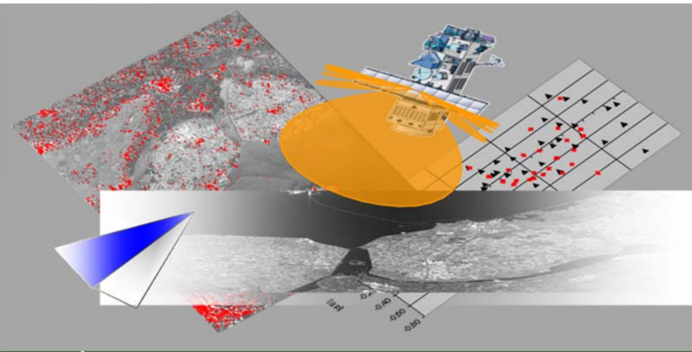

POLITECNICO DI MILANO

Dipartimento di
Elettronica e Informazione

Assimilation of Distributed Targets and PS Information for the Monitoring of Polarimetric SAR Systems

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POLIMI

Project supported by Agenzia Spaziale Italian (ASI), project -1080 - *SAR data Calibration and Validation by Natural Targets*

Outline

- Introduction
  - Traditional polarimetric calibration
- PS-based technique
  - Model
  - Algorithm
  - Integration with external data
    - Calibrated data
    - Distributed targets
- Results on RS2 dataset
- Conclusions

SAR calibration aims to:

- Remove the radiometric and polarimetric distortion from the target signatures
- SAR instrument health status monitoring
  - T/R Modules, Antenna pattern, Power losses

System distortion (without Faraday)

$$\begin{bmatrix} M_{HH} & M_{VH} \\ M_{HV} & M_{VV} \end{bmatrix} = Ae^{j\phi} \cdot \underline{\underline{\begin{bmatrix} 1 & \delta_2 \\ \delta_1 & f_1 \end{bmatrix}}} \cdot \begin{bmatrix} S_{HH} & S_{VH} \\ S_{HV} & S_{VV} \end{bmatrix} \cdot \underline{\underline{\begin{bmatrix} 1 & \delta_3 \\ \delta_4 & f_2 \end{bmatrix}}}$$

RECEIVE MATRIX **TRANSMIT MATRIX**

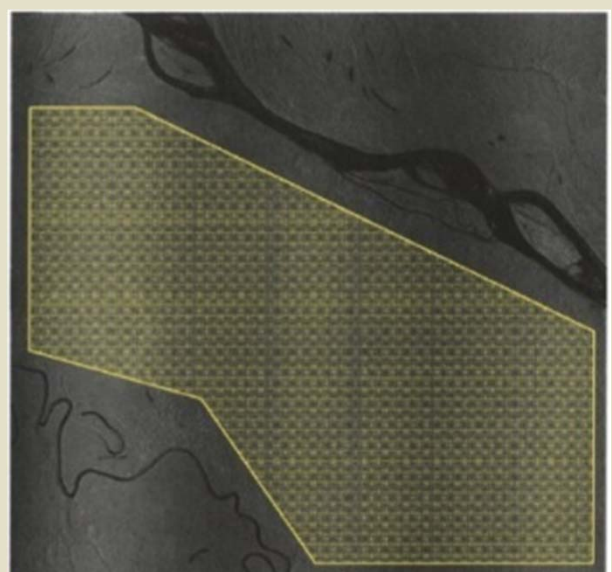
- 6 COMPLEX PARAMETERS
  - IMBALANCES: f_1, f_2 **Typical Requirement: 0.2 dB**
  - CROSS-TALKS (CTs): $\delta_1, \delta_2, \delta_3, \delta_4$ **Typical Requirement: < -30 dB**

Current polarimetric calibration approaches exploit:

A. Network of calibrated active/passive reflectors

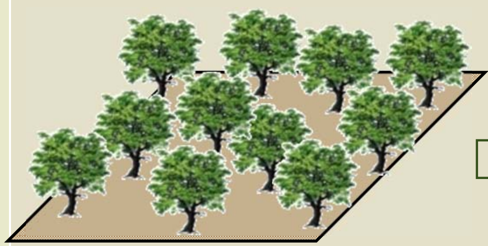
- PARC, corners (3 or more)

B. A Distributed Target (DT)



ISSUES

- A calibration site is expensive to be deployed and then maintained for the whole mission lifetime. Moreover it demands for dedicated acquisitions that can interfere with the mission operations.
- Approaches based solely on DTs (no point calibrators) can provide only partial information



Distributed Target (Azim. Symmetry)

$$\langle S_{hh} S_{hv}^{*} \rangle = \langle S_{vv} S_{hv}^{*} \rangle = 0$$

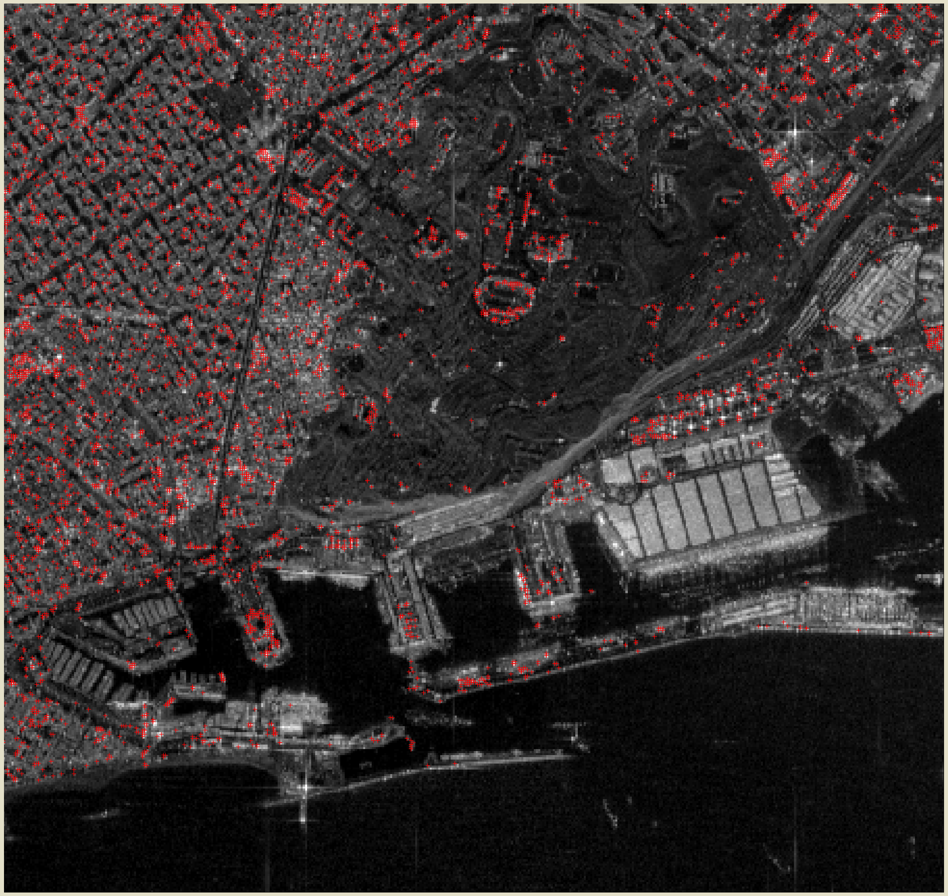
$$\frac{f_1}{f_2}, \delta_1, \frac{\delta_2}{f_1}, \delta_3, \frac{\delta_4}{f_2}, \cancel{f_2, A}$$

- Without a calibrated point target (PT) amplitude and phase imbalances are missing.

Proposed PScal approach

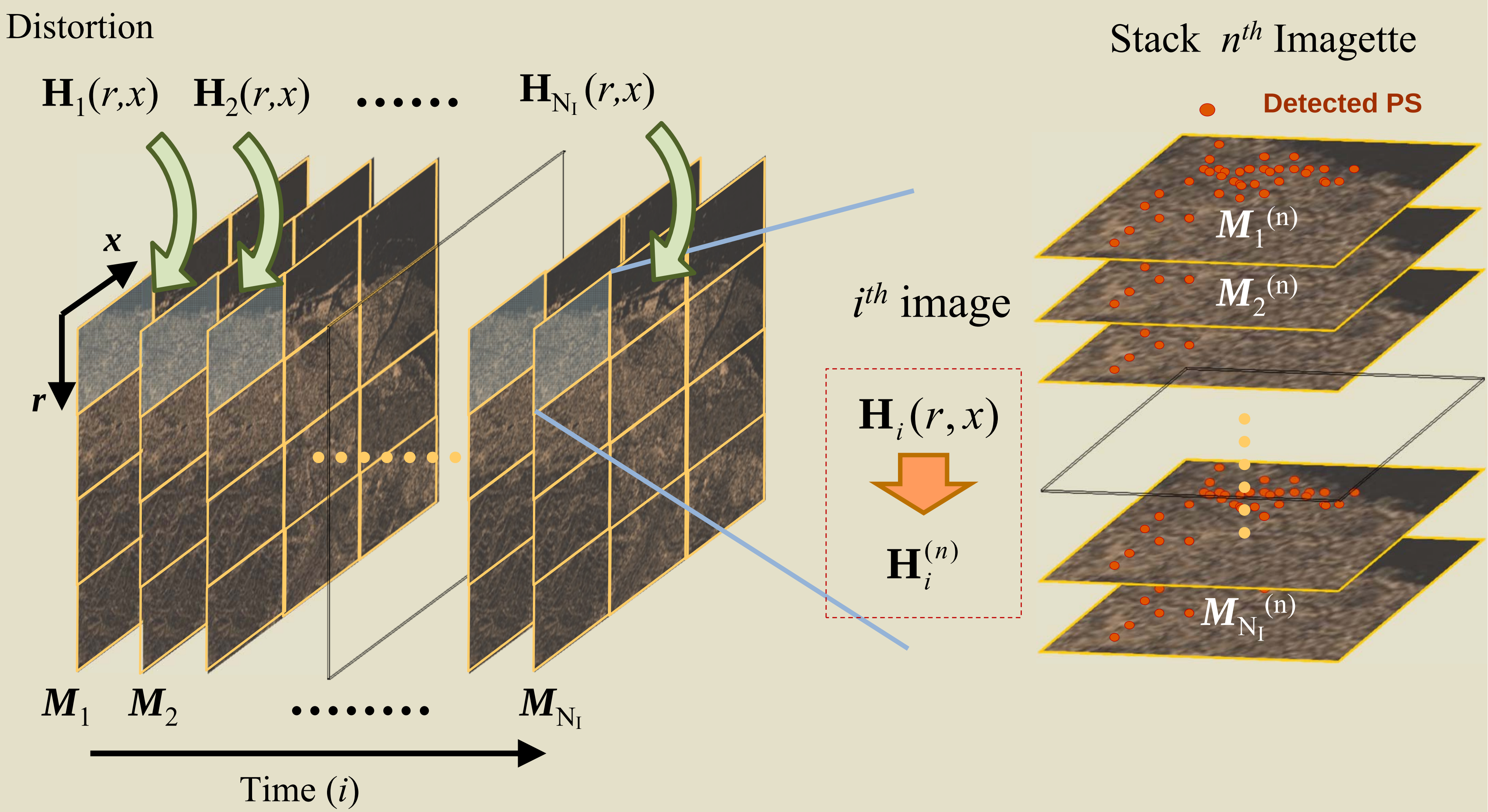
IDEA

- Exploit multiple-image information
- Use the stable targets (Permanent Scatterers) on the scene as if they were calibrated point targets (PT)

