# Schatten Matrix Norm based Polarimetric SAR Data Regularization Application over Chamonix Mont-Blanc

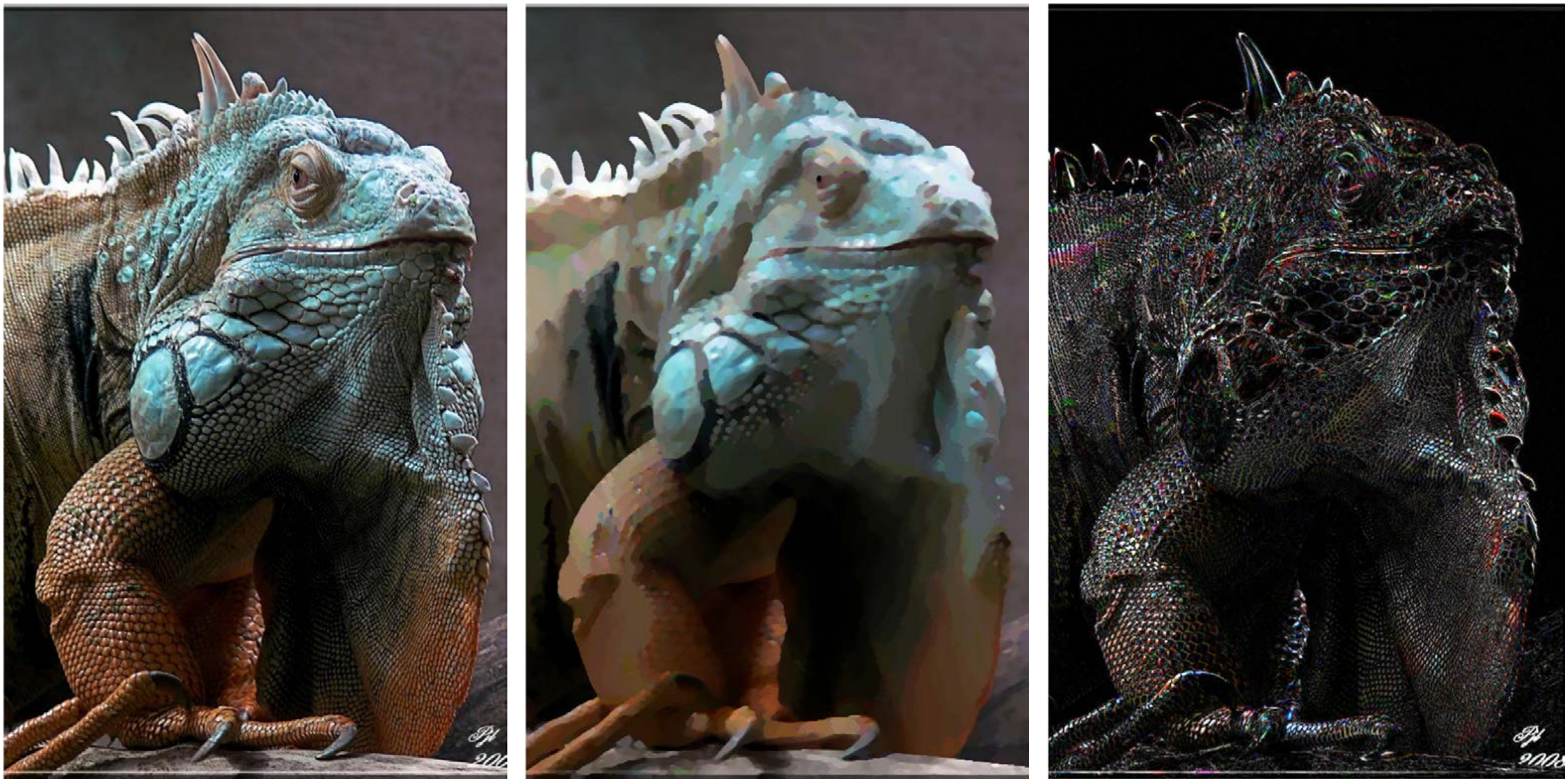
*T. T. Le, A. M. Atto, E. Trouvé*

Thu-Trang.Le@univ-savoie.fr

***Laboratoire d’Informatique, Système, Traitement de l’Information et de la Connaissance (LISTIC)***

***Université de Savoie***

# Regularization, texture separation…
# How to select the appropriate “norm” ?



# Limitations when considering the “L2 norm”

- New approaches developed in optical image filtering exploit **sparsity** in some **transform domain**.

  ➔ which transform for SAR data?
- The best **norm** is the one associated with the data manifold (Example: for a “point” living on the sphere, the best cost function to use should relate to the L2 norm).

  ➔ which norm for PolSAR data?
- **Sparsity** is the domain of the L0 pseudo norm and the Lp norms, for $0 < p <= 1$.

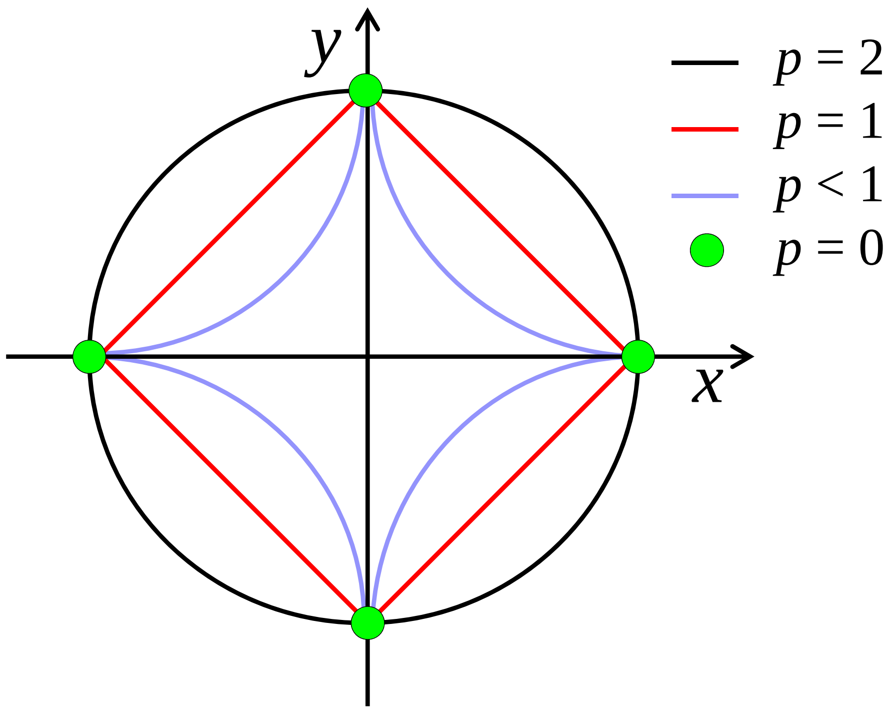
  ➔ which value of “p”?

