution, the wavelet implementation is systematic and allows to deal with the noise in the data.
Keywords: SAR Interferometry, multiresolution analysis, partial differential equations

Introduction

The paper is a short presentation and theoretical comparison of multigrid, finite element, and wavelet methods to solve the partial differential equation problem for applications in Interferometric SAR phase unwrapping.

The phase unwrapping: problem statement

The phase unwrapping is the key step in recovering the terrain elevations from Interferometric SAR data. The problem is to find an estimate of the phase values known the wrapped noisy phase observations. The problem is ill-posed and the solution requires regularization .

Several solutions have been proposed. We refer only the ones based on the minimization of the mean square error between the desired phase gradient and the observations of the wrapped phase gradient. The problem is equivalent to solve a Poisson equation [Ghiglia].

The multigrid algorithms

[Pritt] proposed as a solution for phase unwrapping the weighted least squares method implemented as a multigrid Gauss-Seidel relaxation. The multigrid algorithms are iteratively renewing the solution of the partial differential equation in finer grids using the results from a coarser grid. The multigrid algorithms relies on transforming, by transferring the problem to coarser grids, the low frequency components of the errors into high frequency components which can be removed by the Gauss-Seidel relaxation. The transfer to coarser grids is implemented trough a restriction operator, and the transfer to finer grids with a prolongation operator. These are scale operators similar to wavelet bases of functions but non-systematic mixing the hierarchical decomposition and the resolution steps.

[Fornaro] introduced a finite element method for the phase unwrapping method. The finite element method is computationally efficient in a multigrid implementation, but taking into account the weighting the efficiency is reduced.

The advantage of the multigrid algorithm is the fast convergence and the way to accommodate the weighted least square solution.

The wavelet solution

The wavelet method assumes to decompose the differential operators in a wavelet basis.

[Daubechies] introduced a parameterized family of orthonormal system of functions: the compactly supported wavelets. They are generated from a scaling function and its dual, the wavelet, by dilatations and translations. The elements of the system of functions have compact support and are continuous, the support of the basis functions, due to the rescaling, becomes smaller for larger scaling index. The coefficients of the expansion can be computed with an O(N) algorithm.

[Mallat] demonstrated the multiresolution representation of a given function. Using the scaling and wavelet functions one can represent a function in a system of coarser-finer scales. The Mallat transform consists of convolutions with the filters defining the scaling and wavelet function and downsampling.

[Wells] and [Glowinski] proposed to use the scaling functions at a given scale as finite elements.

[Beylkin] and [Glowinski] introduced a method to solve elliptic differential operators with Dirichlet boundary conditions in the wavelet system of coordinates, by constructing the Green function in O(N) operations. Once the Green function is obtained the solution reduces to a matrix-vector multiplication.

Comparison of the methods

The solution of the partial differential equation using wavelet transforms have several advantages. In the wavelet system of coordinates the differential equations with boundary conditions are characterized by diagonal preconditioner leading to operations with sparse matrices having the condition number O(1), resulting in O(N) algorithms. The condition number is very good, as consequence avoids instabilities, minimizes the errors, and speed up the convergence.

The orthogonality of the wavelet systems allows a systematic and simple mapping in between adjacent scales and also encapsulation of prior knowledge in the solution by disregarding certain wavelet coefficients.

However the wavelets systems are not so easy to compute as finite elements, but the transform is done only once, the number of further iterations compensate this drawback.

Both multigrid and wavelet are using hierarchical decompositions and resolution steps. The wavelet methods for solving the Poisson equation are similar to the multigrid methods, but are using more information: the orthogonal basis of the wavelet decomposition.

The difference of the two methods is in the utilization of the hierarchical decomposition: the multigrid methods mix the hierarchical decomposition and the resolution steps, while the wavelet based method are clearly separated.

Conclusions

The numerical solution of partial differential equation using wavelet decompositions is a new promising field. The emerged methods show a faster convergence, a better accuracy of the solutions and, important for our problem - the SAR interferometry - enable us to deal in a systematic way with the noise of the process.

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