HAROKOPIO UNIVERSITY

esa

→ 4th ADVANCED TRAINING COURSE IN LAND REMOTE SENSING

Radar Polarimetry (1)

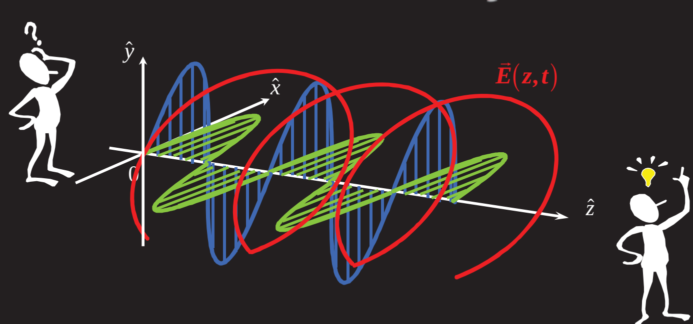
Eric POTTIER

1–5 July 2013 | Harokopio University | Athens, Greece

RADAR POLARIMETRY

- A bit of History
- Airborne and Space-borne Polarimetric SAR Sensors
- Software / Toolbox
- Learning / Training / Results

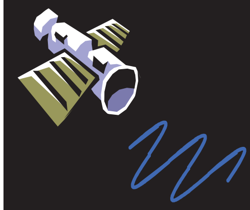
Radar Polarimetry



Radar Polarimetry (Polar : polarisation Metry: measure) is the science of acquiring, processing and analysing the polarization state of an electromagnetic field

Radar Polarimetry deals with the full vector nature of polarized electromagnetic waves

Radar Polarimetry



The POLARISATION information
Contained in the waves backscattered
from a given medium is highly related to:

its geometrical structure
reflectivity, shape and orientation

its geophysical properties such as humidity, roughness, ...

SAR Polarimetry Applications

Forest Vegetation

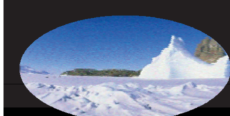
- Forest Height
- Forest Biomass
- Forest Structure
- Canopy Extinction
- Underlying Topography

- Forest Ecology
- Forest Management
- Ecosystem Change
- Carbon Cycle

Agriculture

- Soil Moisture Content
- Soil roughness
- Height of Vegetation Layer
- Extinction of Vegetation Layer
- Moisture of Vegetation Layer

- Farming Management
- Water Cycle
- Desretification



Snow and Ice

- Topography
- Penetration Depth / Density
- Snow Ice Layer
- Snow Ice Extinction
- Water Equivalent

- Ecosystem Change
- Water Cycle
- Water Management

Urban Areas

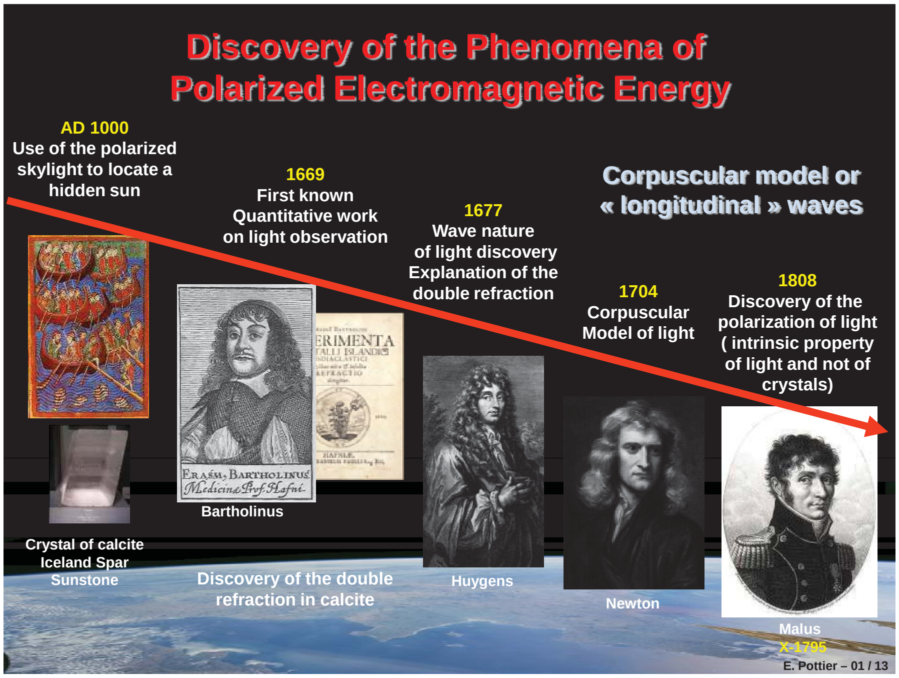
- Geometric Properties
- Dielectric Properties

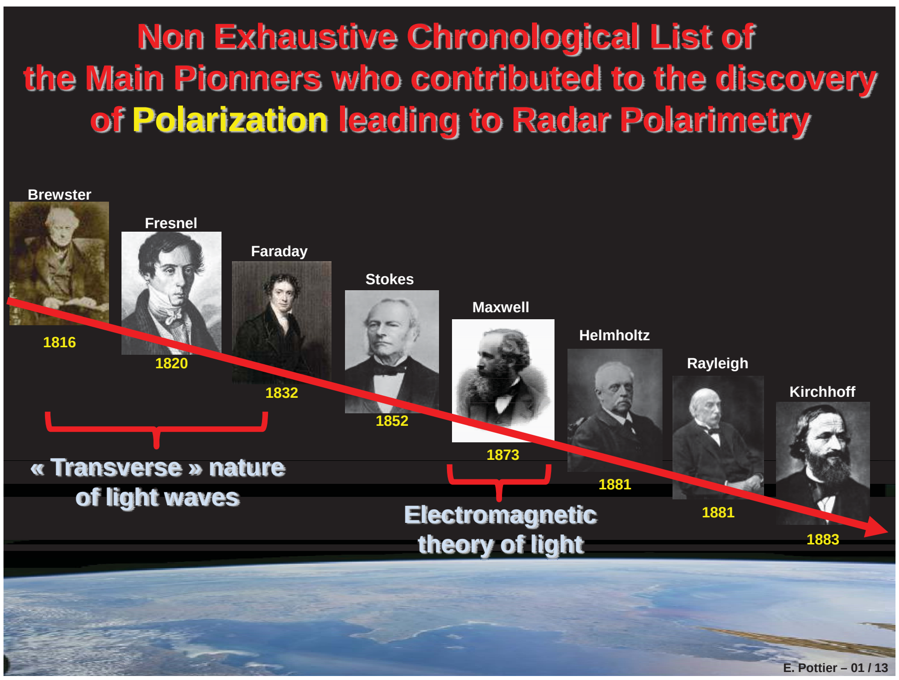
- Urban Monitoring

DLR Courtesy of Dr. I. Hajnsek

A Bit Of History

Radar Polarimetry


Discovery of the Phenomena of
Polarized Electromagnetic Energy
AD 1000
Use of the polarized skylight to locate a hidden sun
1669
First known Quantitative work on light observation
1677
Wave nature of light discovery Explanation of the double refraction
Corpuscular model or « longitudinal » waves
1704
Corpuscular Model of light
1808
Discovery of the polarization of light (intrinsic property of light and not of crystals)
Crystal of calcite
Iceland Spar
Sunstone
ERASM. BARTHOLINUS
Bartholinus
Discovery of the double refraction in calcite
Huygens
Newton
Malus
X-1795
E. Pottier – 01 / 13


Non Exhaustive Chronological List of
the Main Pionners who contributed to the discovery
of Polarization leading to Radar Polarimetry
Brewster
1816
Fresnel
1820
Faraday
1832
Stokes
1852
Maxwell
1873
Helmholtz
1881
Rayleigh
1881
Kirchhoff
1883
« Transverse » nature of light waves
Electromagnetic theory of light
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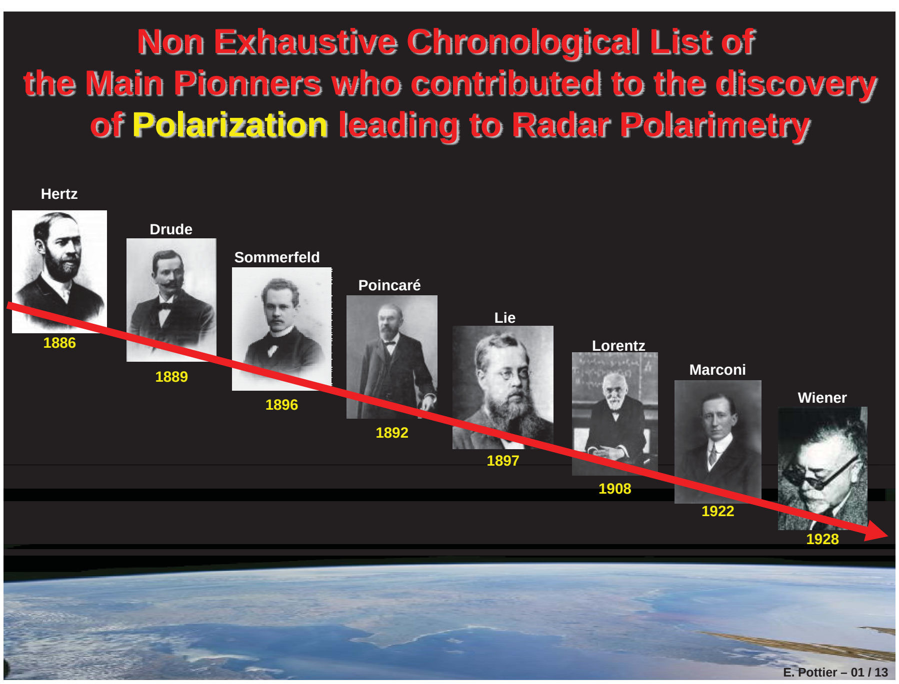
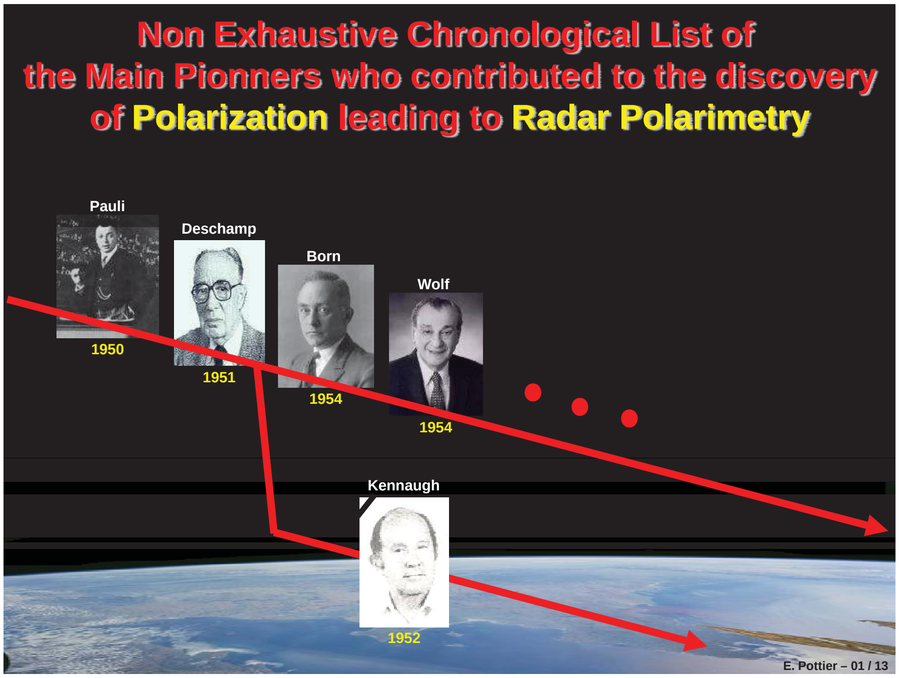

Non Exhaustive Chronological List of
the Main Pionners who contributed to the discovery
of Polarization leading to Radar Polarimetry
Hertz
1886
Drude
1889
Sommerfeld
1896
Poincaré
1892
Lie
1897
Lorentz
1908
Marconi
1922
Wiener
1928
E. Pottier – 01 / 13