# Motivations for "Lp norms, p≤1"

- Example

$$C_p(x,y) = \left(x - x_0\right)^p + \left(y - y_0\right)^p$$

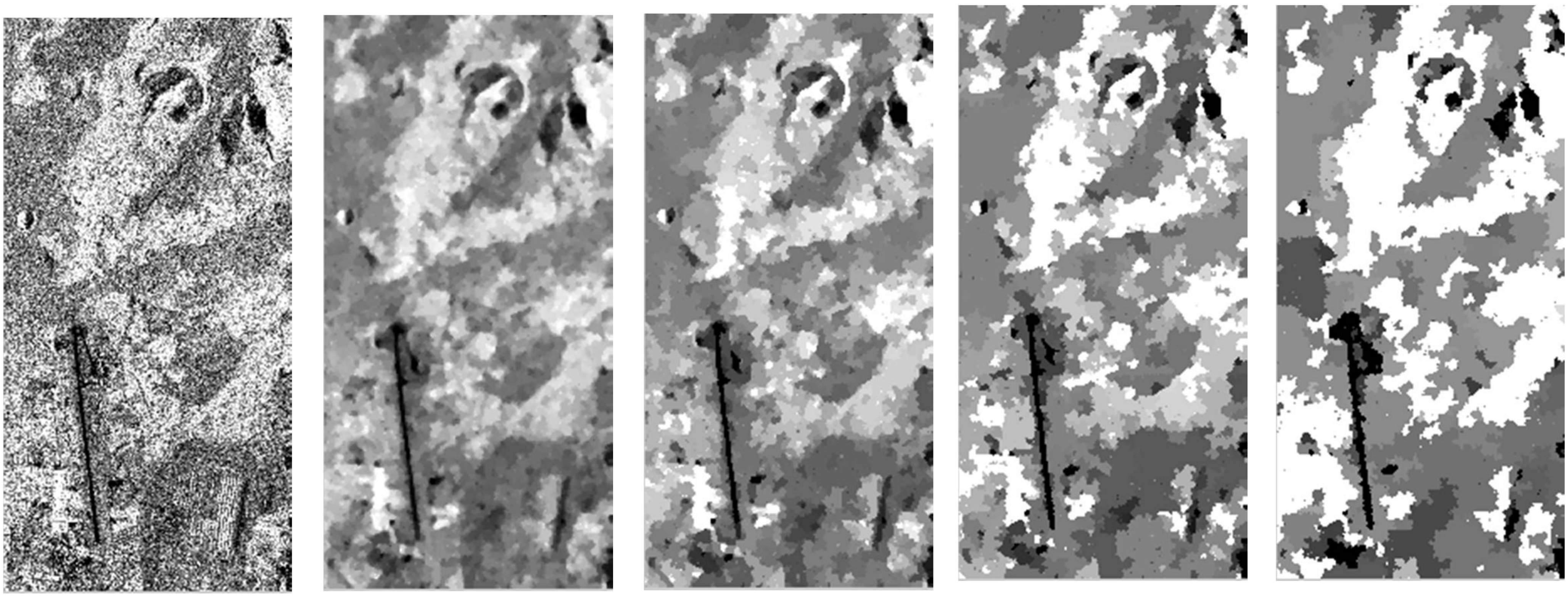
  - $p \geq 1$: $x$ and $y$ can be large
  - $p < 1$: 1 component dominates
    large one: information
    small one: noise → 0




- Difficulties
  - Regularization functionals are usually non-convex for 0<p<1.
  - Regularization methods associate L1 and L2 norms.

# Effect of varying p:

Original | p=1 | p=0.75 | p=0.5 | p=0.25



- Filtering results are optimal in the Lp-space where the data are assumed to live
- $0 \leq p \leq 1$ is the domain of sharp edge preservation

# Overview

- PolSAR data regularisation
  - Conventional approaches
  - Schatten-p norm
- Experimental results
  - Chamonix Mont-Blanc test-site
  - Early results on Radarsat-2 PolSAR data
- Conclusions and perspectives

# Coherent / distributed targets

- Coherent targets: Sinclair matrix / target vector

$$S_2 = \begin{bmatrix} S_{HH} & S_{HV} \\ S_{VH} & S_{VV} \end{bmatrix} \qquad k = \begin{bmatrix} S_{HH} + S_{VV} \\ S_{HH} - S_{VV} \\ 2S_{XX} \end{bmatrix}$$

- Non-coherent (distributed) targets: coherency matrix

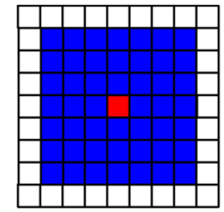
$$T_3 = E\left\{ kk^{*t} \right\} = \begin{bmatrix} T_{11} & T_{12} & T_{13} \\ T_{12}^* & T_{22} & T_{23} \\ T_{13}^* & T_{23}^* & T_{33} \end{bmatrix}$$

- Regularization:
  - Selection of adaptive windows/neighborhoods
  - Choice of the estimator (ML, LLMMSE…)

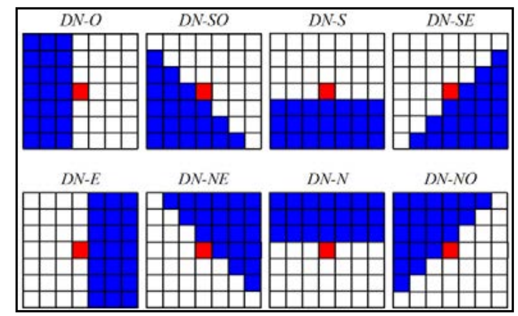
# Spatial adaptive support

- Boxcar windows
  - Fast
  - Edge preservation ?
- Directional neighborhoods
  - Lee filters
  - Only a few directions
- Intensity Driven Adaptive Neighborhoods
  - Intensity reveals discontinuities
  - Time consuming
- Segmentation / classification…
  - Needs robust algorithm, reliable information

