Wave Propagation Model for Coherent Scattering from a Randomly Distributed Target

Don Atwood, Ben Matthiss, Liza Jenkins, Shimon Wdowinski, Sang-Hoon Hong, and Batuhan Osmanoglu

Outline

- Requirements for Coherent Scattering from a Random Volume

- Two Environmental Examples:
  - Mangrove Forests
  - Arctic Frozen Lakes

- Conceptual and Mathematical Description of Virtual Bragg Gratings
  - Dihedral Bragg Grating for Co-Pol
  - Trihedral Bragg Grating for Cross-Pol

- Impact of approach on Polarimetric Decompositions

Requirements for a Coherent Response

1) Forward Scattering from Randomly Distributed Scatterers
   - Mie scattering regime satisfies requirement for Forward Scattering

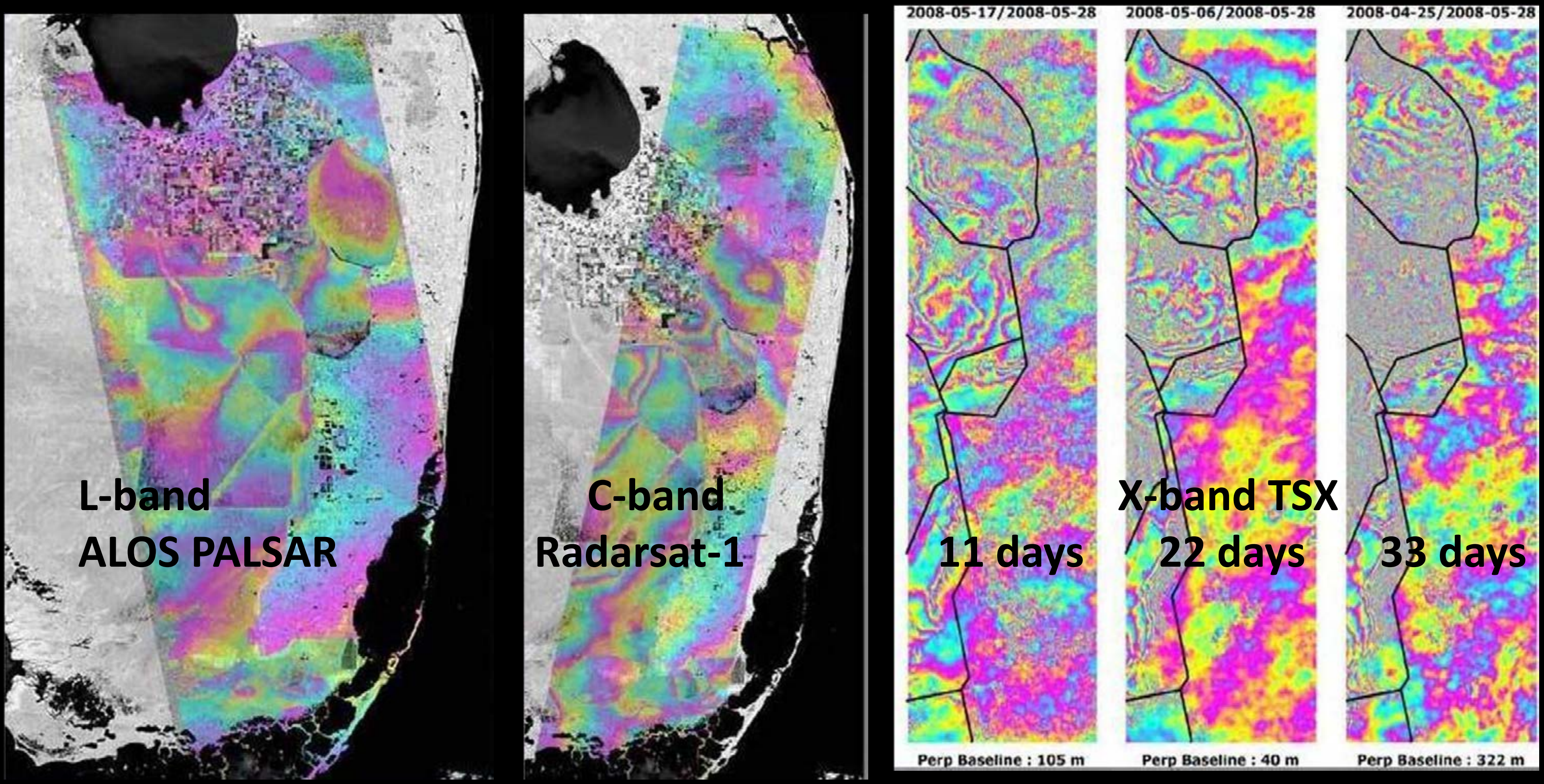
2) A Mirror surface that defines the geometry
   - Coherent Reflection can be turned On and Off by presence of the Mirror

Examples in Nature

- Two examples: Both characterized by random distribution of forward scatterers over a natural mirror
  - Mangroves (Everglades National Park)
  - Frozen Tundra Lakes (Pt. Barrow, Alaska)

Mangrove Characteristics

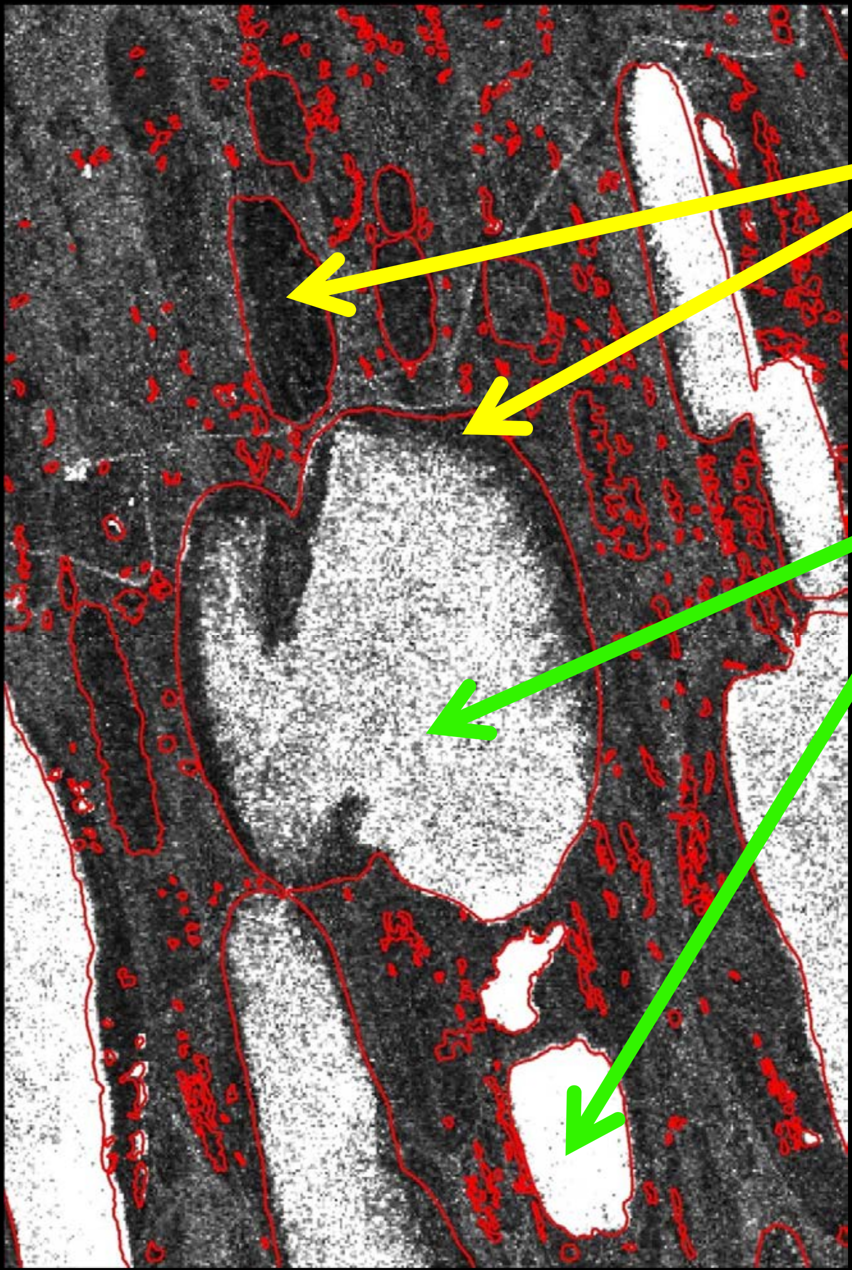


- InSAR used to measure water level in Florida Wetlands
  - Works for L, C, and X-bands

