FRINGE 96

Presentation of an improved Phase Unwrapping Algorithm based on Kalman filters combined with local slope estimation

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Abstract

The paper presents a phase unwrapping algorithm based on an Extended Kalman filter. The Kalman filter exploits a so called „Basic - Slope Model" enabling the filter to incorporate additional local slope information obtained from the sample frequency spectrum of the interferogram by a local slope estimator. The local slope information is then optimally fused with the information directly obtained from real and imaginary part of the interferogram. The paper outlines the principle operation of the phase unwrapping algorithm and explains the cooperation of the Extended Kalman filter with the local slope estimator. At last the efficiency of this phase unwrapping algorithm will be shown by simulations and real InSAR images.

Keywords: Phase unwrapping techniques, phase slope estimation, SAR - interferometry

Introduction

The main problem in calculating digital terrain elevation maps from a SAR interferogram is the unwrapping of the phases. The "measured" phases, calculated directly with the arctan-function from the complex interferogram, are all mapped into the same „baseband" interval (e.g. -,), while any absolute phase offset (an integer multiple of 2) is lost. The searched unambiguous "height" phase, which is a geometrical function of the height, must be generated from the measured phase by phase unwrapping.

In the year 1994 we firstly demonstrated the basic possibility of doing phase unwrapping with Extended Kalman filters. Since then we are working on the improvement of this kind of phase unwrapping algorithms. Based on a new optimised model we were able to remarkably improve the performance of the Kalman filter. The main idea of the new model is to apply a "Basic - Slope - Model", incorporating information which is obtained with an algorithm we would call "Local - Slope - Estimator". With this model, being inexact and partly incorrect, the Kalman filter 'fuses' two kinds of information - the local slope information obtained from a local slope estimator - and the information directly gained from exploiting inphase and quadrature components of the complex interferogram.

The phase unwrapping algorithm

The concept of our phase unwrapping algorithm is shown in figure 1. Starting points are the two coregistered SAR images from which the interferogram, a coherence map and the measured (interferometric) phase is calculated.

In the next step the algorithm computes the necessary parameters for the Kalman filter. The measurement noise variance is calculated from the data of the interferometric amplitude and the coherence map. The measurement noise variance is needed in the observation model of the Kalman filter. Running in parallel the Local - Slope Estimation, which will be described later, calculates the local phase slopes as well as their error variances. With these parameters a very robust and efficient state space model can be built.

In the next step the two dimensional extended linearized Kalman filter optimally combines the information from the local slope estimation given in the state space model, with the interferometric phase observations. The principle operation of a two dimensional Kalman filter for phase unwrapping has been published in (Krämer et. al.,1996a), (Loffeld et. al.,1996) and (Loffeld et. al.,1994) and will not be described in this paper. The state space model will be outlined in the next chapter.

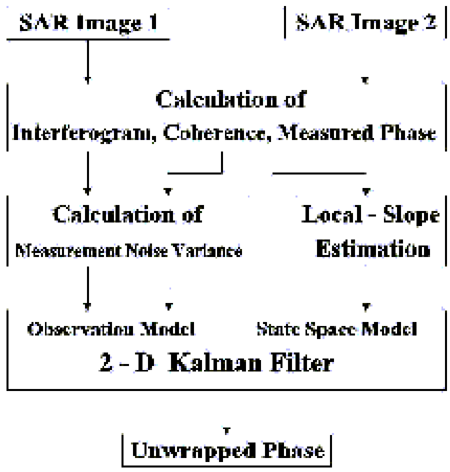


Figure 1: Concept of the phase unwrapping algorithm

The state space model utilised by the Kalman Filter

Due to the Local - Slope Estimation the state vector of the state space model can be simplified to only one dimension and we get following state space model:

$$x(n,m) = \frac{1}{2}\left[x(n-1,m) + s_h(n-1,m) + x(n,m-1) + s_v(n,m-1)\right]$$

where x(n,m) is the searched unwrapped phase and s_h(n,m) and s_v(n,m) are the local phase slopes in horizontal and vertical direction calculated by the local slope estimator. The phase x(n,m) is calculated as the mean of the preceding phase value plus the local phase slope in horizontal direction and the preceding phase value plus the phase slope in vertical direction. (See figure 2)

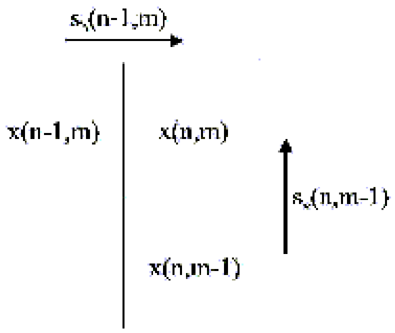


Figure 2: Calculation of a new phase value x(n,m)

The Local - Slope Estimator

The local slope estimation

In the following the local slope estimator will be described in the one dimensional case. The two dimensional case is straight forward.

