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A Modified Wishart Distance Measure and Its Application to PolSAR Change Detection

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Outline

1. Introduction
2. Proposed Method
3. Experimental Results
4. Conclusions

Outline

1.1 Background

Sensor	Area	Polarization	Gray Statistics (real, imaginary)		
			Min	Max	Mean
ESAR	Oberpfaffenhofen ,Germany	VV	(-899.700623, -1821.804321)	(1189.208130, 725.402710)	**(0.008410, -0.006025)**
COSMO-SkyMed	Suzhou,China	HH	(-28844, -23923)	(32766, 28376)	**(0.014537, 0.001744)**
Radarsat-2	Suzhou,China	HV	(-32768, -32768)	(32767, 32767)	**(5.349097, -2.752530)**
TerraSAR-X	Suzhou,China	HH	0 (intensity)	32767 (intensity)	**152.247600 (intensity)**

For the whole SAR image, compared with the gray range, **the gray mean of real and imaginary components is considered as 0**. Therefore, the scattering vector follows a zero mean complex multivariate Gaussian distribution.

1.1 Background

- A sliding window will be employed for most of SAR application (Classification, Change Detection).

- In the small sample size, the scattering vectors will have a long correlation length, and **it is impossible to make their sum into 0 through mutual offset of positive and negative.**

- **Therefore, the 0 mean assumption of scattering vectors in the sliding window should be reconsidered.**

➢ 1.2 Objective

- When the PolSAR scattering vector is an independent nonzero-mean complex circular Gaussian random vector, their coherency matrix will follow a noncentral complex Wishart distribution.

- In this paper, based on the noncentral complex Wishart distribution, we propose a modified Wishart distance measure and apply it to PolSAR change detection.

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2.1 Noncentral Wishart distribution

- A complex multivariate Gaussian distribution is given by

$$f(k) = \frac{1}{\pi^p |\Sigma|} etr\left\{ -\Sigma^{-1} (k-\mu)(k-\mu)^H \right\}$$

and the estimators of the mean and covariance matrix are

$$\overline{\mu} = \sum_{l=1}^{L} k_l , \overline{\Sigma} = \frac{1}{L} \sum_{l=1}^{L} \left(k_l - \overline{\mu} \right) \left(k_l - \overline{\mu} \right)^H$$

- The p×p Hermitian matrix $W = \sum_{i=1}^{L} k_i k_i^H$ will follow a complex Wishart distribution.

$T = W / L$ **: polarimetric coherecy matrix**

➢ 2.1 Noncentral Wishart distribution

➢ Matrix W can be rewritten as

$$W = KK^H \quad \text{with} \quad K = [k_1 \;\; k_2 \cdots k_L]$$

➢ Note that A is a complex multivariate Gaussian matrix

$$A = K^H \in CN_{L\times p}\left(M, I_L \otimes \Sigma\right) \quad \text{where } M = E\left\{A\right\}$$

➢ Therefore, the PDF W is given by

Jacobian determinant

$$p_W\left(W; L, M, \Sigma\right) = J\left(K^H \rightarrow W\right) p_{K^H}\left(K^H; M, \Sigma\right)$$

$$= \frac{\left|W\right|^{L-p}}{\Gamma_p\left(N\right)\left|\Sigma\right|^L} etr\left[-\Sigma^{-1}\left(A-M\right)^H\left(A-M\right)\right]$$

2.1 Noncentral Wishart distribution

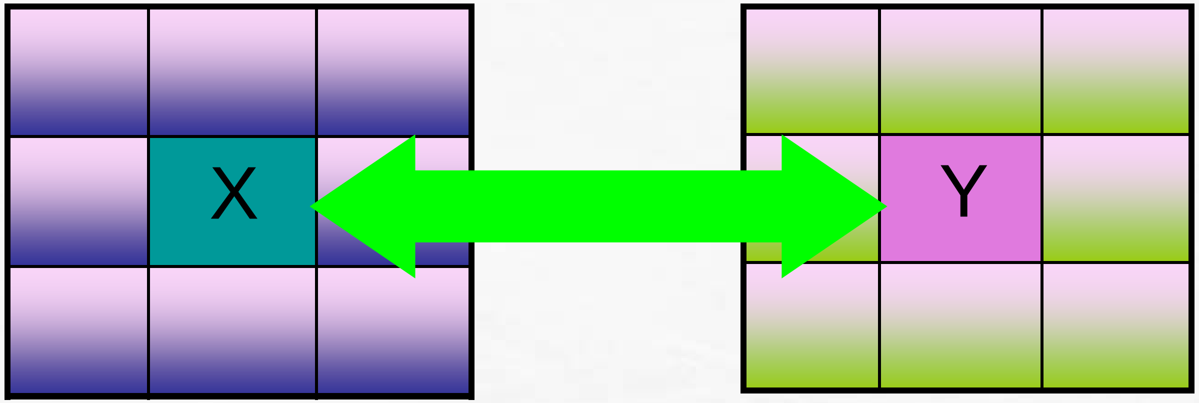
- With $\Omega = \Sigma^{-1} M M^H$, the PDF of W can be recast in the form