Mangrove Characteristics

- Random Vegetation over a water "mirror"
- Remarkably low temporal decorrelation for vegetation
  - Even X-band has coherence lasting months
- Coherence seen for both HH and HV
  - Require an explanation for coherent cross-pol
- Polarimetric decomposition does <u>not</u> indicate Double Bounce (the presumed scattering mechanism)

Frozen Tundra Lakes Characteristics



Grounded pond ice is dark

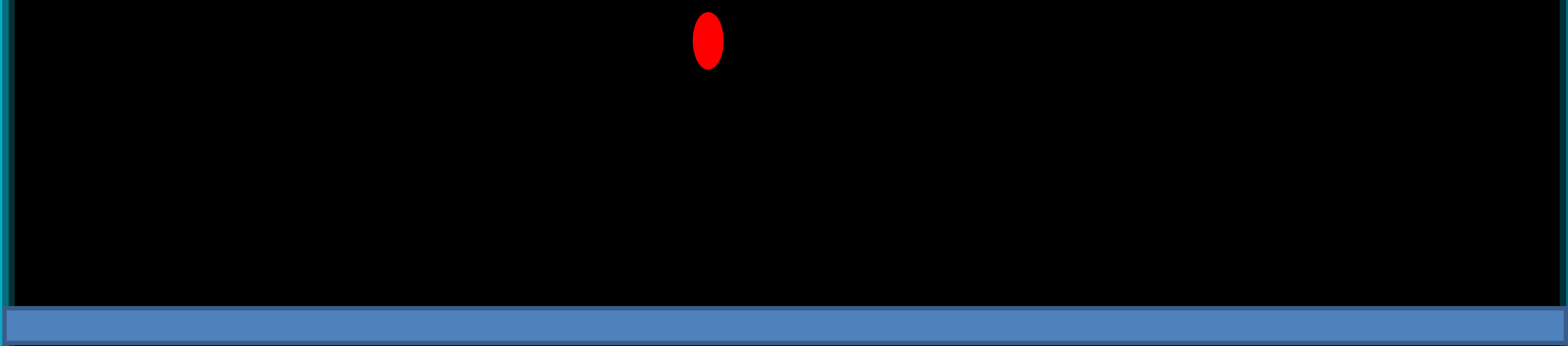
Floating pond ice is bright

- Used to characterize pond bathymetry

Frozen Tundra Lakes Characteristics

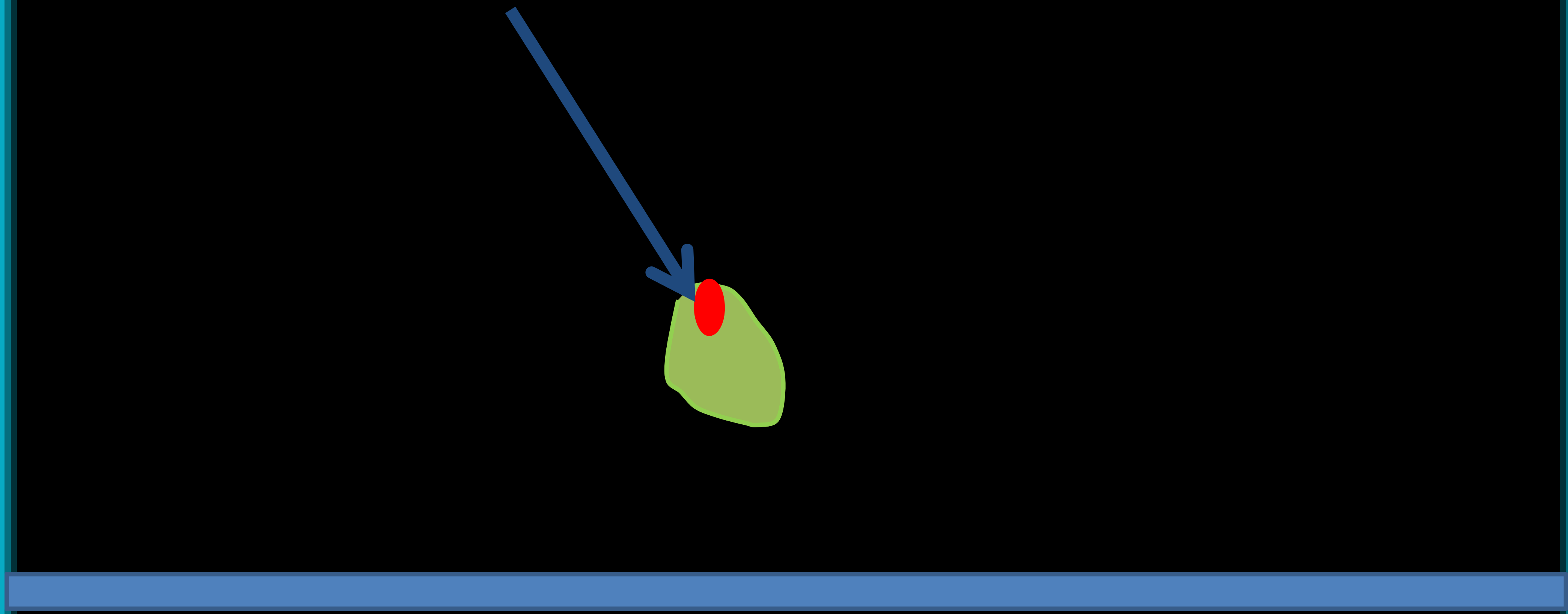
- Random bubbles over an ice/water interface “mirror”
- Single-pass InSAR Coherence of pond ice exceeds that of land
  - Response shown to come from bottom of ice
- Backscatter modulation seen for both HH and HV
  - Require an explanation for coherent cross-pol
- Polarimetric decomposition does <u>not</u> indicate Double Bounce (the presumed scattering mechanism)

Coherence from Randomly Distributed Scatterers



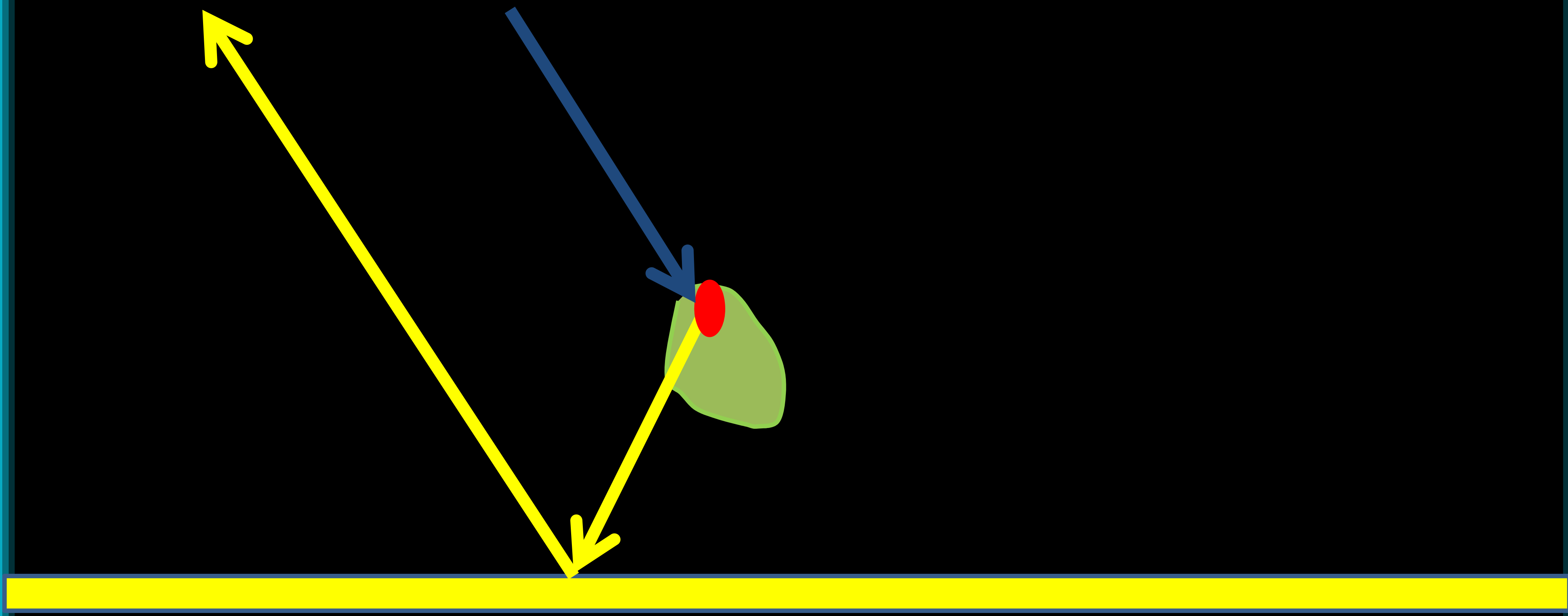
- Individual Scatterer
- A “perfect” Mirror