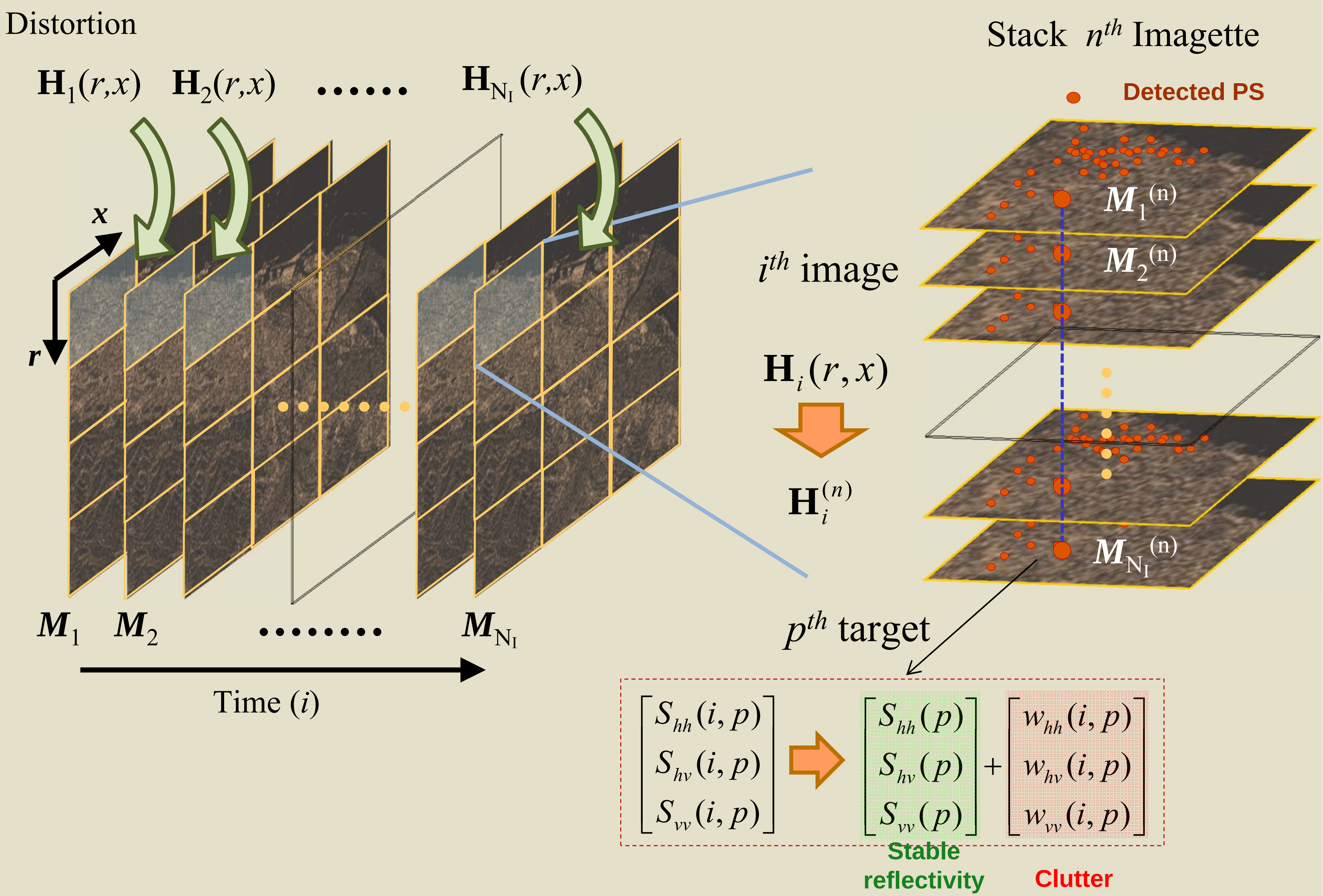


E.g.: 1000 PSs with 10 dB SCR are equivalent to 1 PT with 40 dB SCR

PS-Based Technique
PS model



PS-Based Technique
PS model



PS-Based Technique
PS Model

Stable reflectivity | Clutter | Thermal + Model + al. | PDM 4x3

$$\mathbf{y}_{i,p}=\begin{bmatrix}y_{hh}(i,p)\\y_{hv}(i,p)\\y_{vh}(i,p)\\y_{vv}(i,p)\end{bmatrix}\cong e^{j\varphi(i,p)}\cdot\mathbf{H}_i\cdot\underbrace{\left(\begin{bmatrix}S_{hh}(p)\\S_{hv}(p)\\S_{vv}(p)\end{bmatrix}+\begin{bmatrix}w_{hh}(i,p)\\w_{hv}(i,p)\\w_{vv}(i,p)\end{bmatrix}\right)}_{\mathbf{s}_p+\mathbf{w}_{i,p}}+\begin{bmatrix}n_{hh}(i,p)\\n_{hv}(i,p)\\n_{vh}(i,p)\\n_{vv}(i,p)\end{bmatrix}$$

- Clutter model
  - Geometrical and temporal decorrelation not accounted

    $E[\mathbf{w}(i_1,p)\mathbf{w}(i_2,p)^*]=\mathbf{0}$ with $i_1\neq i_2$

  - ccG behaviour: $\mathbf{w}_{i,p}\sim CN(\mathbf{0},\mathbf{C}_p)$ with

$$\mathbf{C}_p=\begin{bmatrix}\nu_{hh}(p) & \kappa_{hhhv}(p)^* & \kappa_{hhvv}(p)^*\\ \kappa_{hhhv}(p) & \nu_{hv}(p) & \kappa_{hvvv}(p)^*\\ \kappa_{hhvv}(p) & \kappa_{hvvv}(p) & \nu_{vv}(p)\end{bmatrix}$$

Covariances dependent on PS and stationary along the stack

- Intrinsic Ambiguity $\mathbf{H}_i\leftrightarrow\mathbf{s}_p,\mathbf{C}_p$

$$\mathbf{y}_{i,p}=e^{j\varphi(i,p)}\mathbf{H}_i\cdot\left(\mathbf{s}_p+\mathbf{w}_{i,p}\right)+\mathbf{n}_{i,p}=e^{j\varphi(i,p)}\mathbf{H}_i\cdot\mathbf{K}\cdot\mathbf{K}^{-1}\left(\mathbf{s}_p+\mathbf{w}_{i,p}\right)+\mathbf{n}_{i,p}$$

3 by 3 matrix $\mathbf{K}$

Stable reflectivity — Clutter — Thermal + Model + al. — PDM 4x3

$$\mathbf{y}_{i,p} = \begin{bmatrix} y_{hh}(i,p) \\ y_{hv}(i,p) \\ y_{vh}(i,p) \\ y_{vv}(i,p) \end{bmatrix} \cong e^{j\varphi(i,p)} \cdot \mathbf{H}_i \cdot \underbrace{\left(\begin{bmatrix} S_{hh}(p) \\ S_{hv}(p) \\ S_{vv}(p) \end{bmatrix} + \begin{bmatrix} w_{hh}(i,p) \\ w_{hv}(i,p) \\ w_{vv}(i,p) \end{bmatrix} \right)}_{\mathbf{s}_p + \mathbf{w}_{i,p}} + \begin{bmatrix} n_{hh}(i,p) \\ n_{hv}(i,p) \\ n_{vh}(i,p) \\ n_{vv}(i,p) \end{bmatrix}$$

- Clutter model
  - Geometrical and temporal decorrelation not accounted

    $E[\mathbf{w}(i_1,p)\mathbf{w}(i_2,p)^*] = \mathbf{0}$ with $i_1 \neq i_2$
  - ccG behaviour: $\mathbf{w}_{i,p} \sim CN(\mathbf{0},\mathbf{C}_p)$ with

$$\mathbf{C}_p = \begin{bmatrix} \nu_{hh}(p) & \kappa_{hhhv}(p)^* & \kappa_{hhvv}(p)^* \\ \kappa_{hhhv}(p) & \nu_{hv}(p) & \kappa_{hvvv}(p)^* \\ \kappa_{hhvv}(p) & \kappa_{hvvv}(p) & \nu_{vv}(p) \end{bmatrix}$$