Non Exhaustive Chronological List of
the Main Pionners who contributed to the discovery
of Polarization leading to Radar Polarimetry
Pauli
1950
Deschamp
1951
Born
1954
Wolf
1954
Kennaugh
1952
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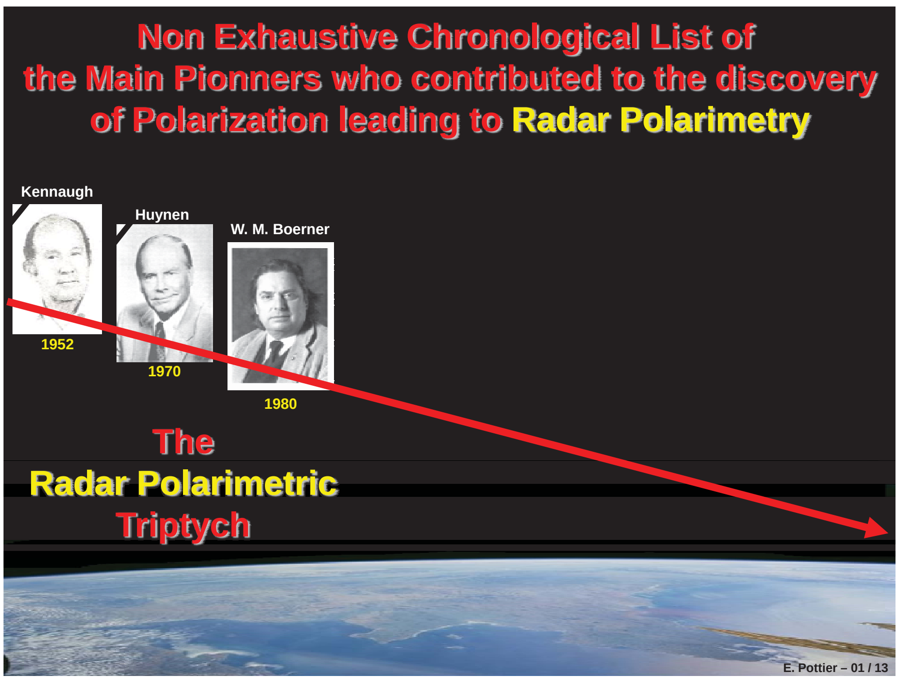
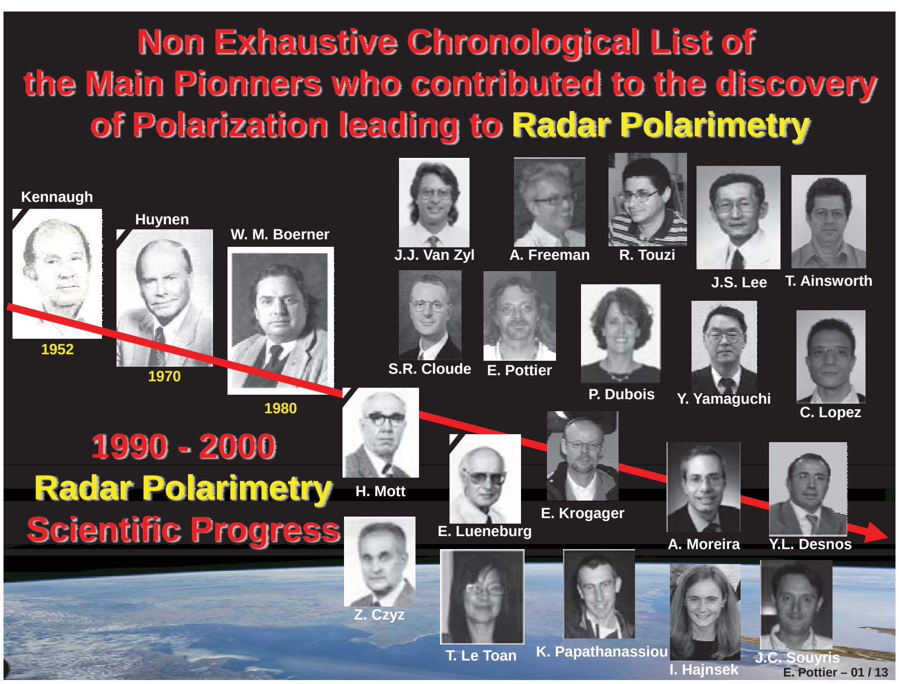


Non Exhaustive Chronological List of
the Main Pionners who contributed to the discovery
of Polarization leading to Radar Polarimetry
Kennaugh
1952
Huynen
1970
W. M. Boerner
1980
The
Radar Polarimetric
Triptych
E. Pottier – 01 / 13



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the Main Pionners who contributed to the discovery
of Polarization leading to Radar Polarimetry
Kennaugh
1952
Huynen
1970
W. M. Boerner
1980
J.J. Van Zyl
A. Freeman
R. Touzi
J.S. Lee
T. Ainsworth
S.R. Cloude
E. Pottier
P. Dubois
Y. Yamaguchi
C. Lopez
1990 - 2000
Radar Polarimetry
Scientific Progress
H. Mott
E. Lueneburg
E. Krogager
A. Moreira
Y.L. Desnos
Z. Czyz
T. Le Toan
K. Papathanassiou
I. Hajnsek
J.C. Souyris
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Polarimetric SAR

Airborne Sensors

Airborne PolSAR Sensors

AES1
InterMap Technologies (D)
GulfStream Commander
X-Band (HH), P-Band (Quad)

AIRSAR
NASA / JPL (USA)
DC8
P, L, C-Band (Quad)

AuSAR - INGARA
D.S.T.O (Aus)
DC3 (97) KingAir 350 (00) Beach 1900C
X-Band (Quad)

DOSAR
EADS / Dornier GmbH (D)
DO 228 (89), C160 (98), G222 (00)
S, C, X-Band (Quad), Ka-Band (VV)

ESAR
DLR (D)
DO 228
P, L, S-Band (Quad)
C, X-Band (Sngl)

EMISAR
DCRS (DK)
G3 Aircraft
L, C-Band (Quad)

MEMPHIS / AER II-PAMIR
FGAN (D)
Transal C160
Ka, W-Band (Quad) / X-Band (Quad)

STORM
UVSQ / CETP (F)
Merlin IV
C-Band (Quad)

PHARUS
TNO - FEL (NL)
CESSNA – Citation II
C-Band (Quad)

PISAR
NASDA / CRL (J)
GulfStream
L, X-Band (Quad)

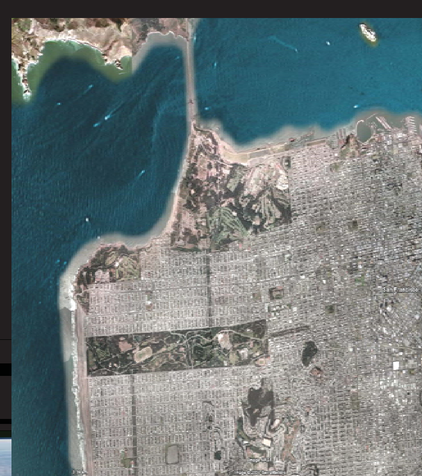
RAMSES
ONERA (F)
Transal C160
P, L, S, C, X, Ku, Ka, W-Band (Quad)

SAR580
Environnement Canada (CA)
Convair CV-580
C, X-Band (Quad)

Airborne PolSAR Sensors

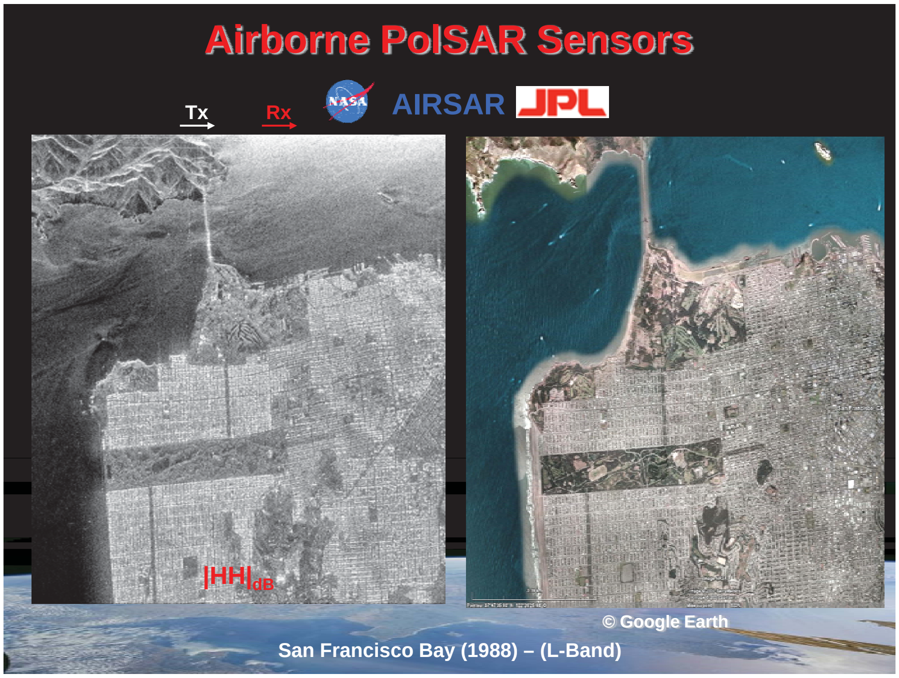
AIRSAR

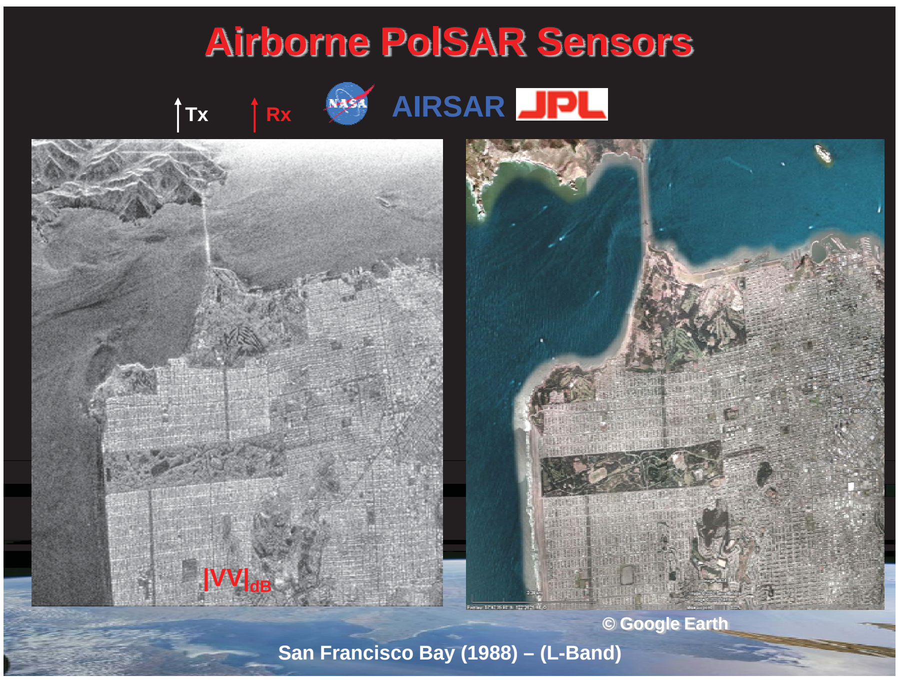
AIRSAR

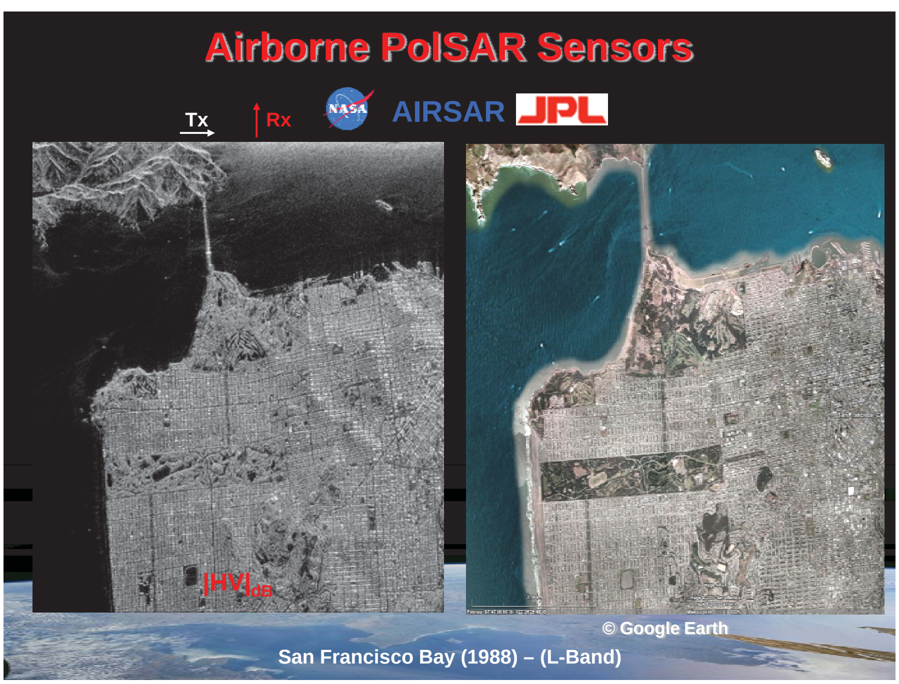


© Google Earth

San Francisco Bay (1988) – (L-Band)


