

Evaluation of the decomposition method based on second- and third-order statistics

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■ Background

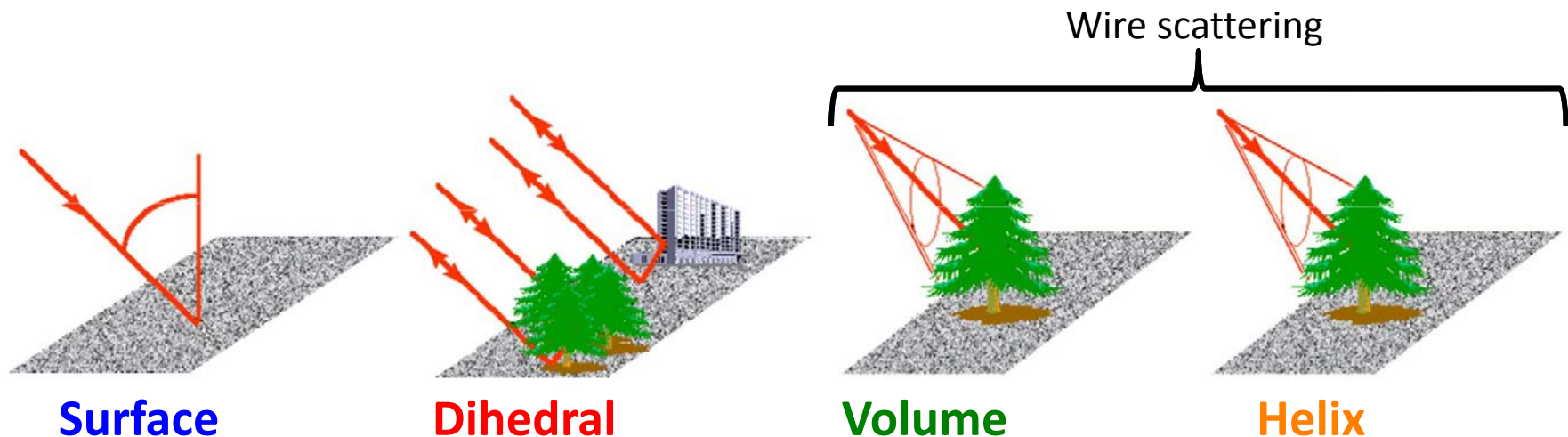
- To better understand the scattering mechanisms from the polarimetric SAR images, there are many researches concerning the decomposition method of the polarimetric SAR data
- Most of them based on the second-order statistics analysis that Freeman suggested for the reflection symmetry condition.
- Yamaguchi added the helix component into Freeman's model and developed 4 component scattering model for the non-reflection symmetry condition.
- Arii developed adaptive model-based decomposition method that could estimate both the mean orientation angle and a degree of randomness for the canopy scattering for each pixel in a SAR image without the reflection symmetry condition.

■ Purpose

- We propose new decomposition method based on second and third-order statistics to estimate the surface, dihedral, volume and helix scattering components without the specific assumptions concerning the model for the volume scattering.
- In addition, we evaluate the method by using simulation data and real data and compare the method with other method.

■ Backscattering model from the ground

We express the backscattering echo from the ground surface as the summation of the surface, dihedral, volume and helix scattering as follows.



$$\begin{pmatrix} a & c \\ c & b \end{pmatrix} = \underbrace{\begin{pmatrix} S_{HH} & 0 \\ 0 & S_{VV} \end{pmatrix}}_{\text{Surface}} + \underbrace{\begin{pmatrix} D_{HH} & 0 \\ 0 & D_{VV} \end{pmatrix}}_{\text{Dihedral}} + \underbrace{\begin{pmatrix} V_{HH} & V_{HV} \\ V_{HV} & V_{VV} \end{pmatrix}}_{\text{Volume}} + \frac{1}{2} \underbrace{\begin{pmatrix} H_{HH} & \pm jH_{HH} \\ \pm jH_{HH} & -H_{HH} \end{pmatrix}}_{\text{Helix}}$$

a : HH polarization SAR data
 b : VV polarization SAR data
 c : HV polarization SAR data
 S_{HH} : HH for the surface scattering
 S_{VV} : VV for the surface scattering

D_{HH} : HH for the dihedral scattering
 D_{VV} : VV for the dihedral scattering
 V_{HH} : HH for the volume scattering
 V_{VV} : VV for the volume scattering
 V_{HV} : HV for the volume scattering
 H_{HH} : HH for the helix scattering

■ Second-order statistics analysis

We calculate the correlation coefficient for each polarization components and express the correlation matrix as the summation of the surface scattering, dihedral scattering, volume scattering and helix scattering component matrix .

$$\begin{aligned}
 & \begin{pmatrix} |a|^2 & \sqrt{2}a^*c & a^*b \\ \sqrt{2}ac^* & 2|c|^2 & \sqrt{2}bc^* \\ ab^* & \sqrt{2}b^*c & |b|^2 \end{pmatrix} = f_s \begin{pmatrix} |\beta|^2 & 0 & \beta^* \\ 0 & 0 & 0 \\ \beta & 0 & 1 \end{pmatrix} + f_d \begin{pmatrix} 1 & 0 & \alpha \\ 0 & 0 & 0 \\ \alpha^* & 0 & |\alpha|^2 \end{pmatrix} \\
 & + \begin{pmatrix} |V_{HH}|^2 & \sqrt{2}V_{HH}^*V_{HV} & V_{HH}^*V_{VV} \\ \sqrt{2}V_{HH}V_{HV}^* & 2|V_{HV}|^2 & \sqrt{2}V_{HV}^*V_{VV} \\ V_{HH}V_{VV}^* & \sqrt{2}V_{HV}V_{VV}^* & |V_{VV}|^2 \end{pmatrix} + \frac{1}{4}f_h \begin{pmatrix} 1 & \pm\sqrt{2}j & -1 \\ \mp\sqrt{2}j & 2 & \pm\sqrt{2}j \\ -1 & \mp\sqrt{2}j & 1 \end{pmatrix} \\
 & \qquad \qquad \qquad \text{Volume} \qquad \qquad \qquad \text{Helix}
 \end{aligned}$$

$$|a|^2 = |\beta|^2 f_s + f_d + |V_{HH}|^2 + \frac{1}{4}f_h \quad (1)$$

$$|c|^2 = |V_{HV}|^2 + \frac{1}{4}f_h \quad (4)$$



$$|b|^2 = f_s + |\alpha|^2 f_d + |V_{VV}|^2 + \frac{1}{4}f_h \quad (2)$$

$$ac^* = V_{HH}V_{HV}^* \mp \frac{1}{4}f_h j \quad (5)$$

$$ab^* = \beta f_s + \alpha^* f_d + V_{HH}V_{VV}^* - \frac{1}{4}f_h \quad (3)$$

$$bc^* = V_{HV}V_{VV}^* \mp \frac{1}{4}f_h j \quad (6)$$

■ Freeman' method and Yamaguchi's method

■ Freeman's method

It was clarified by lots of observations that ac^* and b^*c are close to 0 (reflection symmetry condition) in the forest. From this relationship, Freeman assumed that volume scattering could be expressed in the union sets of thin dipoles and formulized the correlation matrix of the volume scattering.

■ Yamaguchi's method

Yamaguchi add helix scattering term to Freeman's model and can decompose each components in the case of $ac^* \neq b^*c \neq 0$.

Freeman's method

$$\begin{pmatrix} |a|^2 & \sqrt{2}a^*c & a^*b \\ \sqrt{2}ac^* & 2|c|^2 & \sqrt{2}bc^* \\ ab^* & \sqrt{2}b^*c & |b|^2 \end{pmatrix} = f_s \begin{pmatrix} |\beta|^2 & 0 & \beta^* \\ 0 & 0 & 0 \\ \beta & 0 & 1 \end{pmatrix} + f_d \begin{pmatrix} 1 & 0 & \alpha \\ 0 & 0 & 0 \\ \alpha^* & 0 & |\alpha|^2 \end{pmatrix} + f_v \begin{pmatrix} 1 & 0 & 1/3 \\ 0 & 2/3 & 0 \\ 1/3 & 0 & 1 \end{pmatrix}$$

$$+ \frac{1}{4} f_h \begin{pmatrix} 1 & \pm\sqrt{2}j & -1 \\ \mp\sqrt{2}j & 2 & \pm\sqrt{2}j \\ -1 & \mp\sqrt{2}j & 1 \end{pmatrix}$$


Helix

Yamaguchi's method

■ The third-order statistics analysis I

- To solve each parameters without the specific assumptions for the volume scattering, we deduce new relational expressions from the third-order statistics analysis.
- In third-order statistics analysis, we calculate the correlation of the correlation coefficients shown by the red circle.
- abc^*2 is new information that we can't get from the second-order statistics analysis.