Covariances dependent on PS and stationary along the stack

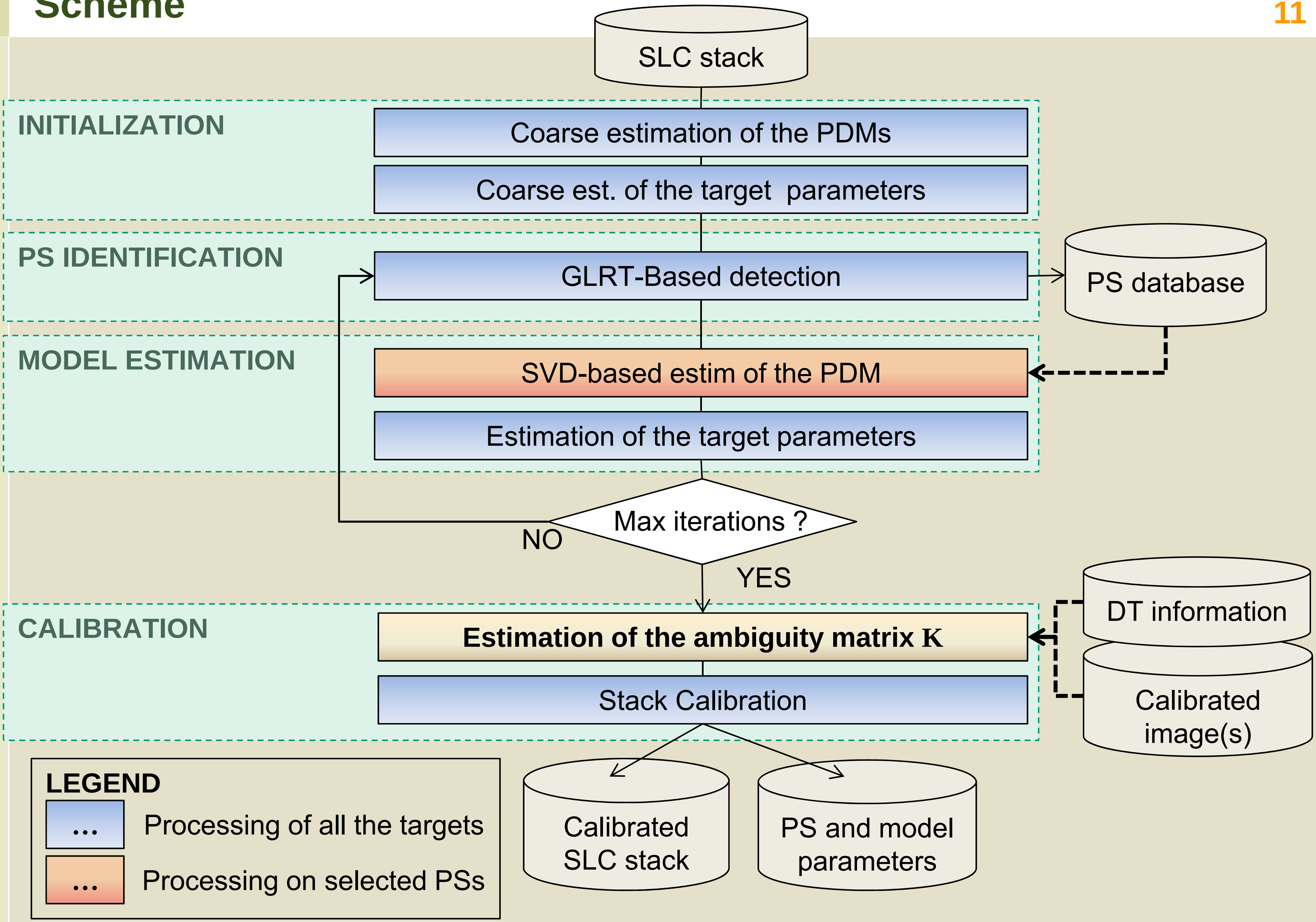
- Intrinsic Ambiguity $\mathbf{H}_i \leftrightarrow \mathbf{s}_p, \mathbf{C}_p$

$$\mathbf{y}_{i,p} = e^{j\varphi(i,p)}\mathbf{H}_i \cdot \left(\mathbf{s}_p + \mathbf{w}_{i,p}\right) + \mathbf{n}_{i,p} = e^{j\varphi(i,p)}\mathbf{H}_i \cdot \mathbf{K} \cdot \mathbf{K}^{-1}\left(\mathbf{s}_p + \mathbf{w}_{i,p}\right) + \mathbf{n}_{i,p}$$

3 by 3 matrix $\mathbf{K}$

PS-Based Technique

Scheme



PS-Based Technique

Resolving the ambiguity

- The PDM estimates before calibration are affected by the $\mathbf{K}$ ambiguity

Polarimetric Distortion Matrix $\hat{\mathbf{H}}_i = \mathbf{H}_i \cdot \mathbf{K}$

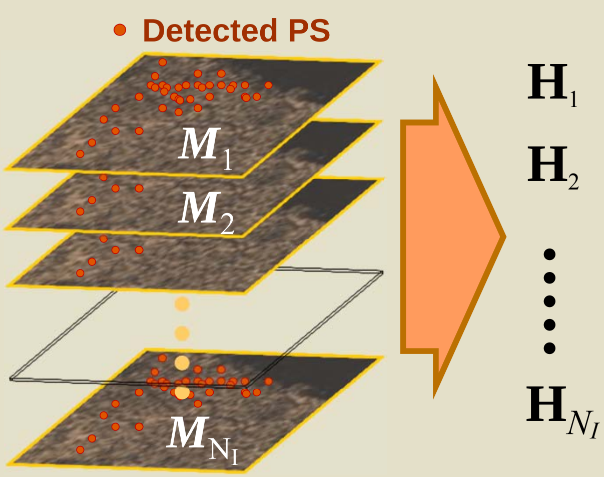
Backscatter vector $\hat{\mathbf{s}}_p = \mathbf{K}^{-1} \cdot \mathbf{s}_p$

$\mathbf{K}$ ambiguity is same for all the stack

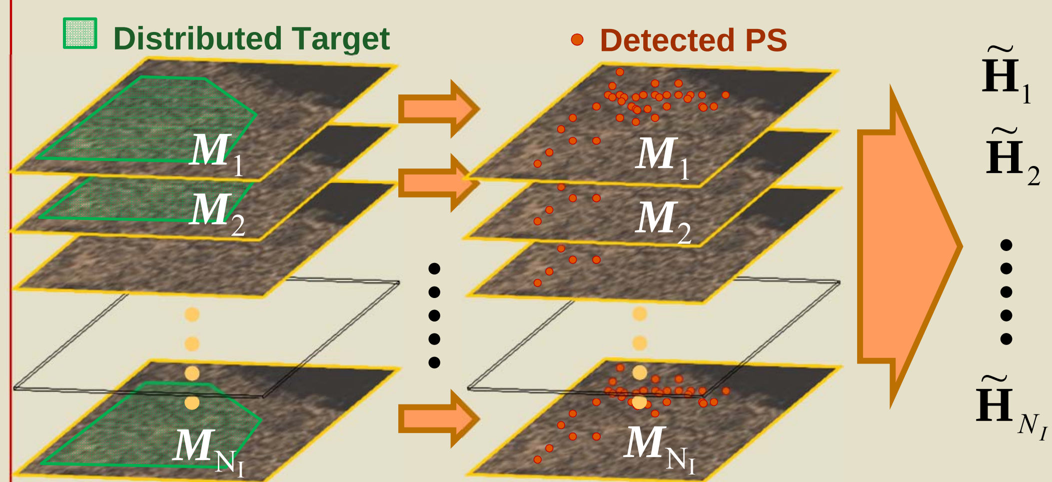
- $\mathbf{K}$ can be estimated:

Completely by ≥1 calibrated image of the stack

$\mathbf{H}_{i_0} = \mathbf{H}_{ref}$



Partially by integration with the **DT** information



PS-Based Technique

Estimation of ambiguity by means of DT

- **K** can be resolved up to a gain A_0 and an imbalance complex factor f_0 common to all the stack estimates

Data calibration is still unfeasible

- Channels are decoupled but still hampered by imbalance

$$S_{HH}^{cal} \cong A_0 \cdot S_{HH}$$
$$S_{HV}^{cal} \cong A_0 f_0 \cdot S_{HV}$$
$$S_{VH}^{cal} \cong A_0 f_0 \cdot S_{VH}$$
$$S_{VV}^{cal} \cong A_0 f_0^2 \cdot S_{VV}$$

A complete temporal monitoring of the distortion is achievable

- PS stability is able to interlink phase and amplitudes of the distortion alongside the stack

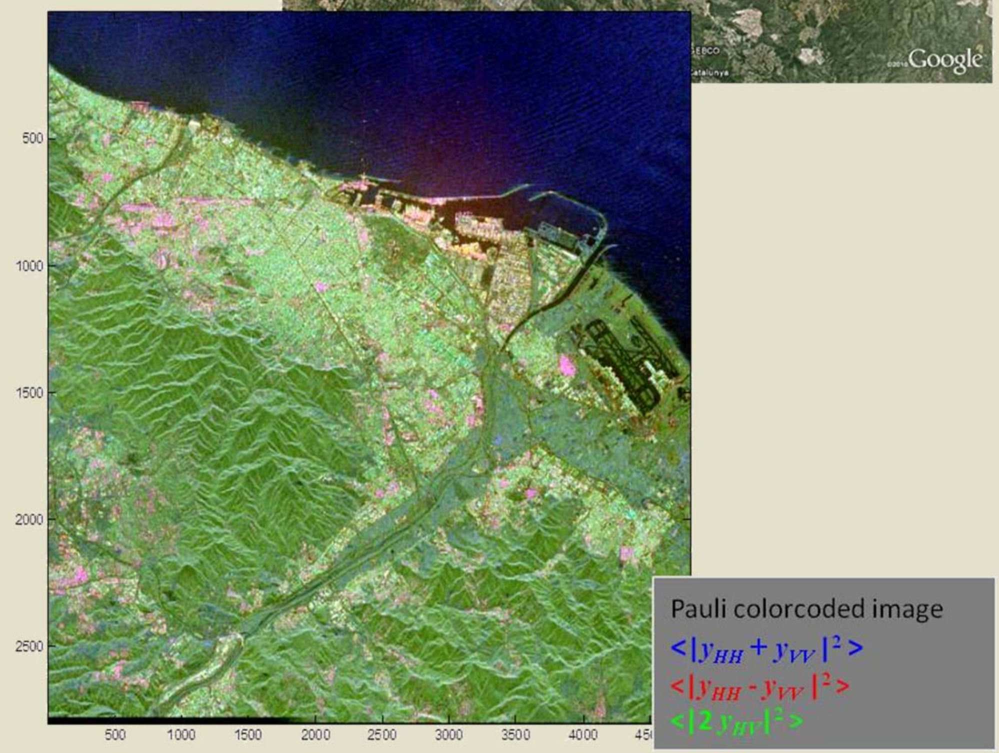
$$\tilde{\mathbf{H}} \Rightarrow \frac{A}{A_0}, \frac{f_1}{f_0}, \frac{f_2}{f_0}, \delta_1, \frac{\delta_2}{f_0}, \delta_3, \frac{\delta_4}{f_0}$$