Airborne PolSAR Sensors
Tx
Rx
AIRSAR
JPL
|HH|dB
© Google Earth
San Francisco Bay (1988) – (L-Band)

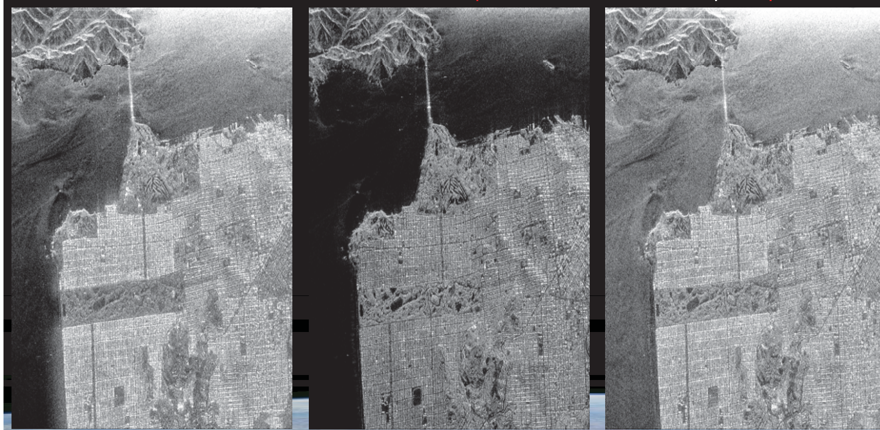

Airborne PolSAR Sensors
Tx
Rx
AIRSAR
JPL
|VV|dB
© Google Earth
San Francisco Bay (1988) – (L-Band)


Airborne PolSAR Sensors
Tx
Rx
AIRSAR
JPL
|HV|dB
© Google Earth
San Francisco Bay (1988) – (L-Band)

Airborne PolSAR Sensors

AIRSAR JPL

Tx Rx Tx Rx Tx Rx

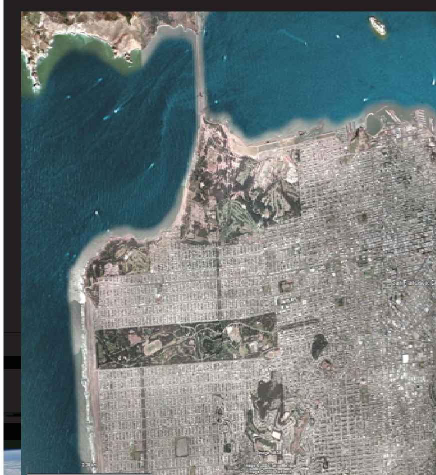


$|HH|_{dB}$ $|HV|_{dB}$ $|VV|_{dB}$

-30dB -15dB 0dB

Airborne PolSAR Sensors

AIRSAR JPL



© Google Earth

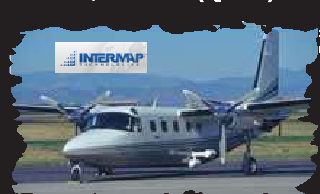
|HH+VV| |HV | |HH-VV|

San Francisco Bay (1988) – (L-Band)

Upcoming Airborne PolSAR Sensors

TopoSAR (AeS-1)

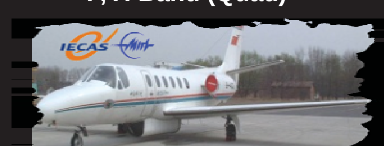
InterMAP
GullStream Commander
P, L-Band (Quad)



IECAS

IECAS - MOTL (C)

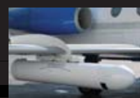
P, X-Band (Quad)

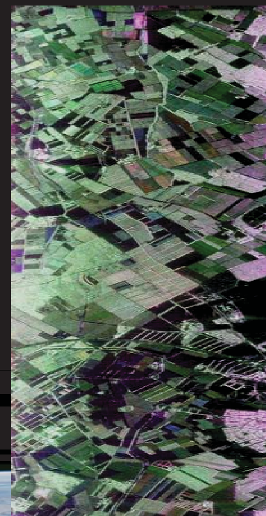


esa

Upcoming Airborne PolSAR Sensors

SETHI
ONERA (F)
Mystere 20
P, L, S, C, X (Quad - Pol)





P-Band Quad-Pol
B=70MHz

(P-Vh, X-Hh, L-Vh)

Upcoming Airborne PolSAR Sensors

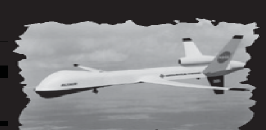
FSAR
DLR (G)
DO 228
S, X (Quad - Pol)

S-Band

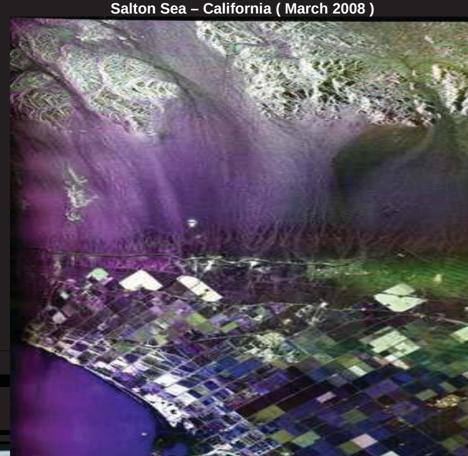
(HH-VV, HV, HH+VV)

Upcoming Airborne PolSAR Sensors

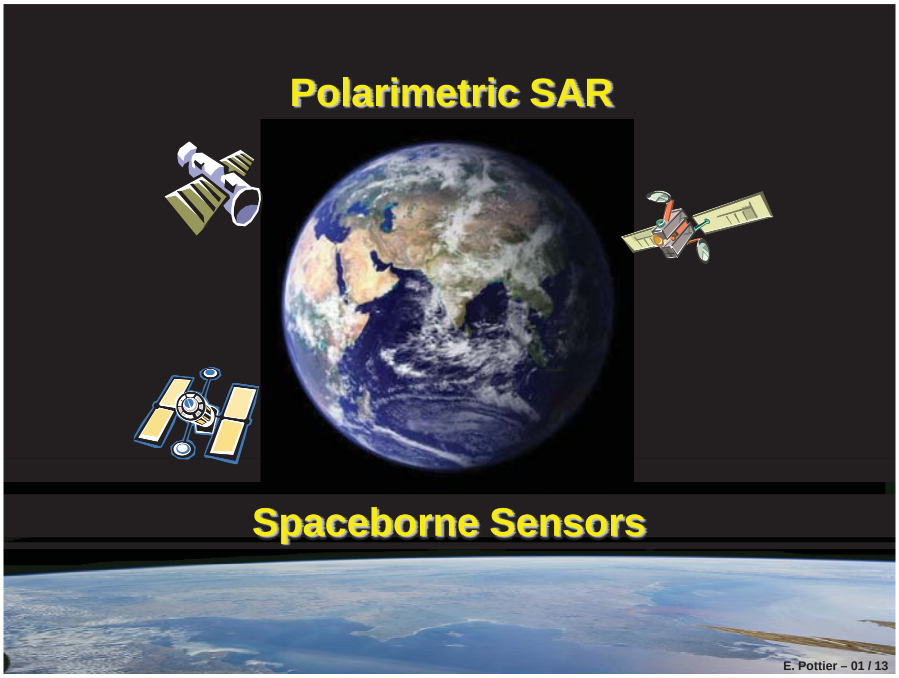
UAVSAR
Gulf-Stream III
L (Quad - Pol)



Salton Sea – California (March 2008)



(HH-VV, HV, HH+VV)


Polarimetric SAR
Spaceborne Sensors
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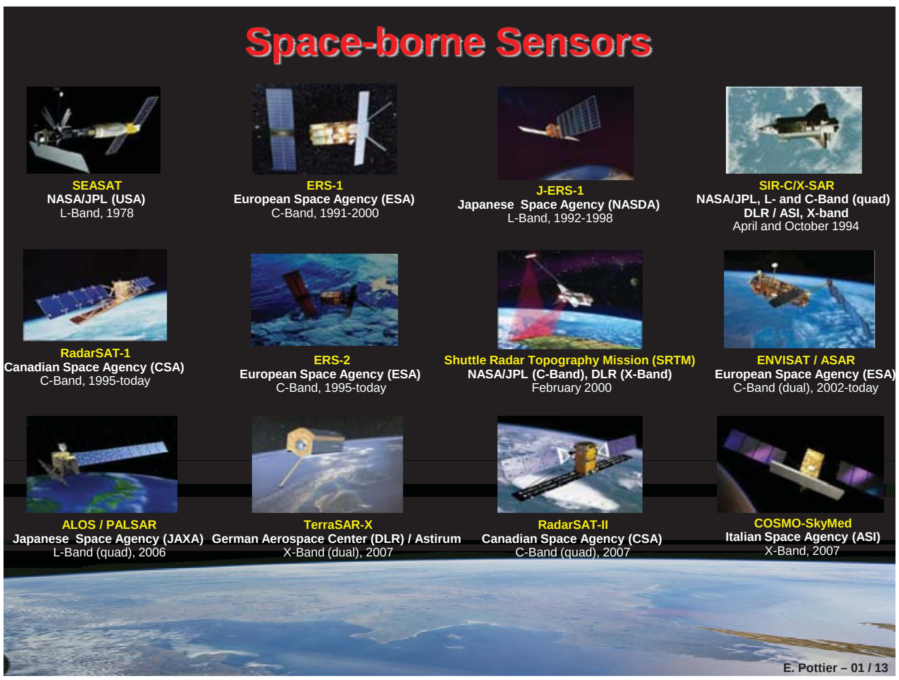
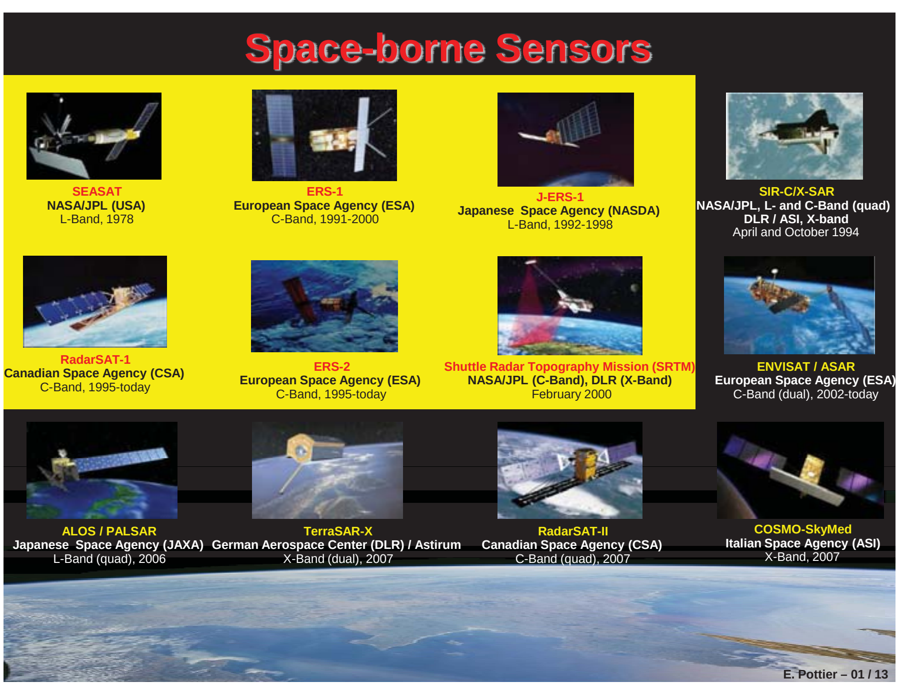

Space-borne Sensors
SEASAT
NASA/JPL (USA)
L-Band, 1978
ERS-1
European Space Agency (ESA)
C-Band, 1991-2000
J-ERS-1
Japanese Space Agency (NASDA)
L-Band, 1992-1998
SIR-C/X-SAR
NASA/JPL, L- and C-Band (quad)
DLR / ASI, X-band
April and October 1994
RadarSAT-1
Canadian Space Agency (CSA)
C-Band, 1995-today
ERS-2
European Space Agency (ESA)
C-Band, 1995-today
Shuttle Radar Topography Mission (SRTM)
NASA/JPL (C-Band), DLR (X-Band)
February 2000
ENVISAT / ASAR
European Space Agency (ESA)
C-Band (dual), 2002-today
ALOS / PALSAR
Japanese Space Agency (JAXA)
L-Band (quad), 2006
TerraSAR-X
German Aerospace Center (DLR) / Astirum
X-Band (dual), 2007
RadarSAT-II
Canadian Space Agency (CSA)
C-Band (quad), 2007
COSMO-SkyMed
Italian Space Agency (ASI)
X-Band, 2007
E. Pottier – 01 / 13


