Product Validation Report

GOCE+ GeoExplore

Document: RP-GOCE+-DNT-05
Issue: 2.1
Date: 10 December 2014
### Document Information Sheet

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<th>Product Validation Report</th>
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<td><strong>Author</strong></td>
<td><strong>Institute</strong></td>
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<td>DGFI, NGU, TNO</td>
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### Document Change Record

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<td>Updated issue for Final Meeting October 2014</td>
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<td>Updated after Final Meeting</td>
<td>Flowchart added in 5.1 Grids at 255 and 225 compared (Figure 5-38)</td>
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1 Introduction
1.1 Purpose
The purpose of this document is to report on the validation of the algorithms that were newly implemented in the GOCE+ GeoExplore study. In addition, the error and validation analysis of the derived products – gravity gradients, gravity anomalies – are qualitatively and quantitatively described. The validation of geophysical models and products is described in the Impact Assessment Report.
1.2 Applicability
This document is part of the List of Deliverables [AD-1] and is to be delivered in draft at MTR and in final form at FR [RD-1].

1.3 Definitions
The term “Agency” is used to indicate the European Space Agency.

The term “Study Team” is used to indicate the persons of the three institutes (DGFI, NGU and TNO) that carry out the STSE – GOCE+ Theme 2 study.

2 Applicable and reference documents

2.1 Applicable documents

2.2 Reference documents
[RD-1] PR-GOCE-DGFI-10/02: Tender, Heterogeneous gravity data combination for Earth interior and geophysical exploration research (Theme 2), Issue 1.0, 14 July 2010


[RD-11]TN-GOCE+DNT-10, LNOF to MRF gradient rotation, Issue 1.1, 08 February 2013

3 Background

GOCE data may improve the understanding and modelling of the Earth’s interior and its dynamic processes, contributing to gain new insights into the geodynamics associated with the lithosphere, mantle composition and rheology, uplift and subduction processes. However, to achieve this challenging target, GOCE should be used in combination with additional data sources: e.g. magnetic, gravity and seismology in situ, airborne and satellite data sets.

The overall objective of the study is to combine GOCE gravity gradients with heterogeneous other satellite gravity information to arrive at a combined set of gravity gradients complementing (near)-surface data sets spanning all together scales from global down to 5 km useful for various geophysical applications and demonstrate their utility to complement additional data sources (e.g., magnetic, seismic) to enhance geophysical modelling and exploration.
4 Validation of algorithms

The purpose of this chapter is to validate the algorithms that were updated or newly implemented for this study. The validation of the products is done in the next chapter.

4.1 Validation of coordinate transformations: From geodetic longitude and latitude to Northing and Easting

In Section 5.1 of the Algorithm Theoretical Basis Document (ATBD) the conversion from geodetic latitude and longitude to Northing and Easting and vice versa are given. The accuracy of the algorithms can be assessed by starting with a set of coordinates and performing the forward and inverse conversions. In closed loop Northing and Easting were computed. That is, Northings and Eastings from NEA margin were used to compute latitude and longitude using the ATBD equations From Northing and Easting to geodetic longitude and latitude. Next these latitudes and longitudes were used to compute Northings and Eastings using the equations in From geodetic longitude and latitude to Northing and Easting. The differences between the Northings and Eastings from the closed-loop are shown in Figure 4-1. The differences are very small (< 1 μm). A similar test was done, but now starting with latitude and longitude, conversion to Northing and Easting and converting back. The differences between start values and computed values are shown in Figure 4-2. Again the differences are very small (< 10^{-11} degree).

![Figure 4-1: Closed-loop simulation using forward and inverse projection: error in m in Easting and Northing](image-url)
Figure 4-2: Closed-loop simulation using forward and inverse projection: difference in longitude and latitude

Another way to validate the implemented algorithms is to compare the numbers after conversion with those from independent software. Geodetic latitude and longitude as well as Northing and Easting are given in the IGMAS+ output files. Using these latitudes and longitudes one can compute Northing and Easting with the ATBD equations. Figure 4-3 shows the differences between Easting computed in this way and the Easting from IGMAS+, and Figure 4-4 shows the differences between Northings. In the area around the central meridian the differences are small, and towards the edges the differences increase.

Figure 4-3: Differences in Easting between coordinates from IGMAS+ and projection given in the ATBD in m. Right colour scale saturated to ±1 m
The other way around, one can computed latitude and longitude with the ATBD equations using the IGMAS+ Northing and Easting as input and these coordinates can be compared with the ones provided in the IGMAS+ output. The differences are shown in Figure 4-5 in longitude and latitude. The differences in latitude and longitude are small in the central area. Towards the edges the differences become larger.

Figure 4-5: Differences in longitude and latitude between IGMAS+ and projection given in the ATBD. Units are degrees
4.2 Validation of gravity gradient computations in MSR

4.2.1 Regional gravity fields from single data sets

In order to validate the computed GOCE gradients in the regional gravity field determination we choose the method of comparing modelled gravity fields from GOCE input data as well as from altimetry data. Hereby we use observations of the satellite altimeter missions ERS-1e and ERS-1f over a time period of one year (April 1994 – March 1995). From these geometric measurements gravity potential values are derived. Figure 4-6 shows the input data sets from GOCE and ERS-1 e/f for a region of 10° x 10° in the South Atlantic. On the left hand side the radial component of the gravity potential V_r from GOCE observations is plotted over a time period of three months (March – May 2011). The reference model ITG-Grace2010 up to d/o 60 is subtracted so that the left graphic shows differential potential values up to the frequency where GOCE is sensitive. That equates approximately to a series expansion until d/o 250. On the right side disturbing potential values in radial direction T_r are computed from ERS-1e/f data at a middle orbit height of 250 km. To compare the values with the GOCE data of the same frequency band, we split the signal within a MRR (multi-resolution representation) into detail signals and focused just on level 7. That equates to a series expansion from d/o 64 up to d/o 127. The potential values are given in Eötvös (1 E = 10^{-9} 1/s²) for both input data sets. The GOCE values are less smoothed than the values from ERS-1e/f data because of variations in the orbit height of several kilometers and because of data errors.

From both input data sets we computed gravity anomalies Δg on the surface of the reference ellipsoid WGS 84. From a reproducing kernel up to the maximum degree l’ = 135 of the series expansion we determine the scaling coefficients within a Gauss-Markov model. Finally we use Blackman scaling functions up to level 7 (d/o 127) on a Reuter grid to model a regional gravity field. Figure 4-7 shows the results obtained from GOCE data on the left side and from ERS-1e/f data on the right hand side. The gravity anomalies Δg vary in both graphics from -7 to +23 mGal. The structures are very similar with a maximum in the northern part of the region, positive anomalies with a north-south run in the eastern part and negative values in the western part with two eastward branches.
We further compare the two regional gravity fields to EGM2008 as a reference model (maximum degree $l_{\text{max}} = 127$). Therefore we compute the differences $\Delta g_{\text{EGM}} - \Delta g_{\text{GOCE}}$ and $\Delta g_{\text{EGM}} - \Delta g_{\text{ERS-1e/f}}$. The resulting structures are shown in Figure 4-8. The variations of ± 20 mGal illustrate that there are some significant deviations between the global ($\Delta g_{\text{EGM}}$) and the regional ($\Delta g_{\text{GOCE}}$, $\Delta g_{\text{ERS-1e/f}}$) gravity field models. But the behavior of the structures is almost identical in both graphics which indicates that the regional models themselves fit well to each other – referred to the global model.

In order to make an independent comparison, one has to choose an uncorrelated reference model. In our case EGM2008 contains observations from ERS-1e/f so that it is not completely independent from the computed gravity anomalies in Figure 4-7. But the statistics from Table 4-1 confirm the
good accordance of the two regional gravity field models. The computed mean and rms (root mean square) values refer to the whole area. The mean deviation to EGM2008 is in both cases with 0.04 and 0.15 mGal very small. The rms values of ± 3.8 and ± 4.0 mGal are larger but among each other in the same order of magnitude.