$$\begin{pmatrix} |a|^2 & \sqrt{2}a^*c & a^*b \\ \sqrt{2}ac^* & 2|c|^2 & \sqrt{2}bc^* \\ ab^* & \sqrt{2}b^*c & |b|^2 \end{pmatrix}$$



$$\begin{pmatrix} \sqrt{2}ac^* & ab^* & \sqrt{2}b^*c \end{pmatrix} \begin{pmatrix} \sqrt{2}a^*c \\ a^*b \\ \sqrt{2}bc^* \end{pmatrix} = \begin{pmatrix} 2|a|^2|c|^2 & \sqrt{2}|a|^2b^*c & 2a^*b^*c^2 \\ \sqrt{2}|a|^2bc^* & |a|^2|b|^2 & \sqrt{2}|b|^2a^*c \\ 2abc^*2 & \sqrt{2}|b|^2ac^* & |b|^2 \end{pmatrix}$$

■ The third-order statistics analysis II

- we express the correlation of the correlation coefficient for each polarimetric data as the summation of the surface scattering, dihedral scattering, volume scattering and helix scattering component matrix.
- We get a new relational equation from this analysis.

$$\begin{pmatrix} 2|a|^2|c|^2 & \sqrt{2}|a|^2b^*c & 2a^*b^*c^2 \\ \sqrt{2}|a|^2bc^* & |a|^2|b|^2 & \sqrt{2}|b|^2a^*c \\ \textcircled{2abc^{*2}} & \sqrt{2}|b|^2ac^* & |b|^2 \end{pmatrix} = f_s^2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & |\beta|^2 & 0 \\ \textcircled{0} & 0 & 0 \end{pmatrix} + f_d^2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & |\alpha|^2 & 0 \\ \textcircled{0} & 0 & 0 \end{pmatrix}$$

$$+ \begin{pmatrix} 2|V_{HH}|^2|V_{HV}|^2 & \sqrt{2}|V_{HH}|^2V_{HV}V_{VV}^* & 2V_{HH}^*V_{VV}^*V_{HV}^2 \\ \sqrt{2}|V_{HH}|^2V_{HV}^*V_{VV} & |V_{HH}|^2|V_{VV}|^2 & \sqrt{2}|V_{VV}|^2V_{HV}V_{HH}^* \\ \textcircled{2V_{HH}V_{VV}V_{HV}^{*2}} & \sqrt{2}|V_{VV}|^2V_{HH}V_{HV}^* & 2|V_{HV}|^2|V_{VV}|^2 \end{pmatrix} + \frac{1}{16} f_h^2 \begin{pmatrix} 2 & \mp\sqrt{2}j & 2 \\ \pm\sqrt{2}j & 1 & \pm\sqrt{2}j \\ \textcircled{2} & \mp\sqrt{2}j & 2 \end{pmatrix}$$

Volume



Helix

$$abc^{*2} = V_{HH}V_{VV}V_{HV}^{*2} + \frac{1}{16} f_h^2 \quad (7)$$

■ The volume and helix scattering components

- We estimate each parameters in the volume scattering and helix scattering by using Eq.(4), (5), (6) and (7).
- As a result, the equation for the helix component is the same formula as one in Yamaguchi's method.
- The surface scattering and dihedral scattering can be estimated from Eq.(3), (4) and (5) by Freeman's estimation method.

$$|c|^2 = |V_{HV}|^2 + \frac{1}{4} f_h \quad (4)$$

$$ac^* = V_{HH} V_{HV}^* \mp \frac{1}{4} f_h j \quad (5)$$

$$b^*c = V_{HV} V_{VV}^* \mp \frac{1}{4} f_h j \quad (6)$$

$$abc^{*2} = V_{HH} V_{VV} V_{HV}^{*2} + \frac{1}{16} f_h^2 \quad (7)$$



$$|V_{HV}|^2 = |c|^2 - \frac{1}{4} f_h$$

$$V_{HH} = \frac{ac^* \pm \frac{1}{4} f_h j}{|c|^2 - \frac{1}{4} f_h}$$

$$V_{HH} = \frac{bc^* \mp \frac{1}{4} f_h j}{|c|^2 - \frac{1}{4} f_h}$$

$$f_h = \mp 2(\text{Im}(ac^*) + \text{Im}(b^*c))$$

■ Evaluation of new method

- To evaluate the proposed method, we simulate polarimetric SAR images as the summation of each component and decompose these polarimetric images into each component by the proposed method.
- In addition, we compare these results with ones estimated by Freeman's method and Yamaguchi's method and remark on the merits and limitations for each method.

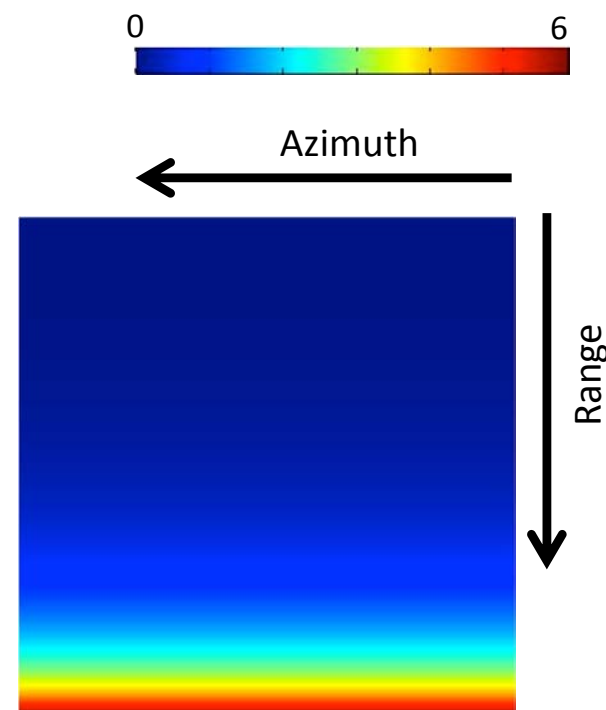
■ Surface scattering model

Surface scattering model (Rice-Bragg model) is shown as follows.

$$S_{HH} = \frac{\cos \theta - \sqrt{\varepsilon - \sin^2 \theta}}{\cos \theta + \sqrt{\varepsilon - \sin^2 \theta}}$$

$$S_{VV} = \frac{(\varepsilon - 1) \{ \sin^2 \theta - \varepsilon (1 + \sin^2 \theta) \}}{(\varepsilon \cos \theta + \sqrt{\varepsilon - \sin^2 \theta})^2}$$

ε is the complex dielectric constant. θ is the incident angle. I calculated surface backscattering as $\varepsilon = 1 + i$ and $\theta = 0 \sim \pi/2$.

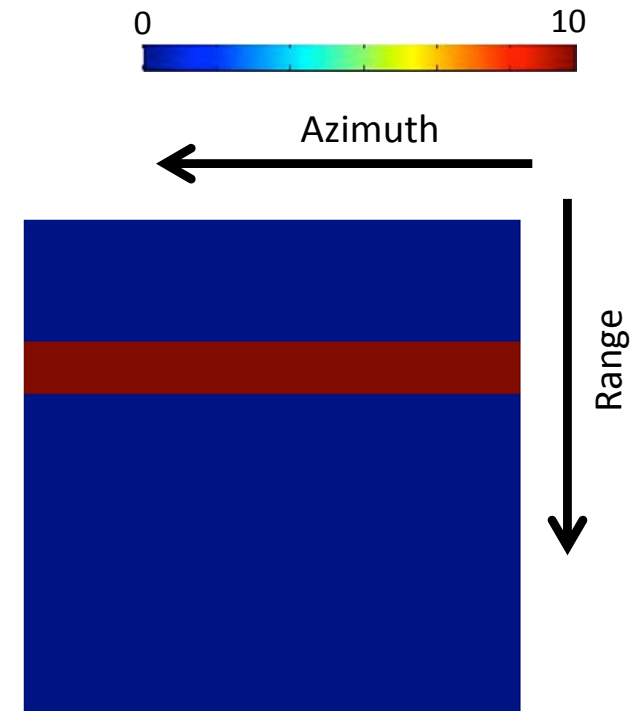


■ Dihedral scattering model

The phase difference between vertical and horizontal polarization is considered as π . Dihedral scattering model is shown as follows.

$$D_{HH} = A_{DHH} \exp(j\theta_{HH})$$

$$D_{VV} = -A_{DVV} \exp(j\theta_{HH})$$



- A_{DHH} and A_{DVV} is the amplitude of the dihedral scattering for the horizontal and vertical polarization.
- θ_{HH} is the incident angle for the horizontal polarization. The incident angle θ_{VV} for the vertical polarization is $(\theta_{HH} + \pi)$.
- We calculated dihedral backscattering as $A_{DHH}=2$, $A_{DVV}=-2.5$ and $\theta_{HH}=\pi/4$.