Space-borne Sensors
SEASAT
NASA/JPL (USA)
L-Band, 1978
ERS-1
European Space Agency (ESA)
C-Band, 1991-2000
J-ERS-1
Japanese Space Agency (NASDA)
L-Band, 1992-1998
SIR-C/X-SAR
NASA/JPL, L- and C-Band (quad)
DLR / ASI, X-band
April and October 1994
RadarSAT-1
Canadian Space Agency (CSA)
C-Band, 1995-today
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NASA/JPL (C-Band), DLR (X-Band)
February 2000
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European Space Agency (ESA)
C-Band (dual), 2002-today
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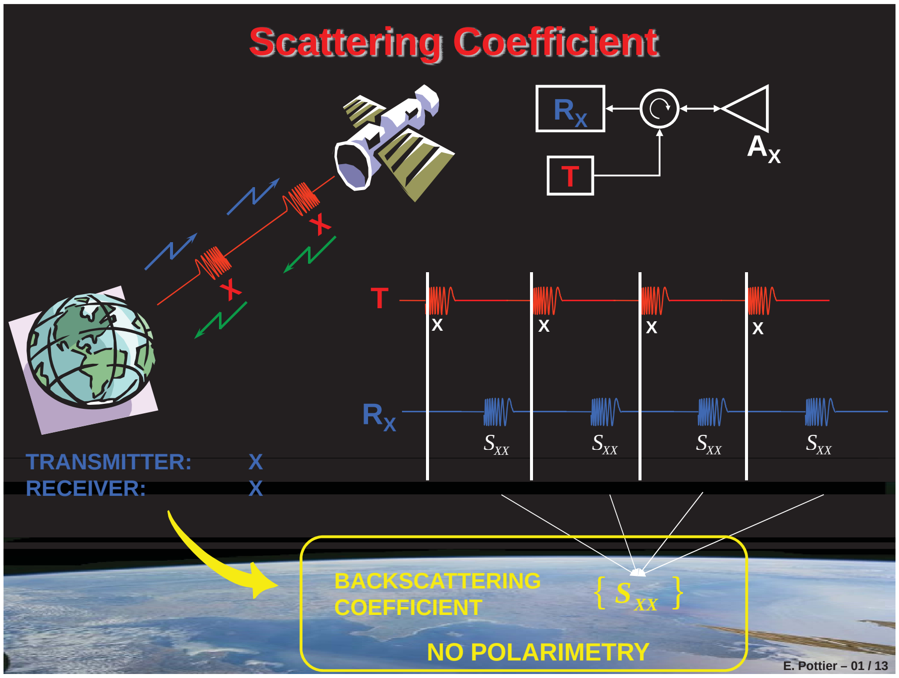
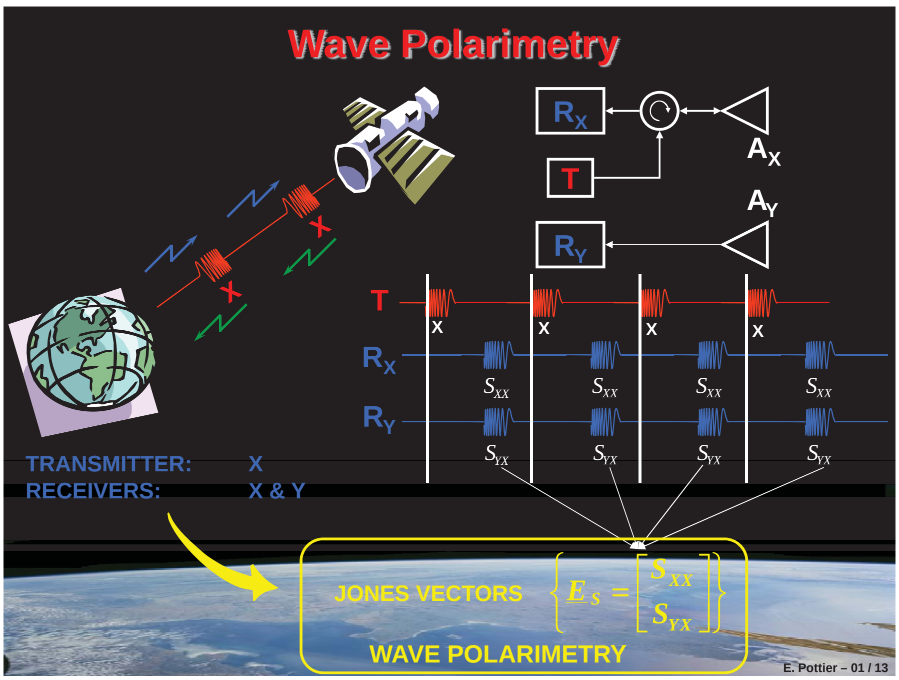
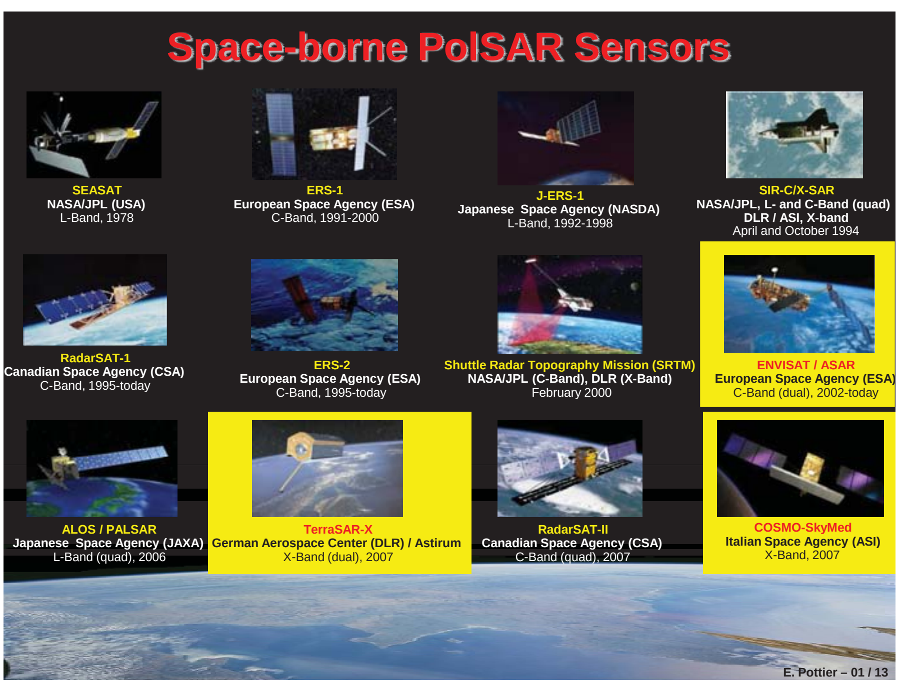
Japanese Space Agency (JAXA)
L-Band (quad), 2006
TerraSAR-X
German Aerospace Center (DLR) / Astirum
X-Band (dual), 2007
RadarSAT-II
Canadian Space Agency (CSA)
C-Band (quad), 2007
COSMO-SkyMed
Italian Space Agency (ASI)
X-Band, 2007
E. Pottier – 01 / 13


Scattering Coefficient
R_X
T
A_X
T
X
R_X
S_{XX}
TRANSMITTER: X
RECEIVER: X
BACKSCATTERING COEFFICIENT
$\{ S_{XX} \}$
NO POLARIMETRY
E. Pottier – 01 / 13


Wave Polarimetry
R_X
T
A_X
A_Y
R_Y
T
X
R_X
S_{XX}
R_Y
S_{YX}
TRANSMITTER: X
RECEIVERS: X & Y
JONES VECTORS
$\left\{ \underline{E}_S = \begin{bmatrix} S_{XX} \\ S_{YX} \end{bmatrix} \right\}$
WAVE POLARIMETRY
E. Pottier – 01 / 13


Space-borne PolSAR Sensors
SEASAT
NASA/JPL (USA)
L-Band, 1978
ERS-1
European Space Agency (ESA)
C-Band, 1991-2000
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NASA/JPL, L- and C-Band (quad)
DLR / ASI, X-band
April and October 1994
RadarSAT-1
Canadian Space Agency (CSA)
C-Band, 1995-today
ERS-2
European Space Agency (ESA)
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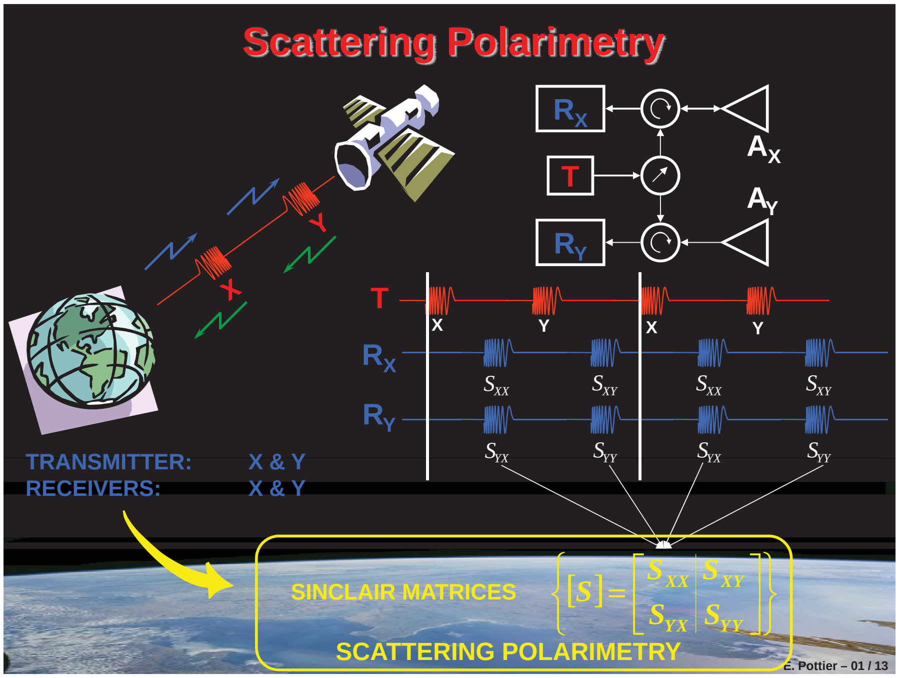
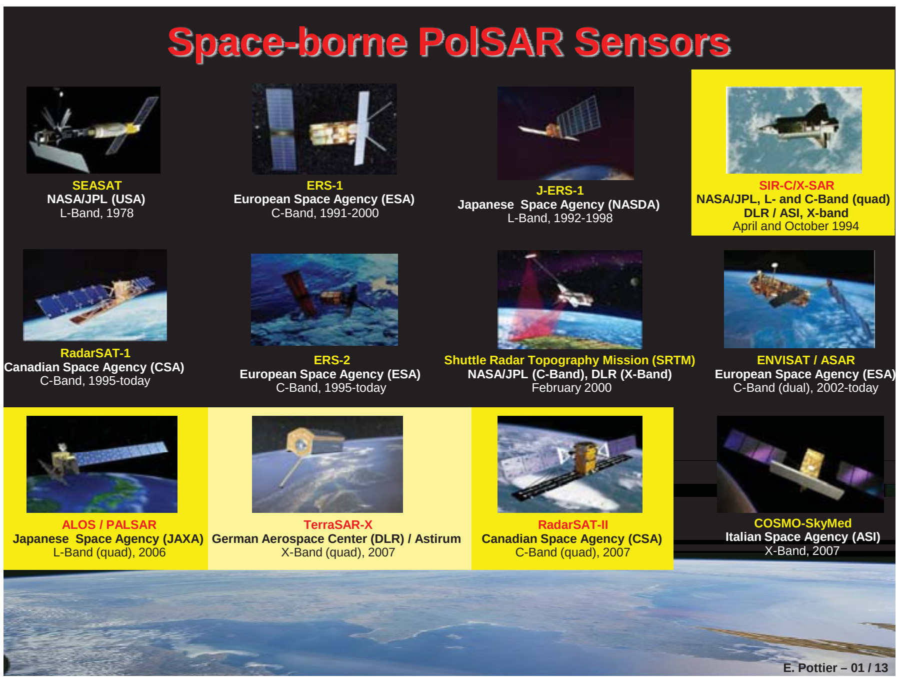
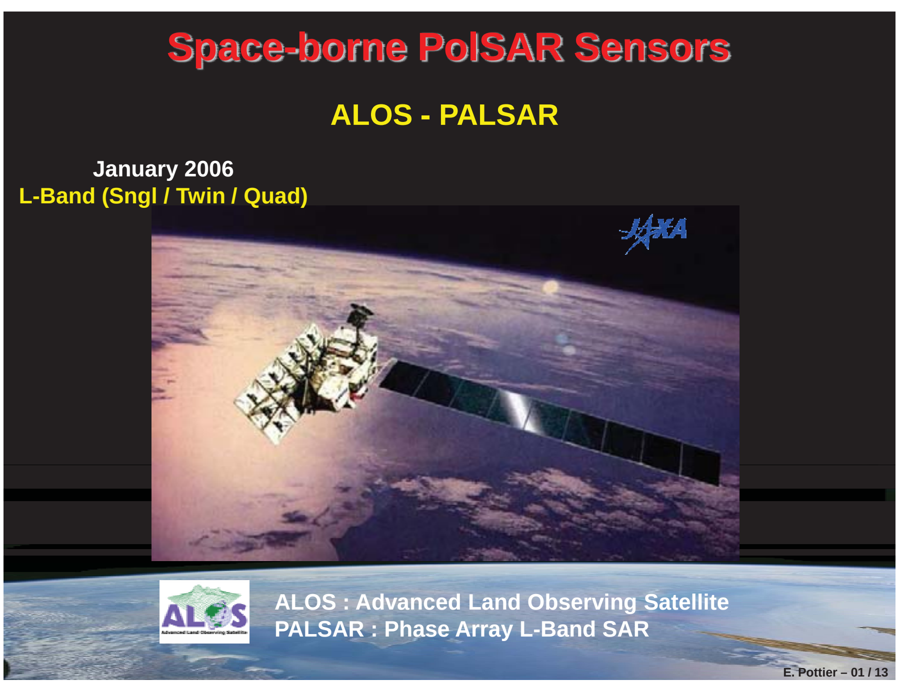
Shuttle Radar Topography Mission (SRTM)
NASA/JPL (C-Band), DLR (X-Band)
February 2000
ENVISAT / ASAR
European Space Agency (ESA)
C-Band (dual), 2002-today
ALOS / PALSAR
Japanese Space Agency (JAXA)
L-Band (quad), 2006
TerraSAR-X
German Aerospace Center (DLR) / Astrium
X-Band (dual), 2007
RadarSAT-II
Canadian Space Agency (CSA)
C-Band (quad), 2007
COSMO-SkyMed
Italian Space Agency (ASI)
X-Band, 2007
E. Pottier – 01 / 13


