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An efficient time-frequency hybrid method for Phase Unwrapping

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Abstract

We present a weighted least-squares solution for phase unwrapping based on the combination of a time-domain and a frequency-domain approach. The former is applied to estimate the original phase gradient only in the unreliable (weighted) areas. The latter, based on the First Green's identity algorithm, is used for robust integration of the overall phase gradient. Presented experiments on real and simulated data validate the proposed algorithm.

Keywords: SAR Interferometry, phase unwrapping

Introduction

Synthetic Aperture Radar Interferometry (IFSAR) is a technique for the generation of high resolution digital elevation models (DEMs) (Q. Lin *et al.*, 1994). This result is achieved by exploiting the phase difference (*unwrapped phase*) of two SAR images of the same area related to two slightly different look angles. Unfortunately, it is only possible to evaluate the restriction of this phase difference to the base interval (*wrapped phase*). Since the altimetry of the observed scene is related to the unwrapped phase it is necessary an operation, usually referred to as phase unwrapping, that allows to retrieve the original phase starting from the wrapped one.

Phase unwrapping (PhU) techniques are usually based on two steps:

1. estimation of the gradient of the unwrapped phase from wrapped data;
2. integration of the computed gradient.

First step is generally carried out by computing the gradient of the unwrapped phase taking the principal value of the gradient of the measured data (Fornaro *et al.*, 1996a). However, in *critical areas* (i.e. layover and large noisy regions), this evaluation is incorrect (Lanari *et al.*, 1996).

A "global integration" operation can be carried out on the estimated gradient in order to limit the propagation of the errors due to the presence of the *critical areas*. This result is achieved by minimizing the distance between the estimated gradient and the desired true gradient of the unwrapped phase (Ghiglia *et al.*, 1994). Equivalent results can be obtained by applying the First Green's identity as discussed in (Fornaro *et al.*, 1996a) and in (Fornaro *et al.*, 1996c). This global integration operation can be implemented in the two-dimensional Fourier domain and achieve high computational efficiency by using Fast Fourier Transform (FFT) codes.

However, in the presence of large noisy zones or very critical layover situations even the use of global integration procedures can cause excessive errors. Therefore these critical areas must be excluded by introducing weighting functions (Ghiglia *et al.*, 1994).

This option needs iterations, thus increasing the required number of FFTs (Ghiglia *et al.*, 1994) if compared to the unweighted case. This shortcoming can be alleviated by implementing the iterative solutions via the Finite Difference (FD) method (Pritt, 1995) or the Finite Elements Method (FEM) (Fornaro *et al.*, 1996b) directly in time domain.

We present in this paper a new method for efficient weighted phase unwrapping based on the combination of a time-domain and frequency-domain approach.

The Algorithm

Iterative time-domain unweighted PhU procedures carry out the unwrapping operation by solving a sistem of linear equations. These approaches can be easly extended to the weighted case because this option simply requires to modify and/or exclude some equations (Ghiglia *et al.*, 1994) and (Fornaro *et al.*, 1996b).

Non-iterative frequency-domain algorithms are, for the unweighted case, more efficient than the iterative procedures because unwrapping operation is carried out via deconvolution implemented by using FFTs codes.

However the extention to the weighted case is, for the frequency-domain approaches, not trivial. In fact these algorithms need the knowledge of the unwrapped phase gradients everywhere, including the unreliable zones excluded by weights. This problem is solved by estimating the missing gradient components via iterations (on the overall domain), thus reducing the computational performance of the procedure (Ghiglia *et al.*, 1994).

We present a new PhU method particularly efficient if the weighted areas are relatively small and sufficiently sparse. The proposed algorithm is hybrid in the sense that combines an iterative time-domain approach with a noniterative frequency-domain one. In particular, we first apply the time-domain weighted FEM algorithm (Fornaro *et al.*, 1996b) to estimate the gradients in the unreliable regions (by solving the phase unwrapping problem only in a small area around the weighted regions) and then carry out the overall integration via First Green's identity based method (Fornaro *et al.*, 1996a).

A block scheme of the algorithm is sketched in Fig. 1.