Coherence from Randomly Distributed Scatterers



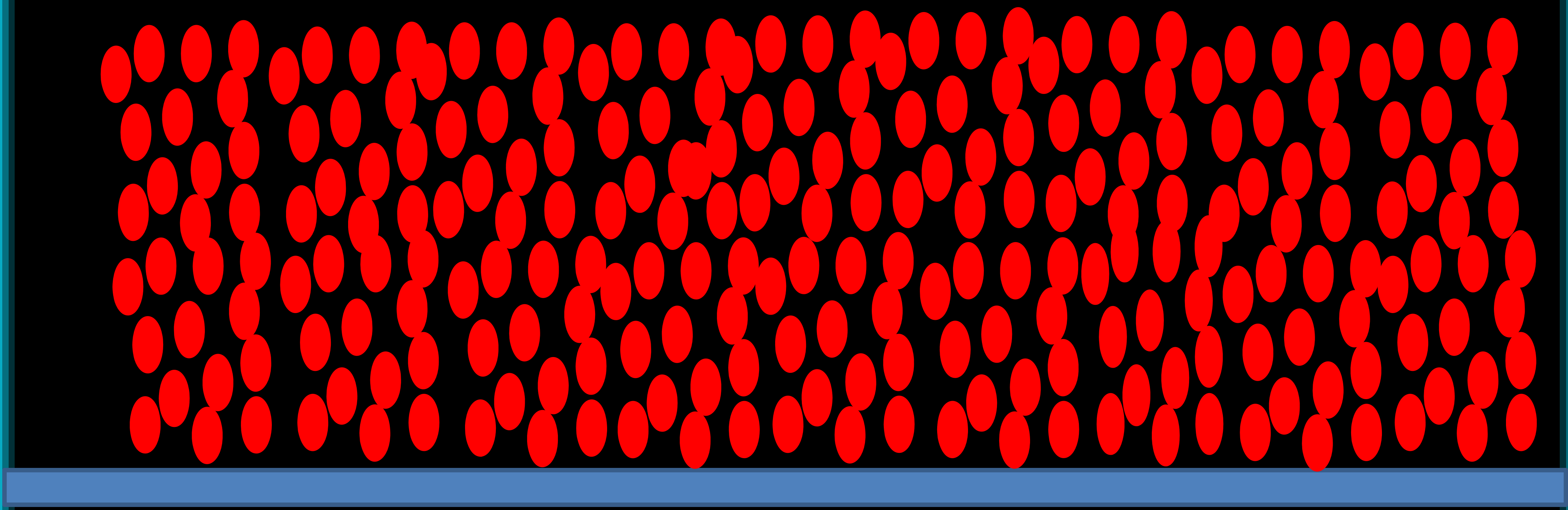
- Requirement of Forward Scattering
- Scattering details depend on case, but not essential to concept

Coherence from Randomly Distributed Scatterers



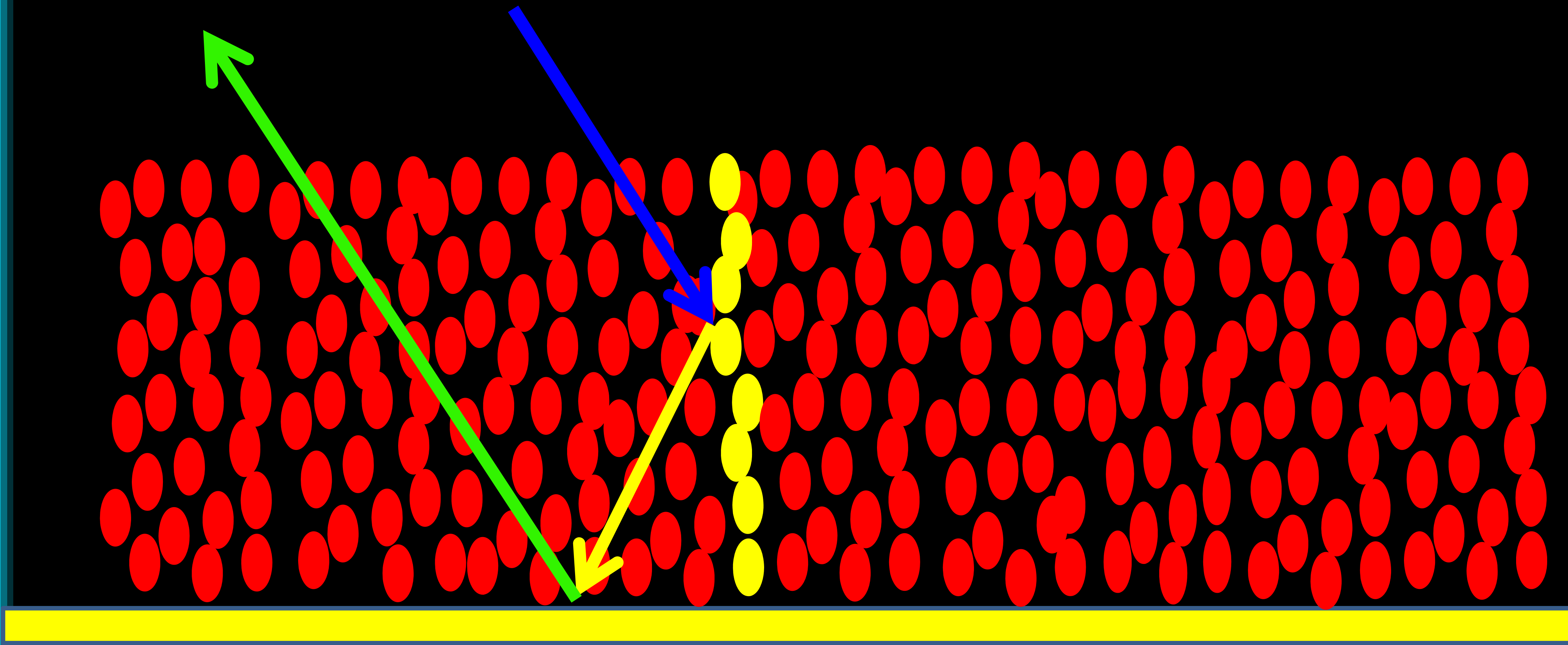
- Select for that scattering event in which photon undergoes reflection at mirror and returns to source
- Angle of Incidence = Angle of Reflection

Coherence from Randomly Distributed Scatterers



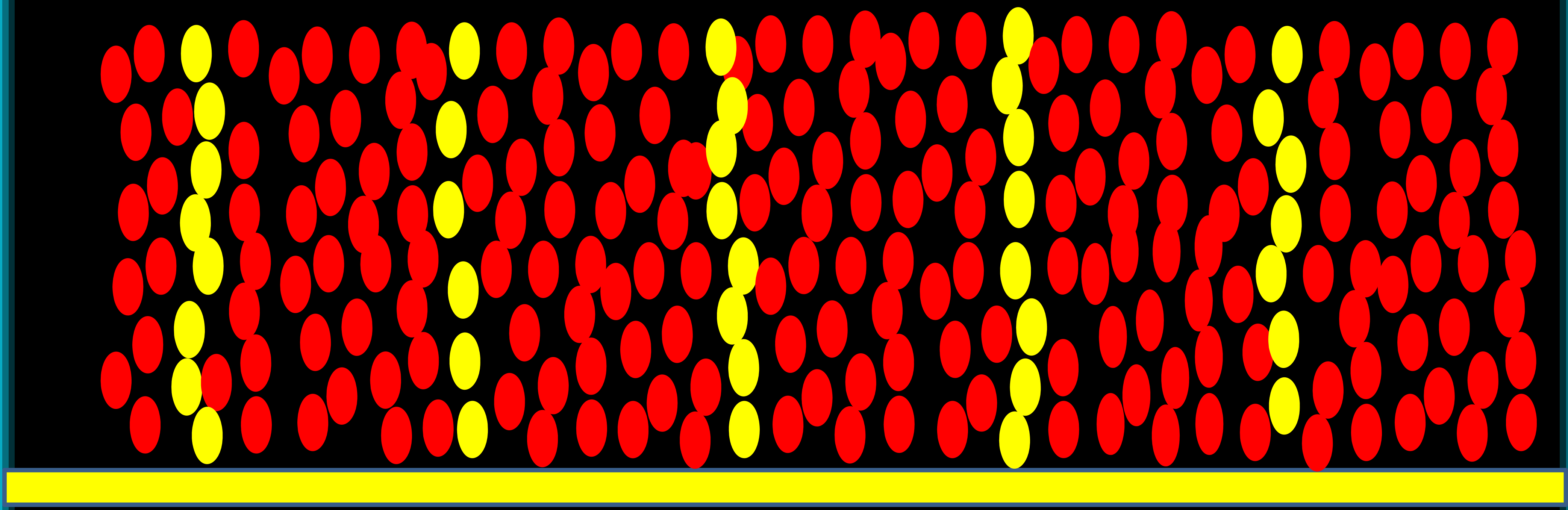
- Consider a large number of randomly distributed scatterers over the mirror