$$p_W\left(W; L, M, \Sigma\right) = \frac{\left|W\right|^{L-p}}{\Gamma_p\left(N\right)\left|\Sigma\right|^L} etr\left(-\Sigma^{-1}W\right) etr\left(-\Omega\right) {}_0F_1\left(L; \Omega\Sigma^{-1}W\right)$$

$$W \in CW_{p\times p}\left(L, \Omega, \Sigma\right)$$

- When $M = 0$, W is the (central) Wishart distribution.
- When $M = 0$, W is the noncentral Wishart (NW) distribution.

➢ 2.2 Test for equality of two NW matrices



Hypothesis Test

$$\begin{cases} H_0 : \mu_X = \mu_Y \ \text{and} \ \Sigma_X = \Sigma_Y \\ H_1 : \mu_X \neq \mu_Y \ \text{or} \ \Sigma_X \neq \Sigma_Y \end{cases}$$

Generalized Likelihood Ratio Test

$$Q = \frac{\max_{H_0} L(W_X, W_Y)}{\max L(W_X, W_Y)}$$

$$D = -\ln Q$$

➢ 2.2 Test for equality of two NW matrices

$$H_0 : \mu_X = \mu_Y \text{ and } \Sigma_X = \Sigma_Y$$

$$H_0^{(1)} : \Sigma_X = \Sigma_Y$$

$$H_0^{(2)} : \mu_X = \mu_Y, \text{ given } \Sigma_X = \Sigma_Y$$

$$Q_1 = \frac{\max_\Sigma L(W_X, W_Y)}{\max_{\Sigma_X, \Sigma_Y} L(W_X, W_Y)}$$

$$Q_2 = \frac{\max_u L(\Sigma)}{\max_{u^{(X)}, u^{(Y)}} L(\Sigma)}$$

$$\boxed{Q = Q_1 Q_2}$$

➢ 2.2 Test for equality of two NW matrices

$$H_0^{(1)} : \Sigma_X = \Sigma_Y \qquad Q_1 = \frac{\max_{\Sigma} L(W_X, W_Y)}{\max_{\Sigma_X, \Sigma_Y} L(W_X, W_Y)}$$

Denominator	$\max_{\Sigma_X,\Sigma_Y} L(W_X, W_Y) = \frac{\lvert W_X\rvert^{L_X - p}}{\Gamma_p(L_X)\lvert\Sigma_X\rvert^{L_X}} \frac{\lvert W_Y\rvert^{L_Y - p}}{\Gamma_p(L_Y)\lvert\Sigma_Y\rvert^{L_Y}},$	$\bar{\Sigma}_X = \sum_{i=1}^{L_X}\left(k_l^{(X)} - \overline{\mu}^{(X)}\right)\left(k_l - \overline{\mu}^{(X)}\right)^H \Big/ L_X$ $\bar{\Sigma}_Y = \sum_{i=1}^{L_Y}\left(k_l^{(Y)} - \overline{\mu}^{(Y)}\right)\left(k_l - \overline{\mu}^{(Y)}\right)^H \Big/ L_Y$
Numerator	$\max_{\Sigma_X,\Sigma_Y} L(W_X, W_Y) = \frac{\lvert W_X\rvert^{L_X - p}}{\Gamma_p(L_X)\lvert\Sigma\rvert^{L_X}} \frac{\lvert W_Y\rvert^{L_Y - p}}{\Gamma_p(L_Y)\lvert\Sigma\rvert^{L_Y}},$	$\bar{\Sigma} = \left(L_X \bar{\Sigma}_X + L_Y \bar{\Sigma}_Y\right) \Big/ \left(L_X + L_Y\right)$

$$Q_1 = \left|\bar{\Sigma}_X\right|^{L_X} \left|\bar{\Sigma}_Y\right|^{L_Y} \Big/ \left|\bar{\Sigma}\right|^{L_X + L_Y}$$