Table 4-1: Statistics of the gravity anomaly differences between EGM2008 (d/o 127) as a reference model and the regional models computed from GOCE and ERS-1e/f data.

<table>
<thead>
<tr>
<th>difference Δg</th>
<th>mean [mGal]</th>
<th>rms [mGal]</th>
</tr>
</thead>
<tbody>
<tr>
<td>EGM – GOCE</td>
<td>0.04</td>
<td>± 3.8</td>
</tr>
<tr>
<td>EGM – ERS-1e/f</td>
<td>0.15</td>
<td>± 4.0</td>
</tr>
</tbody>
</table>

Summing up the results obtained from GOCE and altimetry one can see clearly a very good consistency between both regional models on the one hand and a good match (with larger deviations) with the global EGM2008 on the other hand. From these first validation approaches thus, GOCE gravity gradients show very good results in gravity field solutions compared with altimetry solutions within the GOCE MBW (measurement bandwidth).

4.2.2 Regional gravity fields from a combination of data sets

The combination of the GOCE gravitational gradients with gravity field information from satellite altimetry is done using the so-called potential method. The disturbing potential – which is derived from multiple altimeter missions in a certain region – is evaluated together with GOCE gravity gradients $T_{zz}$. The unknown scaling coefficients are determined within a Gauss-Markov model with the observation equations:

$$\Delta T(x^i) + e(x^i) = \sum_{q=1}^{N_f} d_{j,q} \tilde{\phi}_{j+1}(x^i,x_q),$$

where

- $\Delta T$ ... Potential differences (observations with respect to a background model)
- $x^i$, $x_q$ ... Position vector of the observations $i$ and the grid points $q$
- $e$ ... Measurement errors
- $d_{j,q}$, $\tilde{\phi}_{j+1}$ ... Scaling coefficients (unknowns) and modified scaling functions (spherical basis functions).

More details are given in the ATBD and the same model has been used in Section 4.2.1 to compute regional solutions from GOCE or altimetry. In this section we look at the combination of GOCE and altimetry.

The unknown scaling coefficients are estimated using a reproducing kernel. The regional gravity field modelling is then achieved using Blackman scaling functions, which are located at a pre-defined Reuter grid (grid points $q$). The modified basis functions for the GOCE gradients $T_{zz} = T_{rr}$ and the altimeter data are

$$\tilde{\phi}_{j+1,\text{GOCE}}(x^i,x_q) = \sum_{l=0}^{l_f} \frac{2l+1}{4\pi} \frac{(l+1)(l+2)}{r^2} \left(\frac{R}{r}\right)^{l+1} P_l(x^i,x_q),$$

$$\tilde{\phi}_{j+1,\text{altimetry}}(x^i,x_q) = \frac{1}{r} \sum_{l=0}^{l_f} \frac{2l+1}{4\pi} \left(\frac{R}{r}\right)^{l+1} P_l(x^i,x_q),$$
respectively, with:

- $J$  ... maximum resolution level of the spectral decomposition using MRR
- $l$  ... spherical harmonic degree of the series expansion
- $l_r'$ ... maximum degree of the resolution level $J$
- $r$  ... with $|x| = r$, length of the position vector and $|r| = 1$, unit vector
- $R$  ... mean Earth radius.

The combination is done in the same region of $10^\circ \times 10^\circ$ in the South Atlantic as in Section 4.3.1. The two left panels in Figure 4-9 show the gravity anomalies $\Delta g$ on the WGS84 reference ellipsoid. This is the same as Figure 4-7 and is shown for reference. The right panel (c) in Figure 4-9 shows the results for combination of the two data sets, GOCE and altimetry. In all three cases the gravity anomalies vary between -7 and +23 mGal, and the spatial structures are very similar.

Figure 4-9: Gravity anomalies modeled with Blackman scaling functions ($l' = 127$) on a Reuter grid from GOCE observations (a), from ERS-1e/f observations (b), and from their combination (c).

The nice resemblance between the different results also reflects the validity of the variance component estimation (VCE) method that has been used in the solution. As can been seen from Table 4-2 all three input data sets (ERS-1e, ERS-1f, GOCE) get almost the same weight, which follows from the same order of magnitude of the variance components for $j = 7$. Thus in the combined solution in Figure 4-9c all observation sets contribute to the same amount. The background model ITG-Grace2010s has a much larger variance component, that is, receives much less weight and hardly contributes to the solution.

Table 4-2: Variance components for the data sets in the gravity field modelling for two different resolution levels.

<table>
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<th>Data</th>
<th>VCE ($j = 7$)</th>
<th>VCE ($j = 11$)</th>
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<td>ERS-1e</td>
<td>0.18 e-7</td>
<td>0.42 e-12</td>
</tr>
<tr>
<td>ERS-1f</td>
<td>0.17 e-7</td>
<td>0.41 e-12</td>
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<tr>
<td>GOCE</td>
<td>0.20 e-7</td>
<td>0.15 e-07</td>
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<tr>
<td>Prior information</td>
<td>0.10 e-3</td>
<td>0.31 e-11</td>
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</table>

The combination can also be done at a much higher level, for example for Level $j = 11$ ($l' = 2045$), which is shown in Figure 4-10 for gravity anomalies. The signal amplitude varies with $\pm 100$ mGal much more then for Level $j = 7$, which is caused by the higher signal content in the solution. In addition, the structure of the mid-Atlantic ridge can clearly be recognized in the eastern part of the test area. The three dot-like maxima in the south are islands and seamounts.
Figure 4-10: Gravity anomalies modelled with Blackman scaling functions (l' = 2047) at a Reuter grid from the combination of GOCE and ERS-1e/f observations.

One sees from the variance components in Table 4-2 that for the combined solution at Level j = 11 the GOCE observations have a significant smaller contribution in the gravity field modelling than the ERS-1e/f-observations. The GOCE variance component is with e-7 about 5 orders of magnitude greater than that of ERS-1e/f (e-12) and is therefore much less weighted. This can be explained by the sensitivity of GOCE, which reaches roughly spherical harmonic d/o 250. For higher frequencies GOCE hardly gives information, whereas the resolving power of the altimeter missions ERS-1e/f fully cover the spectral range of Level 11 (until l' = 2045).
5 Validation of derived gravity field products

Three classes of derived gravity field products can be derived, which will all be validated:

1. The first class are enhanced GOCE gravity gradients in the instrument frame (GRF) as well as the local north-oriented frame (LNOF) and the model reference frame (MRF). GOCE gravity gradients are combined with GRACE gravity field information, which enhances the long wavelengths and allows tensor rotation from the GRF to other reference frames.

2. The second class consists of gravity gradients that have been reduced to mean altitude. Using MRR with the enhanced gravity gradients as input a regional representation of the gravity field is computed, which in turn is used to predict gravity gradients at mean orbital altitude (260 km for Saudi Arabia and 270 km for Norway and surroundings).

3. The third class of derived gravity field products consists of gravity anomalies and gravity gradients in the MRF close to the Earth’s surface. Again MRR is used with the enhanced gradients and terrestrial gravity data – including satellite altimetry – as input.

The figure below shows a flow chart of the relations between along-track gravity gradients and regional as well as global gravity gradient grids and how these are validated.

![Flow chart of gravity gradient relations](image)

**Figure:** Relation between along-track gravity gradients, gravity gradients grids and their validation.

### 5.1 Enhanced GOCE gravity gradients

Oliver Baur (IWF) validated the enhanced gravity gradients in the GRF and LNOF. His report is included here.