Coherence from Randomly Distributed Scatterers



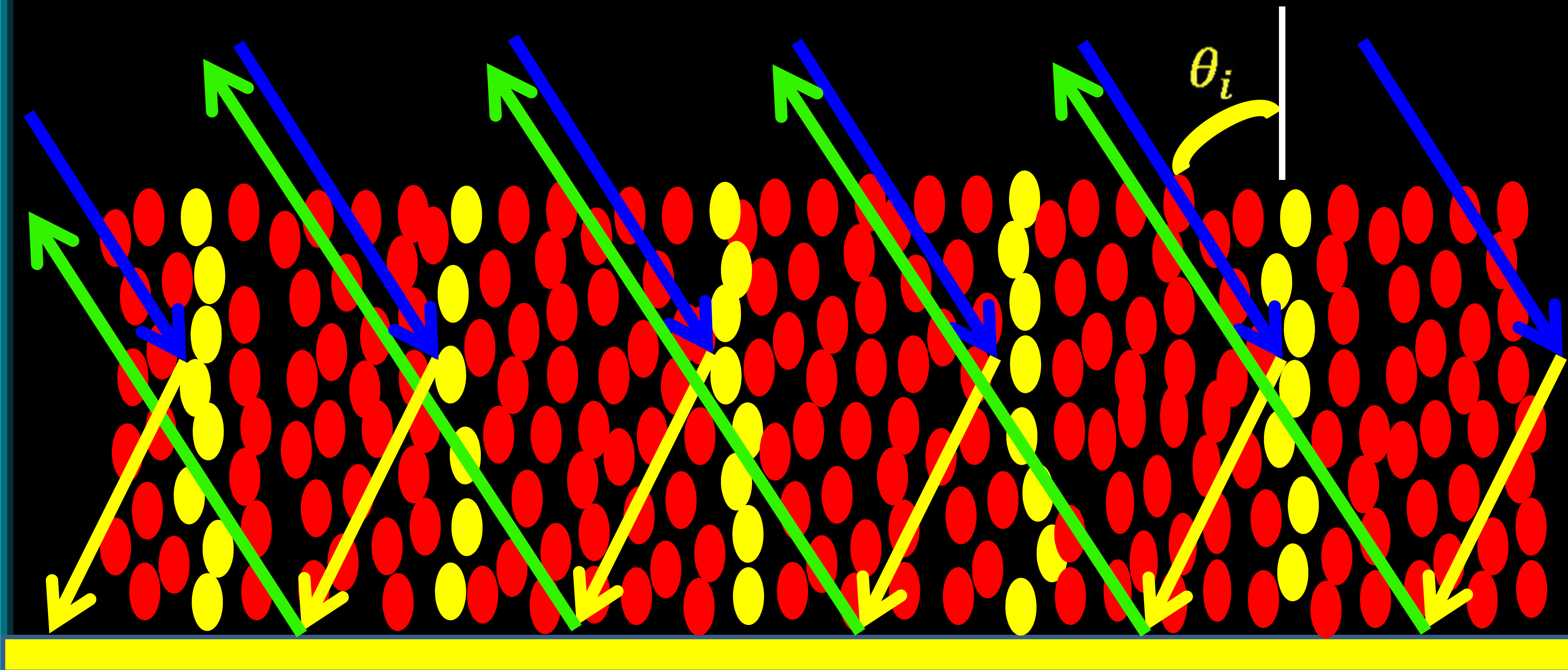
- A *Virtual Dihedral* exists within the random distribution of scatterers, for which there is coherent backscatter
- All microwaves scattering from this diheral are coherent: they have same path length and thus the same phase

Coherence from Randomly Distributed Scatterers



- Consider a *Set of* ***Virtual Dihedrals*** existing within the random distribution of scatterers

Coherence from Randomly Distributed Scatterers



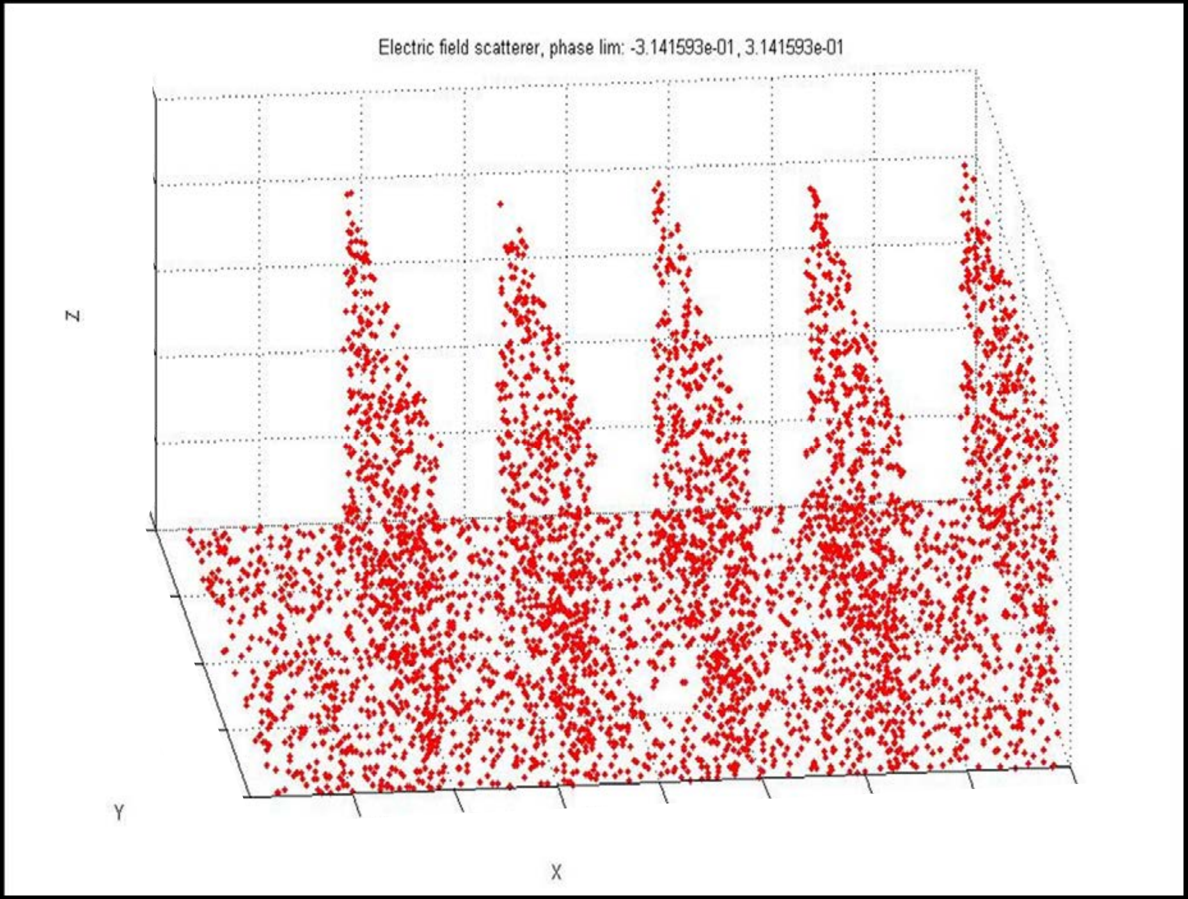
- If the spacing (d) between *Dihedrals* satisfies the Bragg Equation,

$$2d \sin \theta_i = \lambda$$

there will be coherent backscatter from a virtual *Dihedral Bragg Grating*

- Spacing, d, equals 23 cm for L-band at 30° incidence

Coherence from Randomly Distributed Scatterers



Plausibility of Dihedral Bragg Gratings is confirmed with Matlab model

- Introduce random volume of forward scatters over a reflective plane
- Constrain interactions to one scattering and one reflection
- Select for backscatter with a single phase
- Result is set of dihedrals satisfying the Bragg Equation