Scattering Polarimetry
R_X
T
R_Y
A_X
A_Y
X
Y
T
R_X
R_Y
X
Y
X
Y
S_XX
S_XY
S_XX
S_XY
S_YX
S_YY
S_YX
S_YY
TRANSMITTER: X & Y
RECEIVERS: X & Y
SINCLAIR MATRICES
$\{[S] = \begin{bmatrix} S_{XX} & S_{XY} \\ S_{YX} & S_{YY} \end{bmatrix}\}$
SCATTERING POLARIMETRY
E. Pottier – 01 / 13


Space-borne PolSAR Sensors
SEASAT
NASA/JPL (USA)
L-Band, 1978
ERS-1
European Space Agency (ESA)
C-Band, 1991-2000
J-ERS-1
Japanese Space Agency (NASDA)
L-Band, 1992-1998
SIR-C/X-SAR
NASA/JPL, L- and C-Band (quad)
DLR / ASI, X-band
April and October 1994
RadarSAT-1
Canadian Space Agency (CSA)
C-Band, 1995-today
ERS-2
European Space Agency (ESA)
C-Band, 1995-today
Shuttle Radar Topography Mission (SRTM)
NASA/JPL (C-Band), DLR (X-Band)
February 2000
ENVISAT / ASAR
European Space Agency (ESA)
C-Band (dual), 2002-today
ALOS / PALSAR
Japanese Space Agency (JAXA)
L-Band (quad), 2006
TerraSAR-X
German Aerospace Center (DLR) / Astrium
X-Band (quad), 2007
RadarSAT-II
Canadian Space Agency (CSA)
C-Band (quad), 2007
COSMO-SkyMed
Italian Space Agency (ASI)
X-Band, 2007
E. Pottier – 01 / 13


Space-borne PolSAR Sensors
ALOS - PALSAR
January 2006
L-Band (Sngl / Twin / Quad)
JAXA
ALOS
ALOS : Advanced Land Observing Satellite
PALSAR : Phase Array L-Band SAR
E. Pottier – 01 / 13

Space-borne PolSAR Sensors

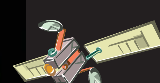
TerraSAR - X

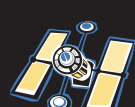
June 2007
X-Band (Sngl / Twin / Quad ?)

DLR EADS ASTRIUM

Space-borne PolSAR Sensors

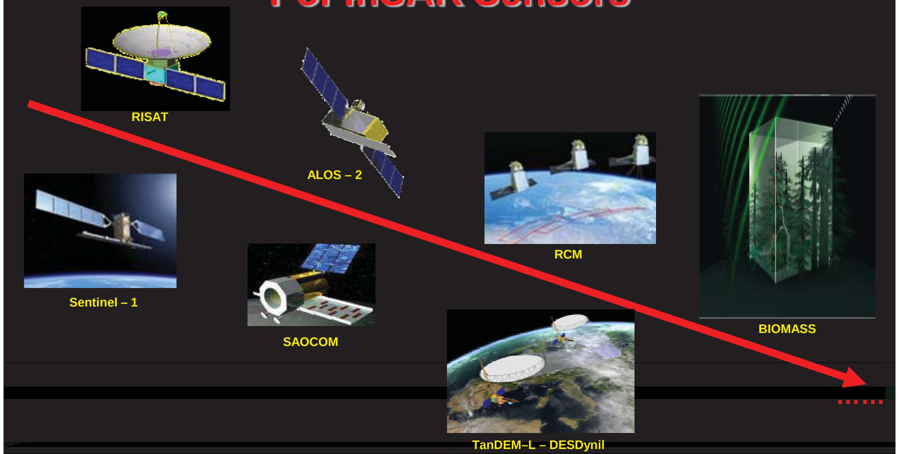
What About The Future ?





From Tomorrow …

Future Space-Borne PolSAR Pol-InSAR Sensors



What About

Software / Toolbox ?

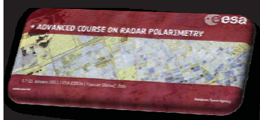
PolSARpro v5.0

The Polarimetric SAR Data Processing and Educational Tool v5.0

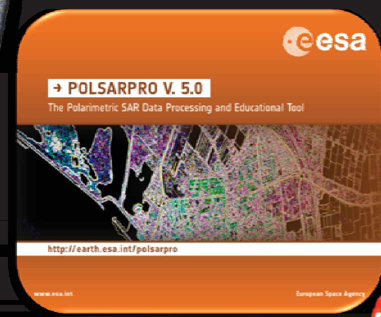
http://earth.esa.int/polsarpro

PolSARpro v5.0

The Polarimetric SAR Data Processing and Educational Tool v□.□

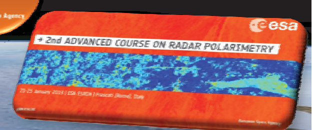


v4.2
Jan. 2011



New!

v5.0
Jan. 2013



PolSARpro v5.0 Software

OPEN SOURCE DEVELOPMENT

The Tool is free download on the Internet from the ESA Web Portal (Earthnet) at :
http://earth.eo.esa.int/polsarpro

PolSARpro v5.0 Software

http://earth.esa.int/polsarpro

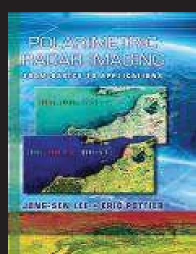
The Web Site provides

- Details of the project
- Access to the tutorial and software
- Information about status of the development
- Demonstration Sample Datasets

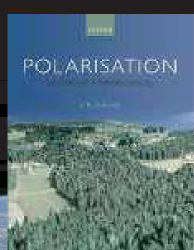
Learning / Training

Next P.I Generations

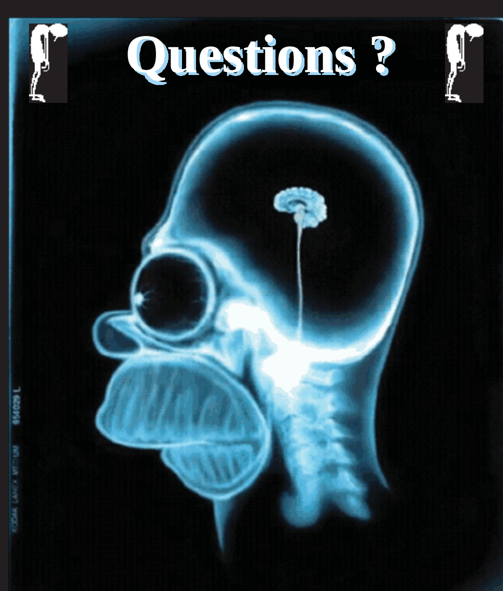
Books On Polarimetric Radar
SAR, Polarimetric Interferometry

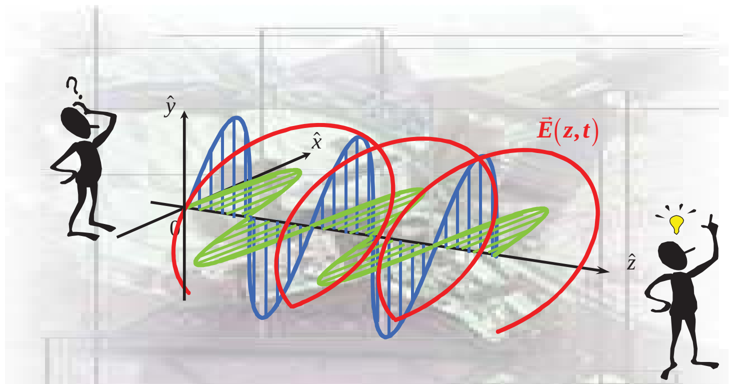


Polarimetric Radar Imaging: From basics to applications
Jong-Sen LEE – Eric POTTIER
CRC Press; 1st ed., February 2009, pp 422
ISBN: 978-1420054972

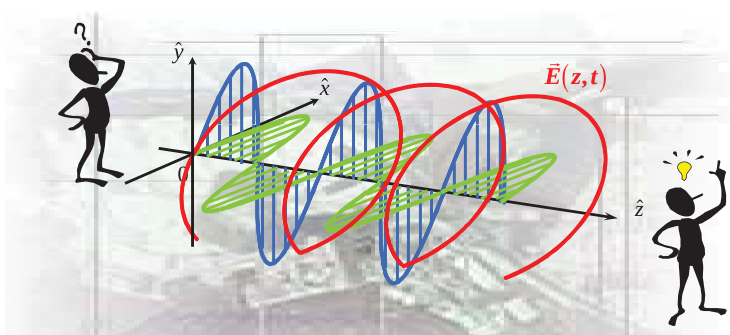


Polarisation: Applications in Remote Sensing
Shane R. CLOUDE
Oxford University Press, October 2009, pp 352
ISBN: 978-0199569731





BASIC CONCEPTS



WAVE POLARIMETRY

4th ADVANCED TRAINING COURSE IN LAND REMOTE SENSING
1–5 July 2013 | Harokopio University | Athens, Greece

PROPAGATION EQUATION

REAL ELECTRIC FIELD VECTOR $\vec{E}(z,t)$

MAXWELL EQUATIONS

MAXWELL – FARADAY EQUATION $\nabla\wedge\vec{E}(z,t)=-\frac{\partial\vec{B}(z,t)}{\partial t}$

MAXWELL – AMPERE EQUATION $\nabla\wedge\vec{H}(z,t)=\vec{J}_T(z,t)$

GAUSS THEOREM $\nabla\cdot\vec{D}(z,t)=\rho(z,t)$

$\nabla\cdot\vec{B}(z,t)=0$

$$\vec{J}_T(z,t)=\vec{J}_C(z,t)+\frac{\partial\vec{D}(z,t)}{\partial t}$$

$$\vec{J}_C(z,t)=\sigma\vec{E}(z,t)$$
$$\vec{B}(z,t)=\mu\vec{H}(z,t)$$
$$\vec{D}(z,t)=\varepsilon\vec{E}(z,t)$$