**Purpose:**
Validation of the gravitational gradients (GG) provided by the GeoExplore team by the GDC team. Validation is performed for the GRF GG and the LNOF GG. The GeoExplore file formats as described in the corresponding DSUM are given in Table 5-1.
Table 5-1: GeoExplore GG file formats (copied from the GeoExplore DSUM)

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Strategy:
The validation is performed in terms of “reproduction” of the GeoExplore GG according to the conceptual line of action applied by GeoExplore. For the sake of simplicity, in the following the GG from GeoExplore are referred to as ‘DGFI GG’, whereas the “reproduced” gradients from GDC are denoted as ‘IWF GG’. Software synchronization has not been envisaged at any stage of the validation process, i.e., the DGFI GG and IWF GG can be considered as independent products. On the other hand, (slight) differences in the processing significantly impact the results, and hence it cannot be expected that the DGFI GG and IWF GG perfectly match each other. All results showed here are based on data from April 2010 (30 days).

Table 5-2 summarizes the GG combination strategy for both the computation of preliminary GG (for GDC internal use only) and the computation of IWF GG.
GOCE data (EGG_NOM_2)  
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<th>reprocessed GG</th>
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<td>ITG-Grace2010s</td>
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</table>

Filter  
| 2\textsuperscript{nd} order Butterworth | 4\textsuperscript{th} order Butterworth |

Cut-off/cut-on frequency  
| 4 mHz (d/o ~20) | 5 mHz (d/o ~25) |

V\textsubscript{xy}, V\textsubscript{yz} handling  
| replaced (TIM_R3) | none |

Combined LNOF GG  
| combined GRF GG rotation | V\textsubscript{xy}, V\textsubscript{yz} replaced (GOCO03S) prior to rotation |

Parameter No. 000 to 003 - GRF (GPS time, latitude, longitude, radial distance)

The differences are uncritical (Figure 5-1).

Figure 5-1: DGFI-IWF differences of GPS time and satellite position.
Parameter No. 004 to 009 - GRF (combined GG)

i) Without consideration of flag information, the differences are up to a few Eötvös (Figure 5-2 and Figure 5-3).

Figure 5-2: DGFI-IWF differences of combined GG (GRF) – flag information not considered.

Figure 5-3: DGFI GG versus IWF GG (GRF), zoom - flag information not considered.
ii) With consideration of the DGFI flag information (elimination of data flagged with values larger than 2), the differences reduce to the level of mE (Figure 5-4). The differences are expected to occur due to (slightly) different data combination strategies (filtering): whereas DGFI interpolates outliers prior to filtering in order to mitigate strong oscillation effects, IWF does not apply any (spline) interpolation.

Figure 5-4: DGFI-IWF differences of combined GG (GRF) – flag information considered.

Using extended flag information (100 samples before and after each GG flagged with values larger than 2 omitted), the differences reduce to the sub-mE level (Figure 5-5).

Figure 5-5: DGFI-IWF differences of combined GG (GRF) – extended flag information considered.
The GOCO03S model has been used to cover the long-wavelength part of the combined GG (cf. Table 5-2). Furthermore, according to the combination strategy (cut-on/cut-off frequency of 5 mHz), the signal of the combined GG reflects the signal contained in the measured GOCE GG within the MBW of 5 mHz to 0.1 Hz. Therefore, the PSD of the differences between the combined GG and synthetic GOCO03S GG should show no significant signal below the cut-off/cut-on frequency of 5 mHz (Figure 5-6). Note that the combined GRF GG $V_{xy}$ and $V_{yz}$ have not been replaced by model gradients. For this reason, the higher PSD differences for $V_{xy}$ and $V_{yz}$ (compared to the other GG) in Figure 5-6 can be attributed to the higher noise level of these combined GG.

![Figure 5-6: PSD of combined GG-GOC03S GG differences (GRF). The dashed red lines indicate the lower and upper bound of the GOCE MBW of 5 mHz and 0.1 Hz, respectively.](image)

The differences in Figure 5-6 can be regarded as the “gain” of GOCE for the geophysical studies. Figure 5-7 presents these differences in the spatial domain.

![Figure 5-7: Spatial representation of combined GG-GOC03S GG differences (GRF) for data covering 1 year. The gray part of the color palette covers the range from -0.1 mE to +0.1 mE.](image)

$V_{zzC}$
**Parameter No. 010 to 015 - GRF (error estimates of combined GG)**

The IWF GG are not equipped with error estimates. Therefore, no validation in terms of comparison is possible. For each DGFI GG, the estimated errors are constant (Figure 5-8).

![Figure 5-8: Error estimates of combined DGFI GG (GRF).](image)

**Parameter No. 016 to 021 - GRF (GG flags)**

The IWF GG are equipped with flags as provided within the EGG_NOM_2 product; DGFI uses a different flagging strategy. Bad or corrupt data is equipped with a flag > 2 by DGFI (Figure 5-9).

![Figure 5-9: DGFI flags and IWF flags (GRF GG).](image)
Parameter No. 022 to 030 - GRF (rotation matrix elements)

The differences are uncritical (Figure 5-10).

Figure 5-10: DGFI-IWF differences of GRF-to-LNOF rotation matrices.
**Parameter No. 000 to 003 - LNOF** (GPS time, latitude, longitude, radial distance)

The differences are uncritical (Figure 5-11).

![Figure 5-11: DGFI-IWF differences of GPS time and satellite position.](image)

**Parameter No. 004 to 009 - LNOF** (combined GG)

i) Without consideration of flag information, the differences are up to a few Eötvös (Figure 5-12 and Figure 5-13).

![Figure 5-12: DGFI-IWF differences of combined GG (LNOF) – flag information not considered.](image)
Figure 5-13: DGFI GG versus IWF GG (LNOF), zoom - flag information not considered.

Figure 5-14: DGFI-IWF differences of combined GG (LNOF) – flag information considered.

ii) With consideration of flag information (elimination of data flagged with values larger than 2), the differences still exceed the mE level (Figure 5-14, Figure 5-15 and Figure 5-16). The differences are expected to occur due to (slightly) different data combination strategies (filtering) in combination with GG rotation: whereas DGFI interpolates outliers prior to filtering in order to mitigate strong oscillation effects, IWF does not apply any (spline) interpolation.
Figure 5-15: DGFI-IWF differences of combined GG (LNOF), zoom1 – flag information considered.

Figure 5-16: DGFI-IWF differences of combined GG (LNOF), zoom2 – flag information considered.
Using extended flag information (200 samples before and after each GG flagged with values larger than 2 omitted; remark: 200 samples with 1 s sampling rate correspond to the cut-off/cut-on frequency of 5 mHz), the differences reduce to the sub-mE level (Figure 5-17).

The GOCO03S model has been used to cover the long-wavelength part of the combined GG (cf. Table 5-2). Furthermore, according to the combination strategy (cut-on/cut-off frequency of 5 mHz), the signal of the combined GG reflects the signal contained in the measured GOCE GG within the MBW of 5 mHz to 0.1 Hz. Therefore, the PSD of the differences between the combined GG and synthetic GOCO03S GG should show no significant signal below the cut-off/cut-on frequency of 5 mHz (Figure 5-18).
Parameter No. 010 to 015 - LNOF (error estimates of combined GG)

The IWF GG are not equipped with error estimates. Therefore, no validation in terms of comparison is possible. For each DGFI GG, the estimated errors are constant (Figure 5-19).

![Figure 5-19: Error estimates of combined DGFI GG (LNOF).](image)

Parameter No. 016 to 021 - LNOF (GG flags)

The IWF GG are equipped with flags as provided within the EGG_NOM_2 product; DGFI uses a different flagging strategy. Bad or corrupt data is equipped with a flag > 2 by DGFI (Figure 5-20).