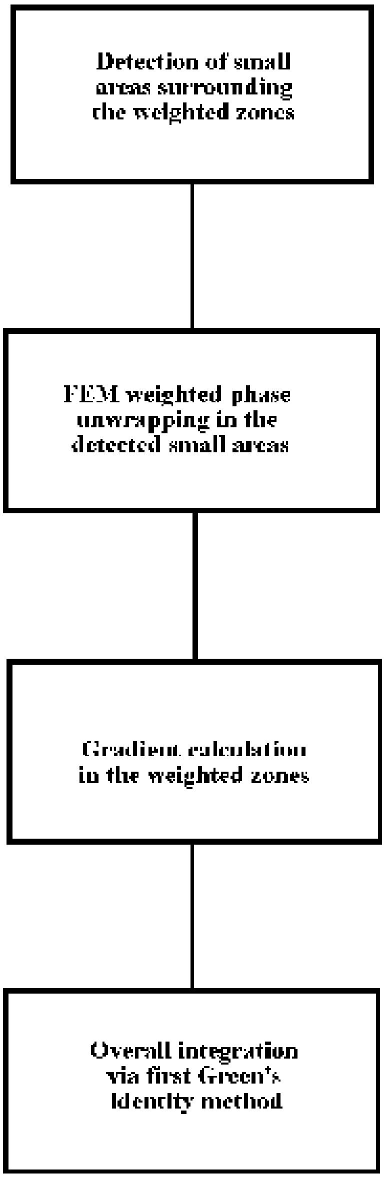

Figure 1: Block scheme of the proposed method

The first step is represented by the individuation of the small areas surrounding the weighted regions. A pictorial example of such localization is shown in Fig. 2 where the continuous lines represent the weighted regions.

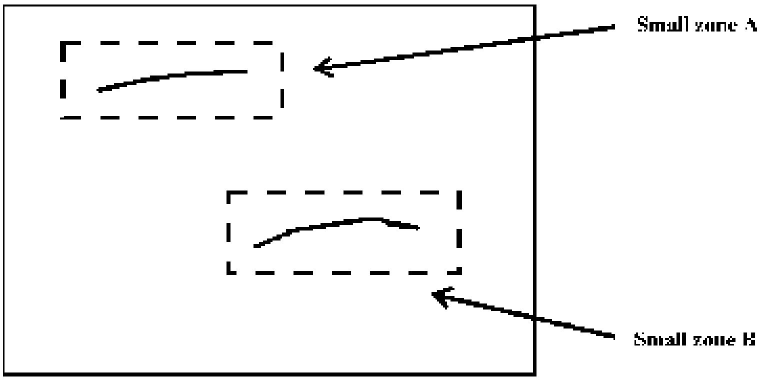

Figure 2: An example of individuation of small zones

Subsequently, a weighted unwrapping (via FEM) operation is carried out in the detected zones (Fornaro *et al.*, 1996b) and then the phase gradient components in the weighted areas are computed. Note that, since we need to compute only gradients, the different constants resulting from the unwrapping procedures applied in the different zones do not play any role.

Once we have evaluated the unknown gradient components in weighted areas, the overall unwrapped phase pattern is computed by applying the direct method discussed in (Fornaro *et al.*, 1996a).

As a final remark we want to stress that the algorithm performance is strongly dependent on two factors:

1. the percentage of the weighted zones with respect to the overall area must be small;
2. a proper selection of the shape of the zones surrounding the weighted areas is necessary in order to avoid an excessive increase of their dimensions.

With respect to the second point we underlines the capability of the FEM to be applied to non-rectangular data grids.

Experimental Results

In order to validate the proposed method, we present in this section experimental results carried out on simulated and real data.

The simulated phase pattern is shown in Figure 3. It represents a piramid of 128 by 128 pixels, with two ledges.

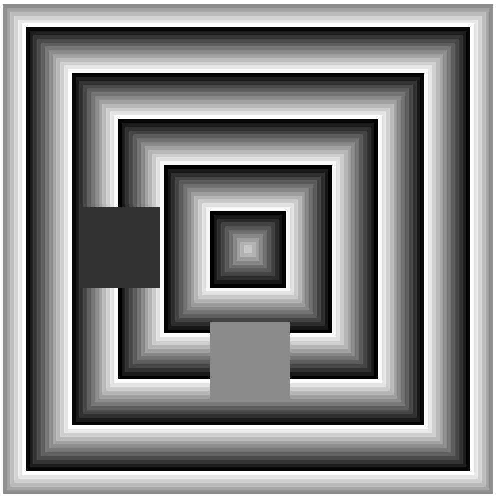

Figure 3: Interferogram of the simulated phase pattern

The weighting function used to unwrap the phase pattern of Figure 3 is shown in Figure 4 (weighted regions are in black).

Figure 4: Weighting function for the simulated phase pattern

We present in Figure 5 a selected small zone wherein the gradient estimation operation via FEM procedure is carried out.