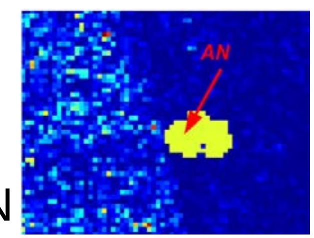
BN 7x7



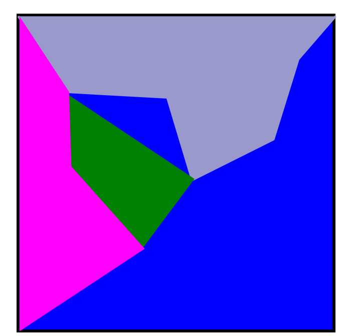
DN




IDAN



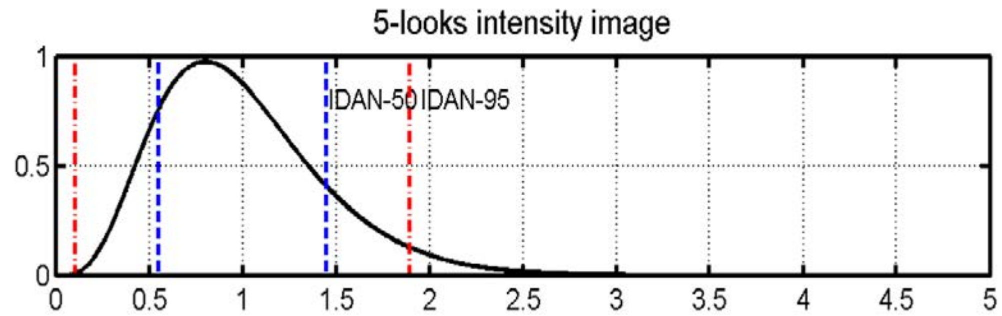

Segmentation

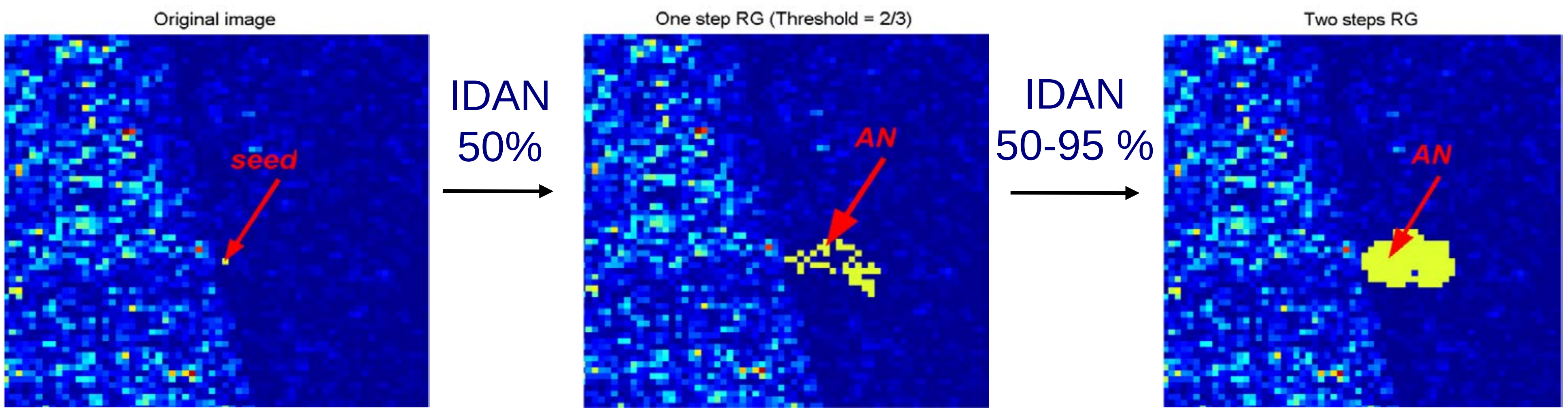


# Adaptive spatial supports

- Intensity Diven Adaptive Neighborhood [Vasile 06]
  - 2-step multivariate region growing
  - Agregation criterium:

$$\sum_{i=1}^{3} \frac{\left\| T_{ii}(m',n') - \hat{T}_{ii}(m,n) \right\|_2}{\left\| \hat{T}_{ii}(m,n) \right\|_2} \leq t.\frac{\sigma_n}{\mu_n}$$



# Estimator selection


- Complex Multi-Looking (ML)

$$\overline{T_{i,j}}(m,n)=\frac{1}{\# AN(m,n)}\sum_{(o,p)\in AN(m,n)} k_i(o,p)k_j^{*t}(o,p)$$

- Locally Linear Minimum Mean Square Error (LLMMSE [Lee 03])

$$T_{i,j}^{LLMMSE}(m,n)=b.T(m,n)+(1-b).\overline{T_{i,j}}(m,n)$$

$0 \leq b \leq 1$ weighting coefficient

# PolSAR data regularization: new approcahes exploiting several Lp norms

- Boxcar spatial neighborhood:

$$I[k,l,r] = \left\{ I(k+i, l+j)_{-r \le i \le r, -r \le j \le r} \right\}$$

- Cost function (Lp error of assuming that the spatial neighborhood of the pixel is constant):

$$f_p(I, I[k,l,r]) = \left\| I - I[k,l,r] \right\|_p$$

- Regularization:

$$\hat{I}(k,l) = \underset{I \in I[k,l,r]}{\arg\min} \left( f_p(I, I[k,l,r]) \right)$$

# PolSAR data regularization: new approaches exploiting several Lp norms

- Different norms exists for measuring the data similarity. For matrices, norms are obtained as:
  - Induced norms:
    describe the effect of the matrix operator on the norms associated with the input vectors
    $$\|M\|_p = \sup_{X \in S} \frac{\|MX\|_p}{\|X\|_p}$$
  - Entrywise norms:
    the matrix is considered as a MxN vector (M: number of rows, N: number of columns).
    $$\|M\|_p = \left( \sum |m_{ij}|^p \right)^{1/p}$$
  - Schatten norms:
    apply on the vector of singular values σ
    $$\|M\|_p = \left( \sum \sigma^p \right)^{1/p}$$
  - …

# Schatten matrix norms

- The Schatten-p norm of a matrix M is defined as the Lp norm of the vector of singular values of matrix M.
  - The Schatten-1 norm is also known as the nuclear norm.
  - The Schatten-2 norm is also called the Frobenius norm.
  - When p tends to infinity, the corresponding norm is called the spectral norm and is given by the largest singular values.

- Advantage of Schatten-p norms:
  singular value decomposition is a parsimonious decomposition in the sense that most of the energy of the matrix data is concentrated on the largest singular values.

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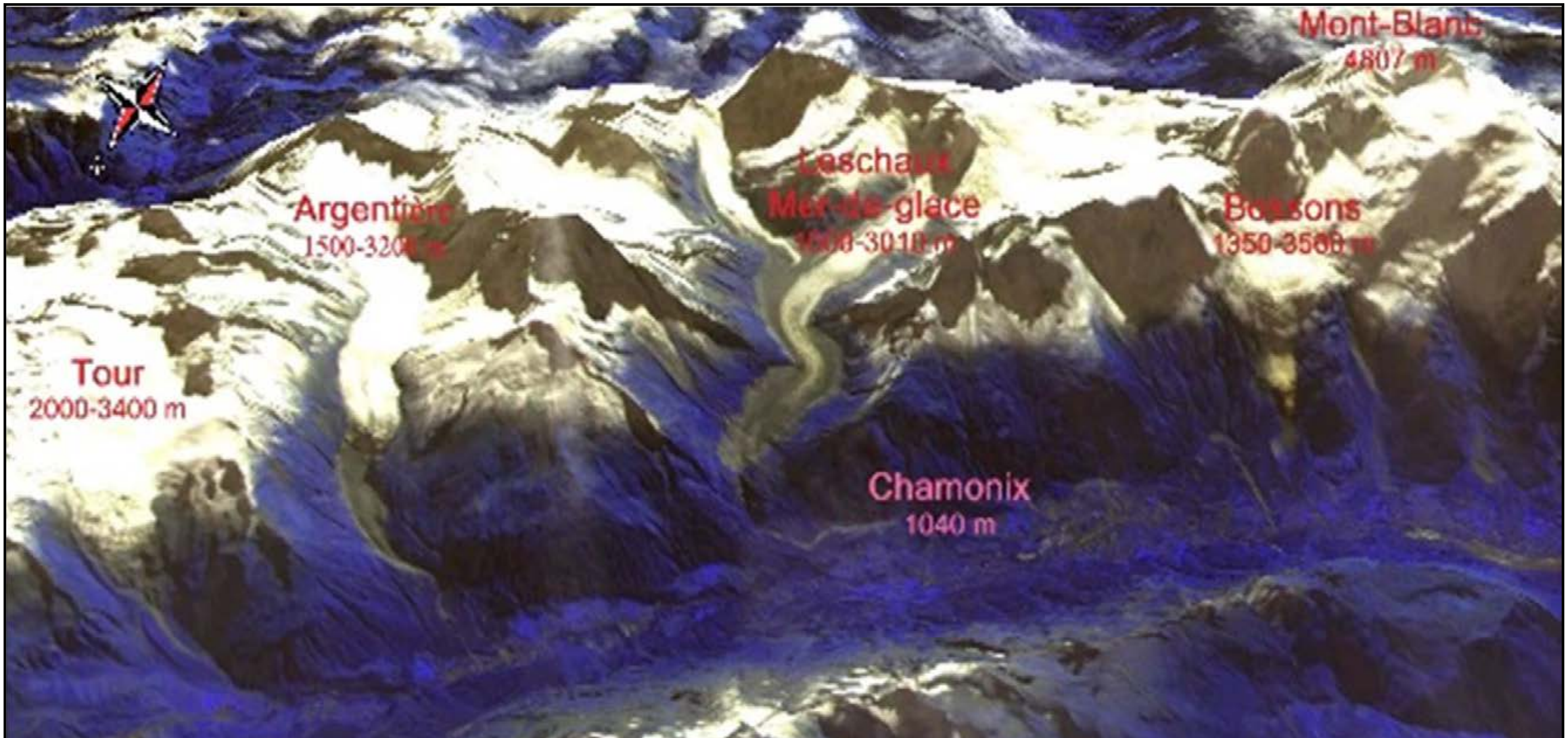
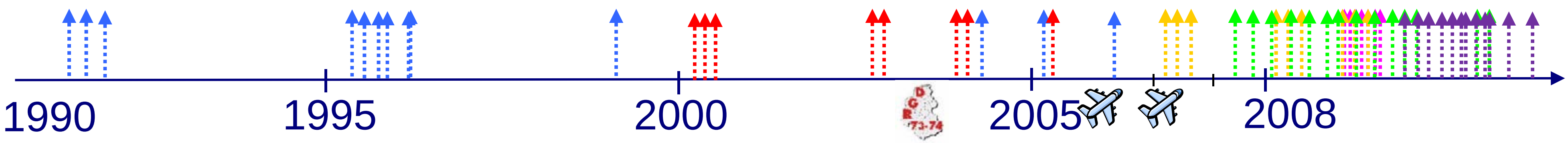
# Chamonix Mont-Blanc test-site