![Figure 5-20: DGFI flags and IWF flags (LNOF GG).](image)

**Overall conclusion:** The differences between DGFI and IWF gradients in the GRF and LNOF are small. The remaining differences are caused by the IWF gradients that are affected more by outliers than the DGFI gradients.
5.2 Gradients at mean orbital altitude using MSR

On the basis of a 3-axis gradiometer GOCE delivers 3-dimensional (3D) information of the Earth’s gravity field. The combination of all 6 GOCE gradients promises a great benefit for geodetic science and applications in the neighbouring fields of solid earth physics, oceanography and glaciology. To study regional gravity field modelling using the full tensor of GOCE gravity gradients, 2 test areas have been defined within the project: one on the Arabian Peninsula (RAK) near the equator with a size of $18^\circ \times 14^\circ$ containing large dessert areas, flat topography and thus little gravity anomaly changes; the other in the North-East Atlantic Margin (NEA) with a size of $23 \times 24^\circ$, containing on- and offshore areas with mountainous regions, a strongly varying bathymetry and thus large gravity anomaly changes (see Figure 5-21).

![Test areas (red bordered) on the Saudi Arabian Peninsula, (a) RAK, and the North-East Atlantic Margin, (b) NEA.](image)

The general procedure of the regional gravity field modelling approach is schematically visualised in Figure 5-22. The measured GOCE gravity gradients (1) at the original orbit heights are used to set up the observation equations at the surface of an reference ellipsoid. Thus, the modelling approach (2) contains a downward continuation term. The advantage is, that in the third step, we can derive any functional of the Earth’s gravity field in any modelling height (3) by projecting the results via upward continuation. Applying downward continuation in the third step would lead to large errors as the signal to noise ratio might be unsufficient/bad.
The first task is to analyse negative effects of the downward continuation in the observation equations. The measured gravity gradients shall be directly used at mean orbit height, see Figure 5-23. The downward continuation term might lead to singularity problems in the computations in case...
of a weak signal-to-noise ratio of the observations. Thus we set up the modelling approach at a mean GOCE orbit height and compute gravity gradients of the disturbing potential at the same height. The general procedure is described in the following four steps:

1. Pre-processing of the GOCE data set
2. Subtraction of a background model
3. Analysis: series expansion in terms of reproducing kernels
4. Synthesis: series expansion in scaling functions

1. Pre-processing of the GOCE data set

   a) Temporal selection
   In this study we use the GOCE Level-2 product. The observation period is divided into 2 parts: The first époque starts after the on-board computer switch from CPU-A to -B in March 2010, as the measurements before and after show different noise behaviour and ends in July 2012. In the following it is denoted as “nominal phase”. The second époque includes the measurement of the lowered orbit phase from August 2012 until end of GOCE mission in September 2013. The aim is to use the full tensor of original GOCE gravity gradients, measured in the Gradiometer Reference Frame (GRF), in order to finally obtain models of the gradients of the disturbing potential T in a Local North-Oriented Frame (LNOF) at a mean orbit height for each test area. The mean orbit height is related to a reference ellipsoid (GRS80 or WGS84) and averages 255 km for the nominal phase and 225 km for the lowered orbit phase.

   b) Spectral selection
   Within the Measurement Bandwidth (MBW) of ~ 5 mHz up to ~ 100 mHz the frequency spectrum is most sensitively detected by the gradiometers. Thus, we apply a filter to extract these accurate measurements within the MBW. The less accurate long wavelengths are removed by implementing a high-pass filter with a cut-on frequency at the lower boundary of the MBW. The long wavelengths then are filled up with model information from the GOCO03S model. Exactly the same model is used for the subtraction of a background model in step 2. Thus, the original GRF observations are not influenced or manipulated by model information within the MBW. The remaining influence of the removed frequencies on the higher frequencies within the MBW is neglected here. As the regional gravity field modelling approach developed in our institute uses base functions which are strongly band-limited, the frequency spectrum is split into several levels j, which are related to specific frequency bands. Due to the decreasing characteristic of the base functions, the highest frequencies within each level are smoothed. To ensure the use of un-smoothed GOCE data we implemented the regional gravity field modelling approach at level 8 (series expansion up to l = 255) and at level 9 (up to l = 511).

2. Subtraction of a background model

   The use of a so-called background model (Vback) has two aspects: (1) The subtraction from the observation data set removes the well-known and -modelled part so that regional gravity field improvements can be modelled from the remaining new signal information. (2) The background model serves as prior information and thus as additional observation to avoid singularity and rank deficiencies in the estimation process (see steps 3 and 4). In our case we use GOCO03S as background model – the same as used for filling up low frequencies. To subtract the model Vback from the observed GOCE gravity gradients Vab, three steps have to be implemented:

   - Computation of the first and second derivatives of the potential Vback, w.r.t. spherical coordinates (r, l, c... radius, geographic longitude and co-latitude), defined in the geocentric Terrestrial Reference Frame TRF: Vr, Vrr, VI, VII, Vc, Vcc, Vrl, Vrc, Vcl.
   - Transformation of the derivatives from the spherical TRF into a Cartesian LNOF w.r.t. Cartesian coordinates (x, y, z): Vxx, Vxy, Vxz, Vyy, Vyz, Vzz.
• Rotation of the tensor components from the LNOF into the GRF to realize the subtraction from the original GOCE observations. The rotation matrix is provided within the GOCE data set.

3. Analysis: series expansion in terms of reproducing kernels

Within the analysis step we set up a functional model which describes the relationship between the measured GOCE gravity gradients and a mathematical expression for the second derivatives of the gravitational potential. The observation equations have to be set up separately for each measurement component and then are combined in a tensor. The unknown coefficients \( d_j \) for each level \( j \) are estimated within a Gauss-Markov Model. The weighting of the different observations is realized using Variance Component Estimation (VCE). The background model GOCO03S is introduced as prior information and serves as additional observation. The following four steps have to be processed:

• Computation of the 1st and 2nd derivatives of the gravitational potential \( V \), w.r.t. the spherical distance \( \psi \) between the observation point and the point where the gravity shall be determined: \( \nabla V, \nabla^2 V \).
• Transformation of the derivatives from the spherical TRF (rlc) into the Cartesian LNOF (xyz): \( V_{xx}, V_{xy}, V_{xz}, V_{yy}, V_{yz}, V_{zz} \).
• Setting up the \( 3 \times 3 \) tensor of observation equations in the LNOF:

\[
\begin{bmatrix}
V_{xx} & V_{xy} & V_{xz} \\
V_{yx} & V_{yy} & V_{yz} \\
V_{zx} & V_{zy} & V_{zz}
\end{bmatrix}
= N_j \begin{bmatrix}
\Phi_{j+1,xx} & \Phi_{j+1,xy} & \Phi_{j+1,xz} \\
\Phi_{j+1,xy} & \Phi_{j+1,yy} & \Phi_{j+1,yz} \\
\Phi_{j+1,xz} & \Phi_{j+1,yz} & \Phi_{j+1,zz}
\end{bmatrix}
\]

• The tensor elements are expressed by series expansions in terms of reproducing kernels based on Legendre polynomials. They have to be adopted for each component and developed until degree \( l = 280 \) (for \( j = 8 \)) and \( l = 560 \) (for \( j = 9 \)), thus higher than the maximum degree of the corresponding level to avoid omission errors. As the tensor is symmetric, it holds \( V_{xy} = V_{yx}, V_{zx} = V_{xz}, V_{yz} = V_{zy} \).
• Rotation of the tensor components from the LNOF into the GRF to realize the use of the original non-rotated and unaffected measurements.

4. Synthesis: series expansion in scaling functions

The estimated coefficients \( d_j \) from the analysis step then are used to model the regional gravity signal on pre-defined grid points. In our study we use regular grids both at the Earth’s surface, and at mean GOCE orbit heights of 255 km for the nominal phase and 225 km for the lowered orbit phase to reduce singularity problems caused by downward continuation. The resolution of the output grids is 0.2°. The modelling approach of the approximation signal \( F_j \) (residual gravity signal up to level \( j \), \( V_{\text{back}} \) subtracted) contains two steps:

• Setting up the synthesis equations for each grid point (according to 1.1), using the estimated coefficients \( d_j \) from the analysis step and a \( 3 \times 3 \) tensor containing modified Blackman scaling functions for each tensor component. The gravitational potential \( V_{ab} \) is defined in LNOF.
• Subtracting the normal potential \( U \) referred to WGS84 from the gravity potential to obtain gravity gradients of the disturbing potential \( T = U - V \): \( T_{xx}, T_{xy}, T_{xz}, T_{yy}, T_{yz}, T_{zz} \).