σ (Conductivity)
μ (Permeability)
ε (Permittivity)

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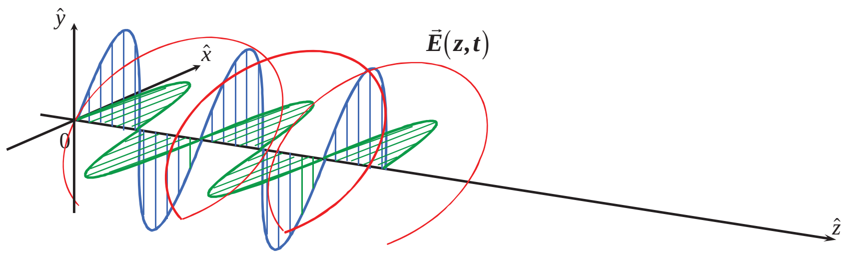
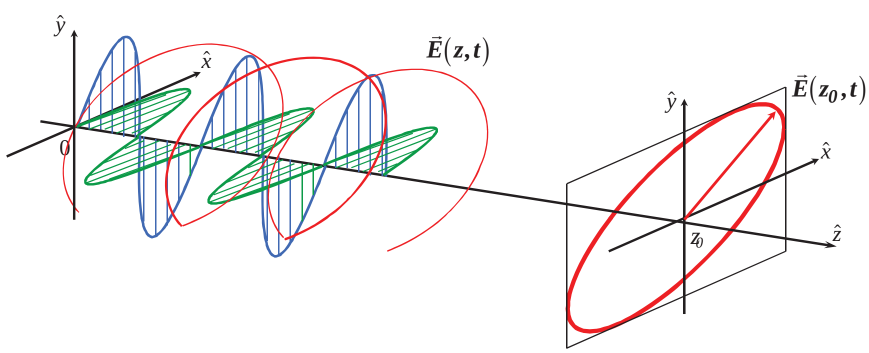
POLARISATION ELLIPSE



REAL ELECTRIC FIELD VECTOR

$$\vec{E}(z,t)=\begin{cases} E_x = E_{0x}\cos(\omega t - kz - \delta_x) \\ E_y = E_{0y}\cos(\omega t - kz - \delta_y) \\ E_z = 0 \end{cases}$$

POLARISATION ELLIPSE

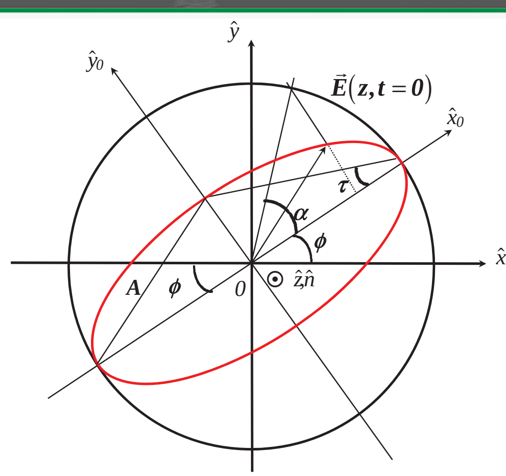


THE REAL ELECTRIC FIELD VECTOR MOVES IN TIME ALONG AN ELLIPSE

$$\left(\frac{E_x}{E_{0x}}\right)^2 - 2\frac{E_x E_y}{E_{0x}E_{0y}}\cos(\delta) + \left(\frac{E_y}{E_{0y}}\right)^2 = \sin^2(\delta)$$

With: $\delta = \delta_y - \delta_x$

POLARISATION ELLIPSE



A : WAVE AMPLITUDE

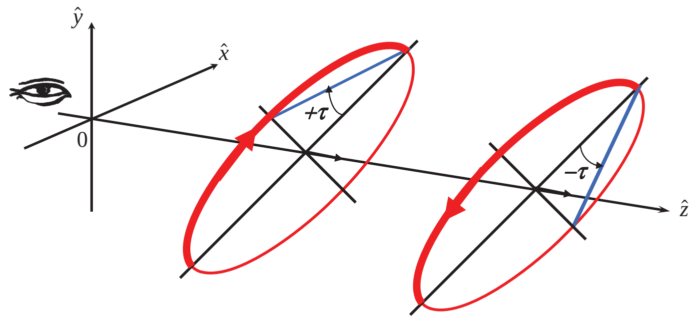
α: ABSOLUTE PHASE

ϕ: ORIENTATION ANGLE $-\frac{\pi}{2} \le \phi \le \frac{\pi}{2}$

τ: ELLIPTICITY ANGLE $0 \le \tau \le \frac{\pi}{4}$

POLARISATION HANDENESS

ROTATION SENSE: LOOKING INTO THE DIRECTION OF THE WAVE PROPAGATION



ANTI-CLOCKWISE ROTATION	CLOCKWISE ROTATION
LEFT HANDED POLARISATION	RIGHT HANDED POLARISATION
⬇	⬇
ELLIPTICITY ANGLE : $\tau > 0$	ELLIPTICITY ANGLE : $\tau < 0$

$$-\frac{\pi}{4} \le \tau \le \frac{\pi}{4}$$

JONES VECTOR

REAL ELECTRIC FIELD VECTOR

$$\vec{E}(z,t) = \begin{cases} E_x = E_{0x}\cos(\omega t - kz - \delta_x) \\ E_y = E_{0y}\cos(\omega t - kz - \delta_y) \\ E_z = 0 \end{cases}$$

PHASOR = JONES VECTOR

$$\underline{E} = \begin{bmatrix} E_x = E_{ox}e^{j\delta_x} \\ E_y = E_{oy}e^{j\delta_y} \end{bmatrix}$$

With: $\vec{E}(z,t) = \Re\left(\underline{E}e^{j(\omega t - kz)}\right)$

GEOMETRICAL PARAMETERS

ABSOLUTE PHASE

$$\alpha = \delta_x$$

AMPLITUDE

$$A = \sqrt{E_{0x}^2 + E_{0y}^2}$$

ORIENTATION ANGLE

$$\tan 2\phi = 2\frac{E_{0x}E_{0y}}{E_{0x}^2 - E_{0y}^2}\cos\delta$$

ELLIPTICITY ANGLE

$$\sin 2\tau = 2\frac{E_{0x}E_{0y}}{E_{0x}^2 + E_{0y}^2}\sin\delta$$

POLARISATION HANDENESS: *Sign(τ)*

JONES VECTOR

Special Unitary Matrices Group and Jones Vector

$$\underline{E}_{(\hat{x},\hat{y})} = \begin{bmatrix} E_x = E_{ox}e^{j\delta_x} \\ E_y = E_{oy}e^{j\delta_y} \end{bmatrix} = Ae^{j\alpha}\begin{bmatrix} \cos(\phi)\cos(\tau) - j\sin(\phi)\sin(\tau) \\ \sin(\phi)\cos(\tau) + j\cos(\phi)\sin(\tau) \end{bmatrix}$$

⬇

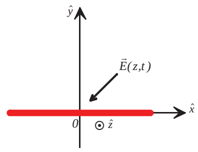
$$\underline{E}_{(\hat{x},\hat{y})} = Ae^{j\alpha}\begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix}\begin{bmatrix} \cos(\tau) \\ j\sin(\tau) \end{bmatrix}$$

⬇

$$\boxed{\underline{E}_{(\hat{x},\hat{y})} = A\begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix}\begin{bmatrix} \cos(\tau) & j\sin(\tau) \\ j\sin(\tau) & \cos(\tau) \end{bmatrix}\begin{bmatrix} e^{j\alpha} & 0 \\ 0 & e^{-j\alpha} \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}}$$

JONES VECTOR

HORIZONTAL POLARISATION STATE

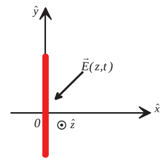


$$\underline{H} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\phi = 0$$

$$\tau = 0$$

VERTICAL POLARISATION STATE

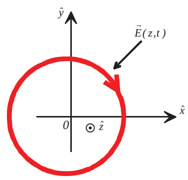


$$\underline{V} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\phi = \frac{\pi}{2}$$

$$\tau = 0$$

LEFT CIRCULAR POLARISATION STATE

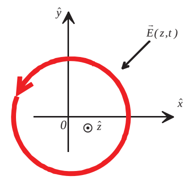


$$\underline{LC} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ j \end{bmatrix}$$

$$-\frac{\pi}{2} \le \phi \le +\frac{\pi}{2}$$

$$\tau = +\frac{\pi}{4}$$

RIGHT CIRCULAR POLARISATION STATE

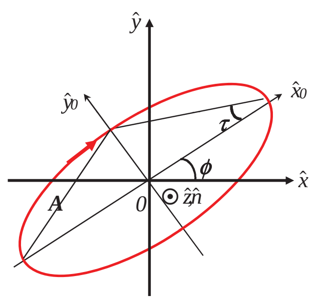


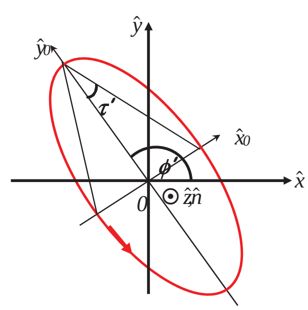
$$\underline{RC} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -j \end{bmatrix}$$

$$-\frac{\pi}{2} \le \phi \le +\frac{\pi}{2}$$

$$\tau = -\frac{\pi}{4}$$

ORTHOGONAL JONES VECTOR





ORTHOGONALITY CONDITIONS

$$(\phi,\tau) \mapsto \begin{cases} \phi' = \phi + \frac{\pi}{2} \\ \tau' = -\tau \end{cases} \Rightarrow$$

CHANGE OF POLARISATION HANDENESS

ELLIPTICAL BASIS TRANSFORMATION

SU(2) : SPECIAL UNITARY TRANSFORMATION MATRIX

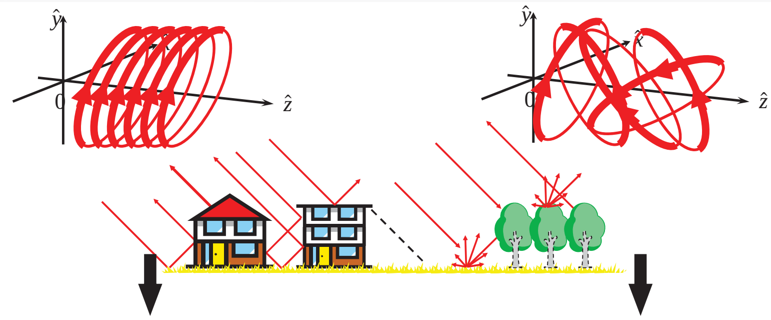
$$[U(\phi,\tau,\alpha)] = \begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix} \begin{bmatrix} \cos(\tau) & j\sin(\tau) \\ j\sin(\tau) & \cos(\tau) \end{bmatrix} \begin{bmatrix} e^{-j\alpha} & 0 \\ 0 & e^{j\alpha} \end{bmatrix}$$

$$\Downarrow$$

ELLIPTICAL BASIS TRANSFORMATION MATRIX

$$\boxed{\begin{aligned} \left[U_{(\underline{A},\underline{A}_\perp)\mapsto(\underline{B},\underline{B}_\perp)}\right] &= [U(\phi,\tau,\alpha)]^{-1} \\ &= \begin{bmatrix} e^{j\alpha} & 0 \\ 0 & e^{-j\alpha} \end{bmatrix} \begin{bmatrix} \cos(\tau) & -j\sin(\tau) \\ -j\sin(\tau) & \cos(\tau) \end{bmatrix} \begin{bmatrix} \cos(\phi) & \sin(\phi) \\ -\sin(\phi) & \cos(\phi) \end{bmatrix} \end{aligned}}$$

PARTIALLY POLARISED WAVES



DETERMINISTIC SCATTERING

COMPLETELY POLARISED WAVE

RANDOM SCATTERING

PARTIALLY POLARISED WAVE

Polarisation Ellipse varies in time
Amplitude, Phase: Random processes

STATISTICAL DESCRIPTION

PARTIALLY POLARISED WAVES

WAVE COVARIANCE MATRIX

$$\langle [J] \rangle = \langle \underline{E}\,\underline{E}^{T*} \rangle = \begin{bmatrix} \langle |E_x|^2 \rangle & \langle E_x E_y^* \rangle \\ \langle E_y E_x^* \rangle & \langle |E_y|^2 \rangle \end{bmatrix}$$