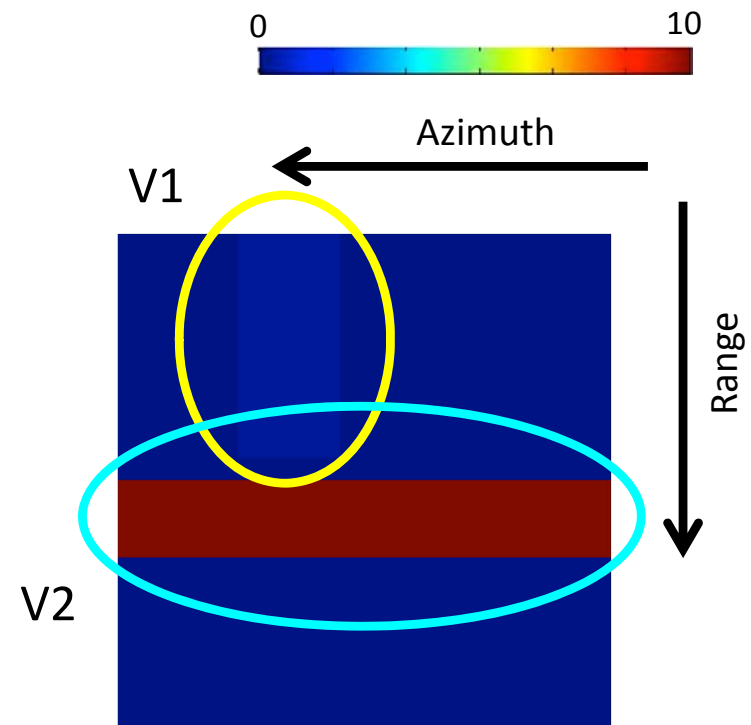
■ Volume scattering model

Volume scattering could be expressed as the union sets of thin cylinder.

$$V_{VH} = -(a - b) \int_0^{2\pi} \cos \theta \sin \theta \cdot f(\theta) d\theta$$

$$V_{HH} = \int_0^{2\pi} (a \cos^2 \theta + b \sin^2 \theta) f(\theta) d\theta$$

$$V_{VV} = \int_0^{2\pi} (a \sin^2 \theta + b \cos^2 \theta) f(\theta) d\theta$$



a and b is the backscattering coefficient from thin cylinder in the horizontal and vertical polarization. $f(\theta)$ and θ is the probability density function and the incident angle. We calculated two volume backscattering as $a_1 = \exp(j*\pi/4)$, $b_1 = -5*\exp(j*\pi/3)$, $a_2 = \exp(j*\pi/4)$, $b_2 = \exp(j*4\pi/3)$ and $f(\theta) = \cos(\theta/2)$.

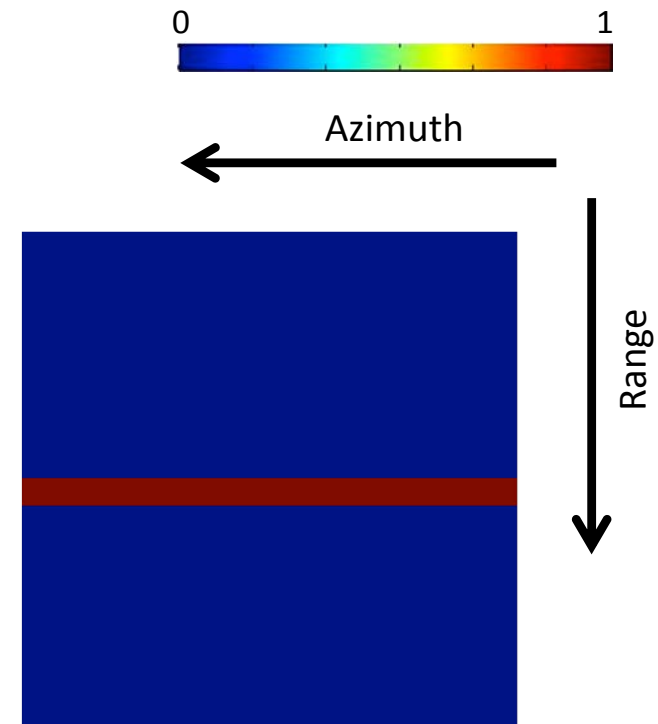
■ Helix scattering model

Dihedral scattering model is shown as follows.

$$h_{HH} = A_{hHH} \exp(j\theta_{HH})$$

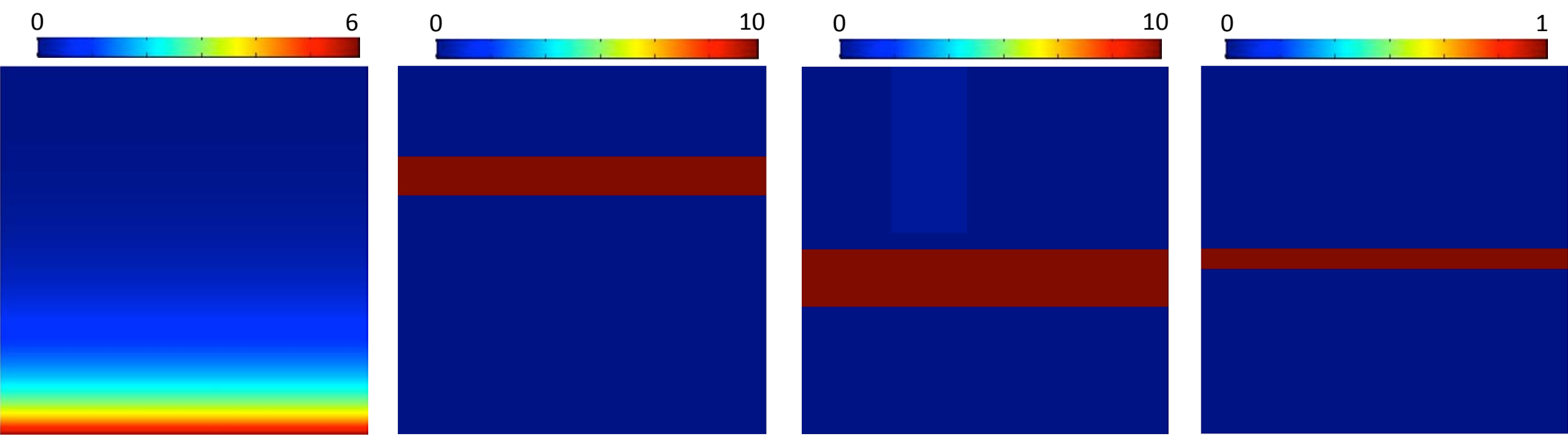
$$h_{VV} = -h_{HH}$$

$$h_{VH} = \pm jh_{HH}$$



- A_{hHH} is the amplitude of the helix scattering for the horizontal. θ_{HH} is the incident angle for the horizontal polarization.
- I calculated helix backscattering as $A_{hHH}=1$ and $\theta_{HH}=\pi/4$.