As output from the synthesis procedure we then obtain the second derivatives of the gravitational disturbing potential for all combinations of the xyz Cartesian coordinates in the LNOF.
5.2.1 Modelling approach and output at mean orbital height

As mentioned above the first task is to study the influence of the downward continuation term in the observation equations. We define a small independent study area of 6° x 5° in the Mediterranean Sea (South Italy and North Sicilia) for different reasons: (1) the area should contain topographical and thus gravitational field variations, (2) the area should not be affected by systematic errors in the yy component of the GOCE observations in polar regions, (3) it further it should not be too close to the equator as the distance of the GOCE satellite tracks becomes very large and (4) it should be small to save computing time. The latter is one of the main aspects defining the parameters for the 4-step modelling approach:

1. We set up our modelling approach using the GOCE observations from the lowered orbital phase from 08/2012 until 09/2013. This time period contains 2 times less measurements than the nominal phase. However, the reduced number of observations is sufficient and delivers even more detailed information from the Earth’s gravity field as the orbit is lower.
2. We subtract the global EGM2008 model up to degree and order 63 from the observations. Thus, we remove gravitational information up to level \( j = 6 \). This background model further serves as prior information. From previous studies we learned that the different global models deliver more or less the same information in the lower degrees (up to \( l \approx 120 \)).
3. In the analysis we develop series expansions in terms of Shannon kernel up to degree \( l = 140 \), so that \( j = 7 \) (\( l = 127 \)) is the highest level. The higher the level the more unknowns have to be estimated. According to Figure 5-23 we set up the modelling approach at a mean orbital height \( h = 225 \) km above a mean Earth sphere. We thus locate the basis functions on a spherical computation grid (Reuter grid) with \( r = 6603.137 \) km.
4. Finally the low-pass filtering Blackman kernel is used for generating the output functionals. We compute second derivatives of the disturbing potential \( T_{ij} \) on a regular grid with a resolution of 0.2°. The output grids are defined at the same ellipsoidal height \( h = 225 \) km, as the computation grids. Thus the whole modelling approach avoids up or downward continuation terms.

Figure 5-24: Study area Italy

The GOCE tensor observations \( V_{ab} \) are treated as six independent measurement groups. The combination is based on variance component estimation (VCE). Each group is relatively weighted to each other group by the inverse of the variance component. Table 5-1 gives the orders of magnitude referred to the a priori information from the background model. The three diagonal components \( V_{xx}, V_{yy}, V_{zz} \) get the smallest variance components, i.e. the highest weights and thus contribute the most to the results. The fourth accurate component \( V_{xz} \) is down-weighted by one order of magnitude relatively to the diagonal elements, but also contributes much to the results. As expected, the two
less accurate components Vxy and Vyz are down-weighted by five orders of magnitude relatively to the components at the main diagonal. The influence of the a priori information is very weak (relative weight 13 orders of magnitude smaller than Vxx, Vyy and Vzz), as we have a large number of GOCE observations and as they are well distributed over the study area.

The results of the gravity gradients at mean orbital height \( h = 225 \) km are shown in Figure 5-25. The six derivatives \( T_{ab} \) of the disturbing potential \( T \) are ordered in the \( xyz \) tensor arrangement. The approximation signals (referenced to the subtracted background model) contain spectral information up to level \( j = 7 \) (degree \( l = 64 \ldots 127 \)). At a mean GOCE altitude of \( h = 225 \) km above reference ellipsoid WGS84 the values vary between \(-0.16\) and \(0.13\) E. The \( T_{zz} \) component contains the largest signal as it is oriented along the main direction of the Earth’s radial gravity field.

Figure 5-26 shows the corresponding results for the complete frequency domain from degree \( l = 0 \ldots 127 \), after restoring the background model. They are denoted as final signals.

\[ j = 7 \]
\[ l = 64 \ldots 127 \]
approximation signals

Figure 5-25: Second derivatives of the disturbing potential \( T \) for the test area South Italy at height \( h = 225 \) km above reference ellipsoid WGS84. The signal content of the approximation signals (referenced to the background model) contains information up to level \( j = 7 \) \((l = 64 \ldots 127)\).
Table 5-1: Orders of magnitude of the estimated variance components for the modelling approach up to level \( j = 7 \) (\( l = 127 \)) at a height of \( h = 225 \) km.

<table>
<thead>
<tr>
<th>Observation group</th>
<th>Order of magnitude of variance component</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_{xx} )</td>
<td>( 10^{13} )</td>
</tr>
<tr>
<td>( V_{xy} )</td>
<td>( 10^{08} )</td>
</tr>
<tr>
<td>( V_{xz} )</td>
<td>( 10^{12} )</td>
</tr>
<tr>
<td>( V_{yy} )</td>
<td>( 10^{13} )</td>
</tr>
<tr>
<td>( V_{yz} )</td>
<td>( 10^{08} )</td>
</tr>
<tr>
<td>( V_{zz} )</td>
<td>( 10^{13} )</td>
</tr>
<tr>
<td>a priori information</td>
<td>1</td>
</tr>
</tbody>
</table>

Figure 5-26: Second derivatives of the disturbing potential \( T \) for the test area South Italy at height \( h = 225 \) km above reference ellipsoid WGS84. The signal content of the final signals (background model has been restored) contains information up to level \( j = 7 \) (\( l = 0 \ldots 127 \)).
One very important validation tool is the Laplace condition. The trace criteria must be fulfilled, i.e. the sum of the diagonal elements of the tensor (Txx + Tyy + Tzz) has to be zero. Computing the trace for both the approximation (Figure 5-25) and the final signal (Figure 5-26) shows that the condition is fulfilled up to the order of magnitude $10^{-16} \text{1/s}^2$, which is the calculation accuracy of the computer using 16 decimal places. For the approximation signal we obtain a mean value of 0.007 μE with standard deviation of +/- 0.208 μE. The final signals have a slightly larger standard deviation of +/- 0.457 μE (mean value 0.018 μE). The largest amplitudes vary between +/- 1 μE. From Figure 5-27 it is clearly visible that the variations have artificial structures. Thus the Laplace condition is fulfilled in the frame of computing accuracy of $10^{-16} \text{1/s}^2$.

5.2.2 Modelling approach at the Earth’s surface and output at mean orbital height

According to Figure 5-22 we set up our modelling approach (2) at the Earth’s surface. We apply the same 4-step regional gravity modelling approach as described in section 5.3.1, but changed the height of the spherical computation grid. In the third step (analysis), the radius of the sphere changes from $r = 6603.137 \text{ km}$ to $r = 6378.137 \text{ km}$. Thus, the observation equations for estimating the unknown coefficients using Shannon scaling functions contain a downward continuation term and further, the observation equations for calculation the output signals at the orbital height of 225km, using Blackman scaling functions, contain an upward continuation term.

The variance component estimation leads to a similar relative weighting of the gravity gradients as described in 5.3.1. Table 5-2 lists the orders of magnitude. They differ from the results in Table 5-1 just in their absolute values. Computing the trace Txx + Tyy + Tzz of the diagonal elements of the output tensor shows a mean value of 0.052 μE with a standard deviation of +/- 3.320 μE. Compared to the trace in section 5.3.1., the values become one order of magnitude larger: $10^{-15} \text{1/s}^2$. However the values are very small and the Laplace condition is fulfilled in the frame of computing accuracy of 16 decimal places, independently from the height of the computation grid.

We conclude that the up- and downward continuation terms in our regional gravity field modelling approach have no significant influence on the results! Thus, the more flexible approach from Figure 5-22 is applied in the following. Setting up the computation grid at the Earth’s surface allows us to compute any functional in any height by upward continuation in the synthesis step! Compared with approach from Figure 5-23 we would be limited to the computation height, there.
Table 5-2: Orders of magnitude of the estimated variance components for the modelling approach up to level \( j = 8 \) \((l = 127)\) at a height of \( h = 0 \) km.