Application to RS2
Case study

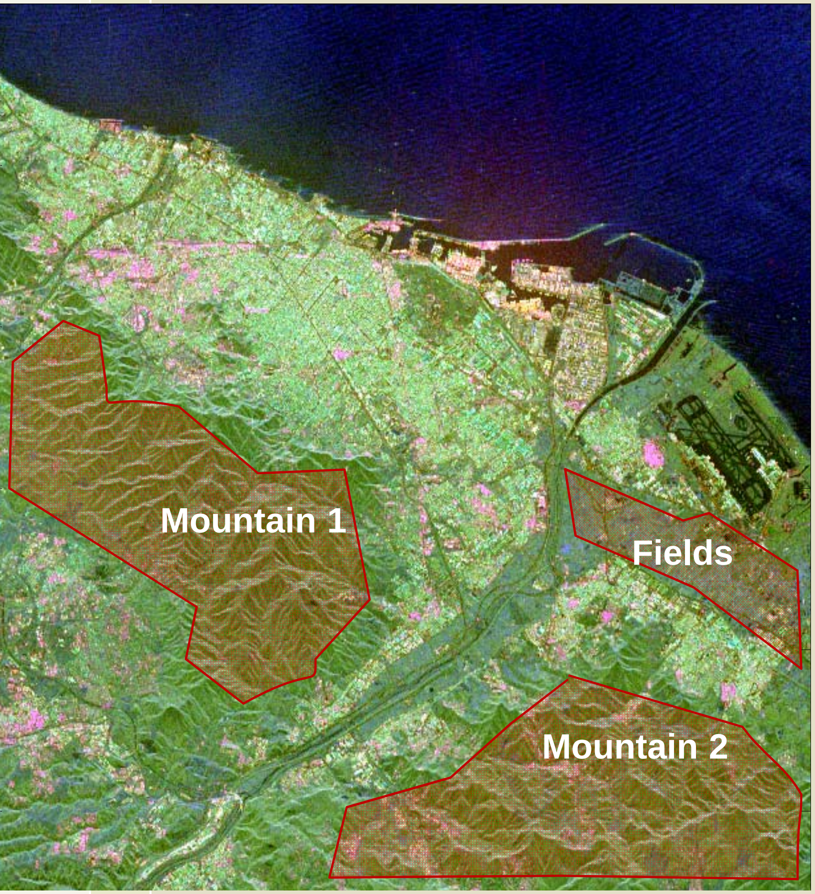
❑ RADARSAT-2 Dataset

Mode	Fine Quad-Pol
Beam	FQ9
Angle	28°-29.8°
Gr. Res.	10.5 – 11.1 m
Width	25 x 25 km

10 images	2008-Apr-12 to 2008-Dec-08
16 images	2010-Feb-13 to 2011-Mar-28



Application to RS2
DT estimates



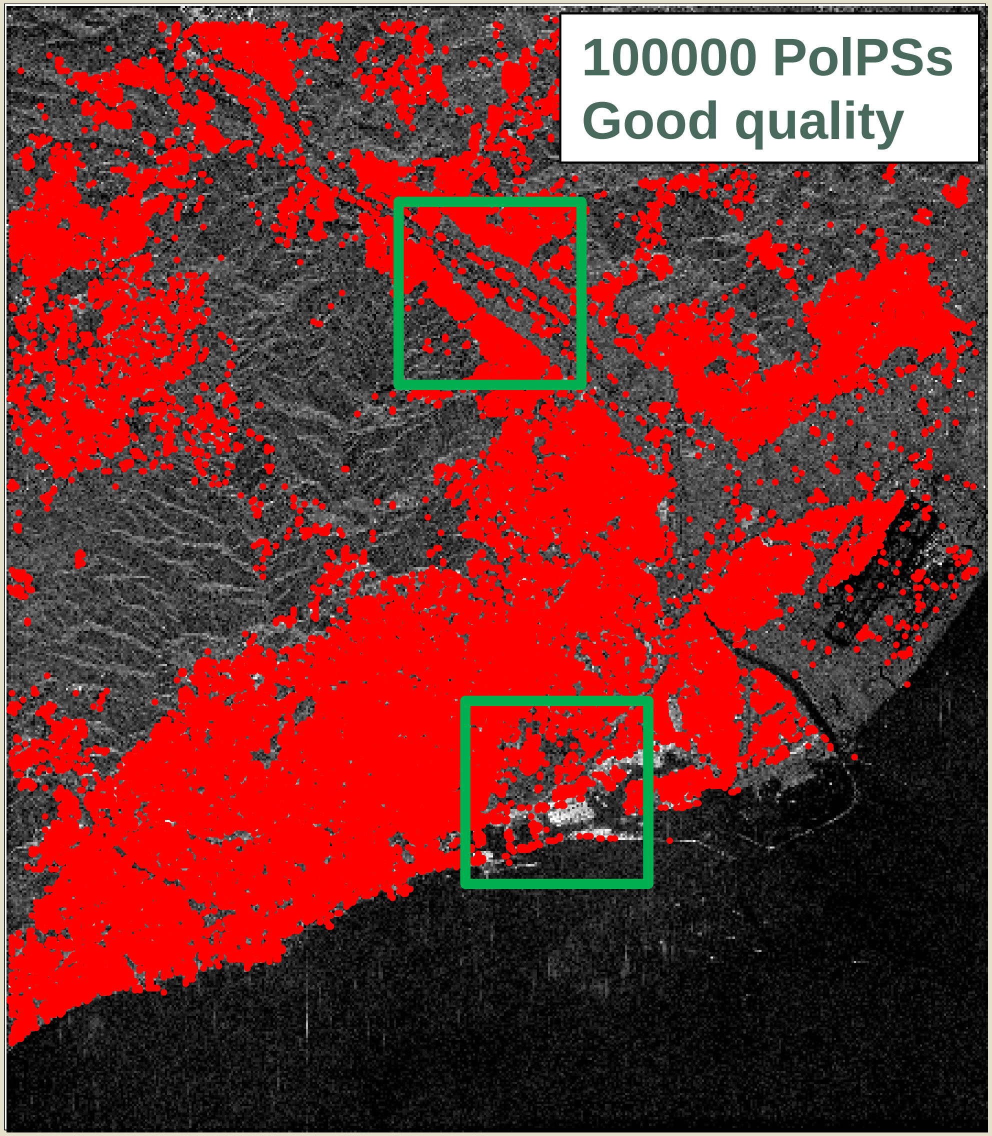
- Constrained to areas which feature orientation symmetry.
- Supervised selection of the most fitted areas

Quegan technique is adopted

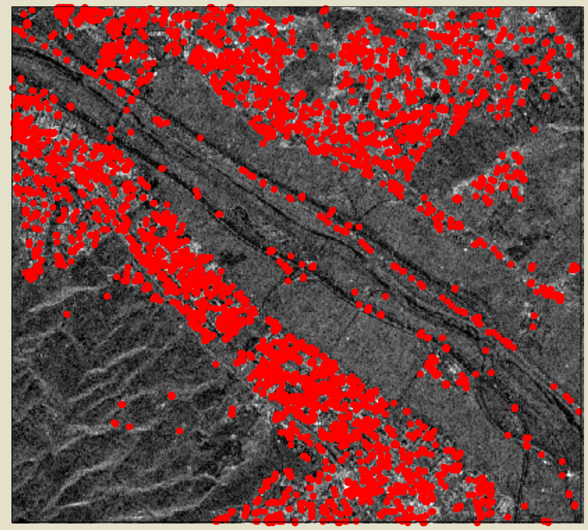
Average values throughout the dataset

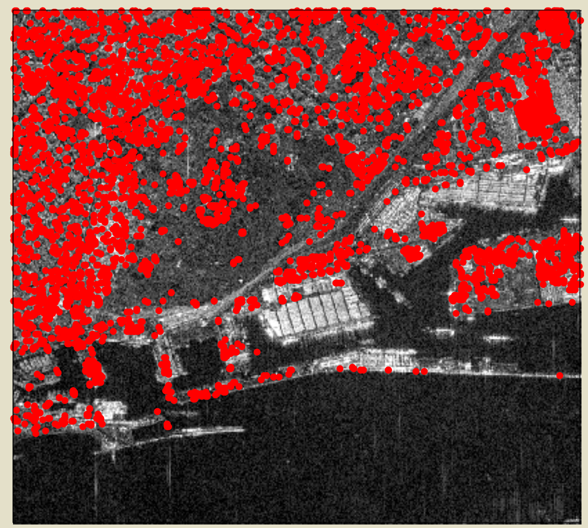
	δ_1 [dB]	δ_2/f_1 [dB]	δ_3 [dB]	δ_4/f_2 [dB]
Mount 1	-46.8	-49.3	-51.5	-52.1
Mount 2	-43.9	-43.2	-47.7	-46
Fields	-44.6	-45.4	-43	-42.9