■ The simulation results

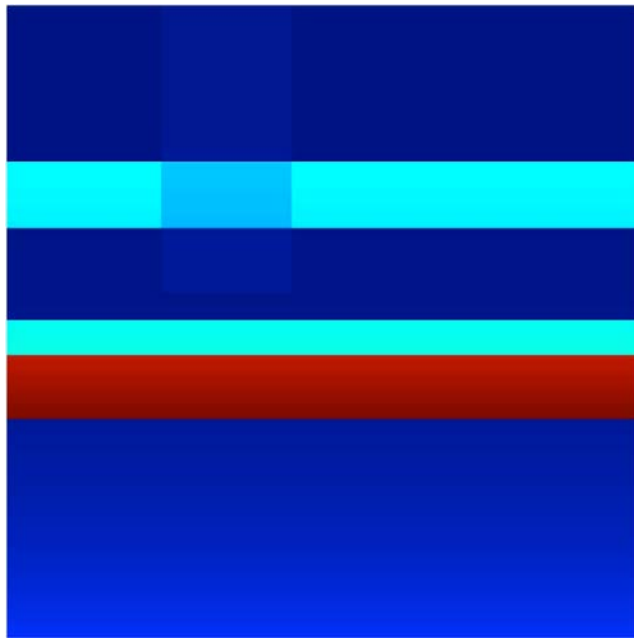


Surface

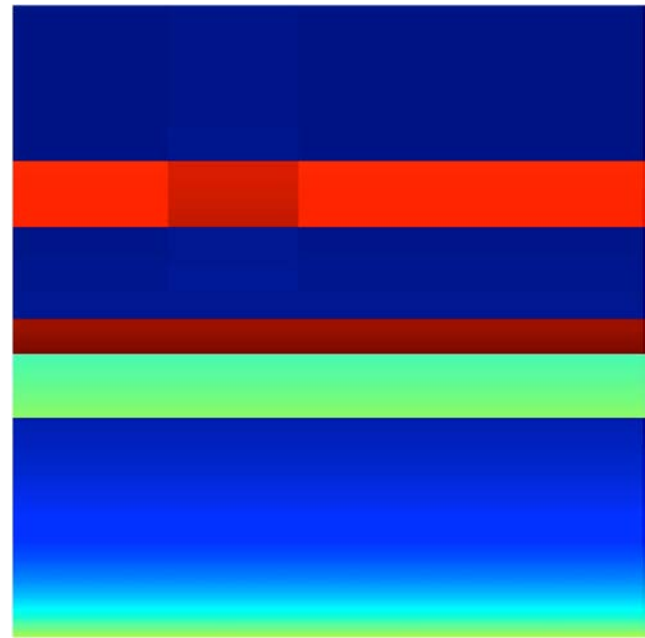
Dihedral

Volume

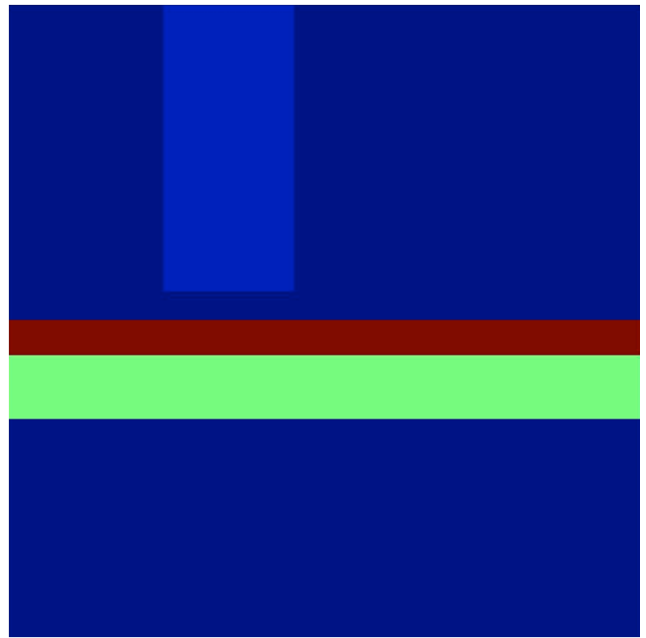
Helix



HH

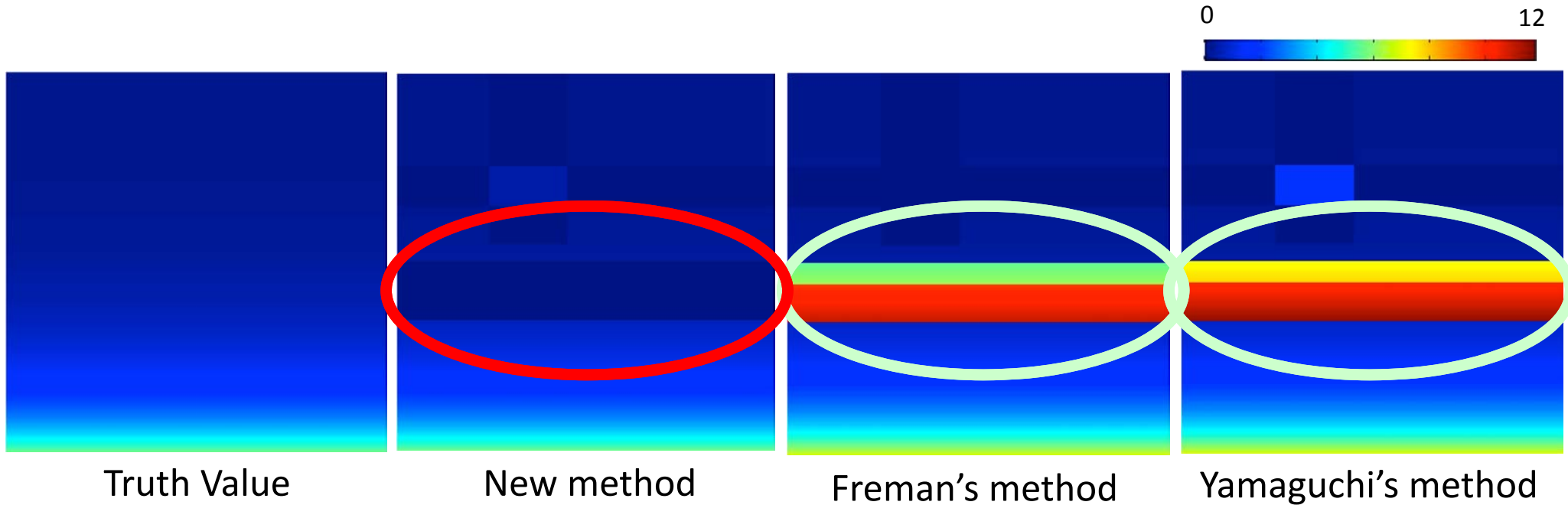


VV



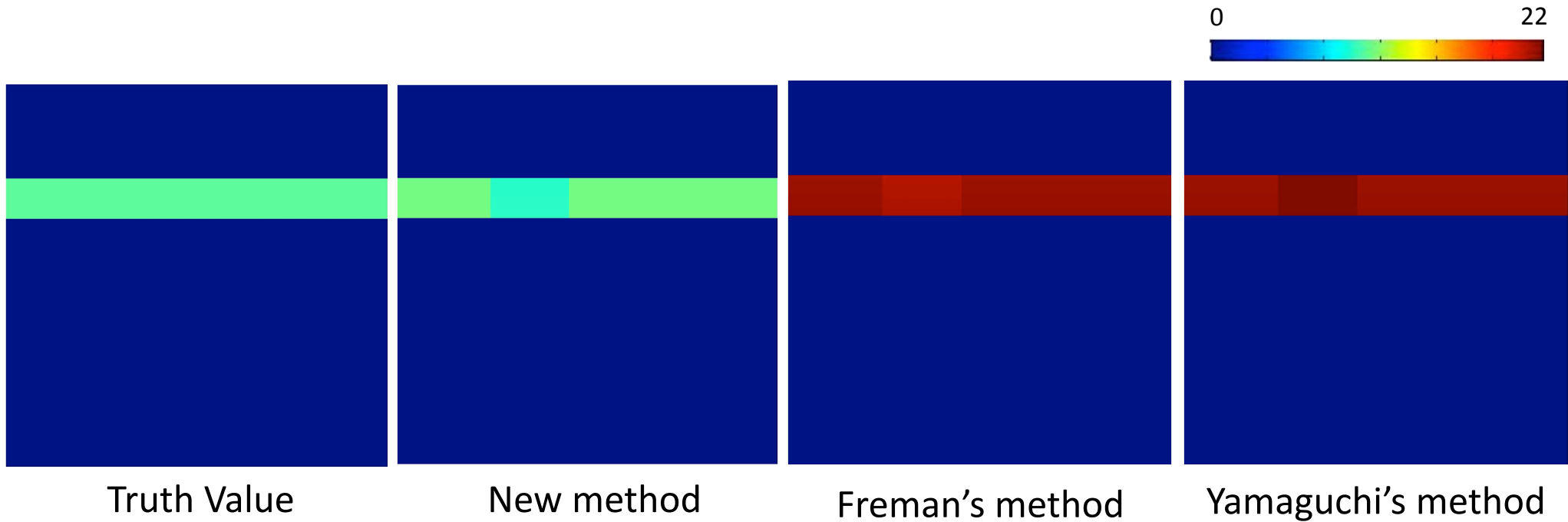
VH

■ Evaluation results of the surface scattering



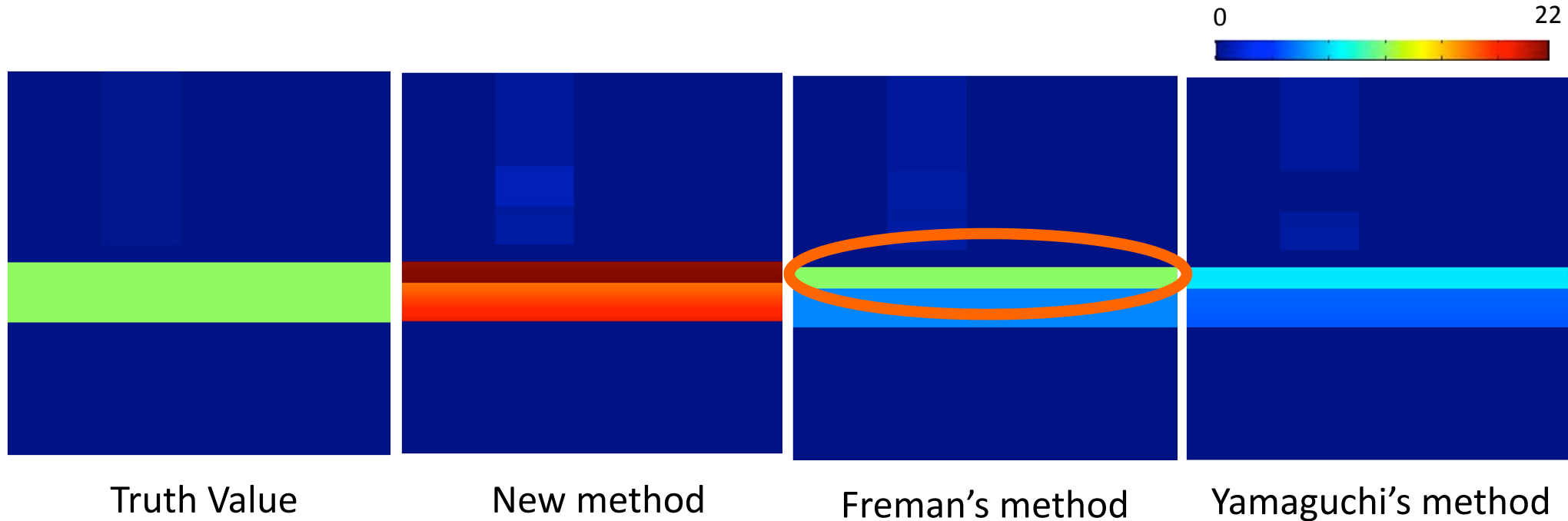
- New method underestimate the surface scattering component when the surface scattering and the volume scattering (the forest area) occur simultaneously.
- Freeman's method and Yamaguchi's method overestimate the surface scattering component in the same area.

■ Evaluation results of the dihedral scattering



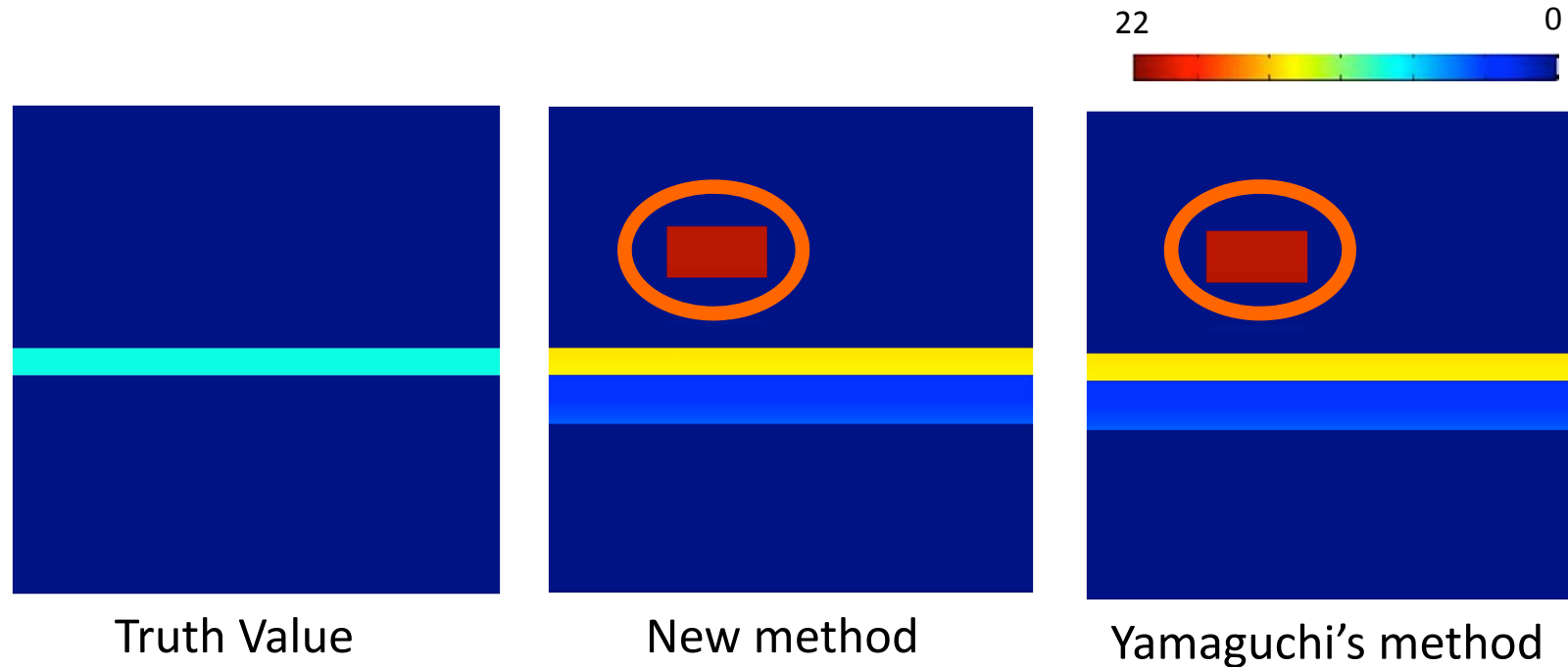
- New method can estimate dihedral scattering component with high accuracy.
- Freeman's and Yamaguchi's method gives an overestimation for the dihedral scattering.

■ Evaluation results of the volume scattering