<table>
<thead>
<tr>
<th>Observation group</th>
<th>Order of magnitude of variance component</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_{xx} )</td>
<td>( 10^{-09} )</td>
</tr>
<tr>
<td>( V_{xy} )</td>
<td>( 10^{-04} )</td>
</tr>
<tr>
<td>( V_{xz} )</td>
<td>( 10^{-08} )</td>
</tr>
<tr>
<td>( V_{yy} )</td>
<td>( 10^{-09} )</td>
</tr>
<tr>
<td>( V_{yz} )</td>
<td>( 10^{-04} )</td>
</tr>
<tr>
<td>( V_{zz} )</td>
<td>( 10^{-09} )</td>
</tr>
<tr>
<td>a priori information</td>
<td>1</td>
</tr>
</tbody>
</table>

Figure 5-28: Sum of the diagonal elements \( T_{xx} + T_{yy} + T_{zz} \) for the approximation signals setting up the modelling approach at the Earth’s surface.

5.2.3 Multi-Scale Representation (MSR)

The Multi-Scale Representation (MSR) or Multi-Resolution Representation (MRR) is a very promising tool that can be applied directly in our regional gravity modelling approach using band-limiting basis functions. It means splitting a target signal in a smoothed version and a number of detail signals, each related to a specific frequency domain. The background is described in chapter 6.3 of the ATBD.

For validating our regional gravity modelling approach, we thus compute gravity gradients at a higher level \( j = 8 \), smooth the target signal down to level 7 by MSR and compare the trace criteria (1) of the level-8, (2) of the smoothed level-7 (obtained from level-8) and (3) of the original level-7 computation.
The results in terms of second derivatives of the disturbing potential contain at level \( j = 8 \) more signal than at level \( j = 7 \). Figure 5-29 shows the functionals in the xyz tensor arrangement. The amplitudes of the Tzz component here vary between -0.30 and + 0.26 \( \mu \text{E} \), i.e. they are two times larger than the smoother amplitudes at level \( j = 7 \). The structures are however the same.

The trace of these level-8 results (1) has a mean value of 0.006 \( \mu \text{E} \) and a standard variation of +/- 0.272 \( \mu \text{E} \). Thus, the Laplace condition is fulfilled in the frame of computing accuracy. The left plot in Figure 5-30 shows the remaining artefacts. The trace of the smoothed level-7 result (2), obtained from level-8 by MSR, has a mean value of 0.001 \( \mu \text{E} \) and a standard deviation of +/- 0.201 \( \mu \text{E} \) (see Figure 5-30, middle plot). Here the Laplace condition is fulfilled as well.

Compared with the previously obtained level-7 result (3) (Figure 5-30, right plot) the structures of the remaining artefacts are similar and all values are smaller than the computing accuracy. This confirms that the Laplace condition is fulfilled, independently from the resolution level. We conclude that our MSR algorithms work correctly.

Figure 5-29: Second derivatives of the disturbing potential T for the test area South Italy at height \( h = 225 \) km above reference ellipsoid WGS84. The signal content of the approximation signals (referenced to the background model) contains information up to level \( j = 8 \) \( (l = 64 \ldots 255) \).
5.2.4 Comparison with the global EGM2008 model

Besides the Laplace condition that has to be fulfilled, we validated our results by comparing them with the global EGM2008 model at the Earth’s surface. We applied the same low-pass Blackman filter to the spherical harmonic model up to degree $l = 255$, so that we can consistently compare our results. The differences between the regional and the global solutions are shown in Figure 5-31 for the diagonal components $T_{xx}$, $T_{yy}$, $T_{zz}$ within the frequency domain of degree $l = 64$ ... 255 at height $h = 0$ km above the reference ellipsoid WGS84. The mean values, standard deviations, as well as the minimum and maximum amplitudes of the differences are listed in Table 5-3. The $zz$ component shows the largest standard variation of 0.5 E. This results from the high information content in radial direction. It is two orders of magnitude smaller compared with the signal variation of the $T_{zz}$ component (-17.9 E ... 13.5 E) of the regional solution at the Earth’s surface. These differences might indicate additional information in the GOCE observations.

Table 5-3: Statistics of the differences at mean orbital height $h = 225$ km between the regional solution at level $j = 8$ and the global EGM2008 model for the components of the main diagonal $T_{xx}$, $T_{yy}$, $T_{zz}$, consistently filtered with Blackman kernel up to degree $l = 255$.

<table>
<thead>
<tr>
<th></th>
<th>Diff $xx$</th>
<th>Diff $yy$</th>
<th>Diff $zz$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean +/- std:</td>
<td>-0.013 +/- 0.304 E</td>
<td>-0.056 +/- 0.353 E</td>
<td>0.069 +/- 0.521 E</td>
</tr>
<tr>
<td>Minmax:</td>
<td>-0.794 ... 0.784 E</td>
<td>-1.147 ... 0.820 E</td>
<td>-1.250 ... 1.524 E</td>
</tr>
</tbody>
</table>

Figure 5-31: Differences between the regional solution at level $j = 8$ and the global EGM2008 model for the components of the main diagonal $T_{xx}$, $T_{yy}$, $T_{zz}$, consistently filtered with Blackman kernel up to degree 255.
5.2.5 Test area NEA

In our studies we focused on the NEA test area as it contains much more gravitational structures and variations. The latest release of GOCE observations (Level-2 products) has been selected for this area with an overlapping margin of \( \sim 0.7^\circ \) to avoid boundary effects within the regional gravity field modelling approach. For NEA we have to take into account the meridian convergence of the tracks and a less accurately measured Vyy component due to systematic errors close to the poles.

![Figure 5-32: Gravity gradients Txx, Tyy, Tzz of the three main diagonal components up to level \( j = 9 \) (l = 511) at a mean orbital height of 225 km based on GOCE observations from the lower orbital phase.](image)

Figure 5-32 shows the gravity gradients of the three main diagonal components Txx, Tyy, Tzz up to level \( j = 9 \) (final signals; degree \( l = 0 \) ... 511) at a mean orbital altitude of 225 km for the test area NEA. We used here the data set from the lower orbital phase. The zz component has the largest amplitudes between -0.6 and + 0.7 E and contains 2 times more information than the xx and the yy component. The structures are very different: in the xx component variations dominate in north-south direction. This component is oriented into the flight direction of the satellite. Due to the high inclination of the satellite (94,6°) the flight tracks take almost the course of the meridians near the poles. Thus the xx component is more sensitive into north-south direction. In contrast, the cross-track oriented yy component is more sensitive into east-west direction and thus shows more structural variations along the latitudes than along the meridians. The sum of the three main diagonal elements finally should be zero to fulfil the Laplace condition. We computed the trace and obtain a mean value of 0.002 μE with a standard deviation of +/- 0.151 μE. Thus the trace is zero in the frame of computing accuracy of \( 10^{-16} \) 1/s². The corresponding graphical representation in Figure 5-33 confirms that the remaining structures have artificial character. We further smoothed the data set by applying MSR. The trace of level \( j = 8 \) (Figure 5-33, right plot) shows similar results: the mean value is 0.000 μE with a standard deviation of +/- 0.147 μE. The Laplace condition thus is fulfilled – independently from the resolution level. The corresponding approximation and detail signals down
to level $j = 8$ (degree $l = 255$) and level $j = 7$ (degree $l = 127$) can be seen in Figure 5-34 for the Txx component, in Figure 5-35 for the Tyy component and in Figure 5-36 for the Tzz component.

Figure 5-33: Trace Txx + Tyy + Tzz for the level-9 results up to degree $l = 511$ (left plot) and the smoothed level-8 results up to degree $l = 255$ (right plot).

