Maximum likelihood SAR tomography based on the polarimetric multi-baseline RVoG model:

Optimal estimation of a covariance matrix structured as the sum of two Kronecker products.

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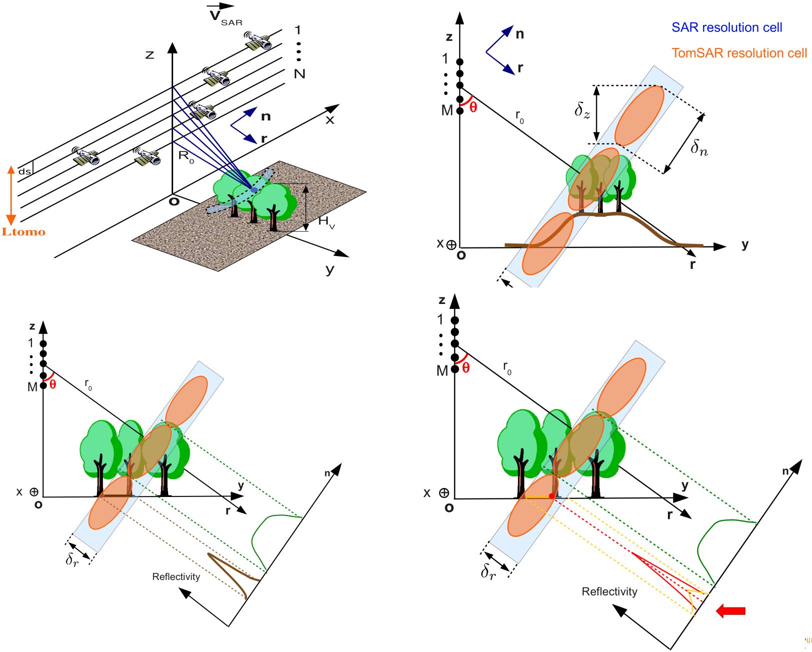

$\vec{V}_{SAR}$
1
N
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x
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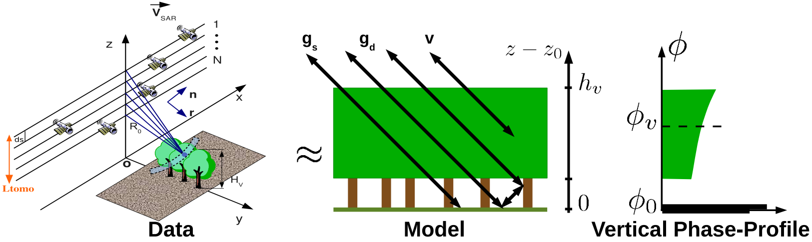
SAR resolution cell
TomSAR resolution cell
M
θ
r_0
δ_z
δ_n
δ_r
Reflectivity

Outline

Single-pol MB-RVOG model



SB case

- Uncorrelated responses

$$y_i(l) = s_{g_i}(l) + s_{v_i}(l),\ I_i = I_g + I_v$$

- Homogeneous media

$$E(s_{x_1} s_{x_2}^*) = I_x \gamma_x,\ \gamma_x = \int f_x(z) exp(jk_z z) \mathrm{d}z$$

- Global InSAR response $\mathbf{y}(l) = [s_1(l), s_2(l)]^T$

$$\mathbf{R}_{SB} = I_g \mathbf{R}_g + I_v \mathbf{R}_v,\ \text{with } \mathbf{R}_x = \begin{bmatrix} 1 & \gamma_x \\ \gamma_x^* & 1 \end{bmatrix}$$

Single-pol MB-RVOG model

MB case

- Point-like scatterer MB-response : $\mathbf{s}(l) = A_c(l)\mathbf{a}(z)$
  $\mathbf{a} = \left[1, exp(jk_{z_2}z), \ldots, exp(jk_{z_{N_s}}z)\right]$
- Global MB-InSAR response $\mathbf{y}(l) == \mathbf{s}_g(l) + \mathbf{s}_v(l)$

$$\mathbf{R}_{MB} = I_g\mathbf{R}_g + I_v\mathbf{R}_v \text{ with } \mathbf{R}_x = \begin{bmatrix} 1 & \gamma_x(k_{z_2}) & \ldots & \gamma_x(k_{z_{N_s}}) \\ & 1 & & \vdots \\ & & \ddots & \vdots \\ & & & 1 \end{bmatrix}$$