- The volume scattering component estimated by new method is double value of simulated truth data.
- The volume scattering component estimated by Freeman's and Yamaguchi's method are half of simulated truth data.
- But, Freeman's method can estimate the volume scattering precisely when the volume scattering and the helix scattering occur simultaneously.

■ Evaluation results of the helix scattering



- The helix scattering component estimated by new method and Yamaguchi's method are in an overestimation.
- Especially, both method overestimates the helix scattering when the volume scattering and the dihedral scattering occur simultaneously.

■ Analysis of UAVSAR data

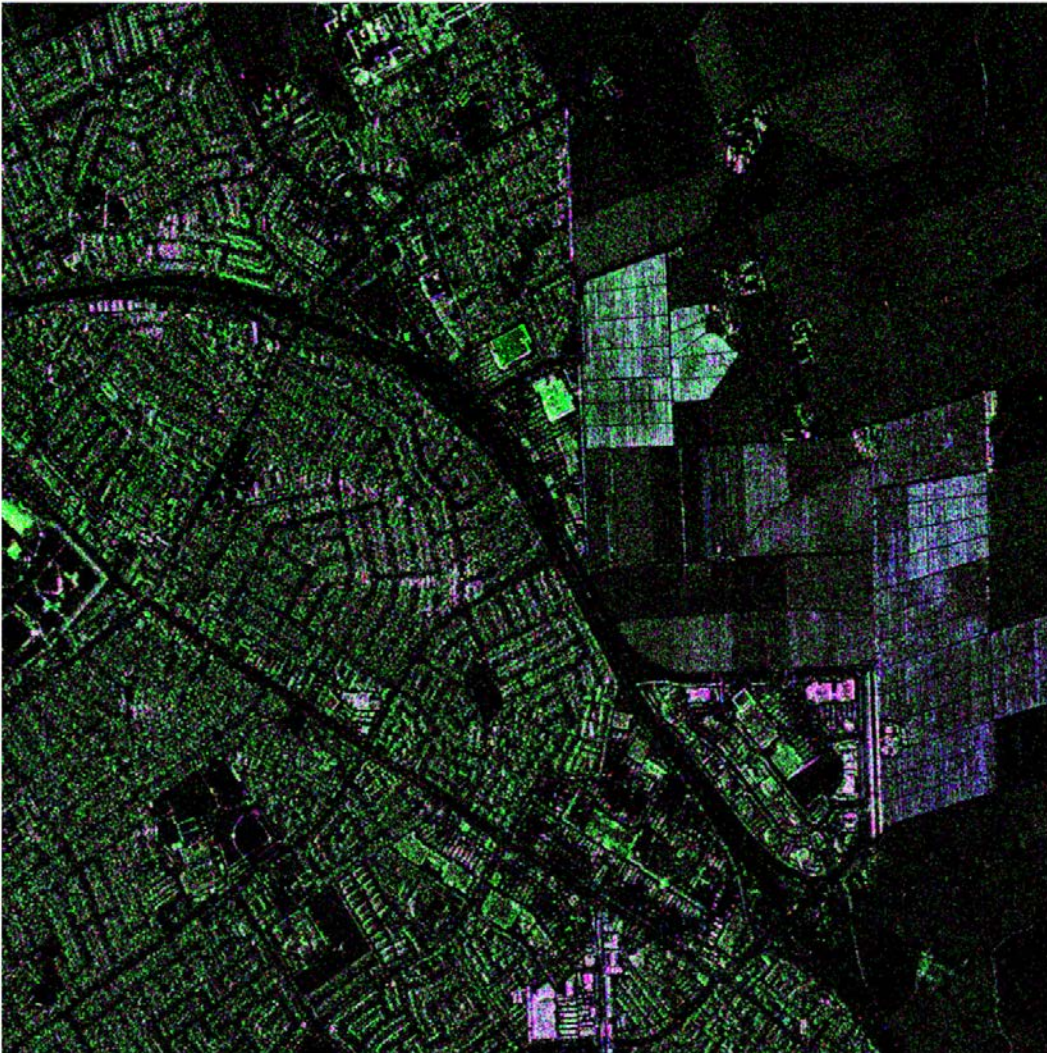
- We evaluate the proposal method by using UAVSAR data.
- Analysis area is Salinas, the south of San Francisco. There are a lot of houses, roads, trees and dry fields.
- We estimate the surface, dihedral, volume and helix scattering component from polarimetric SAR images by the proposal method.