Figure 5: Small zone processed by FEM algorithm

The overall unwrapped phase is shown in Figure 6. The reconstruction operation required about 1:30 minutes of CPU on a IBM Risc machine.

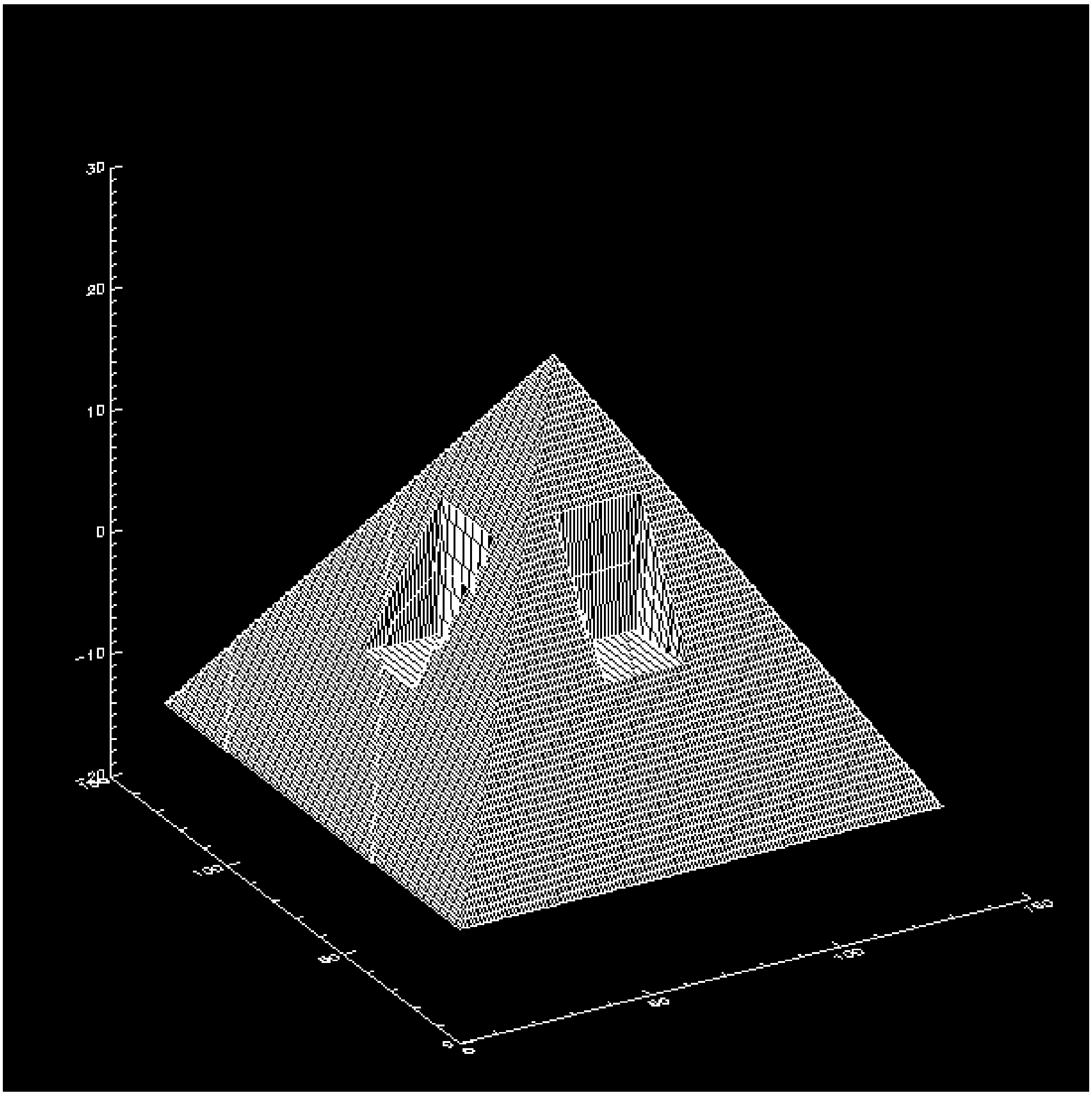

Figure 6: Unwrapped phase pattern

Let us consider the real interferogram relative to the Mt. Etna (Sicilia, Italia) test site (see Figure 7) illuminated by the sensors ERS-1 and ERS-2. Data dimensions are: 800 by 820 pixels.

Figure 7: Interferogram of Mt. Etna ERS1-ERS2 tandem mission

Figure 8 shows the wheighting function applied for the unwrapping operation.