The searched unwrapped phase can be written as follows:

$$\varphi(t) = \varphi(t_0) + \int_{t_0}^{t} \dot{\varphi}(\tau)d\tau = \varphi(t_0) + 2\pi \cdot \int_{t_0}^{t} (f_0 + f_i(\tau))d\tau$$
$$= \varphi(t_0) + \underbrace{2\pi \cdot f_0 \cdot (t - t_0)}_{\text{mean phase variation}} + \underbrace{2\pi \cdot \int_{t_0}^{t} f_i(\tau)d\tau}_{\text{dynamic phase variation}}$$

where

$$\begin{aligned} &\varphi(t): \text{ momentary phase} \\ &\dot{\varphi}(t): \text{ phase derivative} \\ &f_0 \quad : \text{mean instantaneous frequency in some interval } [t_0, t_x) \\ &f_i(\tau): \text{ time varying instantaneous frequency} \end{aligned}$$

In the discrete case with t=(k+1)T and t_0=kT we obtain

$$\varphi(k+1) = \varphi(k) + \underbrace{2\pi \cdot f_0 \cdot T}_{\text{mean phase variation}} + w(k)$$

where $w(k) = \underbrace{2\pi \cdot \int_{t_0}^{t} f_i(\tau)d\tau}_{\text{dynamic phase variation}}$ is the unknown phase variation.

As we can see the phase variation can be decomposed into two parts the mean and dynamic phase variation. The goal of the local slope estimator is to calculate the mean variation of the phase, respectively the frequency f_0, and also the variance of the dynamic phase variation $Ew(k)^2$.

The estimation of the unknown mean phase variation:

The complex interferogram can be written as:

$$
\begin{aligned}
y(t) &= a(t)\cdot e^{j\varphi(t)} + n(t) \\
&= a(t)\cdot \exp j\left(\varphi(t_0) + \underbrace{2\pi\cdot f_0\cdot(t-t_0)}_{\text{mean phase variation}} + \underbrace{2\pi\cdot\int_{t_0}^{t} f_i(\tau)d\tau}_{\text{dynamic phase variation}}\right) + n(t) \\
&= a(t)\cdot e^{j\bar{\varphi}(t_0)}\cdot \underbrace{\exp j\left[2\pi\cdot\int_{t_0}^{t} f_i(\tau)d\tau\right]}_{\text{dynamic phase variation}}\cdot \exp\left[j2\pi\cdot f_0\cdot t\right] + n(t) \\
&= \underbrace{s(t)}_{\substack{\text{phase and amplitude modulated signal}\\ \text{yielding spectral broadening}}} \cdot \underbrace{\exp\left[j2\pi\cdot f_0\cdot t\right]}_{\text{complex harmonic phasor yielding a frequency shift}} + n(t)
\end{aligned}
$$

We notice that the mean phase variation $2\pi f_0 T$ can be observed as a spectral shift f_0 in the interferogram. The complex autocorrelation can be written as:

$$
\begin{aligned}
\varphi_{yy}(\tau) &= E\{y(t)\cdot y(t+\tau)\} \\
&= E\left\{\left[s(t)\cdot e^{j2\pi f_0 t} + n(t)\right]\cdot\left[s(t+\tau)^{*}\cdot e^{-j2\pi f_0(t+\tau)} + n(t+\tau)^{*}\right]\right\} \\
&= E\left\{s(t)\cdot s(t+\tau)\cdot e^{j2\pi f_0\tau}\right\} + \varphi_{nn}(\tau) \\
&= \varphi_{ss}(\tau)\cdot e^{j2\pi f_0\tau} + \varphi_{nn}(\tau)
\end{aligned}
$$

With this equation the power spectral density is:

$$
\begin{aligned}
\Phi_{yy}(f) &= \Phi_{ss}(f) * \delta(f-f_0) + \Phi_{nn}(f) \\
&= \Phi_{ss}(f-f_0) + \Phi_{nn}(f)
\end{aligned}
$$

If the power spectral density is unimodal and approximately symmetric around the mode, then

$$
\Phi_{yy}(f)\Big|_{f=f_0} \approx \max\left\{\Phi_{yy}(f)\right\}
$$

and the spectral shift f_0 can be estimated with the relation

$$
\hat{f}_0 = \arg\max\left\{\Phi_{yy}(f)\right\}
$$

which means, that the spectral shift can be found by seeking the spectral mode of the power spectral density.