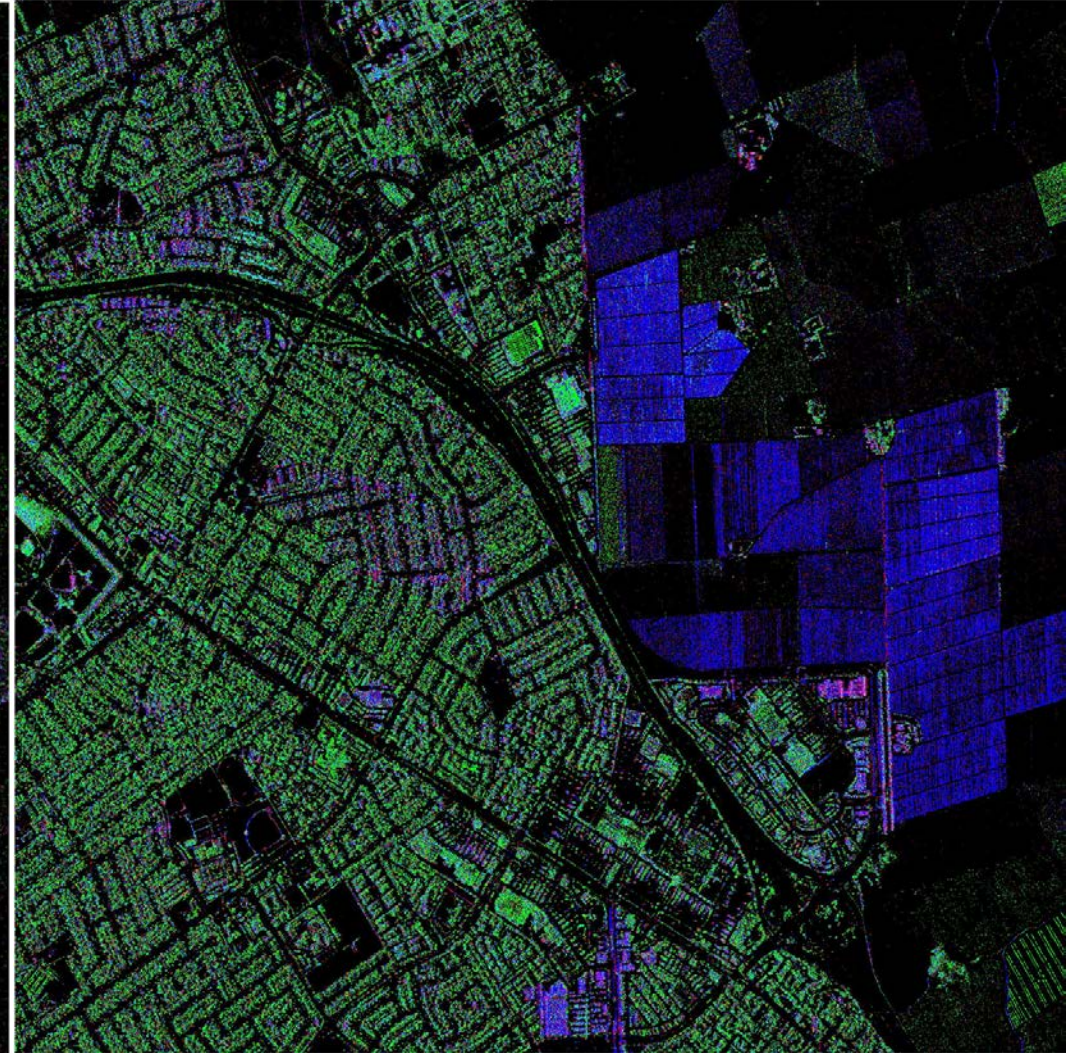
■ Comparison between new method and Yamaguchi's method

- The decomposition result in the proposed method is different from it in Yamaguchi's method in the surface and dihedral scattering dominant area.
- The field is a big difference.