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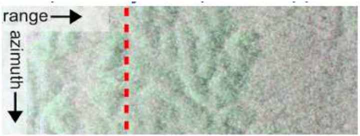
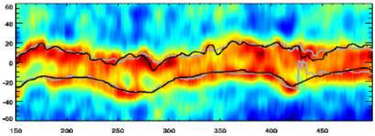
$$\mathbf{R}_{MB} = l_g\mathbf{R}_g + l_v\mathbf{R}_v \text{ with } \mathbf{R}_x = \begin{bmatrix} 1 & \gamma_x(k_{z_2}) & \ldots & \gamma_x(k_{z_{N_s}}) \\ & 1 & & \vdots \\ & & \ddots & \vdots \\ & & & 1 \end{bmatrix}$$



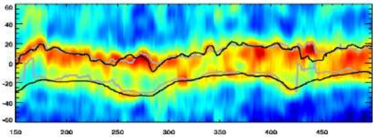
☺ Estimated profiles match LiDAR.

LiDAR —— TomSAR ——

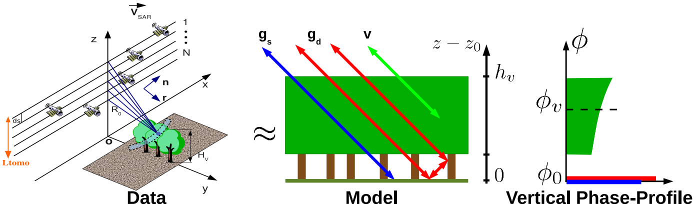
HH



VV



Full-pol MB-RVOG model



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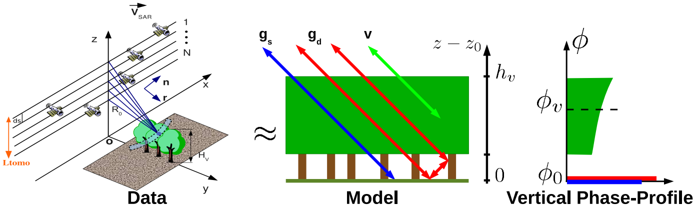
Full-pol MB-RVOG model



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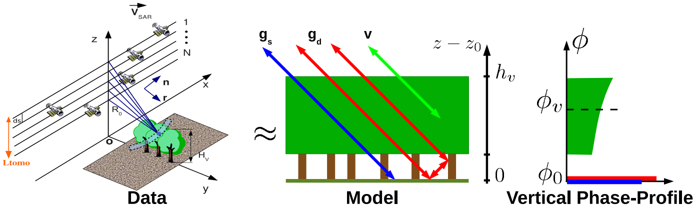

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$$\mathbf{R}_{PS} = \mathbf{C}_g \otimes \mathbf{R}_g + \mathbf{C}_v \otimes \mathbf{R}_v$$

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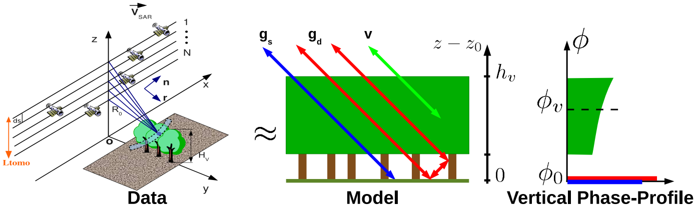

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$$\mathbf{R}_{PS} = \mathbf{C}_g \otimes \mathbf{R}_g + \mathbf{C}_v \otimes \mathbf{R}_v$$