| | Argentière | Mer de Glace – Leschaux |
|---|---|---|
| Localisation | 45°56'15'"N / 7°00'30" E | 45°55'15" N/ 6°55'45" E |
| Surface / Length | 15 (km²)  / 9 (km) | 3,5 (km²)  /  4.7 (km) |
| Mean slope | ~14° (26%) | ~ 9° (17%) |

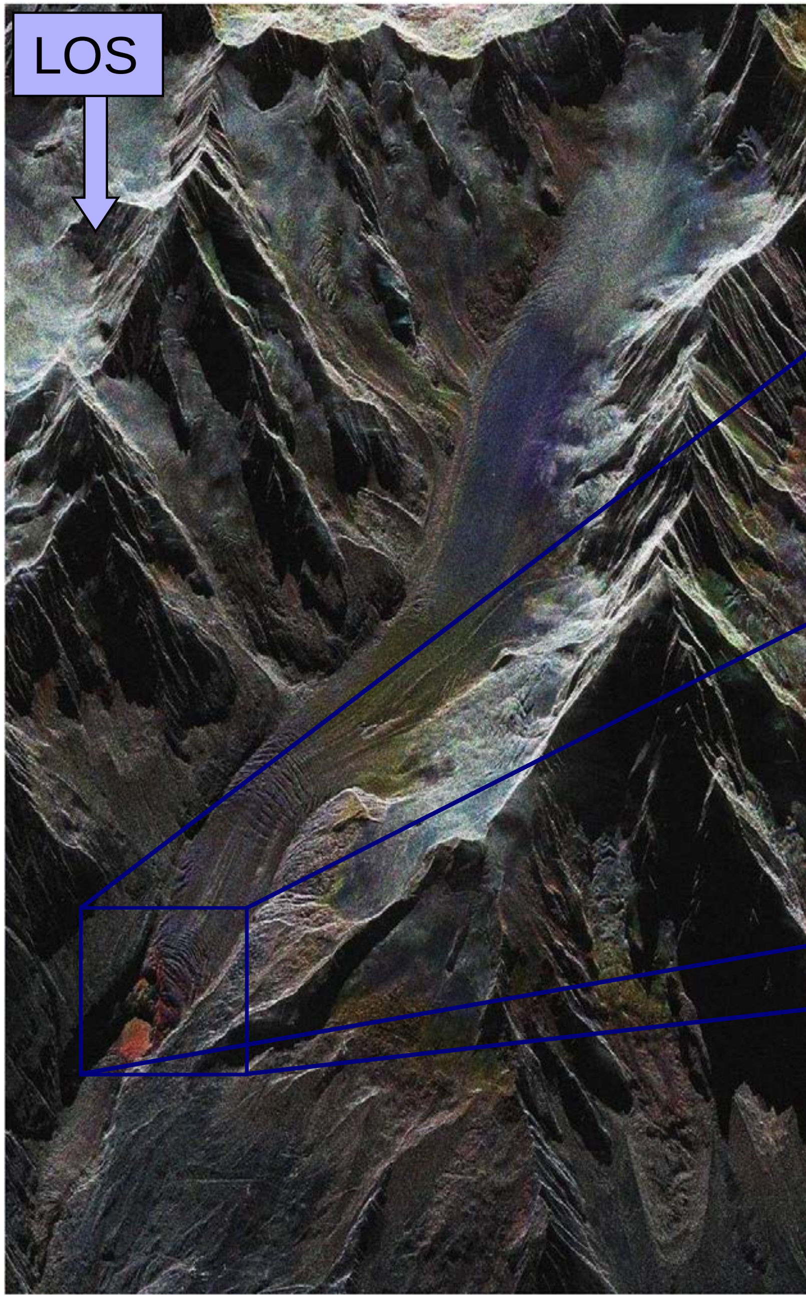
# Chamonix Mont-Blanc database




- Optical data:
  - SPOT 4-5 multispectral (CNES)
  - Access to regional GIS data (RGD 73/74)
- Air-borne SAR data: 2 E-SAR (DLR) campaigns
  - X, C, L and P bands, InSAR, **Pol-InSAR**
  - In-situ measurements: GPS, GPR, snow profiles...
- Space-borne SAR data:
  - ERS-ENVISAT (C band) ➔ ERS Tandem, ENVISAT alt.-pol
  - TerraSAR-X (X band) ➔ 40 single/dual-pol + **3 quad-pol**
  - RadarSAT-2 (C band) ➔ **7 quad-pol**
  - ALOS (L band) ➔ ~30 single/dual + **2 quad-pol**
  - COSMO-SkyMed (X band) ➔68 images…