➢ 2.2 Test for equality of two NW matrices

$$H_0^{(2)} : \mu_X = \mu_Y, \ \text{given } \Sigma_X = \Sigma_Y \qquad Q_2 = \frac{\max_u L(\Sigma)}{\max_{u^{(X)},u^{(Y)}} L(\Sigma)}$$

Denominator	$\max_{u^{(X)},u^{(Y)}} L(\Sigma) = \pi^{-Lp} \lvert \Sigma_\Omega \rvert^{-L},$	$\bar{\Sigma}_\Omega = \frac{1}{L_X + L_Y} \sum_{l=1}^{L_X} \left(k_l^{(X)} - u^{(X)}\right)\left(k_l^{(X)} - u^{(X)}\right)^H + \frac{1}{L_X + L_Y} \sum_{l=1}^{L_Y} \left(k_l^{(Y)} - u^{(Y)}\right)\left(k_l^{(Y)} - u^{(Y)}\right)^H$
Numerator	$\max_u L(\Sigma) = \pi^{-Lp} \lvert \Sigma_\omega \rvert^{-L},$	$\bar{\Sigma}_\omega = \frac{1}{L_X + L_Y} \sum_{l=1}^{L_X} \left(k_l^{(X)} - u\right)\left(k_l^{(X)} - u\right)^H + \frac{1}{L_X + L_Y} \sum_{l=1}^{L_Y} \left(k_l^{(Y)} - u\right)\left(k_l^{(Y)} - u\right)^H$

$$Q_2 = \left|\bar{\Sigma}_\Omega\right|^L \Big/ \left|\bar{\Sigma}_\omega\right|^L \qquad Q_{non-central} = \frac{\left|\bar{\Sigma}_X\right|^{L_X} \left|\bar{\Sigma}_Y\right|^{L_Y}}{\left|\bar{\Sigma}_\omega\right|^{L_X + L_Y}}$$

➢ 2.2 Test for equality of two NW matrices

$$H_0 : \mu_X = \mu_Y \qquad \Sigma_X = \Sigma_Y$$

$$\Rightarrow \quad Q_{non-central} = \frac{\left|\bar{\Sigma}_X\right|^{L_X} \left|\bar{\Sigma}_Y\right|^{L_Y}}{\left|\bar{\Sigma}_\omega\right|^{L_X+L_Y}}$$

$$H_0^{(1)} : \Sigma_X = \Sigma_Y$$

$$\Rightarrow \quad Q'_{non-central} = \left|\bar{\Sigma}_X\right|^{L_X} \left|\bar{\Sigma}_Y\right|^{L_Y} \Big/ \left|\bar{\Sigma}\right|^{L_X+L_Y}$$

$$\xrightarrow{\mu=0} \quad Q_{central} = \frac{\left|W_X\right|^{L_X} \left|W_Y\right|^{L_Y}}{\left|W_X + W_Y\right|^{L_X+L_Y}} \frac{\left(L_X + L_Y\right)^{p(L_X+L_Y)}}{L_X{}^{pL_X} L_Y{}^{pL_Y}}$$

Two special cases of the proposed ratio

2.2 Test for equality of two NW matrices

Distribution	Hypothesis	Distance
Noncentral Wishart	$\mu_X = \mu_Y$ $\Sigma_X = \Sigma_Y$	$D_{non-central} = 2\ln\lvert\bar{\Sigma}_\omega\rvert - \ln\lvert\bar{\Sigma}_X\rvert - \ln\lvert\bar{\Sigma}_Y\rvert$
Noncentral Wishart	$\Sigma_X = \Sigma_Y$	$D'_{non-central} = 2\ln\lvert\bar{\Sigma}\rvert - \ln\lvert\bar{\Sigma}_X\rvert - \ln\lvert\bar{\Sigma}_Y\rvert$
Central Wishart	$\Sigma_X = \Sigma_Y$	$D_{central} = 2\ln\lvert W_X + W_Y\rvert - \ln\lvert W_X\rvert - \ln\lvert W_Y\rvert - 2p\ln 2$