New method



Yamaguchi's method

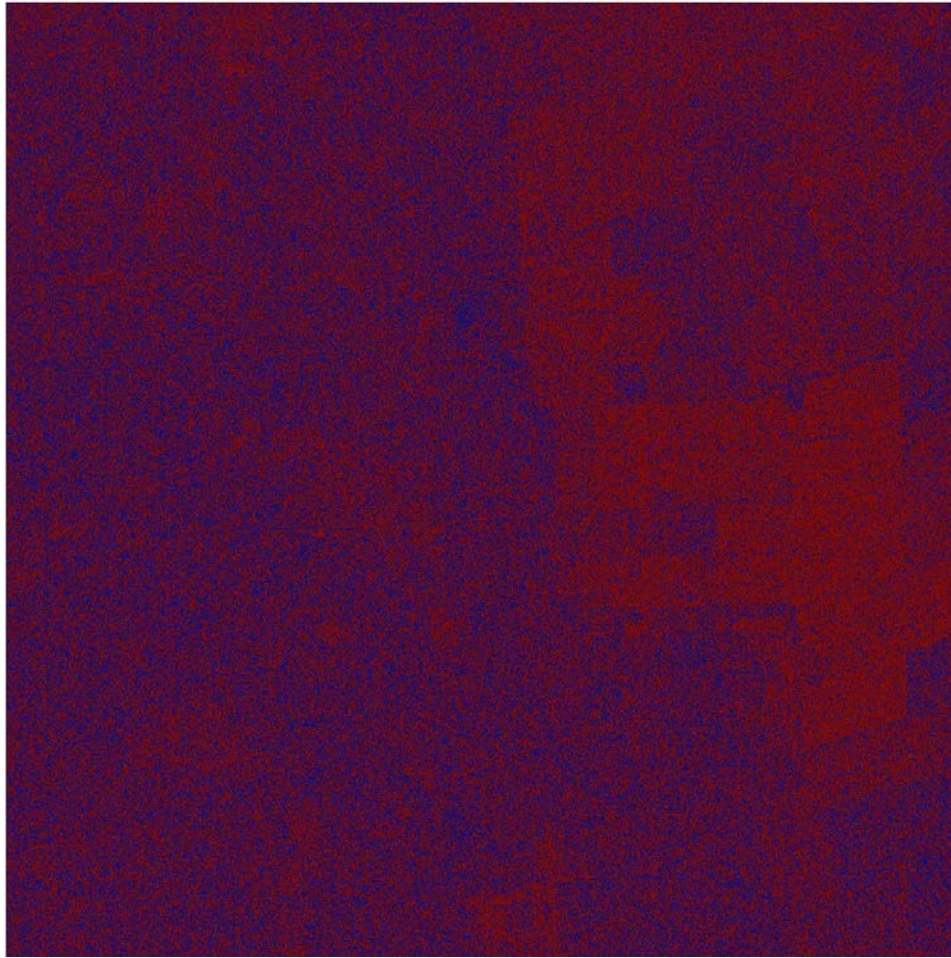


R: Dihedral, G: Volume, B: Surface

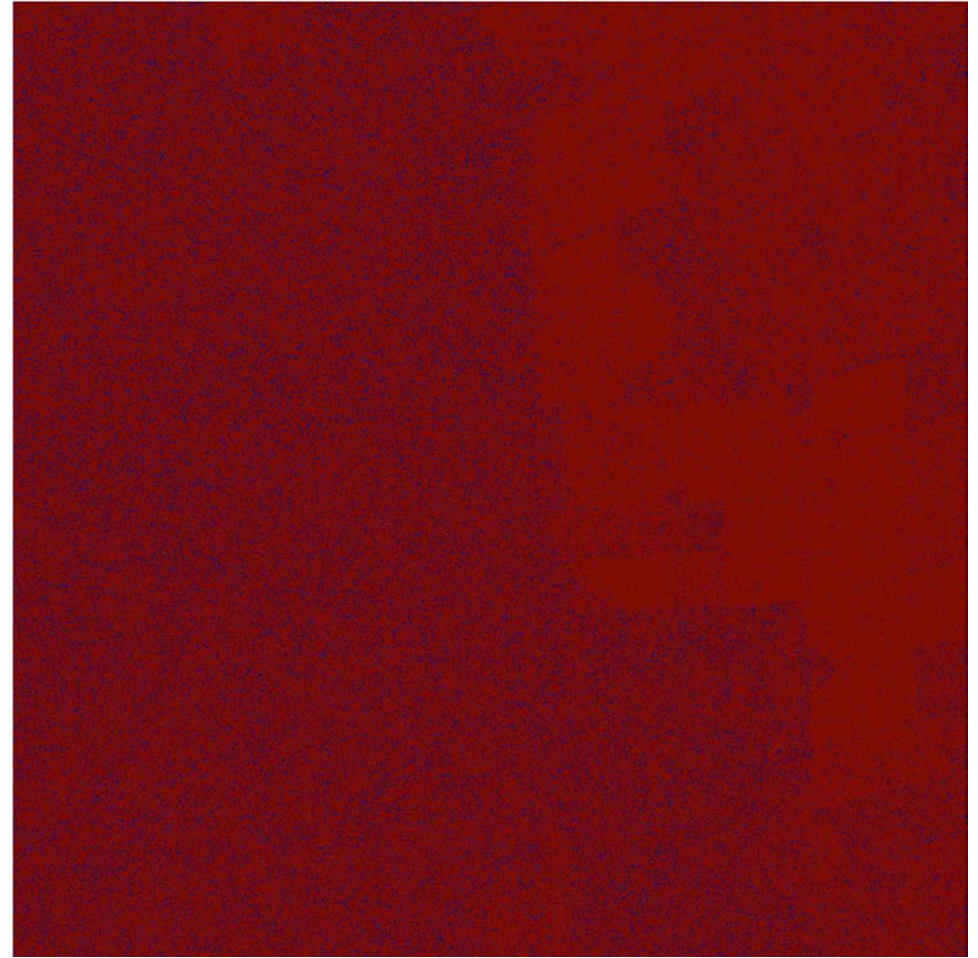
■ Evaluation of estimated value for the surface scattering

- 62% is the positive value in estimated surface scattering by new method.
- 89% is the positive value in estimated surface scattering by Yamaguchi's method.

R:+ , B:-



New method

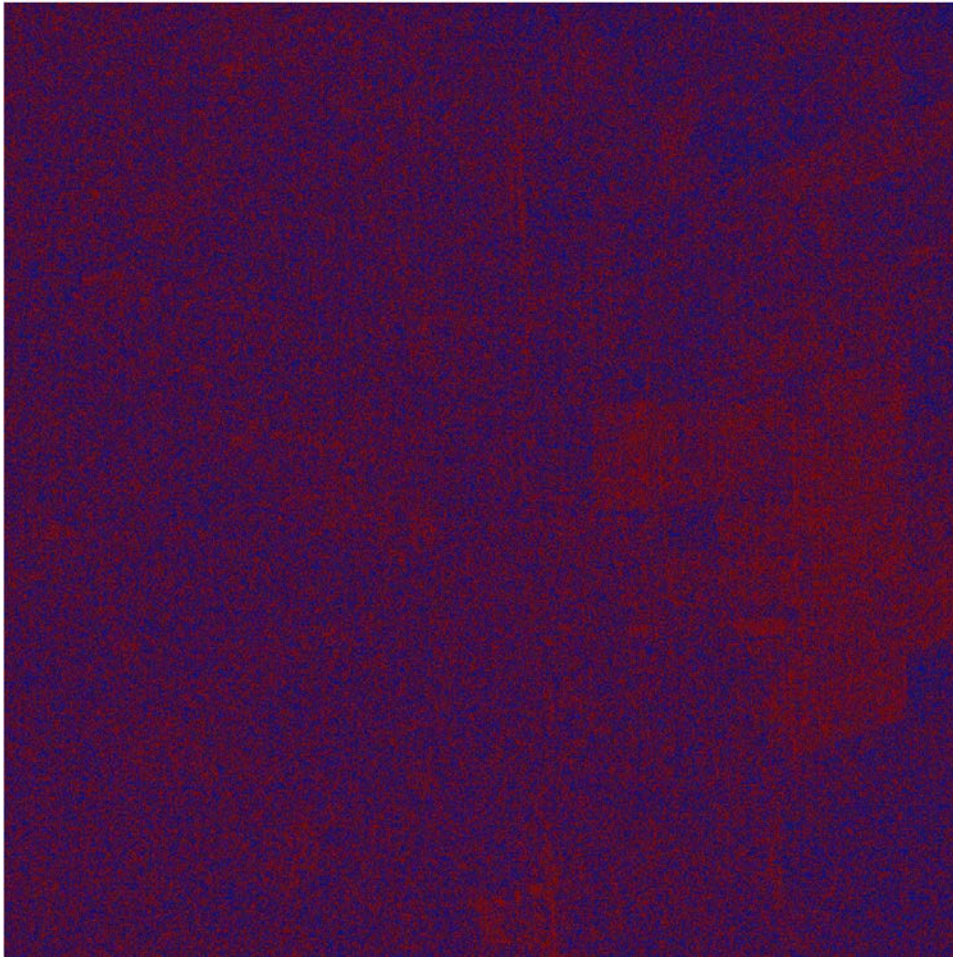


Yamaguchi's method

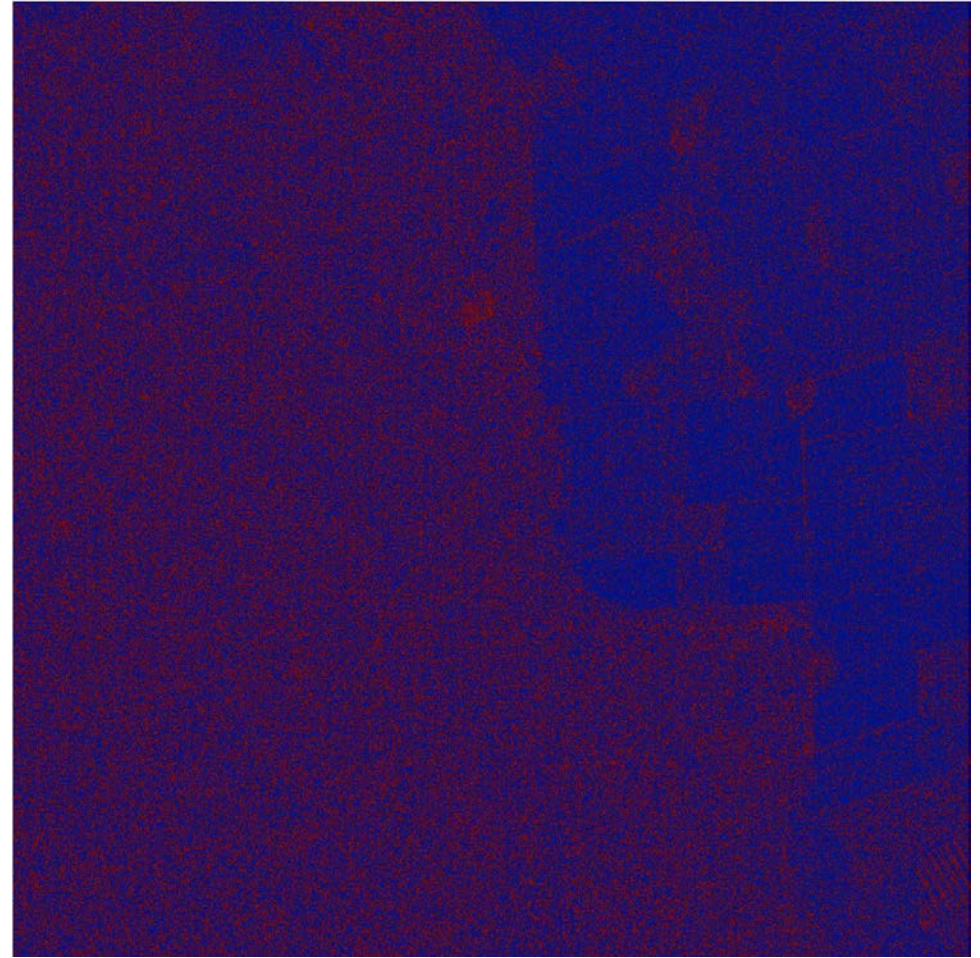
■ Evaluation of estimated value for the dihedral scattering

- 53% is the positive value in estimated dihedral scattering by new method.
- 34% is the positive value in estimated dihedral scattering by Yamaguchi's method.