# Monitoring surface change and displacement

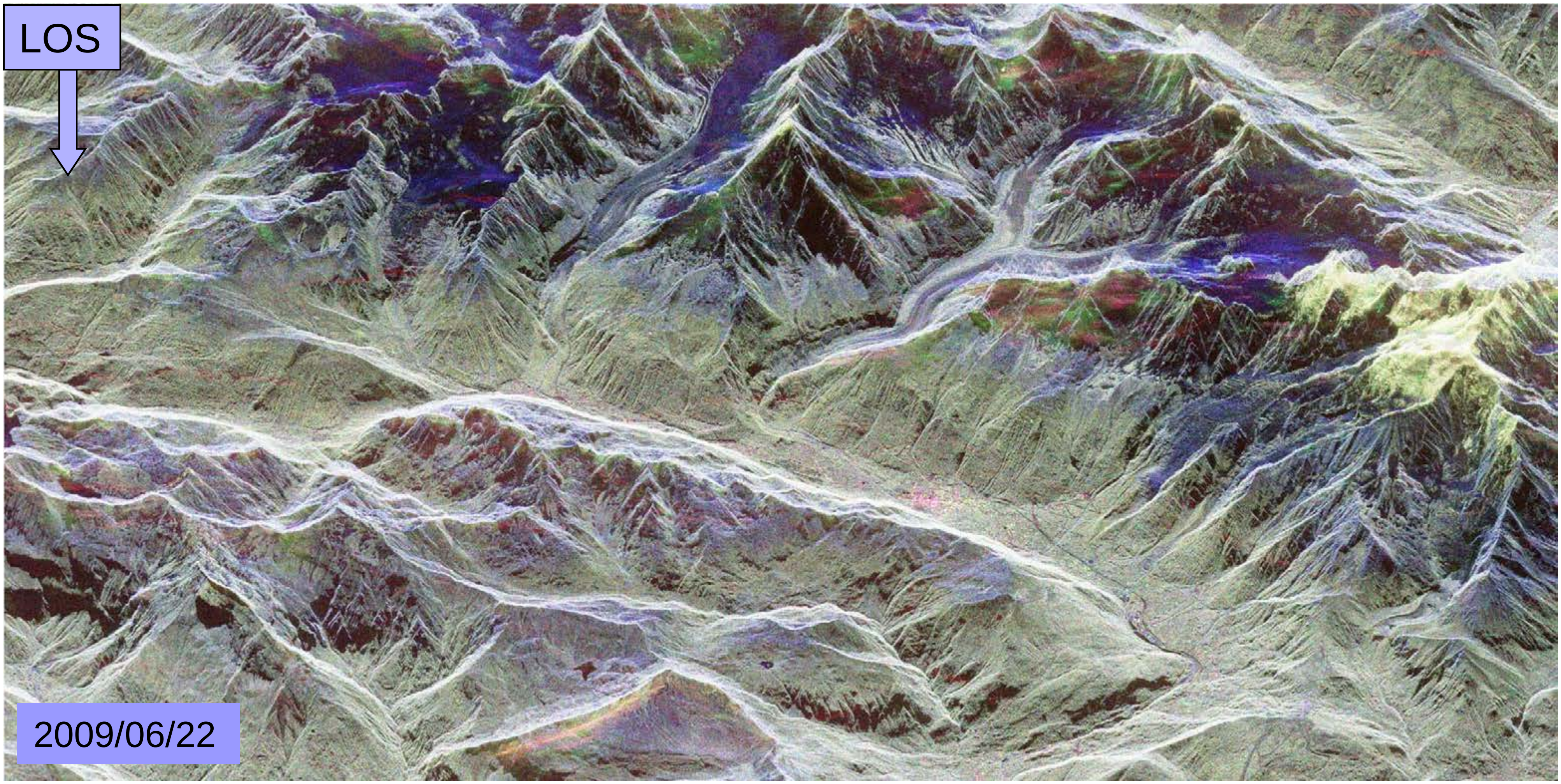
## TerraSAR-X single pol data



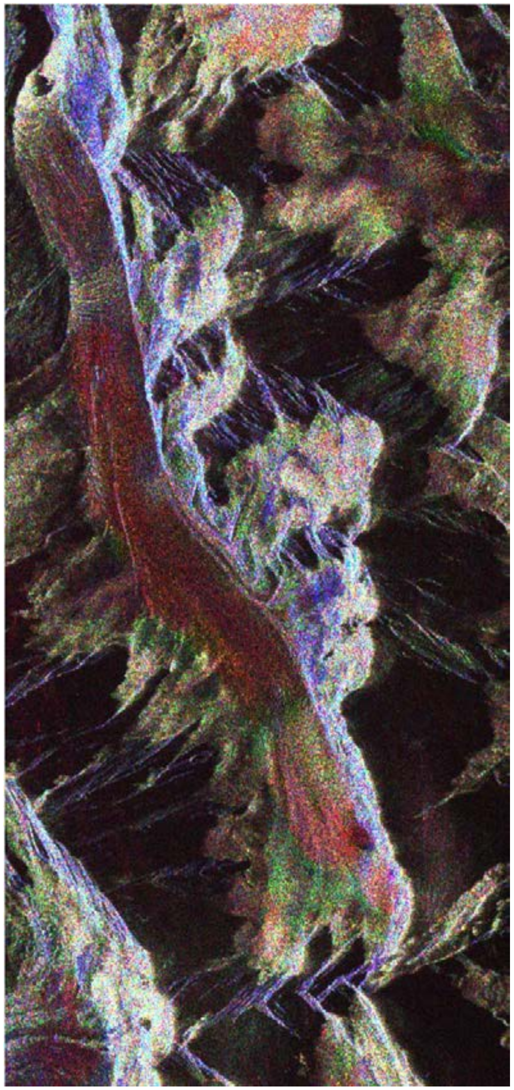

Argentière glacier,
RGB composition:
29/09/08,
10/10/08,
21/10/08

# Monitoring surface change and displacement

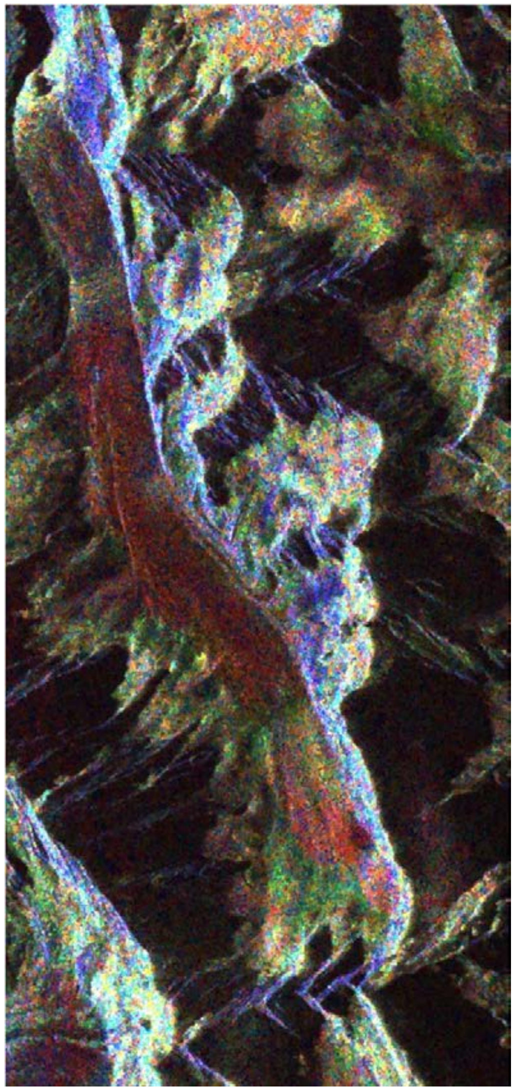
## Radarsat-2 quad-pol data (Pauli basis, R:HH-VV, G:2HV, B:HH+VV)