Application to RS2
PolPS detected



Details





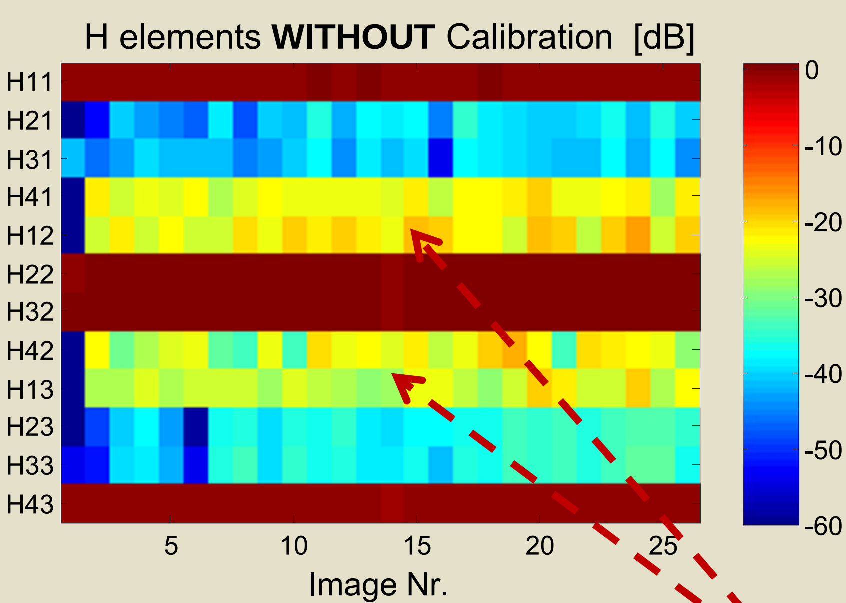
Estimation of ambiguity by means of DT

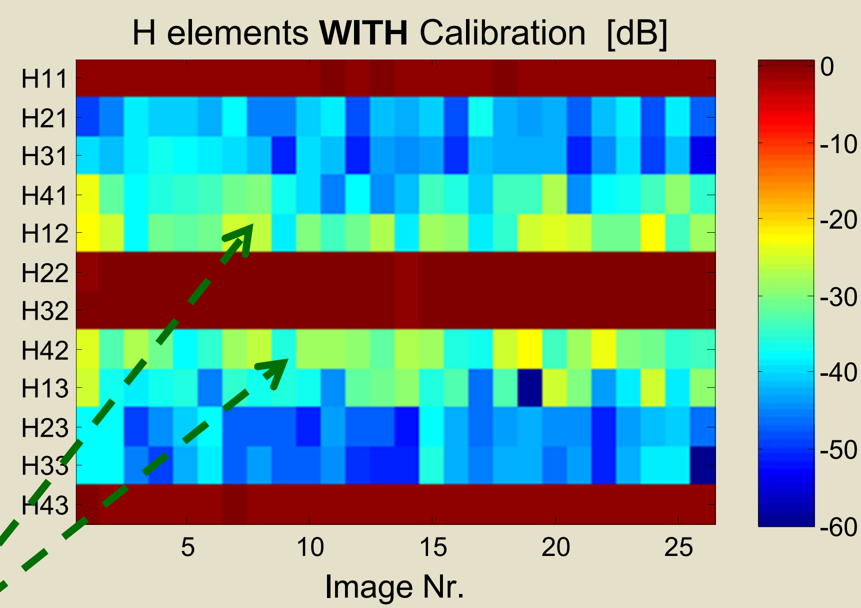
- Example of the calibration effects (on a single imagette)

$$\mathbf{H} = \begin{bmatrix} H_{11} & H_{12} & H_{13} \\ H_{21} & H_{22} & H_{23} \\ H_{31} & H_{32} & H_{33} \\ H_{41} & H_{42} & H_{43} \end{bmatrix}$$

IDEAL PDM

$$\mathbf{H} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



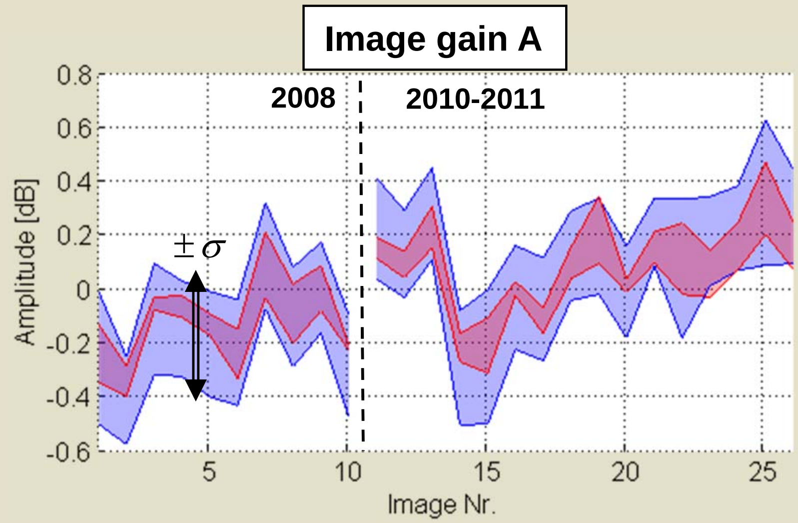


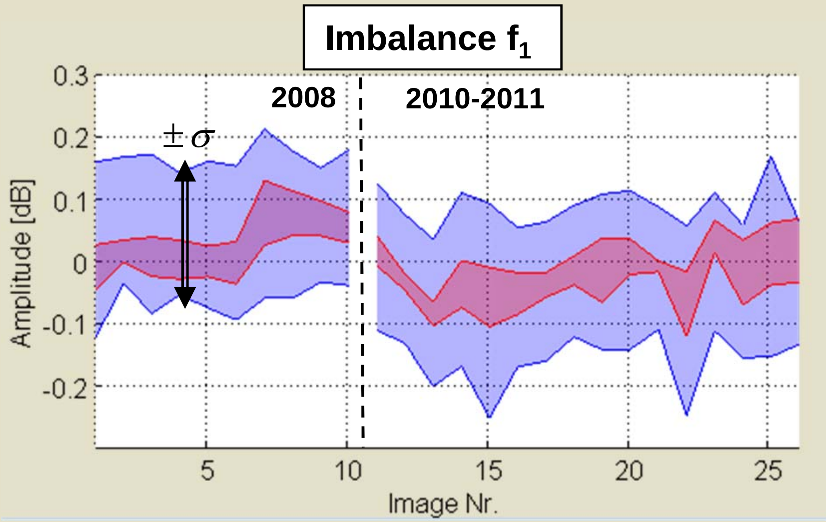
~ From -22 dB to -30/-40 dB

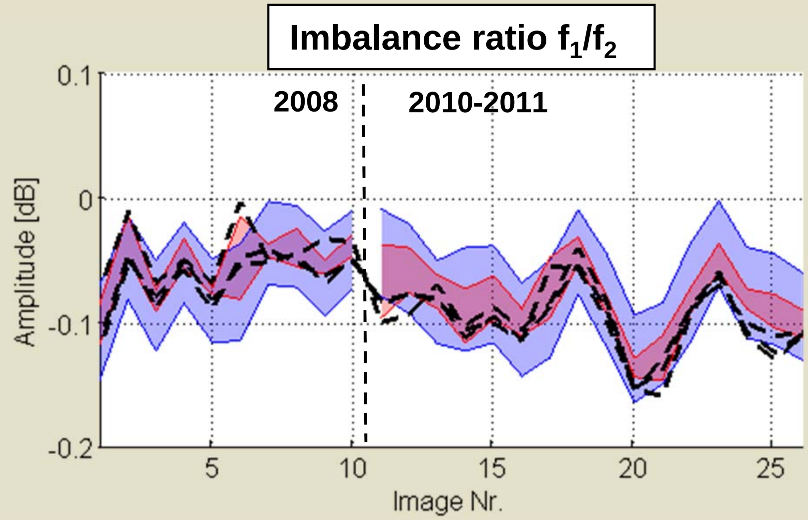
Application to RS2

Amplitude stability on imagettes

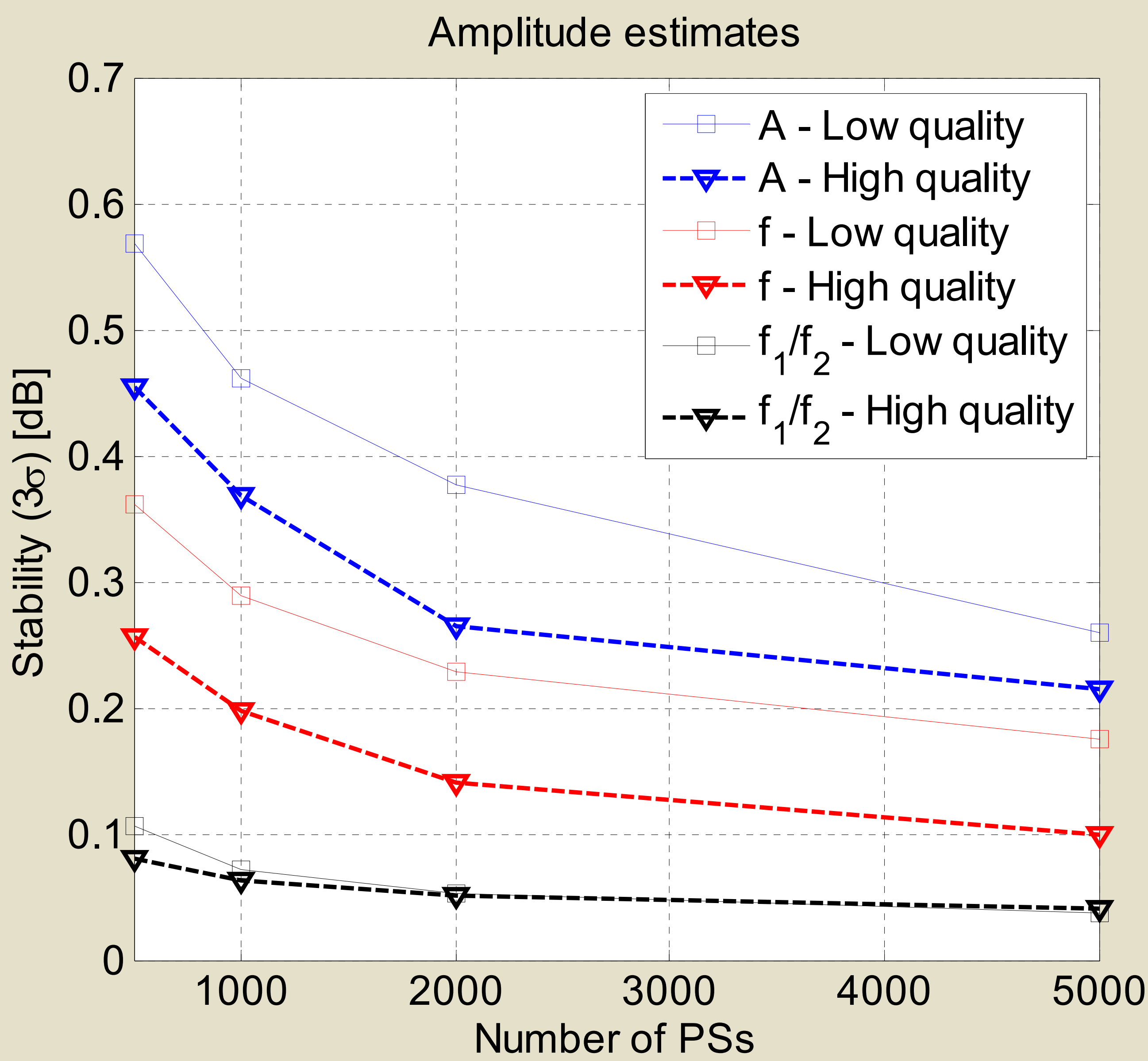
- 500 PS – Low Quality (~ SCR > 8 dB)
- 5000 PS – High Quality (~ SCR > 12 dB)
- Quegan technique