- SB-case $(N_s = 2)$

$$\mathbf{R}_{SP} = \mathbf{R}_g \otimes \mathbf{C}_g + \mathbf{R}_v \otimes \mathbf{C}_v = \begin{bmatrix} \mathbf{C}_g + \mathbf{C}_v & \gamma_g\mathbf{C}_g + \gamma_v\mathbf{C}_v \\ \gamma_g^*\mathbf{C}_g^\dagger + \gamma_v^*\mathbf{C}_v^\dagger & \mathbf{C}_g + \mathbf{C}_v \end{bmatrix}$$

$$\mathbf{R}_{SP} = \begin{bmatrix} \mathbf{C} & \mathbf{C}_{12} \\ \mathbf{C}_{12}^\dagger & \mathbf{C} \end{bmatrix} \Rightarrow \gamma(\mathbf{w}) = \frac{\mathbf{w}^\dagger\mathbf{C}_{12}\mathbf{w}}{\mathbf{w}^\dagger\mathbf{C}\mathbf{w}} = \frac{\gamma_v + \mu(\mathbf{w})\gamma_g}{1 + \mu(\mathbf{w})}$$

MB-RVOG : exactly modeled by an SKP-2 structured covariance matrix

MB-RVOG parameter estimation

SB case

- Coherence line estimation
- GV separation : strong assumptions
  - $|\gamma_g| = 1$, rank$(\mathbf{C}_g) < 3$
  - $f_v(z)$ known

MB case

- SVD SKP-2 decomposition
- GV separation :
  - several solutions
  - accuracy increases with N_s

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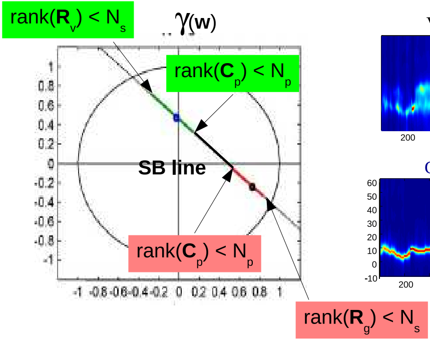

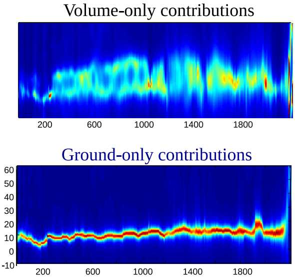

See also : physical interpretation of MB-RVOG JD & SKP decomp.
B. El Hajj-Chehade, L. Ferro-Famil, Poster session Wednesday aft.

Estimation problem statement and objectives

- L independent observations $\{\mathbf{y}(l)\}_{l=1}^{L}$
- Estimate RVOG parameters or its SKP-2 covariance matrix

SB case

- Pol-Sampling $\gamma(\mathbf{w}_i)$, line LS estimate [C & P 2001-3][Flynn & Tabb 2002-3]
- LS estimation (matrix formatting) [LFF et al, 2009-10] [L-M 2012]
- ML estimation [Flynn et al. 2002] and CRLB [Roueff et al. 2011].

MB case

- LS estimation using joint diagonalization [Ferro-Famil et al. 2010]
- SVD based SKP-2 decomposition [Tebaldini et al. 2009, 2010, 2012]

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LS vs ML estimation

- Estimate that best fits the observations (LS) vs
  Estimate most likely to have produced observations (ML).
- LS : generally fast, often has analytical solutions, no pdf assumption
- ML : if UMV estimator exists $\rightarrow$ ML. Can be complex and time consuming

ML estimation of SKP2 covariance matrices

Objectives

- Find $\hat{\mathbf{R}}_{PS-ML} = \arg\max_{\hat{\mathbf{R}}_{PS}} f(\{\mathbf{y}(l)\}_{l=1}^{L}|\mathbf{R}_{PS})$ under the SKP-2 constraint.
- ML (Wishart) concentrated criterion :

$$J(\mathbf{R}_{PS}) = \mathrm{tr}(\mathbf{R}_{PS}^{-1}\hat{\mathbf{R}}_{\mathbf{yy}}) + \log|\mathbf{R}_{PS}|) \text{ with } \hat{\mathbf{R}}_{\mathbf{yy}} = \frac{1}{L}\sum_{l=1}^{L}\mathbf{y}_{PS}(l)\mathbf{y}_{PS}^{\dagger}(l)$$

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- Estimation objective : determine

$$\hat{\mathbf{R}}_{PS-ML} = \arg\min J(\mathbf{R}_{PS}) \text{ with and } \mathbf{R}_{PS} = \mathbf{C}_g \otimes \mathbf{R}_g + \mathbf{C}_v \otimes \mathbf{R}_v$$

- Find a fast and reliable technique

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- Find a fast and reliable technique

Case of a single KP

- No analytical solution.
- Fast and reliable iterative alternate (Flip-Flop) optimization [Lu 2004]
- Asymptotic analytical expression : Weighted LS solution [Werner 2008]

ML estimation of SKP2 covariance matrices

SKP-2 case

- In general $\mathbf{A} \otimes \mathbf{B} + \mathbf{C} \otimes \mathbf{D} \neq \mathbf{E} \otimes \mathbf{F}$
- No existing solution.