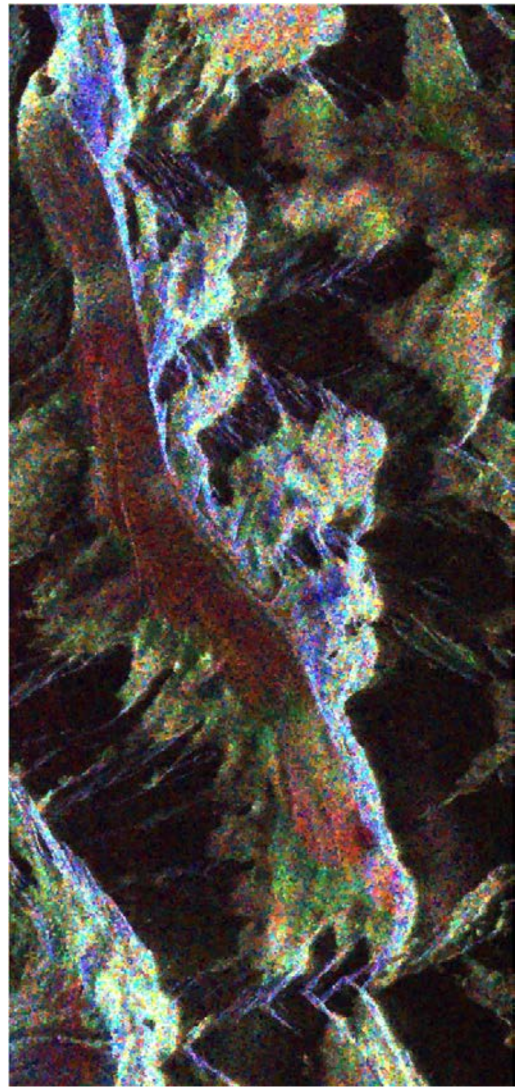
# S2 - Schatten regularization – “p” effect