Wave Propagation within the Random Volume

Wave propagation associated with one such Dihedral Bragg grating can be expressed as:

$$\vec{E} = e^{i\omega t}\left[E_0 e^{i\vec{k_0}\vec{r}} + E_1 e^{i\vec{k_1}\vec{r}} + E_2 e^{i\vec{k_2}\vec{r}} + E_B e^{i\vec{k_B}\vec{r}}\right]$$

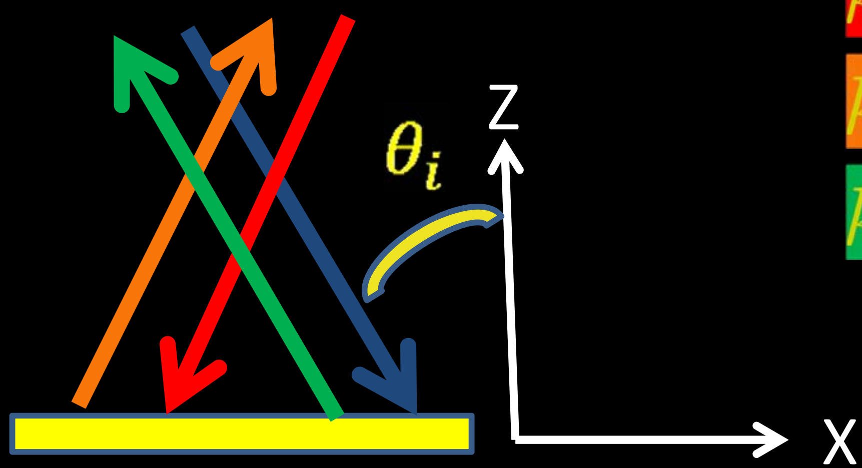
Where

$$\vec{k_0} = -k_0 \cos\theta_i \hat{z} + k_0 \sin\theta_i \hat{x}$$

$$\vec{k_1} = -k_0 \cos\theta_i \hat{z} - k_0 \sin\theta_i \hat{x}$$

$$\vec{k_2} = k_0 \cos\theta_i \hat{z} + k_0 \sin\theta_i \hat{x}$$

$$\vec{k_B} = k_0 \cos\theta_i \hat{z} - k_0 \sin\theta_i \hat{x}$$



Dihedral Bragg Scattering

- We can think of microwaves as "finding the coherent scattering structures" within the random distribution

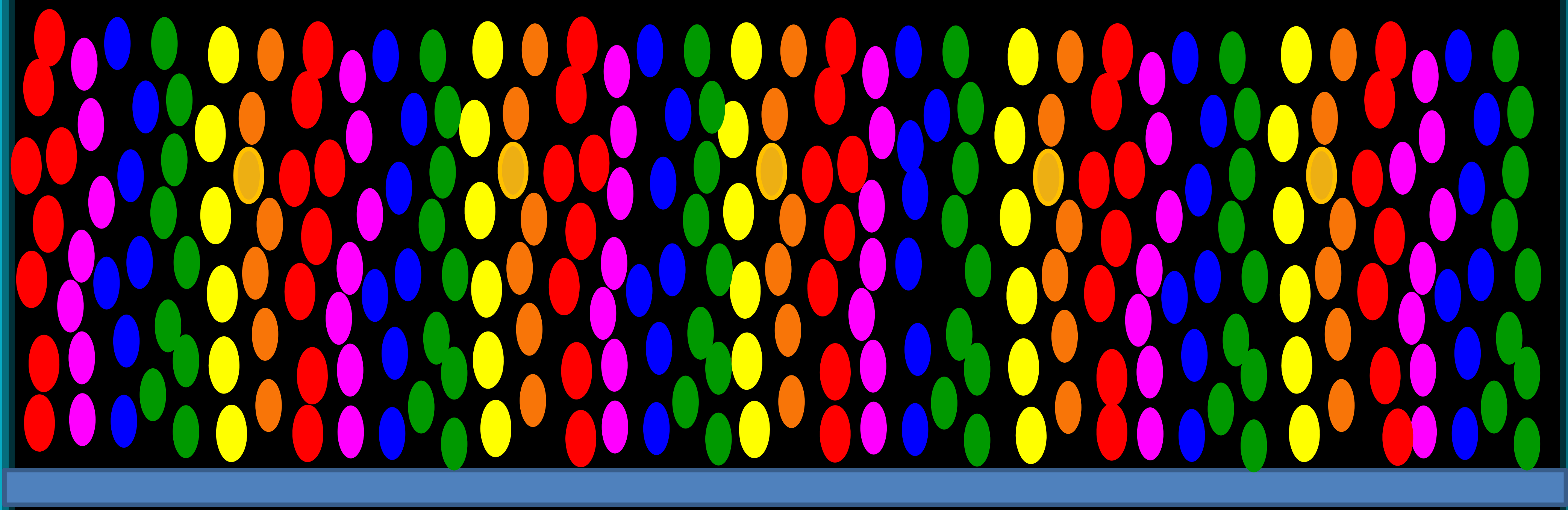
- Analogous to Bragg scattering from the ocean surface, where microwaves backscatter from that component of the wave spectrum that satisfies the Bragg equation:

$$\sigma_{pq} = 8\pi k^4 \left|G_s^{pp}(\theta)\right|^2 \left[\Psi(2\vec{k}_H) + \Psi(-2\vec{k}_H)\right]$$

where $\Psi(\vec{k})$ is the Fourier transform of the wave spectrum,
and $2\vec{k}_H = 2k\sin(\theta)$ is the Bragg wavenumber

Jackson, C. R., & Apel, J. R. (2004). *Synthetic aperture radar marine user's manual*. US Department of Commerce. Chapter 4: Microwave Scattering from the Sea, by Donald Thompson

Dihedral Bragg Scattering



For total return, must account for all such Bragg gratings

Computing the Backscatter Intensity

The complex amplitude associated with one such dihedral Bragg grating is

$$A(0)e^{ikr_0}$$

Summing over a total of N+1 such gratings yields a total amplitude given by:

$$A_T = \sum_{i=0}^{N} A(X_i)e^{i2k(r_0+\sin\theta X_i)}$$

Where θ is the incidence angle and X_i is the distance along the ground, which ranges from 0 to d (the grating spacing).

- From the Bragg formula , the phase, $2k\sin\theta\, X_i$, ranges from 0 to 2π.

Computing the Backscatter Intensity

Skipping a step or two, the backscatter intensity from the resolution cell can be written as:

$$I_T = \sum_0^N I_i + \sum_{i<>j} A_i A_j e^{i\Delta_{ij}} = \sum_0^N I_i + \sum_{i>j} 2A_i A_j \cos(\Delta_{ij})$$

where $\Delta_{ij} = 2k\sin\theta(X_i - X_j.)$ Assuming random values for A_i and A_j, we see that the sum of cosines tends to zero. Thus the total intensity is simply the incoherent sum of intensities from all the virtual Bragg gratings. Normalizing for area, the Normalized Radar Cross Section becomes:

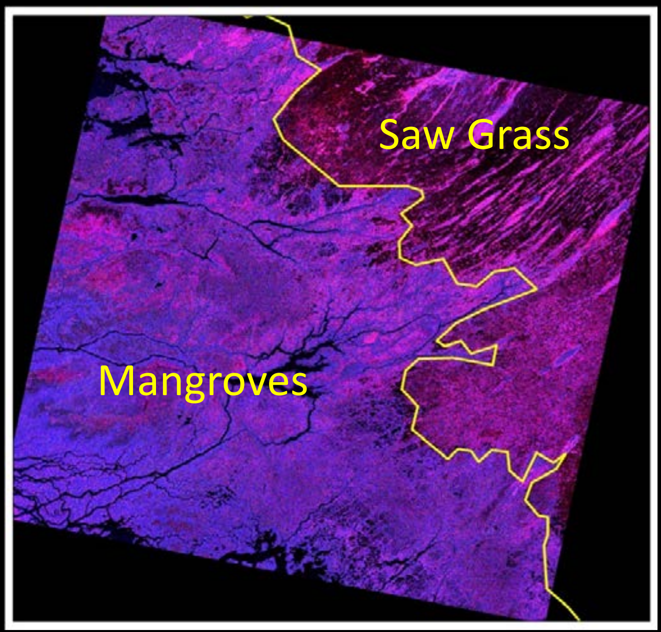
$$\sigma^0 = \sum_i \sigma_i^0$$

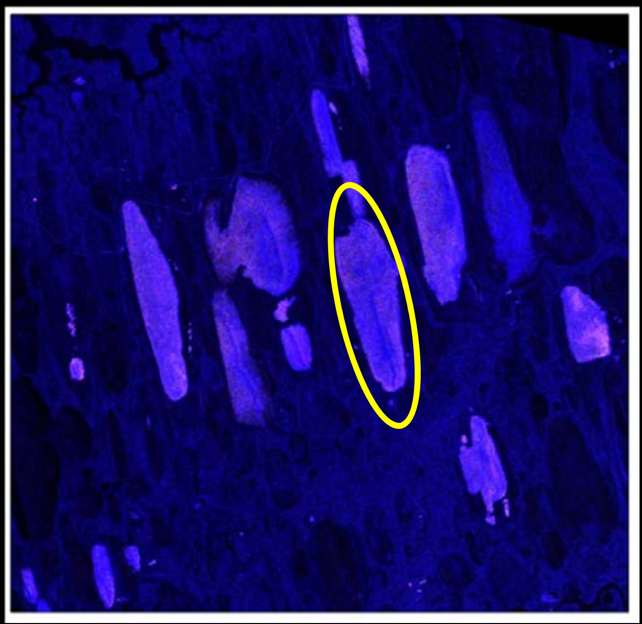
Polarimetric Decomposition

Most journal articles allude to an unspecified "Double Bounce" in explaining:

- the interferometry present in Mangroves
- the return from frozen lakes with liquid water beneath the ice

Yet, decompositions (e.g. Pauli, Van Zyl. And Yamaguchi) fail to show a strong Double Bounce component , dominated by the T22 coherency term



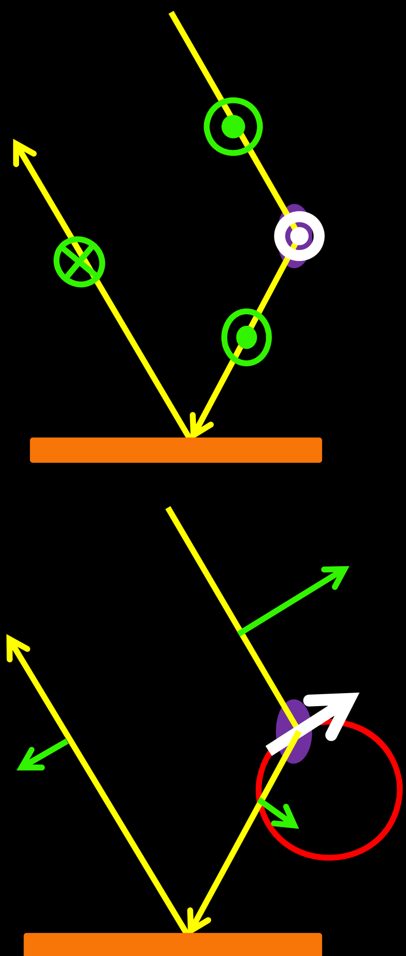


Pauli decompositions (HH-VV HV HH+VV) of mangroves and yellow Ecotone (left) and frozen lakes (right)

Polarimetric Characterization of Double Bounce

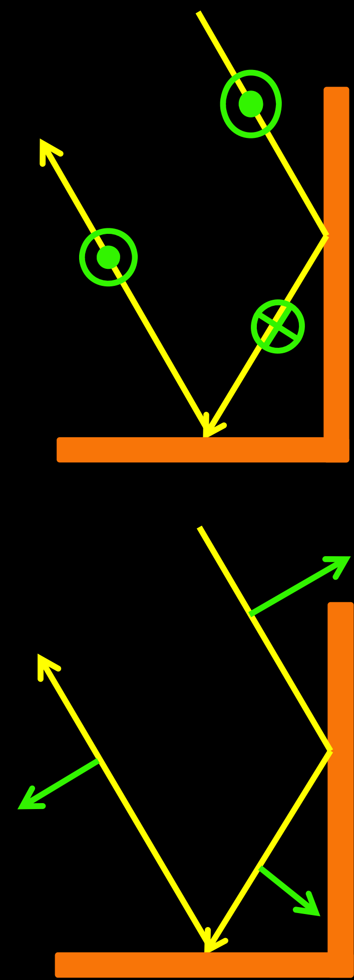
Assume that the individual scatterers behave as Hertzian Dipoles

(Al-Kahachi, N. and Papathanassiou, K., 2012)



The backscatter phases are opposite for Horizontal Polarization

The backscatter phases are the same for Vertical Polarization



Virtual Dihedral

“Classic” Dihedral

Polarimetric Characterization of Double Bounce

While Double Bounce for the “Classic” Dihedral is characterized as:

HH - VV

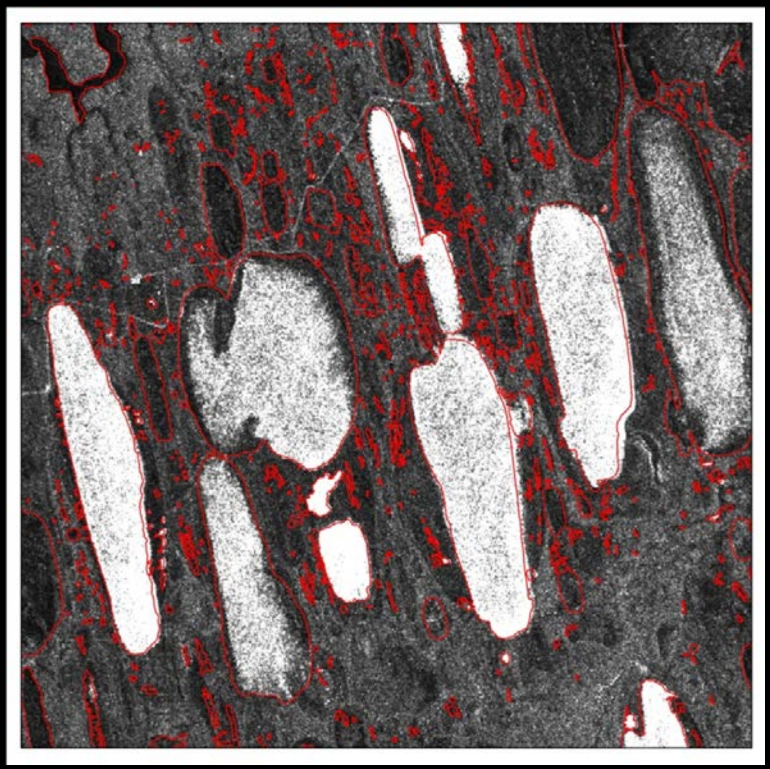
The Double Bounce for the Virtual Dihedral is characterized as:

HH + VV

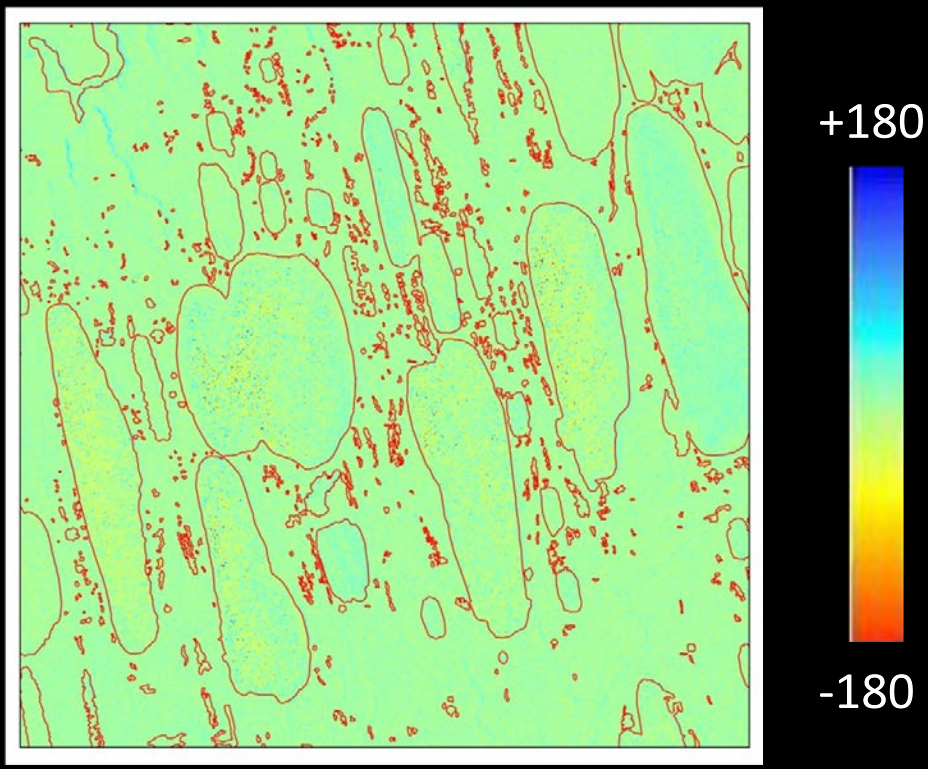
Thus, Virtual Dihedrals look like Surface Scattering