Figure 5-34: MSR of the Txx component. The upper plots show the smoothed approximation signals from level $j = 9$ ($l = 511$) on the left, down to level $j = 8$ ($l = 255$) in the middle, down to level $j = 7$ ($l = 127$) on the right. The lower plots show the differential detail signals for level $j = 8$ ($l = 128 \ldots 255$) on the left and level $j = 7$ ($l = 64 \ldots 127$) on the right.
Figure 5-35: MSR of the Tyy component. The upper plots show the smoothed approximation signals from level \( j = 9 \) (\( l = 511 \)) on the left, down to level \( j = 8 \) (\( l = 255 \)) in the middle, down to level \( j = 7 \) (\( l = 127 \)) on the right. The lower plots show the differential detail signals for level \( j = 8 \) (\( l = 128 \ldots 255 \)) on the left and level \( j = 7 \) (\( l = 64 \ldots 127 \)) on the right.

To benefit from this multidimensional measurement system, the combination of all 6 GOCE gradients and additionally the consistent combination with other gravity observations mean an innovative challenge for regional gravity field modelling. Especially the flexible handling and combination of the 3D measurements promise a great benefit for geophysical applications from GOCE gravity gradients, as they contain information on radial as well as on lateral gravity changes.
Figure 5-36: MSR of the Tzz component. The upper plots show the smoothed approximation signals from level $j = 9$ ($l = 511$) on the left, down to level $j = 8$ ($l = 255$) in the middle, down to level $j = 7$ ($l = 127$) on the right. The lower plots show the differential detail signals for level $j = 8$ ($l = 128 \ldots 255$) on the left and level $j = 7$ ($l = 64 \ldots 127$) on the right.

5.3 Gradients at mean orbital altitude using tesseroids

Gravity gradients at mean orbital altitude of 225 km and 255 km above the ellipsoid have been computed using the tesseroid approach as described in the ATBD. The grids in 255 km have been computed using the data from the nominal phase, whereas the grids in 225 km have been computed using data from the lower orbit phase. Validation results are presented here for the whole globe as well as for selected regions.

5.3.1 Global comparisons

In all global comparisons mainly Tzz differences are shown and the color scales have been saturated to $[-6 \ 6]$ mE.

The figure below shows the global differences at 225 km between the tesseroid vertical gravity gradients and three different global models:

- EGM2008 contains GRACE data, satellite altimeter data and terrestrial gravity data
- EIGEN6C3 contains GRACE and GOCE data, satellite altimeter data and terrestrial gravity data
- DIR R5 contains GRACE and GOCE data from the whole mission

Over land the differences with EGM2008 are extremely large, which is caused by the poor terrestrial data in EGM2008. But also over the oceans larger differences occur: south-west of South Africa the differences are likely caused by ocean currents that have not been sufficiently removed from
EGM2008. The differences with EIGEN6C3 and DIR R5 – both containing GOCE data – are much smaller. In both difference plots, however, trackiness is observable, probably caused by a sub-optimal combination of GRACE and GOCE in both models. Over land somewhat larger differences occur for EIGEN6C3, which is again caused by poor terrestrial gravity data.

Figure 5-37: Global differences at 225 km between tesseroid vertical gradient and global models. From top to bottom: EGM2008, EIGEN6C3 and DIR R5.

When we use the spherical harmonic coefficients that were determined with the lower orbit data and compute with these the gravity gradients at 255 km and compare these with the gridded data at 255 km, then we see that the differences consist mainly of noise (Figure 5-38 for XY and ZZ). The
gradient grids at 225 and 255 km are therefore consistent. The differences for the other gradients are not shown as the spatial pattern is equal for each gradient (more noise towards the south). The noise level differs from gradient to gradient, where XY and ZZ are the extreme cases.

Figure 5-38: Global differences at 255 km between XY and ZZ from gridded data and upward continued data from 225 km.

5.3.2 Philippines
When we zoom in on the Philippines and surroundings, the differences between the tesseroid solution and GOCO03s contain additional signal picked up by the tesseroids, but not by GOCO03s because it has an omission error above spherical harmonic degree L = 250 and because regularization starts to play a role from degree L = 200 onward (see Figure below).

Figure 5-39: Differences at 225 km and 255 km between tesseroid gradients and GOCO03s for the Philippines and surrounding area.
Figure 5-40: Geoid signal and error power plot as function of SH degree. Yellow: EIGEN6C3 signal; red: GOCO03s signal (from ICGEM website).

EGM2008 predicts an omission error of roughly 2 mE at 225 km for Tzz above degree L = 250, whereas it is 12 mE above L = 200 (see Figure below). This suggests that the additional tesseroid signal for Tzz of around 6 mE is for the largest part explained by the regularization in GOCO03s from degree 200 to 250. As the tesseroids have a size of 0.5°, the maximum resolvable degree in terms of spherical harmonics is roughly degree L = 360. EGM2008 predicts that at 225 km the omission error in the vertical gravity gradient is at the level of 0.1 mE or less.

Figure 5-41: Omission error at 225 km for the Philippines and surrounding area as predicted by EGM2008.

It is also noted that there are no visual differences between the full signal at satellite height between the different models discussed here, as is obvious from the figure below.
The differences between the tesseroid solution and global models are 6 mE or less over the oceans and much larger over land for EGM2008. In the differences with DIR R5 and EIGEN63C GRACE stripes show up and/or potential problems with satellite altimetry close to the coast.

Finally, the tesseroid results are validated with MSR results. The differences between the MSR, Level 9 results and GOCO03s have a similar spatial pattern and amplitude as the differences between tesseroids and GOCO03s, which is to be expected. The differences between the two regional results are small compared with the residual signal and are caused by different data weighting, differences in the methods, etc.
When we zoom in on Hawaii and surroundings, the differences between the tesseroid solution and global models are 6 mE or less. Comparing the EGM2008 and EIGEN63C differences, the former seem to contain systematic errors. In the differences with DIR R5 GRACE stripes show up. With respect to GOCO03s there is additional signal. This is picked up by the tesseroids, but not by GOCO03s because it has an omission error above spherical harmonic degree \( L = 250 \) and because regularization starts to play a role from degree \( L = 200 \) onward (see above).
Figure 5-45: Differences at 225 km between tesseroid vertical gradient and global models for Hawaii and surroundings. Clockwise: EGM2008, EIGEN6C3, GOCO03s and DIR R5.

The figure below shows the differences between GOCO03s and tesseroid gradients (xx, yy and zz) at two different heights. In addition, topography and bathymetry contour lines are shown. Not only is there additional signal in the tesseroid gradients for the Hawaiian Islands, but also for sea mountains to the North West. Naturally, the additional signal at 255 km is smaller than at 225 km.

Figure 5-46: Differences at 225 and 255 km between tesseroid gradients and GOCO03s for Hawaii.
5.3.4 Caribbean and Arabian Peninsula
Results for the Caribbean are shown below. Again there is a clear correlation between residual signal and topography/bathymetry.

Figure 5-47: Differences at 225 km and 255 km between tesseroid gradients and GOCO03s for the Caribbean.
Results for the Arabian Peninsula are shown below. Also here is correlation between residual signal and topography/bathymetry. In general, the residual signals are much smaller and the correlation is not always obvious. See e.g. xx@225km between 24° to 32° N and 32° to 48° E.

Figure 5-48: Differences at 225 km and 255 km between tesseroid gradients and GOCO03s for the Arabian Peninsula and surrounding area.