ML estimation of SKP2 covariance matrices

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Exact joint diagonalization of two matrices

- $\mathbf{A}$, $\mathbf{B} > \mathbf{0}$ and Hermitian can be exactly jointly diagonalized [Yeredor 2005] :
$$\exists \mathbf{W} : \mathbf{W}\mathbf{A}\mathbf{W}^{\dagger} = \mathbf{D}_{\mathbf{A}},\ \mathbf{W}\mathbf{B}\mathbf{W}^{\dagger} = \mathbf{D}_{\mathbf{B}},\ \text{in general } \mathbf{W}\mathbf{W}^{\dagger} \neq \mathbf{I}$$

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- In the SKP-2 case, $\mathbf{R}_{PS} = \mathbf{C}_g \otimes \mathbf{R}_g + \mathbf{C}_v \otimes \mathbf{R}_v$
$$\exists \mathbf{W} = \mathbf{W}_\mathbf{C} \otimes \mathbf{W}_\mathbf{R} :$$
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ML criterion minimization over $\mathbf{D}$

- Inserting $\mathbf{D}, \mathbf{W}$ into $J(\mathbf{R}_{PS})$

$$J(\mathbf{D}, \mathbf{W}) = \text{tr}(\mathbf{D}^{-1}\mathbf{W}\hat{\mathbf{R}}_{\mathbf{yy}}\mathbf{W}^\dagger) + \log|\mathbf{D}| - \log|\mathbf{W}\mathbf{W}^\dagger|$$

- Minimum obtained for : $\hat{\mathbf{D}} = \text{diag}(\mathbf{W}\hat{\mathbf{R}}_{\mathbf{yy}}\mathbf{W}^\dagger)$

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- Minimum obtained for : $\hat{\mathbf{D}} = \operatorname{diag}(\mathbf{W}\hat{\mathbf{R}}_{\mathbf{yy}}\mathbf{W}^\dagger)$
- Resulting concentrated ML criterion
$$J(\mathbf{W}) = \log|\operatorname{diag}(\mathbf{W}\hat{\mathbf{R}}_{\mathbf{yy}}\mathbf{W}^\dagger)| - \log|\mathbf{W}\mathbf{W}^\dagger|$$

ML estimation of SKP2 covariance matrices

ML criterion minimization over $\mathbf{W}$: formulation

- Explicit ML criterion $J(\mathbf{W_C}, \mathbf{W_R}) = \log|\mathrm{diag}(\tilde{\mathbf{D}})| - \log|\mathbf{W}\mathbf{W}^\dagger|$
  with $\tilde{\mathbf{D}} = \mathbf{W}\hat{\mathbf{R}}_{\mathbf{yy}}\mathbf{W}^\dagger$ and $\mathbf{W} = \mathbf{W_C} \otimes \mathbf{W_R}$

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  with $\tilde{\mathbf{D}} = \mathbf{W}\hat{\mathbf{R}}_{\mathbf{yy}}\mathbf{W}^\dagger$ and $\mathbf{W} = \mathbf{W_C} \otimes \mathbf{W_R}$
- Conditional criterion : known spatial diagonalizer

$$J(\mathbf{W_C}|\mathbf{W_R}) = \log|\sum_{i=1}^{N_s} \mathrm{diag}(\mathbf{W_C}\tilde{\mathbf{D}}_{\mathbf{C}_i}\mathbf{W_C})| - \log|\mathbf{W_C}\mathbf{W_C}^\dagger|$$

- Conditional criterion : known polarimetric diagonalizer

$$J(\mathbf{W_R}|\mathbf{W_C}) = \log|\sum_{i=1}^{N_p} \mathrm{diag}(\mathbf{W_R}\tilde{\mathbf{D}}_{\mathbf{R}_i}\mathbf{W_R})| - \log|\mathbf{W_R}\mathbf{W_R}^\dagger|$$