Estimating of the variance of the dynamic phase variation

The variance $E\{w(k)^2\}$, where w(k) denotes the difference between nominal (mean) phase variation and total phase variation, is identical with the driving noise covariance Q(k) which is needed by the Kalman filter.

The variance ${}_{f0}^{2}(k)$ of the variation between the estimated mean slope variation and the true mean slope variation can be obtained from the spectral bandwidth of the interferogram, by calculating the squared spectral bandwidth as the second central moment

$$
\begin{aligned}
\hat{E}\{f_0(k)\} &= \int_{-\infty}^{\infty} f\cdot\tilde{\Phi}yy(f,k)df \\
\hat{E}\{f_0^{2}(k)\} &= \int_{-\infty}^{\infty} f^{2}\cdot\tilde{\Phi}yy(f,k)df \\
&\text{and} \\
\sigma_{f0}^{2}(k) &= E\{f_0^{2}(k)\} - E\{f_0(k)\}^{2}
\end{aligned}
$$

where

$$
\tilde{\Phi}yy(f,k) = \frac{\Phi_{yy}(f,k)}{\int_{f=-\infty}^{\infty}\Phi_{yy}(f,k)df}
$$

is the normalised power spectral density from a section of y(k) bounded by the interval [k-N/2,k+N/2].

We are now able to calculate the driving noise variance. Starting with

$$\sigma_{f0}^2(k) = E\left\{\left[f_0(k) - \hat{f}_0(k)\right]^2\right\} = E\left\{\left[\frac{1}{N+1}\sum_{i=k-\frac{N}{2}}^{k+\frac{N}{2}} f(i) - \hat{f}_0(k)\right]^2\right\}$$

where $\hat{f}_0(k)$ is the estimated mean frequency in the interval [k-N/2,k+N/2] we get under the assumption of $E\{f(i)\cdot f(j)\} = \sigma_f^2(i)\cdot\delta(i,j) + E\{f(i)\}\cdot E\{f(j)\}$ the following solution for the error variance of the spectral shift:

$$\sigma_{f0}^2(k) = \frac{\overline{\sigma_f^2}(k)}{N+1}$$

where $\overline{\sigma_f^2}(k)$ is the ensemble average of all individual variances within the interval [k-N/2,k+N/2]. Finally we get the desired driving noise variance:

$$\begin{aligned} Q(k) &= (2\pi T)^2 \cdot \overline{\sigma_f^2}(k) \\ &= (2\pi T)^2 \cdot (N+1)\cdot \sigma_{f0}^2(k) \end{aligned}$$

Results

The capability of the filter will be shown by a fractally simulated phase image and an ERS1/2 scene from Egypt.

We will begin with the fractally simulated phase image, which is shown in figure 3. Starting with this phase a measured phase is generated by superimposing white Gaussian noise onto the complex image and wrapping the result. The measured phase, which we got for a signal to noise ratio of -7.2dB corresponding to a coherence value of 0.4, is shown in figure 4. The result of the phase unwrapping is depicted in figure 5. If we rewrap this result again (figure 6), we can compare the result with the measured phase. We see that the noise has been cancelled completely and that neither additional fringelines occur nor any fringelines are missing, which is an important requirement for error free phase unwrapping.

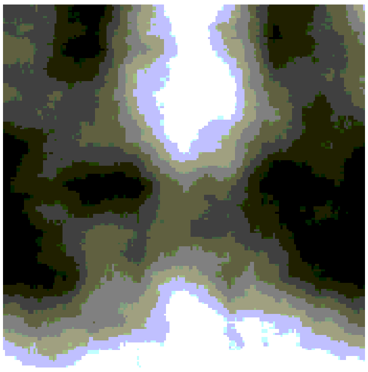
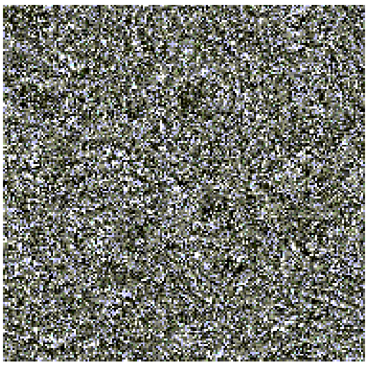