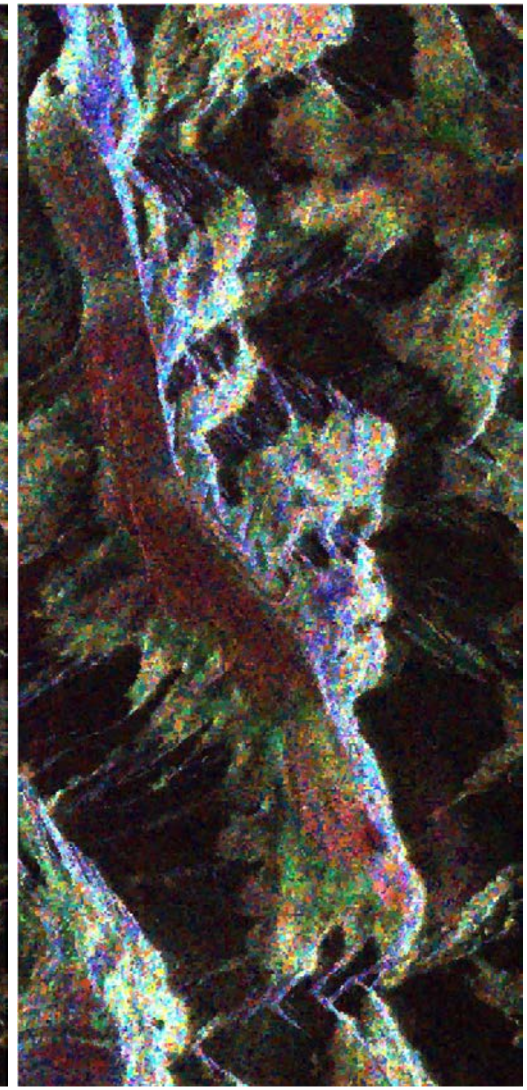
**Original, S2**



**S2, Lp2, W3x3**

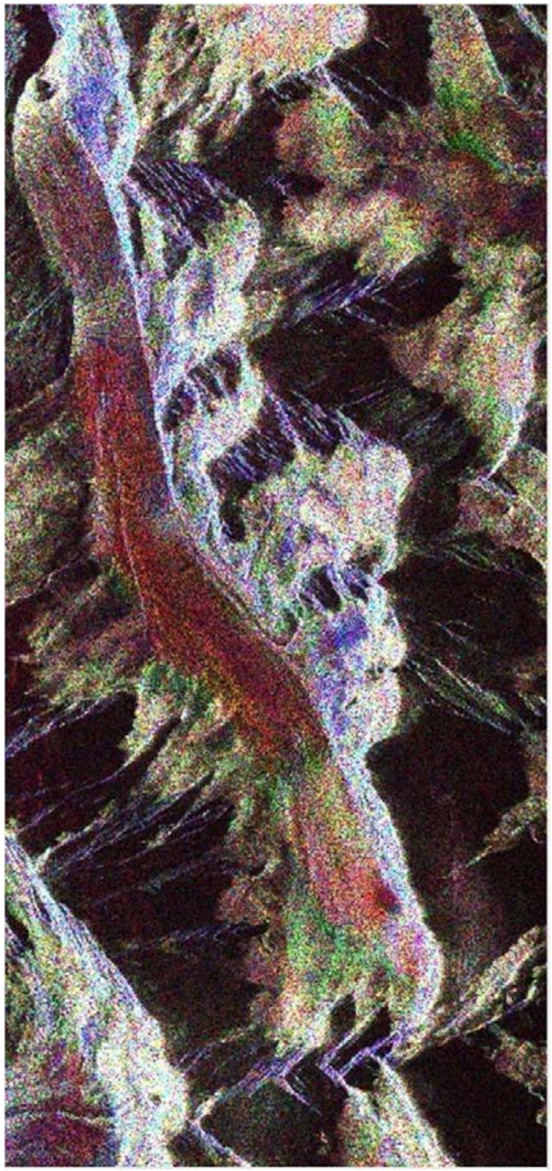


**S2, Lp1, W3x3**

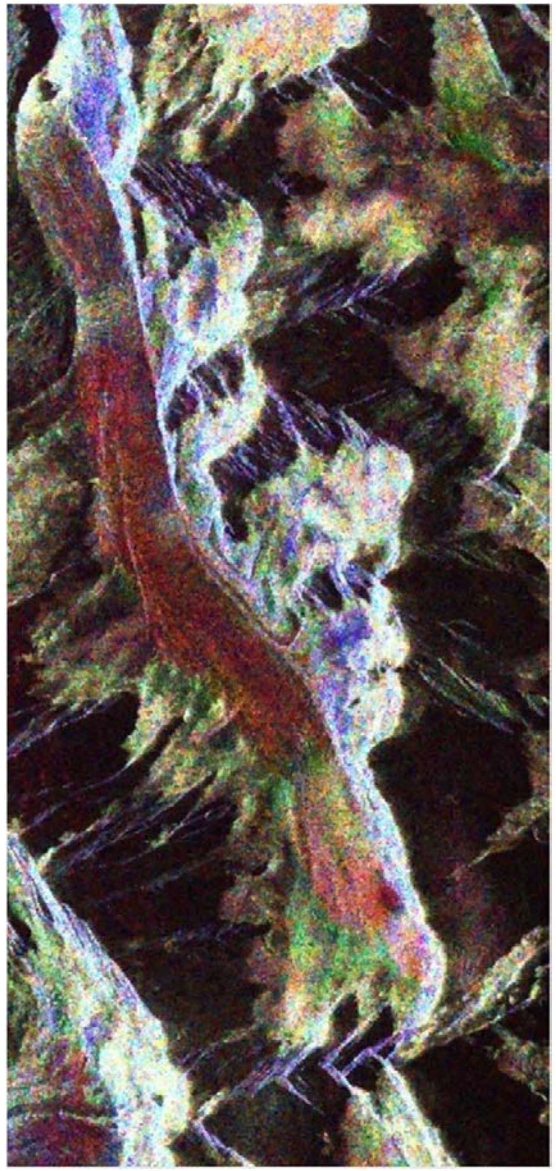


**S2, Lp0.5, W3x3**

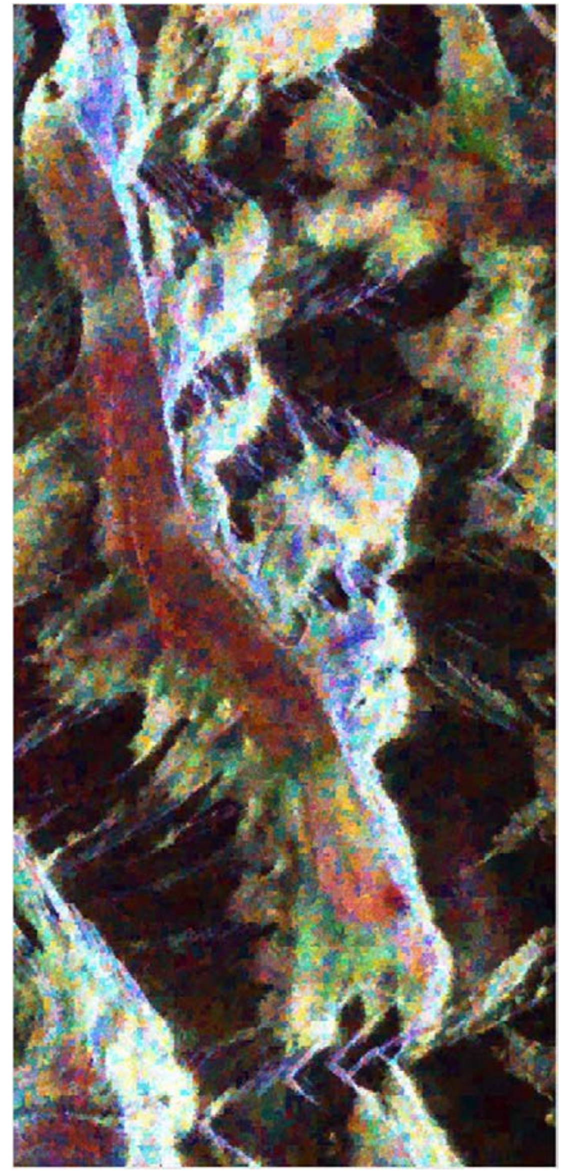
# S2 - Schatten regularization – W size effect



**Original, S2**



**S2, Lp2+Lp1+Lp0.5, W3x3**

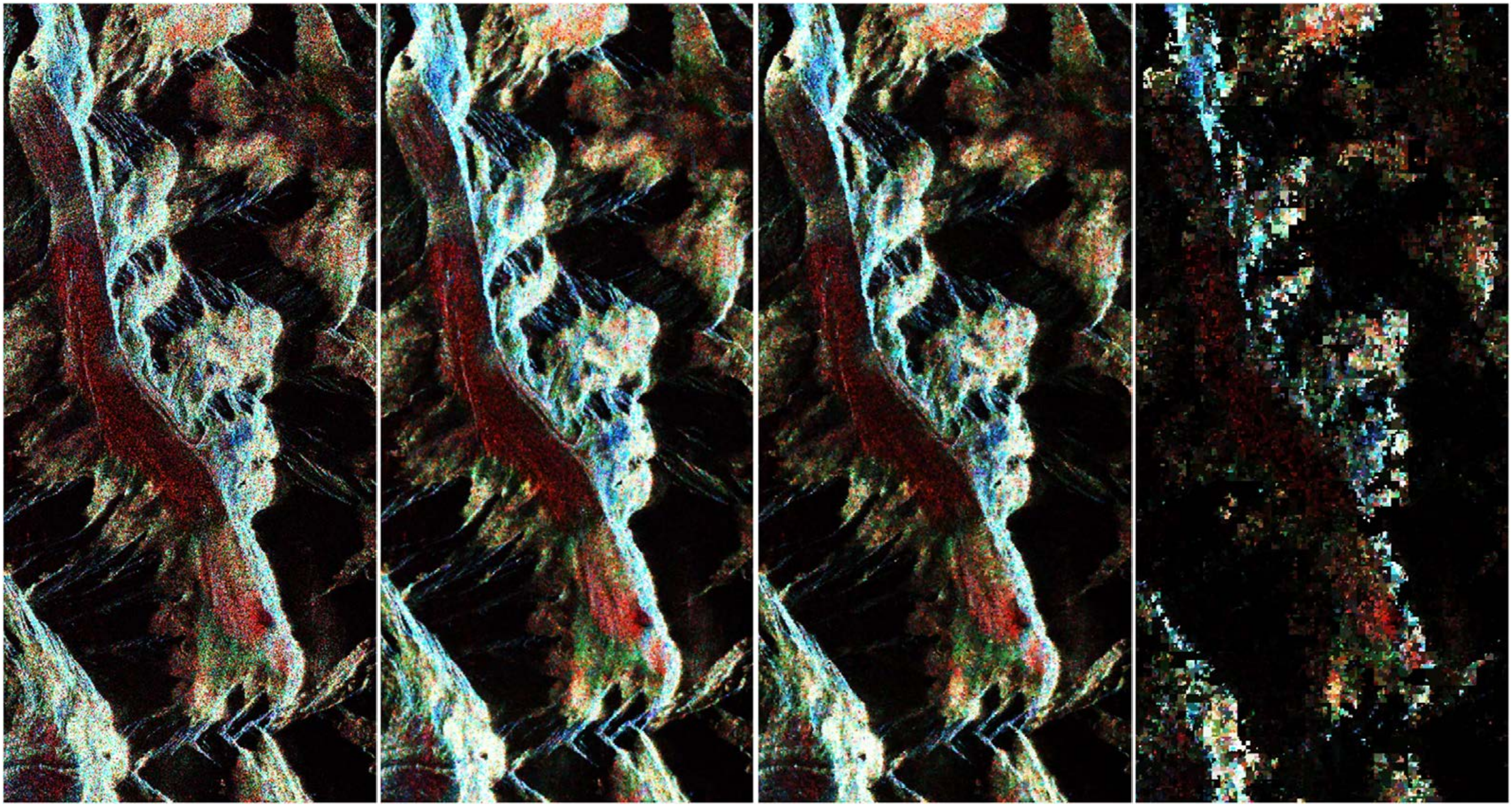


**S2, Lp2+Lp1+Lp0.5, W5x5**

## Slide 20

| | |
|---|---|
| **A5** | On montre l'influence du support spatial sur S2 car T3 est trop filtré pour permettre l'utilisation d'un grand voisinage<br>Atto, 24/01/2013 |
| **A6** | L'image du milieu (w=3x3) est franchement bien (avis personnel) : on voit des choses qui sont pas forcement visibles sur originale (gauche) et en plus, tous les "filaments" reliant les grandes structures sont bien restitués<br>Atto, 25/01/2013 |