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ML criterion minimization over $\mathbf{W}$: joint diagonalization

- $J(\mathbf{W_C}|\mathbf{W_R})$ or $J(\mathbf{W_R}|\mathbf{W_C})$ $\Rightarrow$ **efficiently** minimized
  modified Pham's technique [Pham 2001]
- Iterative constrained joint diagonalization of $\{\tilde{\mathbf{D}}_{\mathbf{C}_i}\}$ or $\{\tilde{\mathbf{D}}_{\mathbf{R}_i}\}$
- Convergence is proven, generally very fast (4 iterations)

ML estimation of SKP2 covariance matrices : algorithm

Step 0 Initialisation

- Compute $\hat{\mathbf{R}}_{\mathbf{yy}} = \frac{1}{L}\sum_{l=1}^{L} \mathbf{y}_{PS}(l)\mathbf{y}_{PS}^{\dagger}(l)$, set $\mathbf{W}_{\mathbf{C}}(0), \mathbf{W}_{\mathbf{R}}(0)$, $n = 0$

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Step 1 Diagonalization

- Compute $\tilde{\mathbf{D}}(n+1) = \mathbf{W}(n)\hat{\mathbf{R}}_{\mathbf{yy}}\mathbf{W}^{\dagger}(n)$, $n = n+1$

ML estimation of SKP2 covariance matrices : algorithm

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Step 2 Minimization using Pham's technique

- $\mathbf{W}_{\mathbf{C}}(n) = \arg\min_{\mathbf{W}_{\mathbf{C}}} J(\mathbf{W}_{\mathbf{C}}|\mathbf{W}_{\mathbf{R}}(n-1)),\ \mathbf{W}_{\mathbf{R}}(n) = \arg\min_{\mathbf{W}_{\mathbf{R}}} J(\mathbf{W}_{\mathbf{R}}|\mathbf{W}_{\mathbf{C}}(n-1))$

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Step 3 Convergence ?

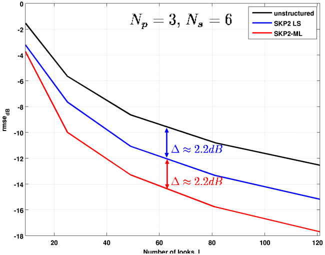
- **If no** : Go to **Step 1**

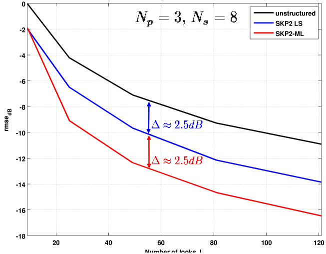
Step 4 Identification

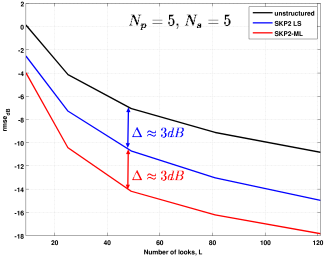
- $\tilde{\mathbf{D}} = \mathrm{diag}(\hat{\mathbf{W}}\hat{\mathbf{R}}_{\mathbf{yy}}\hat{\mathbf{W}}^{\dagger}),\ \hat{\mathbf{R}}_{PS-ML} = \hat{\mathbf{W}}^{-1}\tilde{\mathbf{D}}\hat{\mathbf{W}}^{-\dagger}$
- From $\hat{\mathbf{R}}_{PS-ML}$, identify $\hat{\mathbf{C}}_g, \hat{\mathbf{R}}_g,\ \hat{\mathbf{C}}_v, \hat{\mathbf{R}}_v$

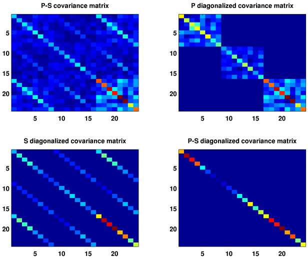
Global relative rms error

$$rmse = \frac{\mathrm{E}\ ||\mathbf{R}_{PS} - \hat{\mathbf{R}}_{PS}||_F^2}{||\mathbf{R}_{PS}||_F^2}, \text{ with } \hat{\mathbf{R}}_{PS} = \hat{\mathbf{R}}_{\mathbf{yy}}, \hat{\mathbf{R}}_{PS-LS}, \hat{\mathbf{R}}_{PS-ML}$$