<u>DIAGONAL ELEMENTS</u> : INTENSITIES ON EACH OF THE 2 ORTHOGONAL COMPONENTS OF THE WAVE

<u>OFF-DIAGONAL ELEMENTS</u> : CROSS-CORRELATIONS BETWEEN THE 2 ORTHOGONAL COMPONENTS OF THE WAVE

$$Trace([J]) = \langle |E_x|^2 \rangle + \langle |E_y|^2 \rangle = A^2$$ TOTAL WAVE INTENSITY

THE WAVE COVARIANCE MATRIX IS A
2x2 HERMITIAN POSITIVE SEMI-DEFINITE MATRIX

PARTIALLY POLARISED WAVES

EIGENVALUES DECOMPOSITION

$$\langle [J] \rangle = [U_2] \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} [U_2]^{-1} = \lambda_1 \underline{u}_1 \underline{u}_1^{T*} + \lambda_2 \underline{u}_2 \underline{u}_2^{T*}$$

➡ 2 ORTHOGONAL EIGENVECTORS $[U_2] = [\underline{u}_1, \underline{u}_2]$

➡ 2 REAL EIGENVALUES

$$\lambda_1 = \frac{1}{2}\left\{ \langle g_0 \rangle + \sqrt{\langle g_1 \rangle^2 + \langle g_2 \rangle^2 + \langle g_3 \rangle^2} \right\}$$

$$\lambda_2 = \frac{1}{2}\left\{ \langle g_0 \rangle - \sqrt{\langle g_1 \rangle^2 + \langle g_2 \rangle^2 + \langle g_3 \rangle^2} \right\}$$

PARTIALLY POLARISED WAVES

PARTIALLY POLARISED WAVES DESCRIPTORS

Degree of Polarisation

$$DoP = \frac{\sqrt{\langle g_1 \rangle^2 + \langle g_2 \rangle^2 + \langle g_3 \rangle^2}}{\langle g_0 \rangle} = \frac{\lambda_1 - \lambda_2}{\lambda_1 + \lambda_2} = \left(1 - \frac{4\,det([J])}{Trace^2([J])} \right)$$

Polarised Wave Power / Total Wave Power

Anisotropy

$$0 \le DoP \le 1$$

Wave Entropy

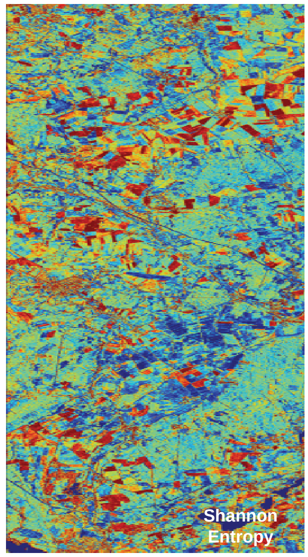
$$0 \le H \le 1$$

$$H = -\sum_{i=1}^{i=2} p_i \log_2(p_i)$$

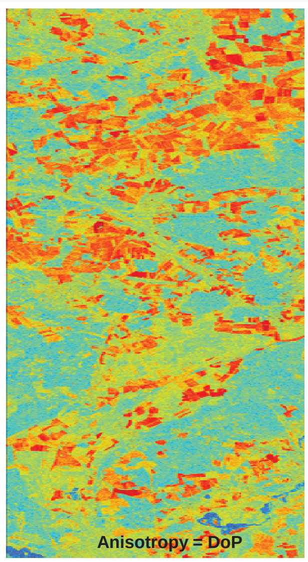
With: $p_i = \frac{\lambda_i}{\lambda_1 + \lambda_2}$

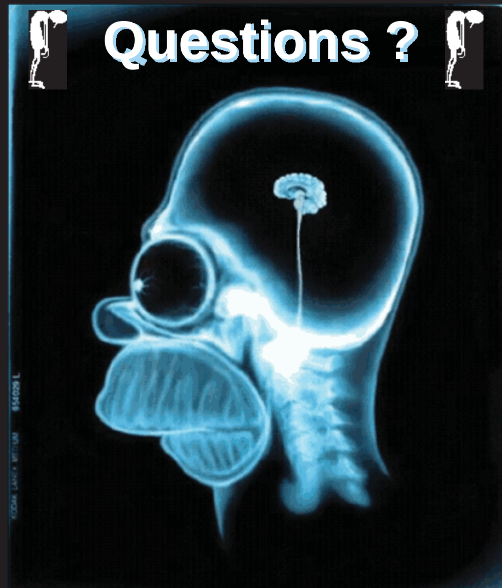
Degree of randomness, statistical disorder

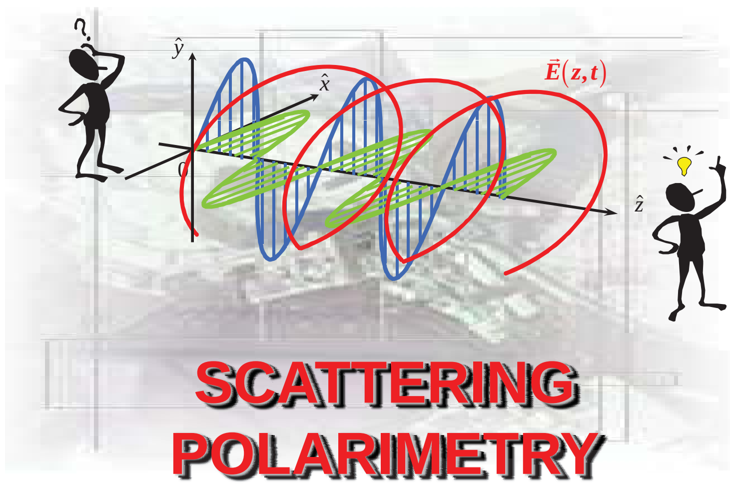
PARTIALLY POLARISED WAVES



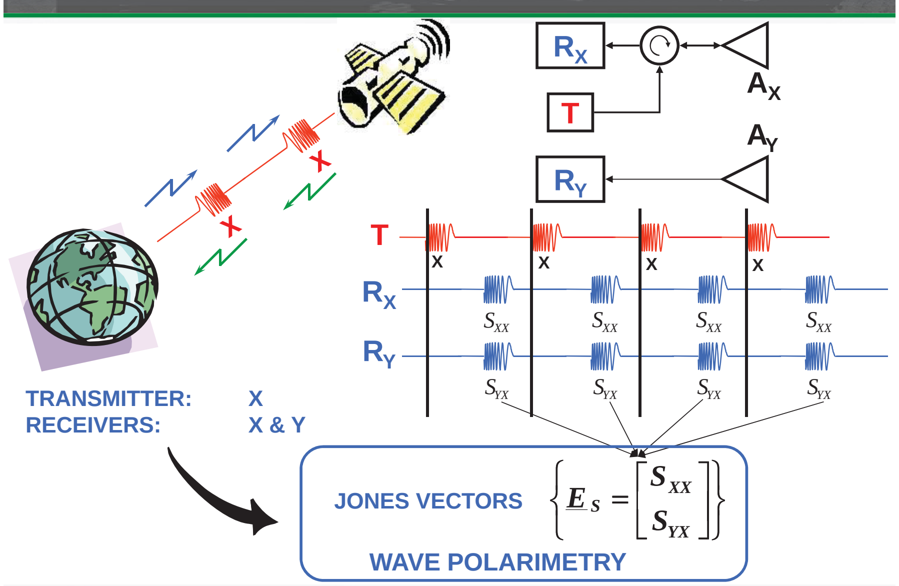
PARTIALLY POLARISED WAVES

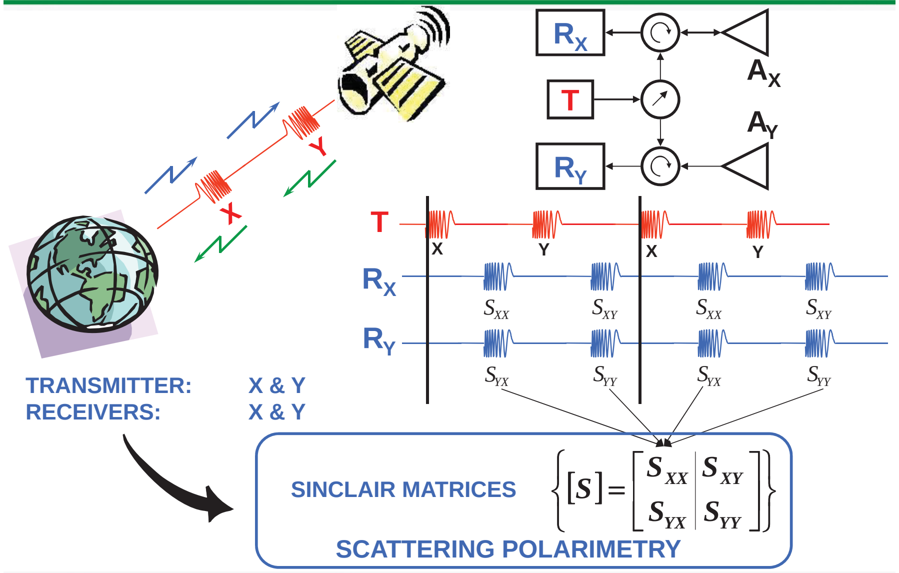


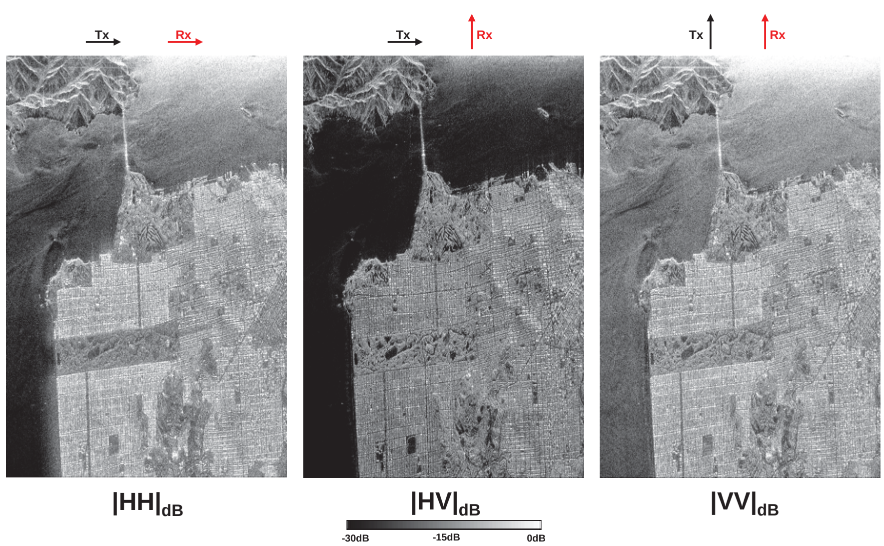




WAVE POLARIMETRY







Sinclair Color Coding

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|HH| |HV| |VV|

$$\left[S_{(B,B_\perp)}\right]=\left[U_{(A,A_\perp)\mapsto(B,B_\perp)}\right]^T\left[S_{(A,A_\perp)}\right]\left[U_{(A,A_\perp)\mapsto(B,B_\perp)}\right]$$

CON-SIMILARITY TRANSFORMATION

$\left[U_{(A,A_\perp)\mapsto(B,B_\perp)}\right]$ SU(2) SPECIAL UNITARY ELLIPTICAL BASIS TRANSFORMATION MATRIX

$$\begin{aligned}\left[U_{(A,A_\perp)\mapsto(B,B_\perp)}\right] &= \left[U(\phi,\tau,\alpha)\right]^{-1}\\ &= \underbrace{\begin{bmatrix} e^{j\alpha} & 0 \\ 0 & e^{-j\alpha}\end{bmatrix}}_{[U_2(-\alpha)]}\underbrace{\begin{bmatrix} \cos(\tau) & -j\sin(\tau) \\ -j\sin(\tau) & \cos(\tau)\end{bmatrix}}_{[U_2(-\tau)]}\underbrace{\begin{bmatrix} \cos(\phi) & \sin(\phi) \\ -\sin(\phi) & \cos(\phi)\end{bmatrix}}_{[U_2(-\phi)]}\end{aligned}$$

ELLIPTICAL BASIS TRANSFORMATION esa

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(H,V) POLARISATION BASIS

|HH+VV| |HV | |HH-VV|

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ELLIPTICAL BASIS TRANSFORMATION esa