# T3 - Schatten regularization – p effect



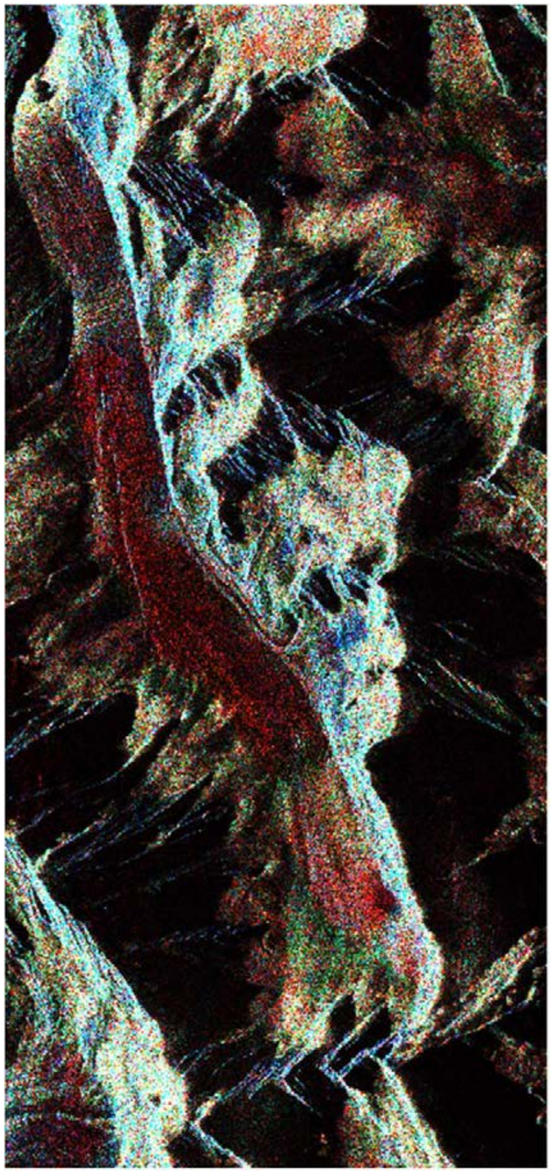
Original, T3 | T3, Lp2, W3x3 | T3, Lp1, W3x3 | T3, Lp0.5, W3x3

**Slide 21**

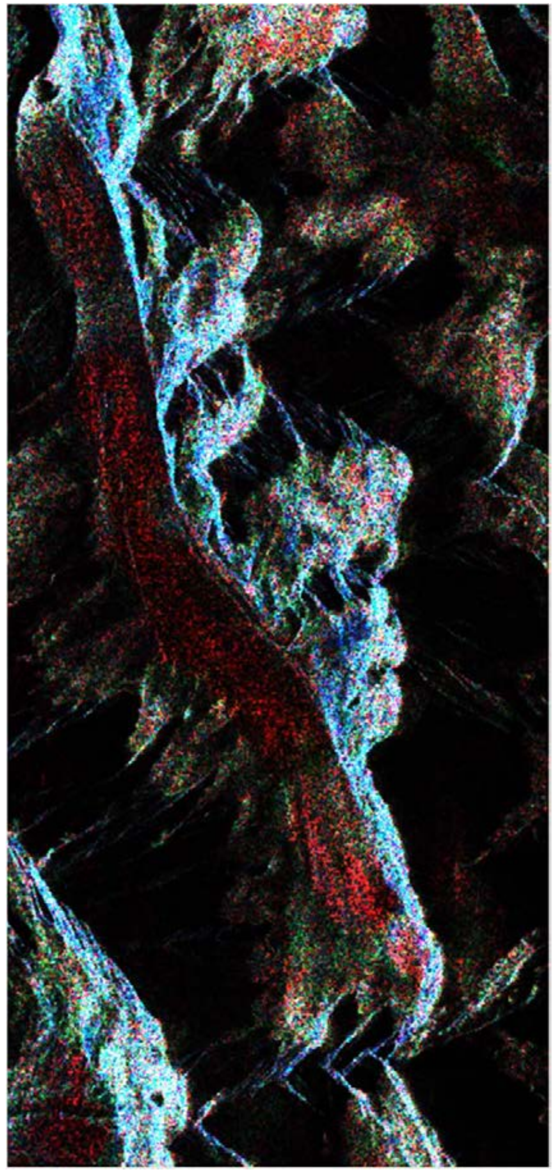
**A3** Lorsque p decroit, seuls les point de plus forte occurence et quasi-invariants par transformation orthogonale sont retenus.... on met en evidence de structures pas forcément visibles à partir des images originales
Atto, 24/01/2013

**A4** Pour info, le voisinage est de 3x3
Atto, 24/01/2013

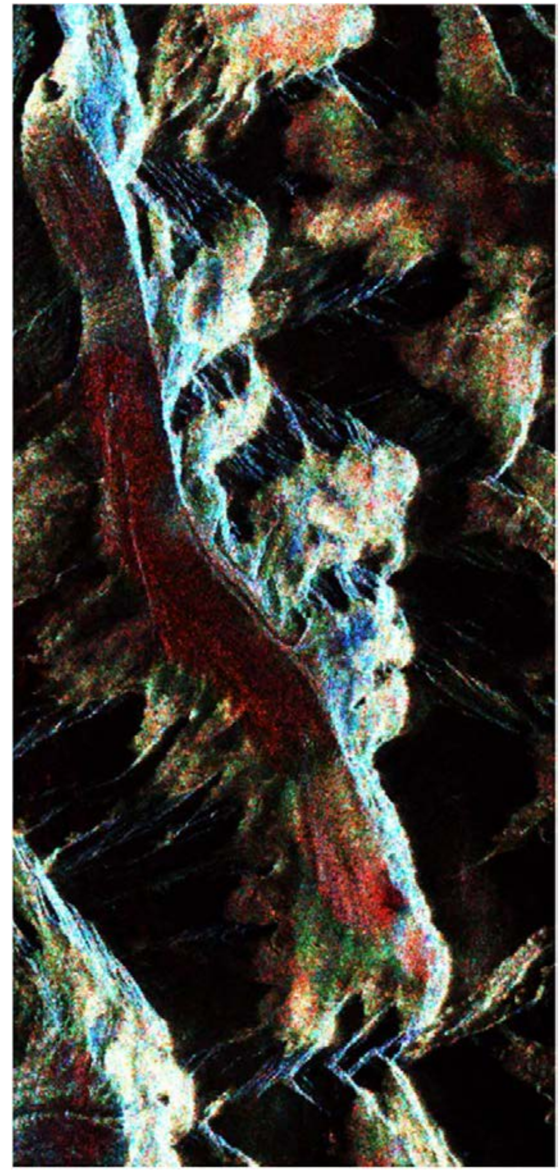
# T3 - fusion of different p (2, 1, 0,5)



**T3, Original**



**T3, IDAN**



**T3, P2+P1+P0,5, W3x3**

**Slide 22**

| | |
|---|---|
| **A1** | T3 etant deja pre-filtré (par nature) avant notre filtrage, l'influence de la taille de fenetre n'est pas approprié pour T3 : on va plutot le montrer sur S2<br>Atto, 24/01/2013 |
| **A2** | Notre originalité: on utilise les propriétés des "p" pour avoir un meilleur rendu en rehaussant les basses frequences selon differents "p" et en tenant compte des hautes fréquences (teture.... on a pas de bruit)!<br>Atto, 24/01/2013 |

# T3 - Schatten regularization – Time series

**T3, Original**

**T3, Filtered, p=1, W=3x3**

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# Conclusions and perspectives

- New approaches for PolSAR data regularization
  - ➔ Use of Matrix based Lp norms
  - ➔ “p<1” to increase sparsity
- Limitations – Further work
  - Performance assessment
  - Initial transform for SAR / PolSAR data
  - Tuning of “p” ➔ fusion
  - Applications of different decomposition / coding
    ➔ classification: land-use, snow, firn, ice …
    ➔ feature detection / tracking
- Acknowledgments:
  - DLR for TerraSAR-X data (project MTH0232)
  - CSA for the RadarSAT-2 data (SOAR #3399)

# A synthetic view of the problem…