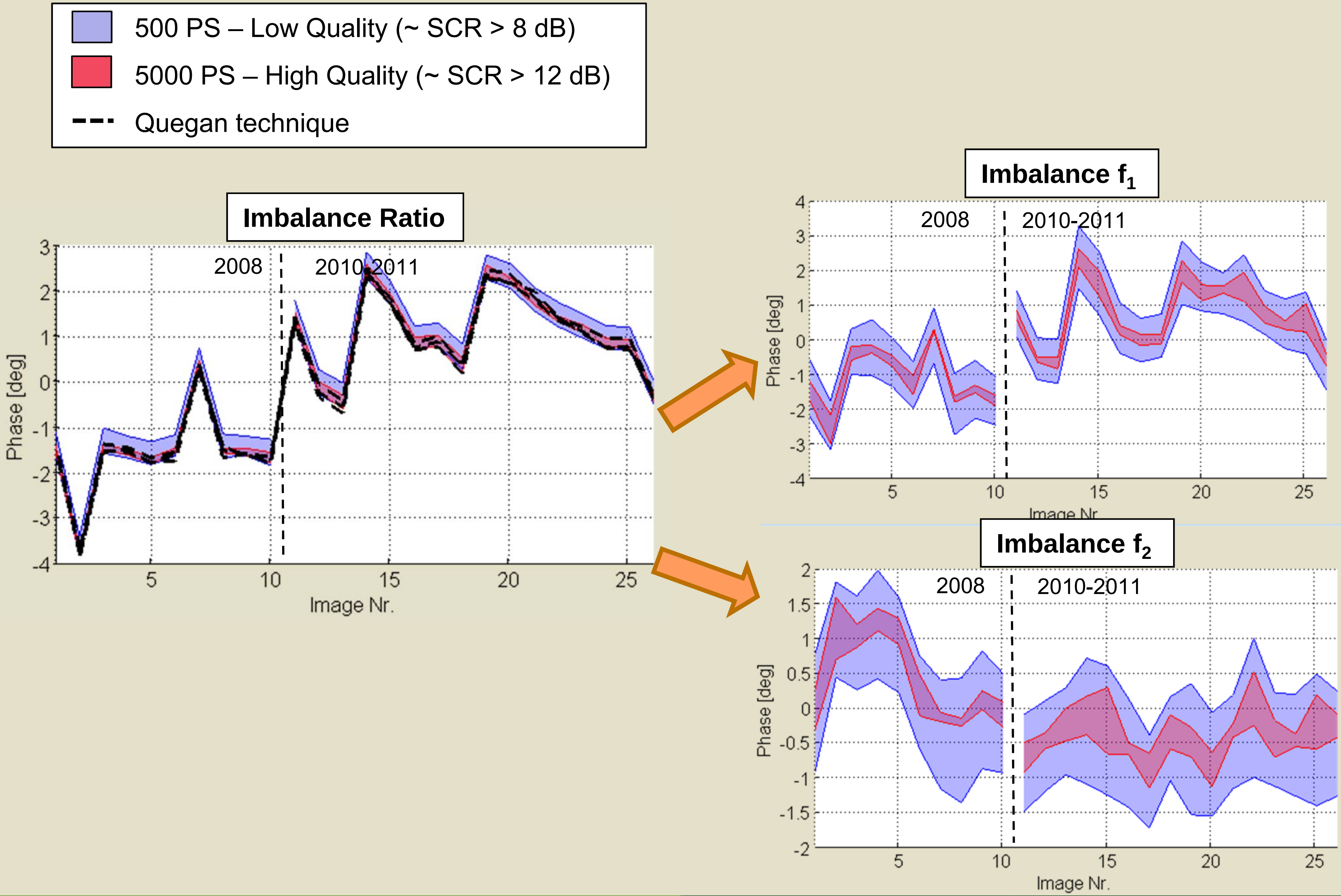




Application to RS2
Amplitude stability on imagettes

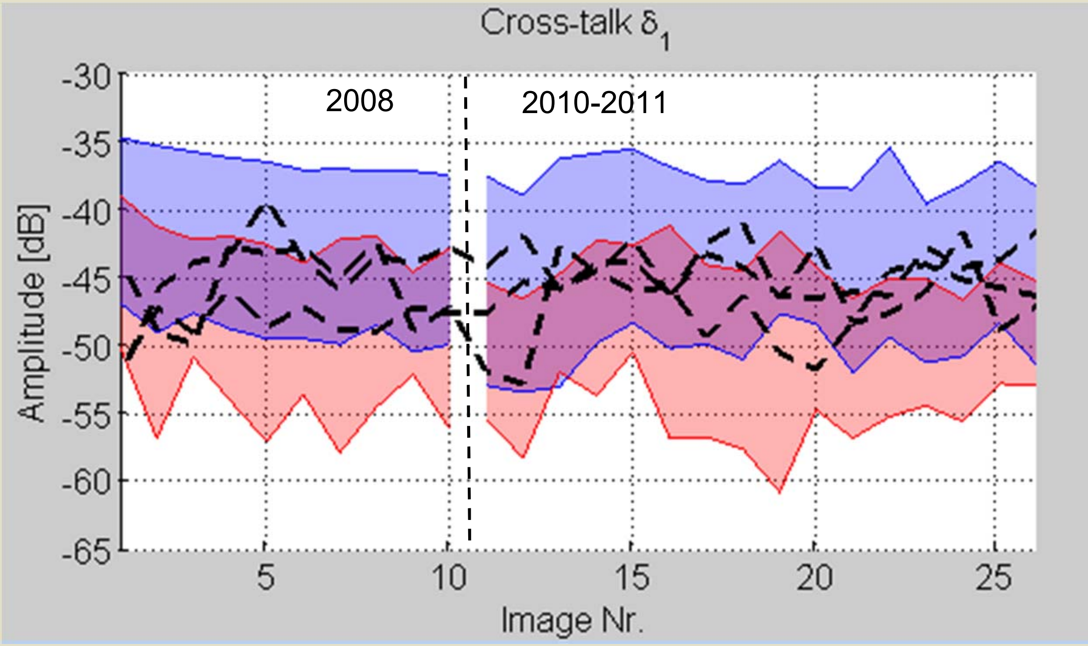


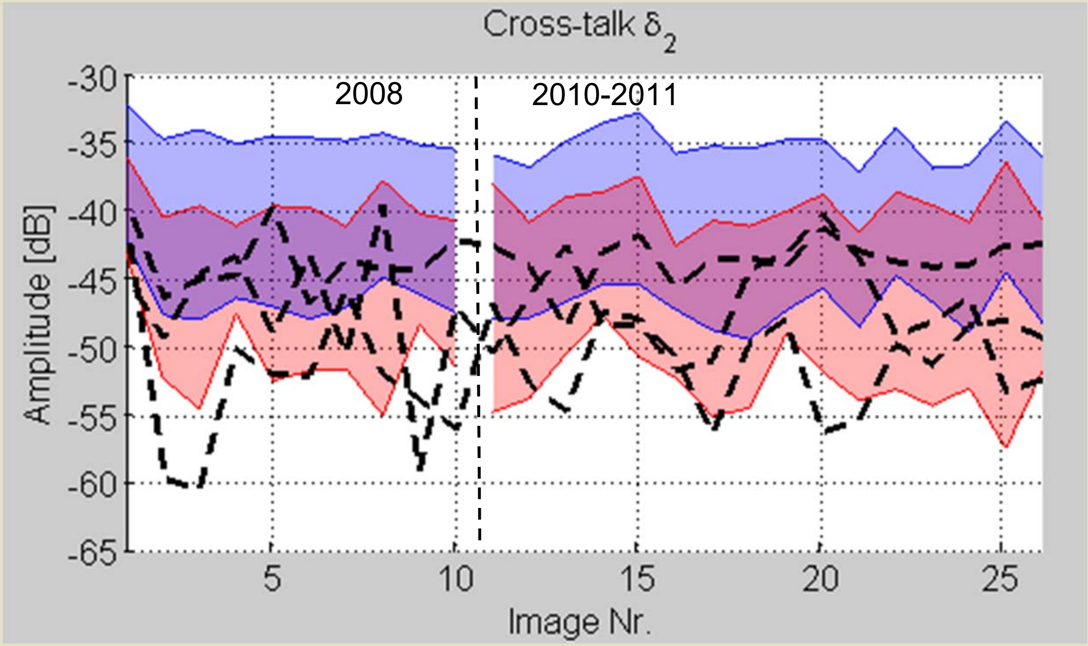
Application to RS2
Phase stability on imagettes



Application to RS2
CT results

- CROSS-TALKS
  - Hard test for PolPSCal
  - The number and quality of PSs must provide higher stability than the RS2 CT level (< - 40 dB) to be effective





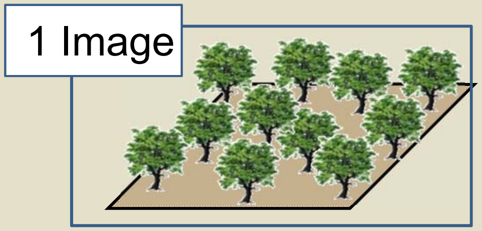
500 PS – Low Quality (~ SCR > 8 dB)
5000 PS – High Quality (~ SCR > 12 dB)
Quegan technique