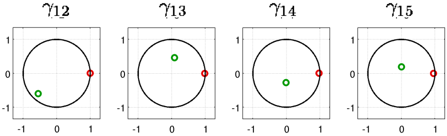






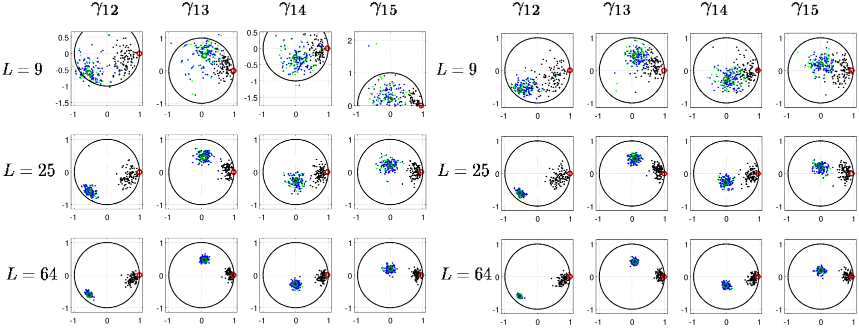
Coherence locations in the complex plane

$N_p = 3, N_s = 5,$ Ground : rank$(\mathbf{C}_g) = 2$, Volume : full rank, minimum phase



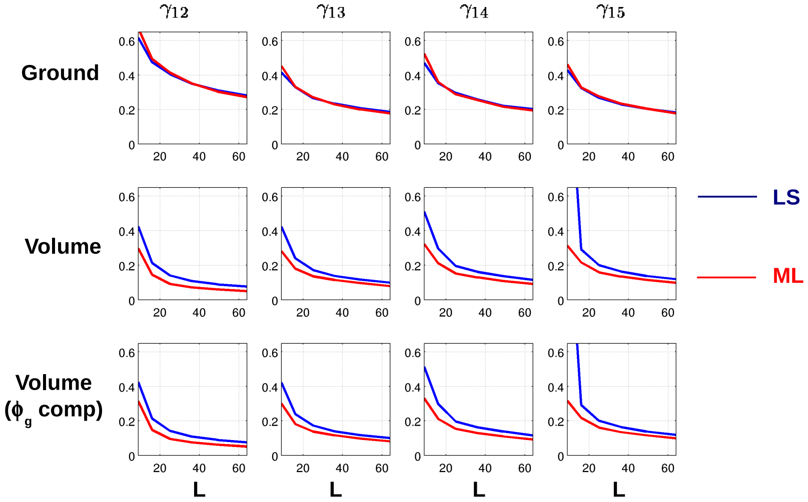
SKP-2 Least Squares

SKP-2 Maximum Likelihood



Coherence wise rms error

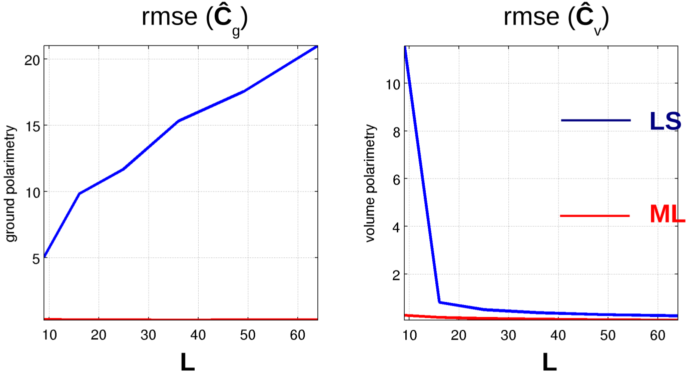
$$rmse(\gamma_{ij}) = E(||\gamma_{ij} - \hat{\gamma}_{ij}||^2)$$



Equal performance for low rank (and DoF) $\mathbf{R}_g$, well chosen solution.