Figure 8: Weighting function for the interferogram of Figure 7

Figure 9 is dedicated to the achieved uwwrapped phase pattern. This result was obtained in about 35 minutes of CPU.

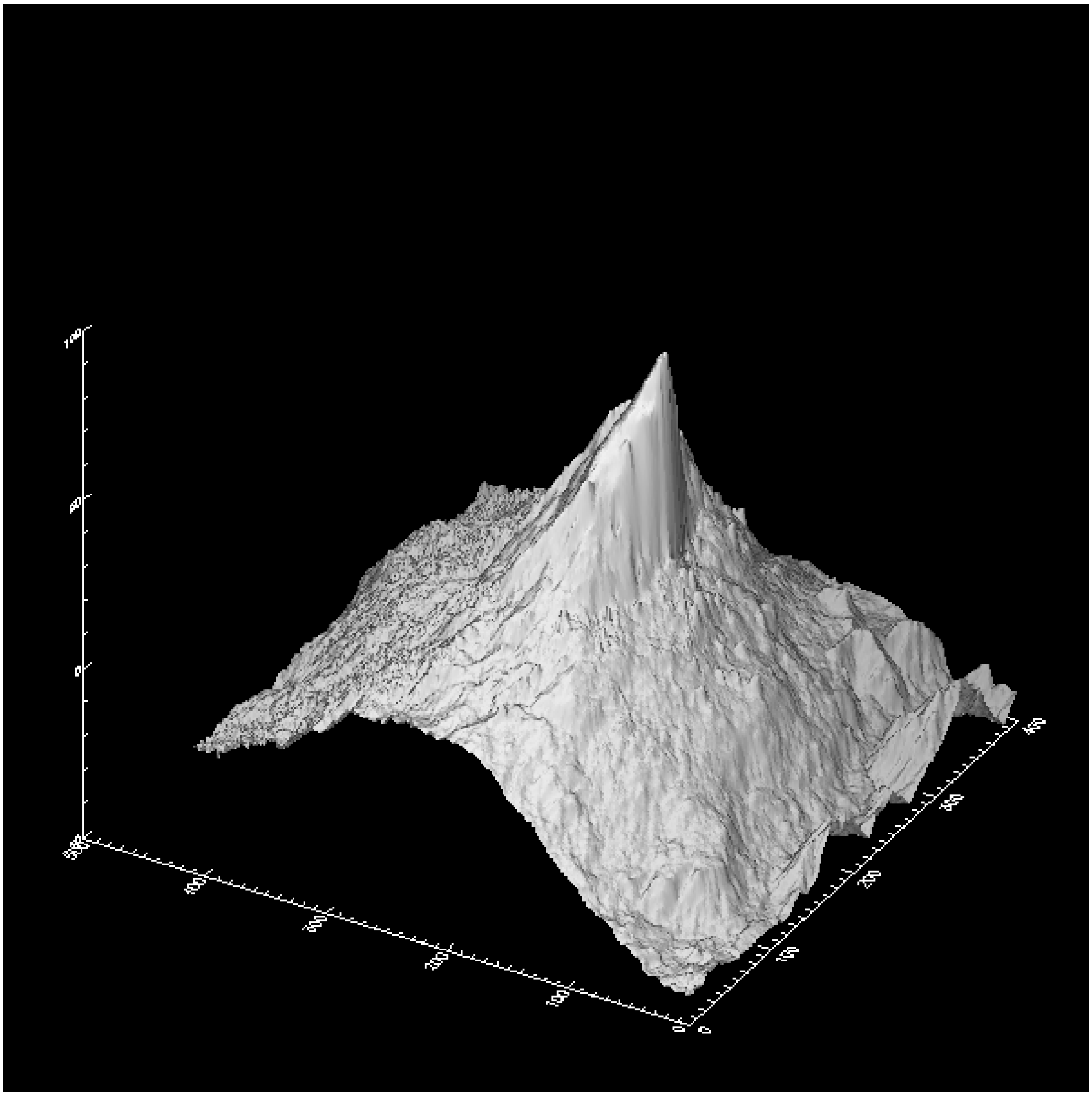

Figure 9: 3-D view of the unwrapped phase

Conclusions

A new method for weighted phase unwrapping, particularly efficient in the case of relatively small and sparse weighted areas, has been presented. It combines a time-domain FEM appproach used to estimate the unknown gradient components in the unrealiable (weighted) areas to a frequency-domain algorithm for robust integration of the overall phase gradient.

A number of experiments have been presented in order to validate the proposed method.

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