Conclusions

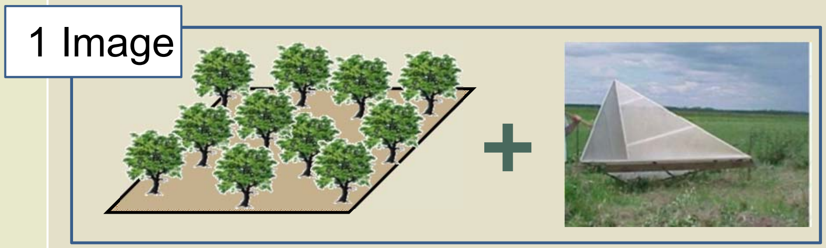
- An algorithm for a joint radiometric and polarimetric calibration based on the natural stable scatterers in the scene has been proposed.
- Full calibration requires a single calibrated image out of the whole stack.
- System distortion fluctuations (temporal monitoring) can be assessed without a calibrated target just through assimilation with DT
  - Technique stability was tested on a 26-images RS2 dataset registering values of stability below 0.3 and 0.2 dB (3σ) for radiometric gain and imbalances.
  - Cross-talk estimation also managed to provide good results. The - 40 dB level was indeed effectively achieved by high quality imagettes.
- Extension to other systems is indeed feasible. The potential for carrying out Faraday compensation seems for instance one the most promising directions for further research

Thank you

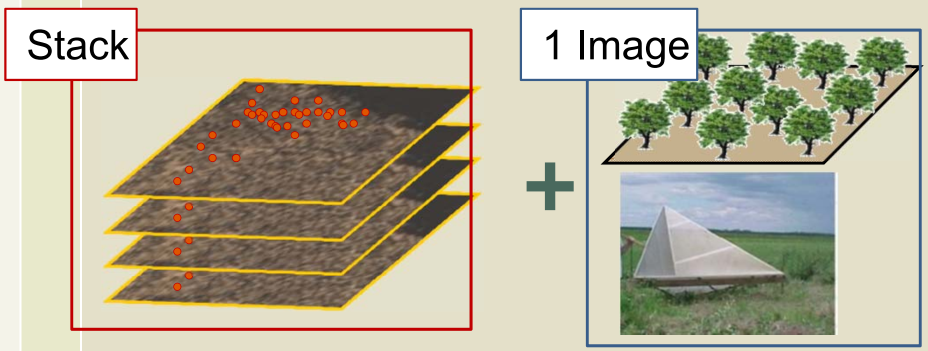
Summary



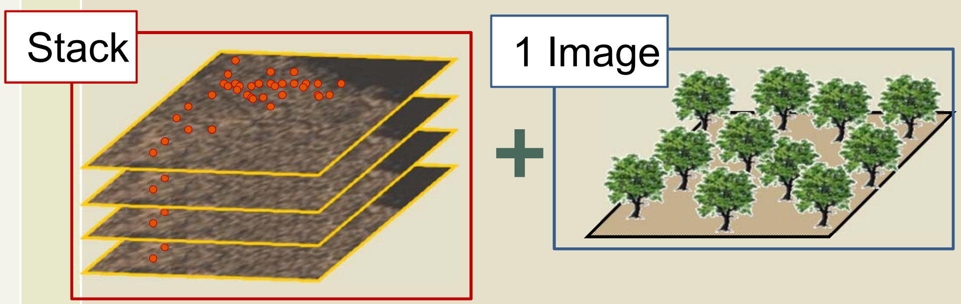
Partial monitoring of the distortion



Full monitoring and calibration



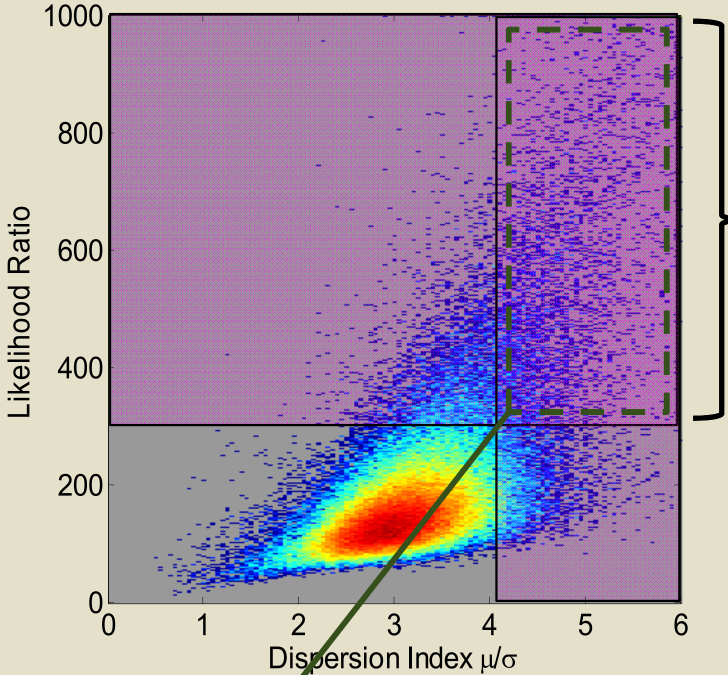
Full monitoring and calibration

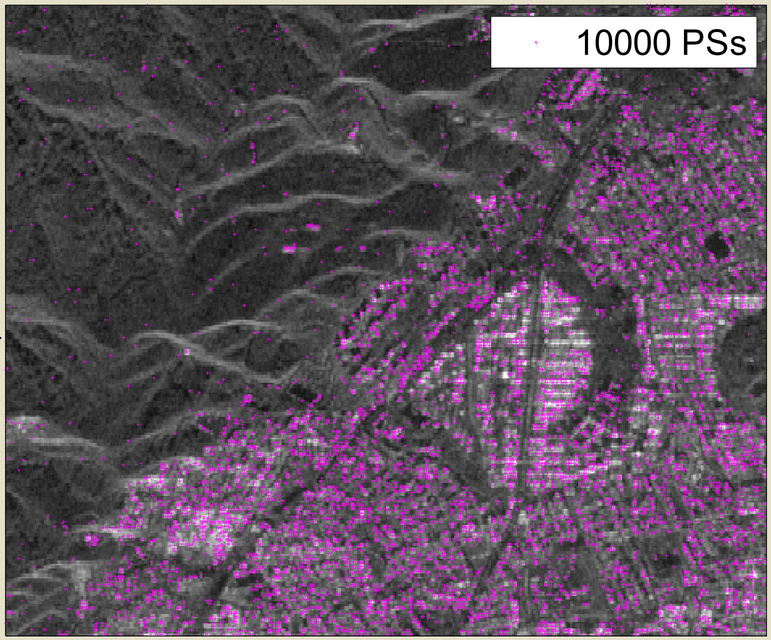


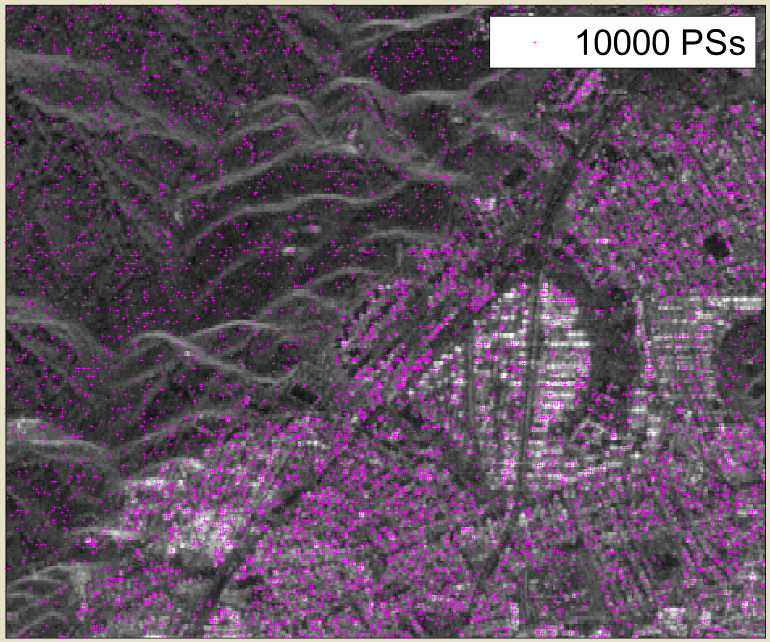
Full temporal monitoring of the distortion
(only partial data calibration)

PS-Based Technique
PS detection

- Best 10000 PS are compared







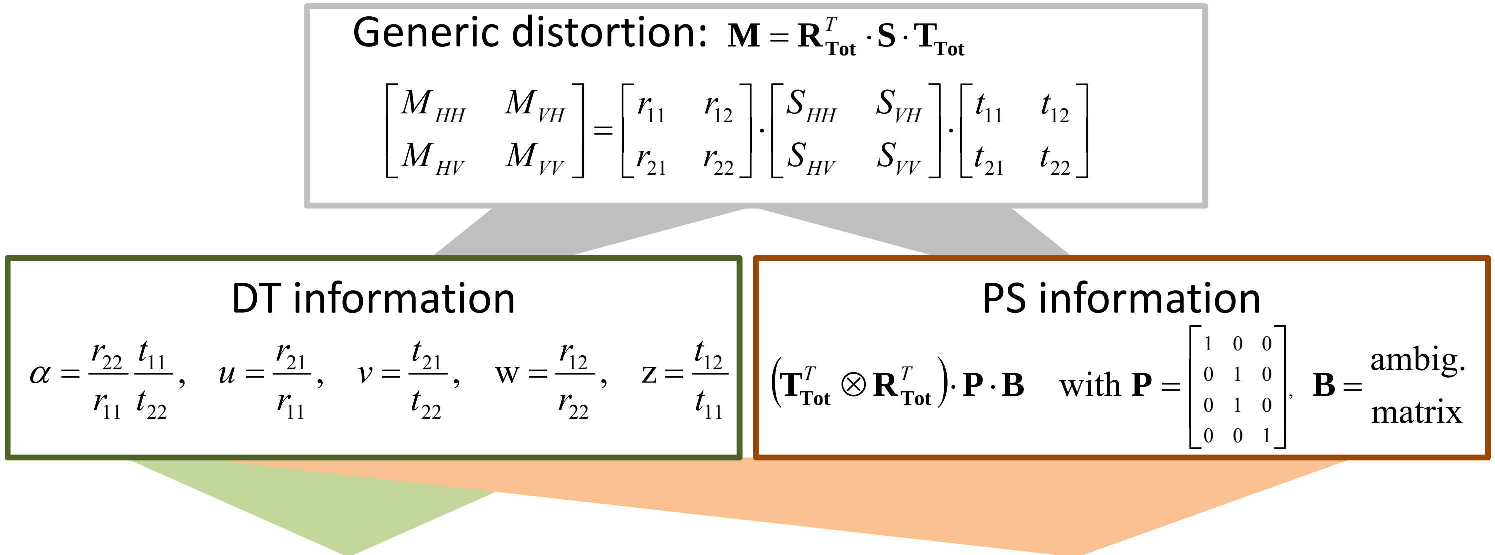
- Common PS: 52 %

Result

- Best quality is reached with the GLRT selection method

LEGEND

DT	Distributed target
PS	Permanent Scatterer
FRA	Faraday rotation angle
CT	Cross-talk