Polarimetric rms error

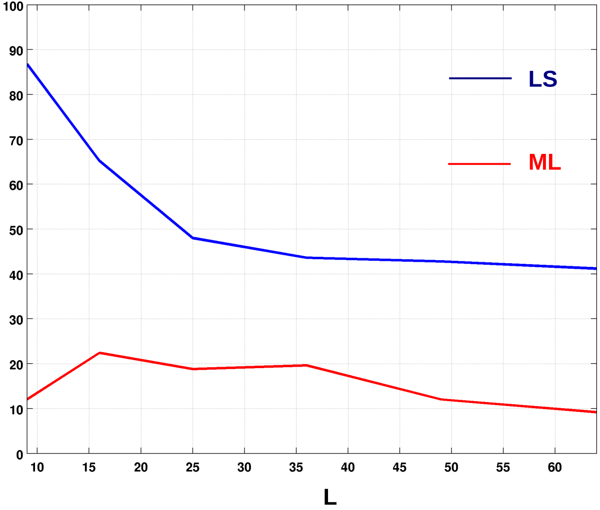
$$rmse_i = \frac{\mathrm{E}\ ||\mathbf{C}_{PS_i} - \hat{\mathbf{C}}_{PS_i}||_F^2}{||\mathbf{C}_{PS_i}||_F^2}$$



Large LS error over low rank $\mathbf{C}_g$

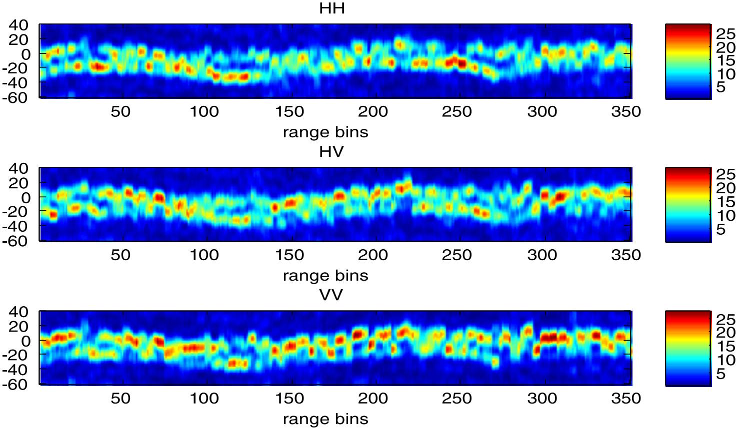
MB-RVOG model validity vs estimation

$$\frac{\text{numb. unfeasible exp.}}{numb.exp}100$$



Tomography at P band of a TROPISAR profile (ONERA/SETHI)

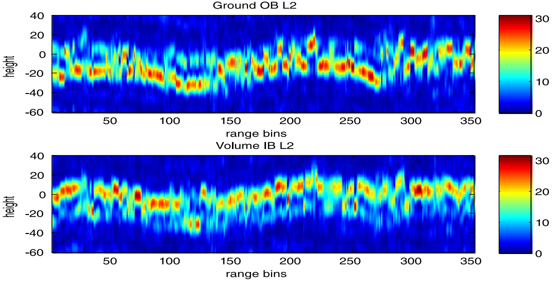
$$N_p = 3, N_s = 6 \; L = 16 < N_s N_p$$



LS & ML SKP-2 Tomography

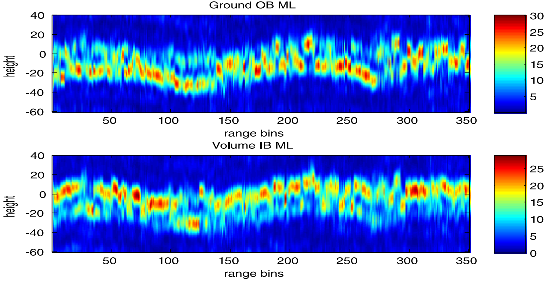
$$N_p = 3, N_s = 6 \quad L = 16 < N_s N_p$$

LS



Unfeasible Points 43 %

ML



Unfeasible Points 6 %

Conclusion

Numerical ML estimation of the MB-RVOG model

- Rigorous and fast
- Adapted to POL-inSAR and TOMO-SAR
- Could also be used in medical imaging, image processing ...

Performance

- Fewer looks needed
- Significant variance reduction $LS \rightarrow ML \equiv$ unformated $\rightarrow LS$
- Better estimation of polarimetric features, not only spatial ones