Polarimetric Characterization of Double Bounce

C-band Polarimetry shows that no phase shift exists between HH and VV



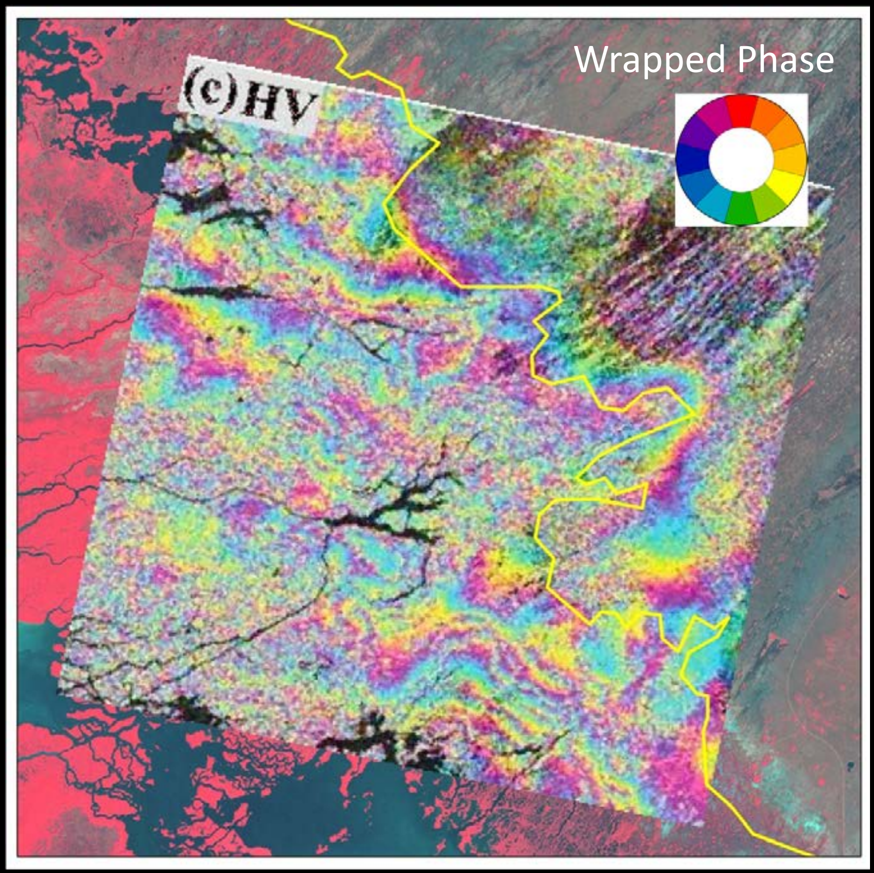
HH (same as VV)