5.4 Gravity field products at the Earth’s surface

To obtain gravity field products at the Earth’s surface we set up the regional gravity field modelling approach described in 5.2 at a height of 10 km above the reference ellipsoid WGS84 or GSR80. Depending on the heights of the measurements that we use, an up- or downward continuation has
to be implemented. In this study we compare and combine GOCE gravity gradients measured at a mean orbit height of 270 km within the NEA area and free-air anomalies (FA) measured at the Earth’s surface. The downward continuation of the GOCE observations amplifies the signal especially of the high frequent topography induced signal. For geophysical applications as lithospheric modelling these fine structures are not desired. Further the amplification of noise and omission errors is a disadvantage which might be reduced if we implement a near-surface gravity field modelling at 10 km height instead of representing the Earth’s gravity field directly at the reference ellipsoid. Especially the combination with free-air anomalies of high frequency content makes the modelling approach much more stable. Further the amplification of noise and singularity problems caused by the downward continuation can lead to instabilities. Avoiding this effect we introduce a background model \( \text{V}_{\text{back}} \) (GOCO03S up to degree/order 250) as prior information. Figure 5-48 summarises the regional gravity field modelling approach using GOCE gravity gradients measured at mean orbit height (1), combined with free-air anomalies (2) on a Reuter grid at 10 km height (3). The NGU FA dataset contains values on a regular grid [RD-7]. The output products (gravity anomalies \( Dg \), gravity gradients of the disturbing potential \( T_{ij} \)) are also provided on grids at 10 km height (4). While the computation procedure is done in the GRF to enable the use of the original GOCE gravity gradients, the output grids are given in the Earth-related LNOF.

![Diagram of gravity field modelling](image.png)

**Figure 5-49:** Procedure of using GOCE gravity gradients (1) combined with free air anomalies (2) to obtain regional gravity field models (3) at 10 km height above the Earth’s surface. Different functionals of the Earth’s gravity field than can be derived (4).

### 5.4.1 Gravity anomalies

The gravity anomalies are displayed on a regular grid at 10 km height using series expansion in terms of Blackman scaling functions up to degree \( l = 511 \) (level \( j = 9 \)). In Figure 5-50 the results for the test area NEA are presented (a) using only GOCE observations, (b) using only FA, (c) using both GOCE and FA observations. As GOCE gradients have their highest accuracy in the middle frequency domain (MBW 5...100 mHz) the errors increase for degree \( l > 230 \). That means that the signal to noise ratio is smaller than 1. The structures in Figure 5-50 (a) with amplitudes bigger than +/- 200 mGal thus show mainly noise which has been amplified using the downward continuation. In contrast the structures in Figure 5-49 (b) show gravity anomalies obtained from free-air anomalies with amplitudes between +/- 100 mGal. This signal (order of amplitudes and local distribution) seems to be realistic and
significant. Mountainous areas as in the South of Norway have the largest positive magnitudes while the Gulf of Bothnia has the largest negative anomalies. A comparison to the global EGM2008 model would approve the outcome.

In the resulting plot of the combination of GOCE gravity gradients with free-air anomalies in Figure 5-49 (c) the FA dataset gets a higher relative weight than the GOCE observations due to its higher sensitivity in the frequency domain of level $j = 9$ (up to degree $l = 511$). Thus the structures are dominated by FA observations and look very similar to (b). However, GOCE contributes the most signal in the mid frequency domain and thus delivers essential information. Low frequencies are filled up with a priori information of the GOCO03S background model. Thus the regional gravity field is represented in all spectral domains from the combination of a global model, very accurate GOCE measurements and high-resolution free-air anomalies.

In the South between $54^\circ$ and $55^\circ$ latitude we see noisy structures with higher amplitudes. In this stripe only GOCE observations and the prior information of the background model are available. As the background model GOCO03S is only available up to degree/order 250 it gets a lower relative weight than GOCE gradients and thus does not contribute in this region. Otherwise in (b) the background model is the only available gravity field information so that the missing FA anomalies here are filled up with model information. Figure 5-51 shows the distribution of the FA dataset (grey lines) compared with the modelling area (green bordered region) and the Reuter grid points (red crosses). Table 5-6 gives an overview of the relative weighting of the observations. The variance components (VC) are manually set and define inverse weights. The relatively high VCs of the Vxy and
Vyz GOCE tensor observations thus mean a low weighting as they contain systematic errors. The diagonal elements Vxx and Vzz and the free-air anomalies get the highest weights while Vback is relatively down-weighted by two orders.

**Table 5-6: Relative weighting of observations (IN) in terms of variance components (VC). Used are the reciprocal values.**

<table>
<thead>
<tr>
<th>IN</th>
<th>order of VC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vback</td>
<td>E-07</td>
</tr>
<tr>
<td>Vxx</td>
<td>E-09</td>
</tr>
<tr>
<td>Vxy</td>
<td>E-01</td>
</tr>
<tr>
<td>Vxz</td>
<td>E-07</td>
</tr>
<tr>
<td>Vyy</td>
<td>E-04</td>
</tr>
<tr>
<td>Vyz</td>
<td>E-01</td>
</tr>
<tr>
<td>Vzz</td>
<td>E-09</td>
</tr>
<tr>
<td>FA</td>
<td>E-09</td>
</tr>
</tbody>
</table>

We further apply the Multi-Scale Representation for the combination of GOCE gravity gradients and FA in order to reduce the noisy effects of GOCE observations at level $j = 9$. We smooth the combined solution down to level $j = 8$ (degree $l = 255$) and thus remove high-frequent signal but especially high-frequent noise in the southern part of the study area where the FA data are missing. Figure 5-52 shows gravity anomalies obtained from the combination of GOCE and FA: in the left plot we see the smoothed approximation signal at level $j = 8$, in the middle plot the smoothed final signal at level $j = 8$ (background model restored) and in the right plot again the level-9 solution for comparison.
5.4.2 Gravity gradients

Analogously to computing gravity gradients grids at mean orbital height (cf. chapter 5.2), we compute those grids at near Earth’s surface, at 10 km height above a reference ellipsoid (here GRS80). As the downward continuation amplifies especially high frequencies, the GOCE-only solution up to level $j = 9$ (degree $l = 511$) is dominated by noisy structures. Figure 5-53 shows exemplarily the output of two components, $T_{zz}$ and $T_{xx}$. Both the oscillating structures and the high amplitudes indicate clearly noise.

Figure 5-52: MSR/MRR of the combined level-9 solution (right plot) expressed in gravity anomalies at height $h = 10$ km above the reference ellipsoid GRS80. The right and the middle plot show the approximation and the final signal of the smoothed level-8 solution, removing high-frequent signal but especially high-frequent noise of the GOCE observations in the south.

Figure 5-53: Gravity gradients $T_{zz}$ (left plot) and $T_{xx}$ (right plot) from GOCE observations modelled at a height of $h = 10$ km above the reference ellipsoid GRS80 up to level $j = 9$ (background model GOCO03s up to degree and order 63 subtracted).
Figure 5-54: Gravity gradients from GOCE observations up to level $j = 8$ at a mean height of 10 km above the reference ellipsoid GRS80, subtracted by the background model GOCO03s (d/o 63).

Smoothing the level-9 solutions down to level $j = 8$ via MSR removes the high-frequent parts, especially the noise but also some signal parts. However, the level-8 gravity gradient grids (Figure 5-54) show variations between $\pm 10$ E for the approximation signals, referenced to the subtracted background model GOCO03s (d/o 63).

In contrast, Figure 5-55 shows the gravity gradient grids of the combined solution from GOCE + FA observations at level $j = 9$ (up to degree $l = 511$). In the southern part where FA data are missing, the noisy structures are clearly visible. But in the main part of the study area, the FA data deliver high-frequent information, so that we obtain more detailed structures and less smoothed variations with amplitudes up to $\pm 20$ E, thus two times higher than the amplitudes at level $j = 8$. The influence and contribution of the free-air anomalies is indicated by the variance components (VC). They obtain a VC which is two orders of magnitude lower than the VC of the four accurate GOCE gradients $V_{xx}$, $V_{xz}$, $V_{yy}$ and $V_{zz}$. Thus they are down-weighted by two orders of magnitude relatively to the GOCE observations.
Figure 5-55: Gravity gradient grids of GOCE + FA data at h = 10 km above the reference ellipsoid GRS80 up to level j = 9 (degree l = 511), subtracted by the background model GOCO03s (d/o 63).