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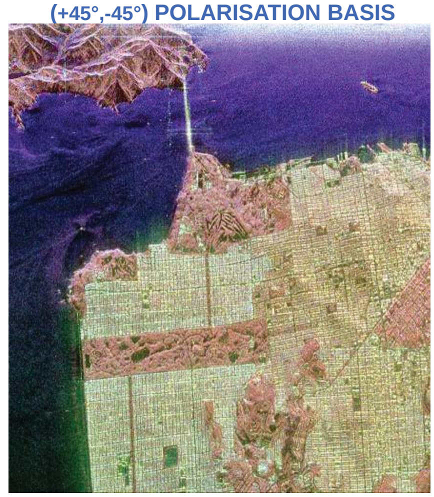
(+45°,-45°) POLARISATION BASIS



|AA+BB| |AB | |AA-BB|

With: A=Linear +45°, B=Linear –45°

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ELLIPTICAL BASIS TRANSFORMATION

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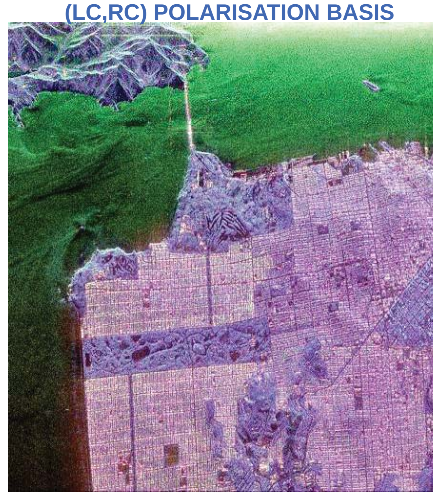
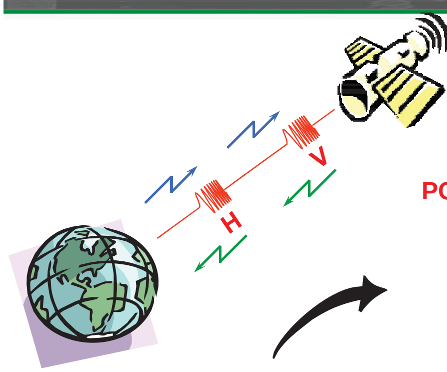
(LC,RC) POLARISATION BASIS



|LL+RR| |LR | |LL-RR|

SINCLAIR MATRIX



TRANSMITTER: H & V
RECEIVERS: H & V

POLARIMETRIC DESCRIPTORS

[S] SINCLAIR Matrix

$$[S]=\begin{bmatrix} S_{HH} & S_{HV} \\ S_{VH} & S_{VV} \end{bmatrix}$$

$\underline{k}$ Target Vector

[T] 3x3 COHERENCY Matrix

TARGET VECTOR

VECTORIZATION OF [S]

$$[S]=\begin{bmatrix} S_{HH} & S_{HV} \\ S_{HV} & S_{VV} \end{bmatrix} \Rightarrow \underline{k}=V([S])=\frac{1}{2}Trace([S][\psi])$$

SET OF 2x2 COMPLEX MATRICES
FROM THE PAULI MATRICES GROUP

$$[\psi_P]=\left\{\sqrt{2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \sqrt{2}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \sqrt{2}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\right\}$$

TARGET VECTOR

$$\underline{k}=\frac{1}{\sqrt{2}}\left[S_{HH}+S_{VV} \quad S_{HH}-S_{VV} \quad 2S_{HV}\right]^T$$

COHERENCY MATRIX

TARGET VECTOR $\underline{k}$

$$\underline{k} = \frac{1}{\sqrt{2}}\left[S_{HH}+S_{VV} \quad S_{HH}-S_{VV} \quad 2S_{HV}\right]^T$$

↓

COHERENCY MATRIX [*T*]

$$[T] = \underline{k}\cdot\underline{k}^{*T} = \begin{bmatrix} 2A_0 & C-jD & H+jG \\ C+jD & B_0+B & E+jF \\ H-jG & E-jF & B_0-B \end{bmatrix}$$

HERMITIAN MATRIX - RANK 1

A0, B0+B, B0-B : HUYNEN TARGET GENERATORS

↓

[*T*] is closer related to Physical and Geometrical Properties of the Scattering Process, and thus allows a better and direct physical interpretation

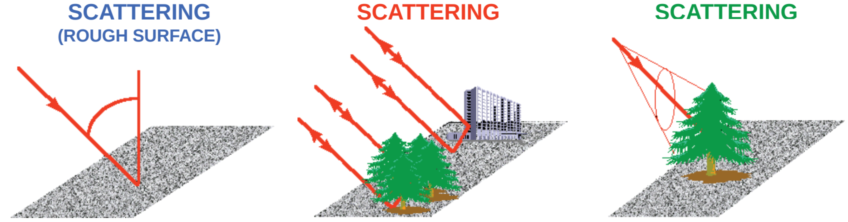
TARGET GENERATORS

PHYSICAL INTERPRETATION

SINGLE BOUNCE SCATTERING (ROUGH SURFACE)

DOUBLE BOUNCE SCATTERING

VOLUME SCATTERING

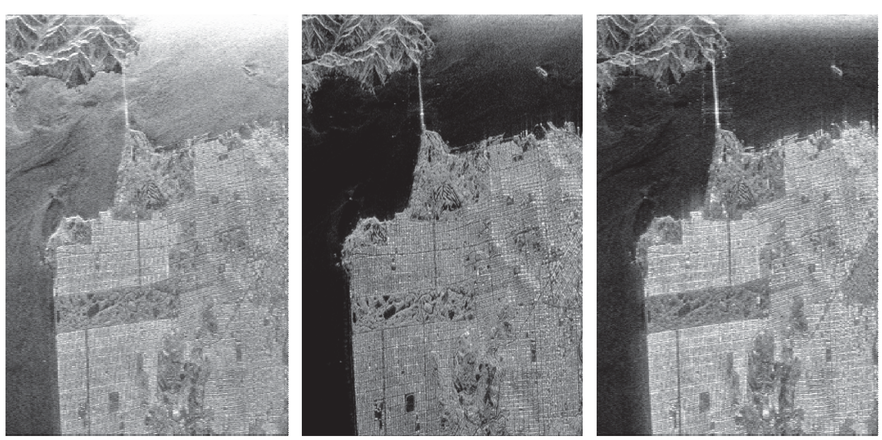


$$T_{11} = 2A_0 = \left|S_{XX}+S_{YY}\right|^2$$

$$T_{33} = B_0 - B = 2\left|S_{XY}\right|^2$$

$$T_{22} = B_0 + B = \left|S_{XX}-S_{YY}\right|^2$$

TARGET GENERATORS



$|HH+VV|_{dB}$ $|HV|_{dB}$ $|HH-VV|_{dB}$

TARGET GENERATORS

(H,V) POLARISATION BASIS

© Google Earth

|HH+VV| |HV | |HH-VV|

ELLIPTICAL BASIS TRANSFORMATION

SINCLAIR MATRIX

$$\underline{E}^{s}_{(A,A_\perp)} = \left[S_{(A,A_\perp)}\right] \underline{E}^{i}_{(A,A_\perp)} \qquad \underline{E}^{s}_{(B,B_\perp)} = \left[S_{(B,B_\perp)}\right] \underline{E}^{i}_{(B,B_\perp)}$$

$$\left[S_{(B,B_\perp)}\right] = \left[U_{(A,A_\perp)\mapsto(B,B_\perp)}\right]^{T} \left[S_{(A,A_\perp)}\right] \left[U_{(A,A_\perp)\mapsto(B,B_\perp)}\right]$$

CON-SIMILARITY TRANSFORMATION

COHERENCY MATRIX

$$\left[T_{(B,B_\perp)}\right] = \left[U_{3(A,A_\perp)\mapsto(B,B_\perp)}\right] \left[T_{(A,A_\perp)}\right] \left[U_{3(A,A_\perp)\mapsto(B,B_\perp)}\right]^{-1}$$

SIMILARITY TRANSFORMATION

$\left[U_{3(A,A_\perp)\mapsto(B,B_\perp)}\right]$ U(3) SPECIAL UNITARY ELLIPTICAL BASIS TRANSFORMATION MATRIX

ELLIPTICAL BASIS TRANSFORMATION

SPECIAL UNITARY SU(2) GROUP

$$[U_2] = \underbrace{\begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix}}_{[U_2(\phi)]} \underbrace{\begin{bmatrix} \cos(\tau) & j\sin(\tau) \\ j\sin(\tau) & \cos(\tau) \end{bmatrix}}_{[U_2(\tau)]} \underbrace{\begin{bmatrix} e^{-j\alpha} & 0 \\ 0 & e^{j\alpha} \end{bmatrix}}_{[U_2(\alpha)]}$$

SPECIAL UNITARY SU(3) GROUP

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(2\phi) & \sin(2\phi) \\ 0 & -\sin(2\phi) & \cos(2\phi) \end{bmatrix}}_{[U_3(2\phi)]} \underbrace{\begin{bmatrix} \cos(2\tau) & 0 & j\sin(2\tau) \\ 0 & 1 & 0 \\ j\sin(2\tau) & 0 & \cos(2\tau) \end{bmatrix}}_{[U_3(2\tau)]} \underbrace{\begin{bmatrix} \cos(2\alpha) & -j\sin(2\alpha) & 0 \\ -j\sin(2\alpha) & \cos(2\alpha) & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{[U_3(2\alpha)]}$$

TARGET EQUATIONS

POLARIMETRIC GOLDEN NUMBER

POLARIMETRIC TARGET DIMENSION

POLARIMETRIC TARGET DIMENSION

$$[S] = \begin{bmatrix} S_{XX} & S_{XY} \\ S_{YX} & S_{YY} \end{bmatrix}$$

5 DEGREES OF FREEDOM

$$|S_{XX}|, |S_{XY}|, |S_{YY}|$$

$$\phi_{XY-XX}, \phi_{YY-XX}$$

COHERENCY MATRIX $[T]$

9 HUYNEN REAL PARAMETERS

(*A0, B0, B, C, D, E, F, G, H*)

TARGET MONOSTATIC POLARIMETRIC « *DIMENSION* »

II

5

9 - 5 = 4 TARGET EQUATIONS

TARGET EQUATIONS

PURE TARGET – MONOSTATIC CASE

$$[T] = \underline{k} \cdot \underline{k}^{*T} = \begin{bmatrix} 2A_0 & C - jD & H + jG \\ C + jD & B_0 + B & E + jF \\ H - jG & E - jF & B_0 - B \end{bmatrix}$$

3x3 HERMITIAN MATRIX - RANK 1

9 PRINCIPAL MINORS = 0

$$2A_0(B_0 + B) - C^2 - D^2 = 0 \qquad 2A_0(B_0 - B) - G^2 - H^2 = 0$$

$$-2A_0 E + CH - DG = 0 \qquad B_0^2 - B^2 - E^2 - F^2 = 0$$

$$C(B_0 - B) - EH - GF = 0 \qquad -D(B_0 - B) + FH - GE = 0$$

$$2A_0 F - CG - DH = 0 \qquad -G(B_0 + B) + FC - ED = 0$$

$$H(B_0 + B) - CE - DF = 0$$

TARGET EQUATIONS

$$[S]=\begin{bmatrix} S_{XX} & S_{XY} \\ S_{YX} & S_{YY} \end{bmatrix}$$

5 DEGREES OF FREEDOM

$$|S_{XX}|,|S_{XY}|,|S_{YY}|$$

$$\phi_{XY-XX},\phi_{YY-XX}$$

COHERENCY MATRIX $[T]$

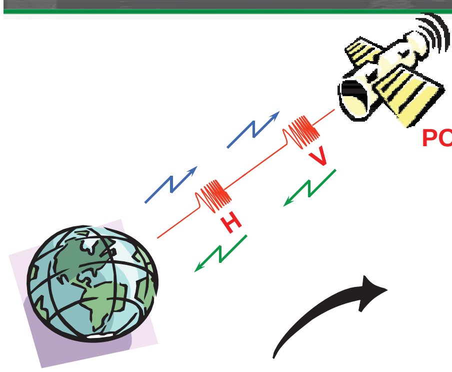
9 HUYNEN REAL PARAMETERS
(*A0, B0, B, C, D, E, F, G, H*)

TARGET MONOSTATIC POLARIMETRIC « *DIMENSION* »
=
5

9 - 5 = 4 TARGET EQUATIONS

$$\begin{aligned} 2A_0(B_0+B) &= C^2+D^2 \\ 2A_0(B_0-B) &= G^2+H^2 \\ 2A_0E &= CH-DG \\ 2A_0F &= CG+DH \end{aligned}$$

REMOTE SENSING



POLARIMETRIC DESCRIPTORS

[S] SINCLAIR Matrix

$$[S]=\begin{bmatrix} S_{HH} & S_{HV} \\ S_{VH} & S_{VV} \end{bmatrix}$$

$\underline{k}$ Target Vector

[T] 3x3 COHERENCY Matrix

TRANSMITTER: H & V
RECEIVERS: H & V

POLARIMETRIC REMOTE SENSING