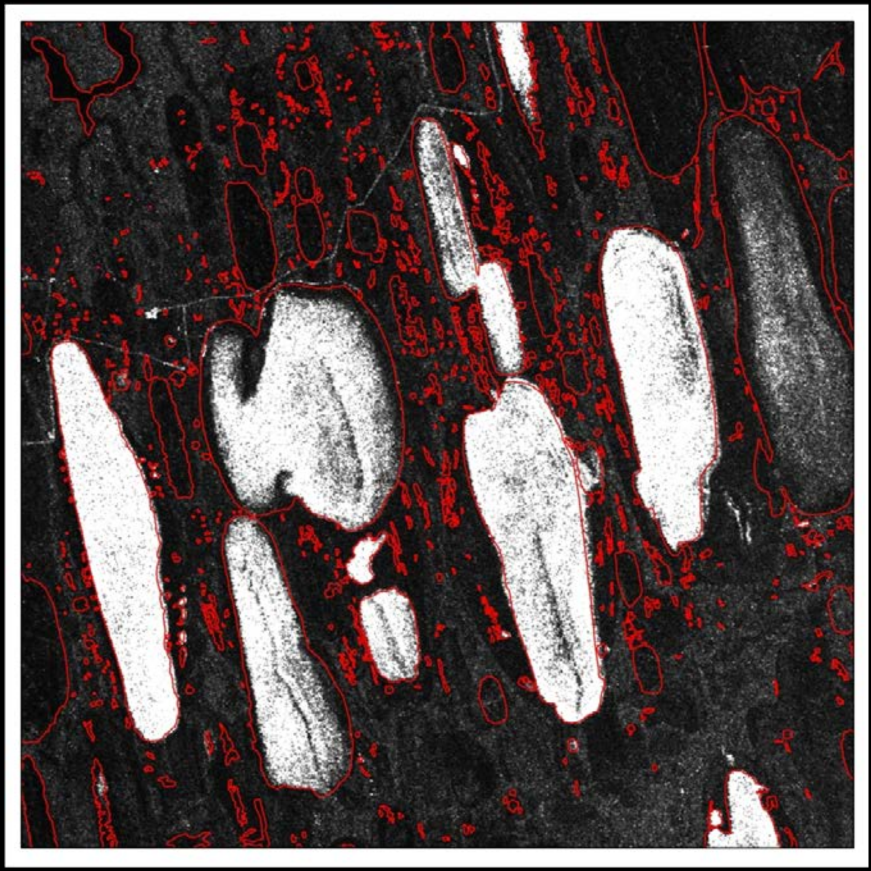
HH / VV Phase Difference

Coherent Cross-Pol

Cross-Pol Coherence exists in both environmental examples



HV Interferogram in Mangrove



HV Signal in Frozen Lakes

Mechanism for Coherent Cross-Pol

- Cross-Pol mechanism exhibits high coherence and very strong backscatter

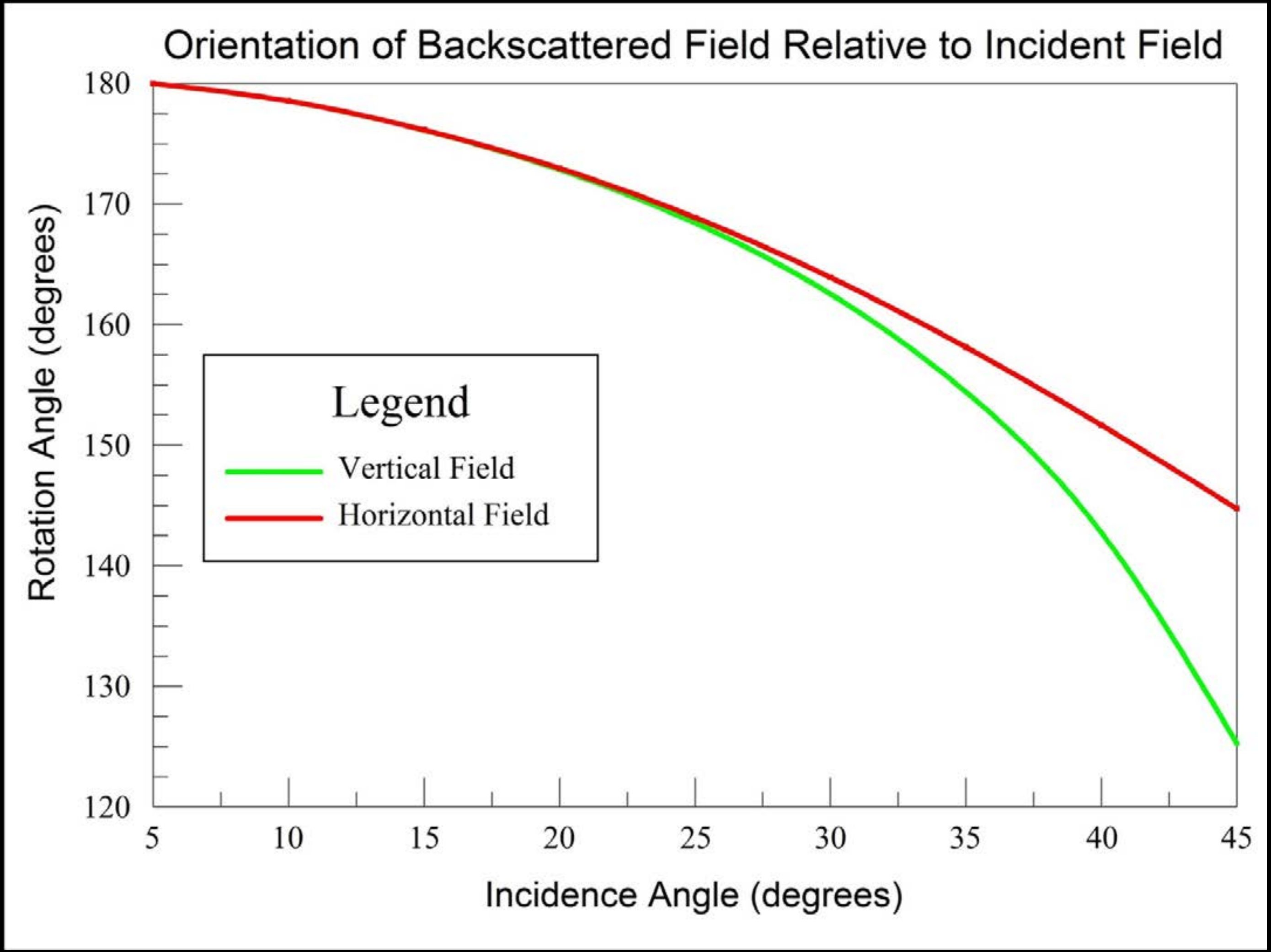
- Effect is turned on/off by the presence of a mirror below random volume of scatterers

- Any explanation must yield a coherent response <u>and</u> must rotate the polarization

Mechanism for Coherent Cross-Pol

- Effect can be explained by a Trihedral Bragg Grating: Microwaves scatter from diffuse targets twice and from the bottom mirror once
  - A fixed path length assures the coherence
  - Scattering from Hertzian Diploes rotates the polarization
- Effect is completely analogous to the Dihedral Bragg case

Coherent Rotation of Polarization

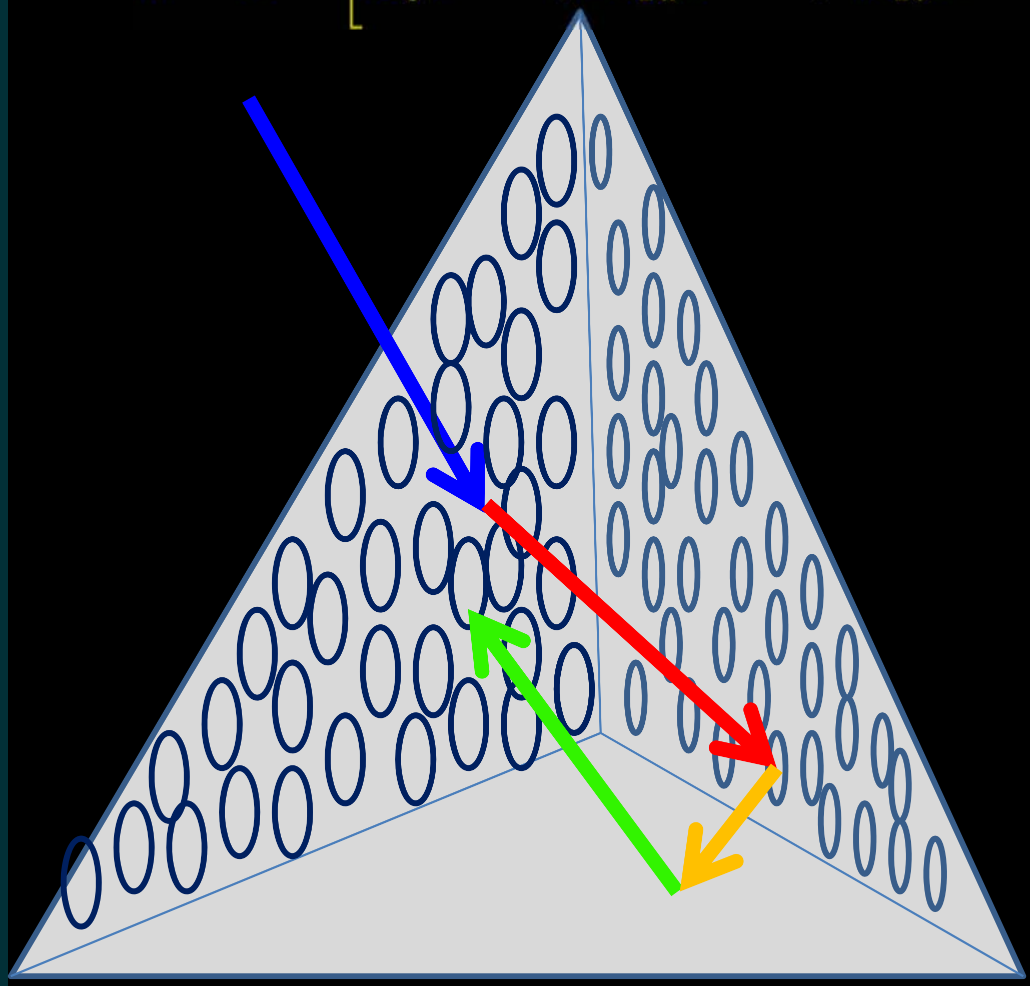


Unlike a metallic trihedral, the Hertzian dipoles of the distributed scatterers serve to rotate the electric field; introducing a coherent cross-pol term

Wave Propagation within the Random Volume

Wave propagation associated with one such trihedral Bragg grating can be expressed as:

$$\vec{E} = e^{i\omega t}\left[E_0 e^{i\vec{k}_0\vec{r}} + E_{1a} e^{i\vec{k}_1\vec{r}} + E_{1b} e^{-i\vec{k}_1\vec{r}} + E_{2a} e^{i\vec{k}_2\vec{r}} + E_{2b} e^{-i\vec{k}_2\vec{r}} + E_B e^{i\vec{k}_B\vec{r}}\right]$$



$$\vec{k_0} = -k_0 \cos\theta_i \hat{z} - \frac{k_0}{\sqrt{2}} \sin\theta_i (\hat{x} + \hat{y})$$

$$\vec{k_1} = -k_0 \cos\theta_i \hat{z} - \frac{k_0}{\sqrt{2}} \sin\theta_i (\hat{x} - \hat{y})$$

$$\vec{k_2} = -k_0 \cos\theta_i \hat{z} + \frac{k_0}{\sqrt{2}} \sin\theta_i (\hat{x} + \hat{y})$$

$$\vec{k_B} = +k_0 \cos\theta_i \hat{z} + \frac{k_0}{\sqrt{2}} \sin\theta_i (\hat{x} + \hat{y})$$

Summary and Conclusions

- A new scattering mechanism for a random volume of scatterers over a "mirror" has been introduced
- Like surface scattering from the ocean surface, it can be characterized as a form of Bragg Scattering
  - Co-pol backscatter can be viewed as double bounce from a *Dihedral Bragg Grating*
  - Cross-pol backscatter can be viewed as triple bounce from a *Trihedral Bragg Grating*
- The coherent response dominates over the millions of possible incoherent responses (due to the N^2 dependence)

Summary and Conclusions

- Virtual Dihedrals have a phase change that is inconsistent with the standard decomposition for Double Bounce (HH-VV)
- Two real-world environmental examples have been chosen: Ice bubbles over an ice/water interface and mangrove trees over water
- The new model serves to explain:
  - Strong Co-Pol and Cross –Pol Backscatter responses
  - Interferometric Coherence (with low temporal decorrelation)
  - Anomalous decomposition for the assumed Double Bounce behavior
  - Coherence in the HV signal which is typically seen as incoherent Volume Scattering

Questions, Comments?

Don Atwood
dkatwood@alaska.edu
(907) 474-7380