R:+ , B:-



New method



Yamaguchi's method

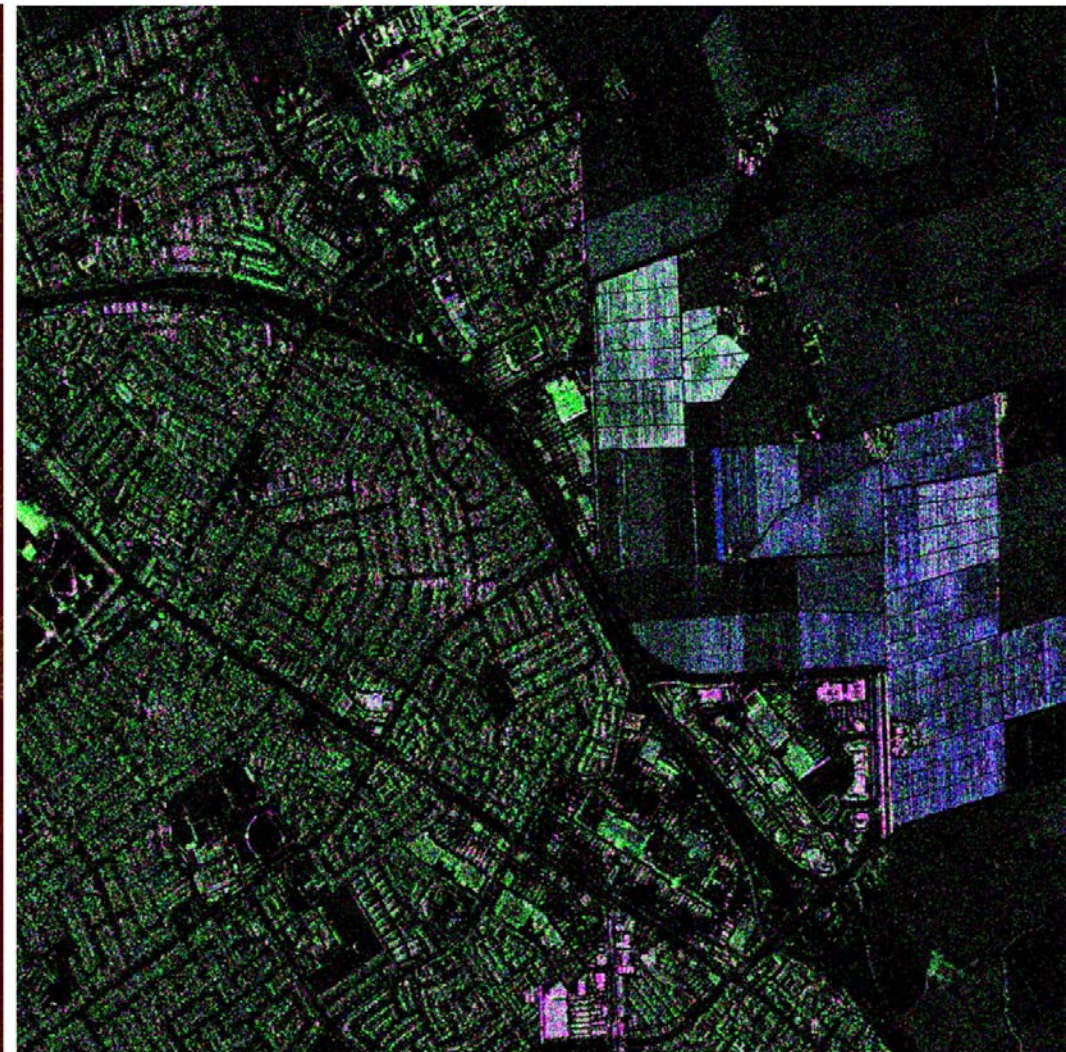
■ The decomposition result for HH and VV component

The field in the HH result **➔** The dihedral scattering

The field in the VV result **➔** The surface and dihedral scattering

HH component

VV component



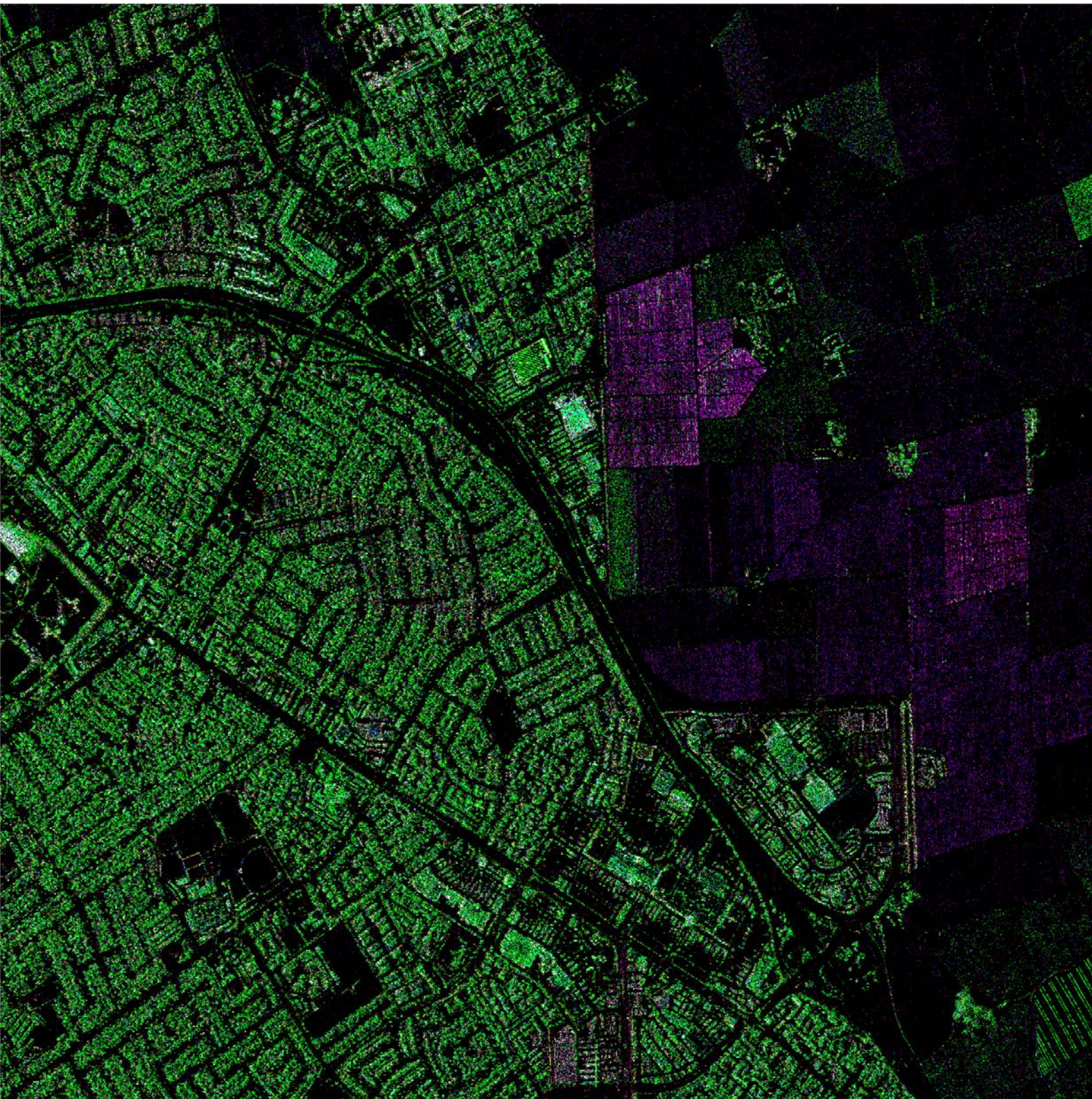
R: Dihedral, G: Volume, B: Surface

New Decomposition
(Volume component)

R:HH

G:VH

B:VV



■ Conclusion

- The proposed method is able to estimate each polarization component for the volume scattering without specific a priori volume scattering assumptions.
- To better understand the back scattering mechanisms from the polarimetric SAR images, we clarified each method's merits and limitations including the estimation accuracy by analyzing simulated data.
 - New method can estimate the dihedral scattering precisely.
 - Freeman's method can estimate the volume scattering precisely when the volume and helix scattering occur simultaneously.
 - The surface scattering is underestimated by new method and overestimated by Freeman's and Yamaguchi's method.
- Further research and fieldwork are needed to better quantify and understand these results fully.