Figure 3: Original phase Figure 4: Measured phase

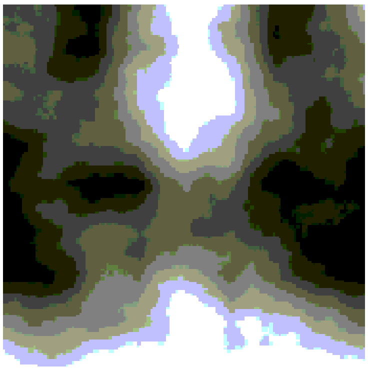



Figure 5: Unwrapped phase Figure 6: Unwrapped phase

In the following pictures we see the phase unwrapping result of a real ERS1/2 interferogram of a part of Egypt. Figure 7 shows the measured phase and the images 8 and 9 the coherence of the interferogram. As we can see there are large regions of very low coherences in that interferogram.

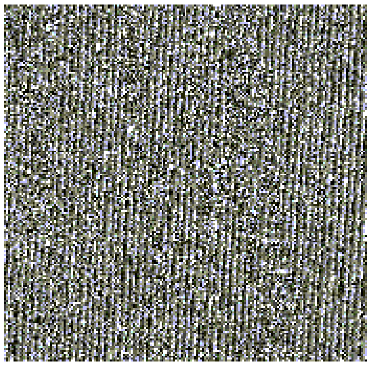


Figure 7: Measured phase

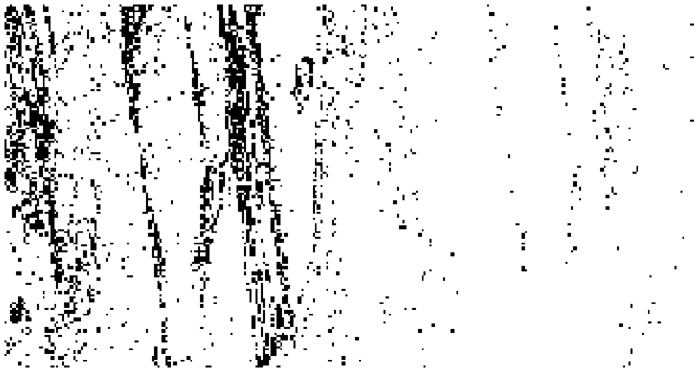
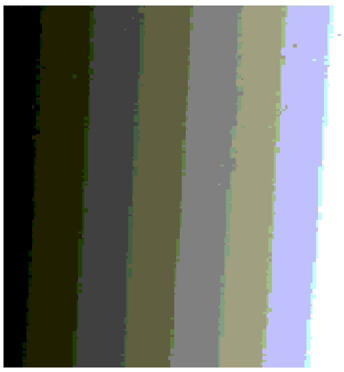


Figure 8: Places with coherence Figure 9: Places with coherence lower than 0.4 lower than 0.1

Figure 10 presents the result we got from the phase unwrapping algorithm. To examine the errors we have rewrapped this result again (fig. 11). We see that some little error propagations occurr, but only in very large regions of low coherence and even there the errors do not always occur. As we can see this kind of phase unwrapping algorithm works very well even there are regions of coherences near zero if this regions are not to large.



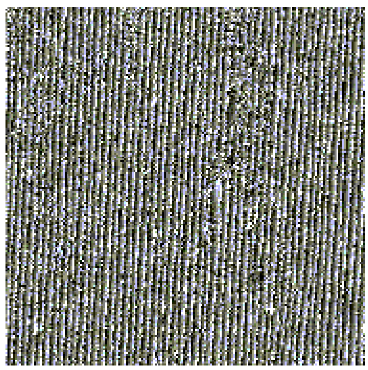


Figure 10: Unwrapped phase Figure 11: Unwrapped phase

Conclusions

A method to calculate local slope variations has been presented. The results of this local slope estimation was used to improve the state space model of a phase unwrapping algorithm based on an extended Kalman filter. The results show that this combination yields a very robust phase unwrapping algorithm which works down to a coherence value of 0.4 without error propagation, but also it is able to cross limited regions of coherence down to zero, if these regions are not too large.

Further work will be concentrated to improve this phase unwrapper in a way, that should the occasion of a error propagation arise, this error propagation should be limited on a small area in the image.

The work herein has been motivated and funded by the DARA (project „Rapid", grand number 50 EE 9431) which is greatly appreciated.

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