The comparison of three Wishart Distances

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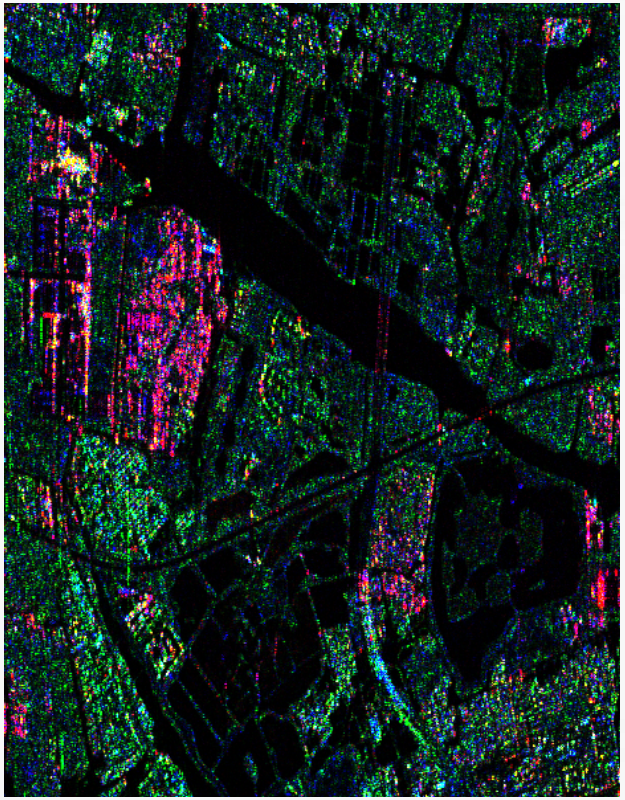
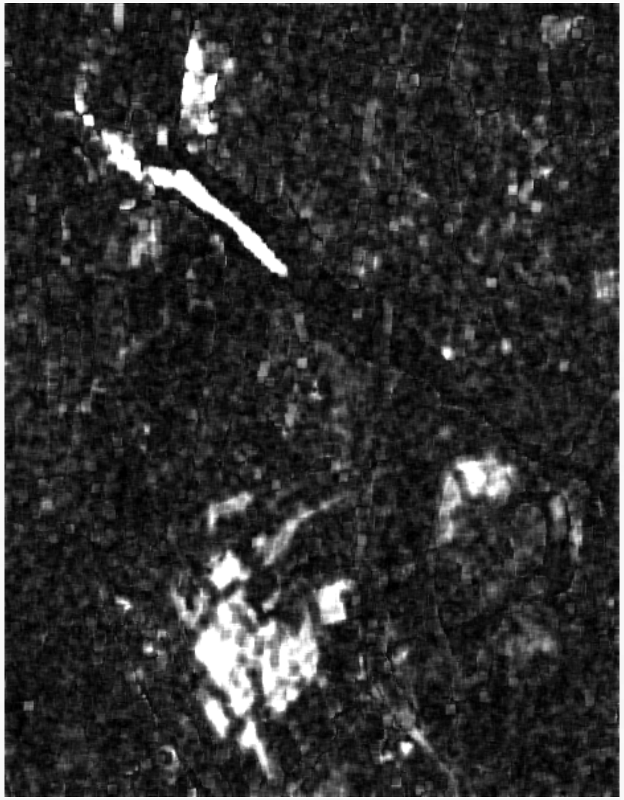
➢ 3.1 Experimental data

Sensor: RADARSAT-2
Time 1: 2009-04-09

Resolution: 10m×8m
Time 2: 2010-06-15



➢ 3.2 Experimental results



$D_{non-central}$

$D'_{non-central}$

$D_{central}$

➢ 3.2 Experimental results

- ➢ All the changed areas are detected successfully by the three distances.
- ➢ Unfortunately, the proposed modified Wishart distance has an advantage over the central Wishart distance **only in some small changed areas**.
- ➢ The difference of covariance matrix may be the main reason for SAR change detection.

➢ 3.2 Experimental results

Distance	Mean	Standard deviation	The equivalent number of looks
$D_{non-central}$	**0.605321**	0.492301	**1.512**
$D'_{non-central}$	0.415514	0.472729	0.773
$D_{central}$	0.415013	0.474311	0.765

- The proposed distance gives the maximum mean and ENL, which will contribute to the extraction of changed areas.

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1

Compared with the Wishart distance, the proposed non central Wishart distance will give a higher quality difference map for change detecton.

2

It seems that all changed areas can be detected by each distance. Therefore, further comparison, such as ROC curve and detection rate, should be added in the final version.

3

Some other related applications , such as classification, and segmentation, can be applied by replacing central Wishart distance with the proposed distance.

Thank you!