Scenarios	**DT** Monitoring		**DT+PS** monitoring	
	Capabilities	Issues	Capabilities	Issues
Null FRA Full model: $A, f_1, f_2, \delta_1, \delta_2, \delta_3, \delta_4$	The parameter set $\{f_1/f_2, \delta_1, \delta_2/f_1, \delta_3, \delta_4/f_2\}$ is unambiguously retrieved.	An unknown radiometric and phase factor between between the different polarizations must be accounted. It is related to $f_{1,2}$, which can be non-stationary along time.	The $f_{1,2}$ uncertainty is resolved up to a complex factor common for the whole stack. Full temporal system monitoring and Intra-stack calibration are now possible	Inter-stack data processing still requires the common scale factor information that can only be provided by external calibrators.
Non-null FRA CTs reciprocal: $A, f_1, f_2, \delta_1 = \delta_3, \delta_2 = \delta_4, \Omega$	All the parameters can be retrieved with accuracy depending on the FRA and on the imbalance ratio phase (see section ref:sec).	When Faraday is null or close to 0 or the imbalance ratio phase is $\approx \pm \pi/2$ the quality of the estimates is poor.	All the stack distortion can be retrieved when at least one DT estimate has good quality.	When good quality DT estimates cannot be found the user should refer to the previous scenario .
Non-null FRA Full model: $A, f_1, f_2, \delta_1, \delta_2, \delta_3, \delta_4, \Omega$	The distortion model is intrinsically ambiguous. The DT-based parameter set $\{\alpha, u, v, w, z\}$ can be nonetheless computed. The parameters become functions of Ω as well.	Nor system monitoring nor accurate Faraday estimation is possible.	Improvements are eventually possible only if additional model constraints are accounted, such as slow distortion fluctuations along the time-series (hint for future research).	To be investigated.

Estimation of ambiguity by means of DT

- **K** can be resolved up to an imbalance factor

$$\mathbf{K}_{res} = A_0 \begin{bmatrix} 1 & & \\ & f_0 & \\ & & f_0^2 \end{bmatrix} \qquad \mathbf{K}' = \mathbf{K} \cdot \mathbf{K}_{res}$$

is also a valid solution for **K**

- So that the estimated PDMs are:

$$\tilde{\mathbf{H}}_i = \hat{\mathbf{H}}_i \cdot \mathbf{K}^{-1} = \frac{A}{A_0} \cdot \begin{bmatrix} 1 & \delta_2 + \delta_4 & \delta_2\delta_4 \\ \delta_1 & f_1 + \delta_1\delta_4 & \delta_4 f_1 \\ \delta_3 & f_2 + \delta_2\delta_3 & \delta_2 f_2 \\ \delta_1\delta_3 & (\delta_1 + \delta_3) f_2 & f_1 f_2 \end{bmatrix} \cdot \begin{bmatrix} 1 & & \\ & f_0^{-1} & \\ & & f_0^{-2} \end{bmatrix}$$

A complete temporal monitoring of the distortion is achievable

$$\tilde{\mathbf{H}} \Rightarrow \frac{A}{A_0}, \frac{f_1}{f_0}, \frac{f_2}{f_0}, \delta_1, \frac{\delta_2}{f_0}, \delta_3, \frac{\delta_4}{f_0}$$

- PS stability is able to interlink phase and amplitudes of the distortion alongside the stack

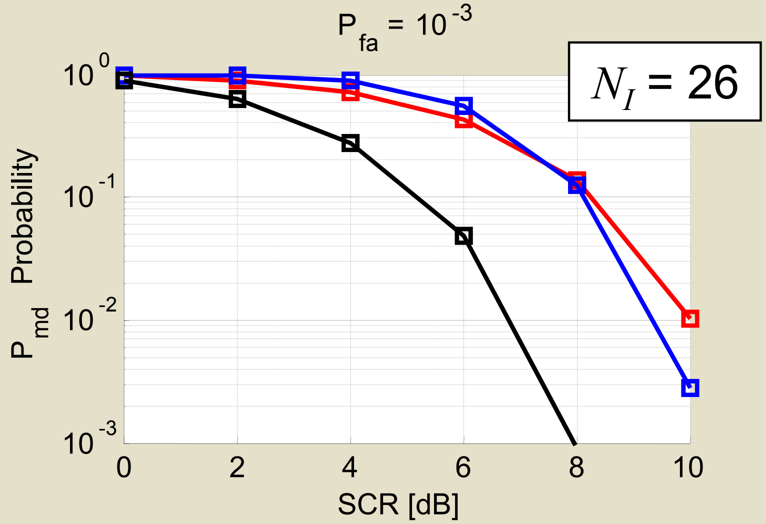
Data calibration is still unfeasible

- Channels are decoupled but still hampered by imbalance

$$M_{HH}^{cal} \cong A_0 \cdot M_{HH}$$
$$M_{HV}^{cal} \cong A_0 f_0 \cdot M_{HV}$$
$$M_{VH}^{cal} \cong A_0 f_0 \cdot M_{VH}$$
$$M_{VV}^{cal} \cong A_0 f_0^2 \cdot M_{VV}$$

PolPSCal Technique
PS detection

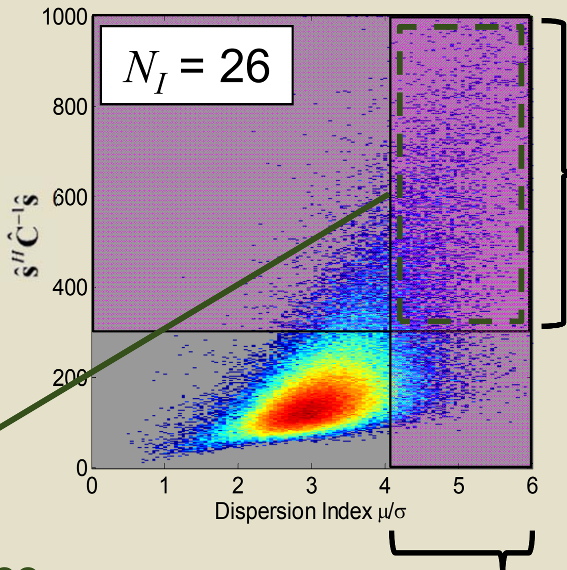
- Theoretical performance

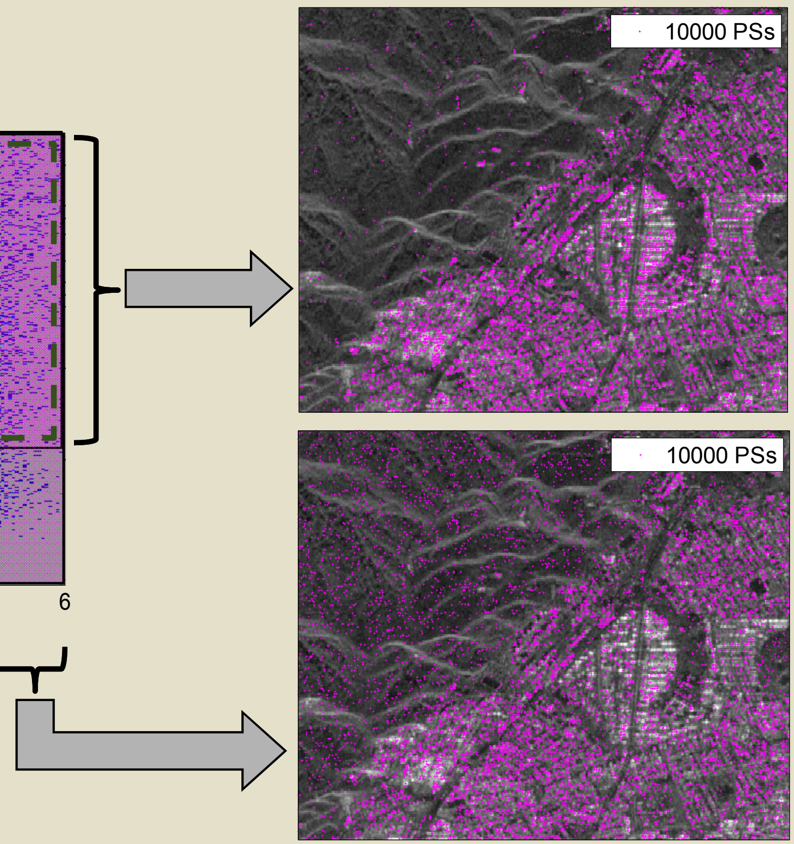


Probabilities of false alarm (Pfa) and missing detection (Pmd)

Based on Montecarlo simulations:
- 1-Pol Dispersion: μ / σ
- SPAN Dispersion: $\mu_{\sqrt{SPAN}} / \sigma_{\sqrt{SPAN}}$
- Likelihood Ratio: $\log(1+ \mathbf{s}^H\mathbf{C}^{-1}\mathbf{s})$

- Application to real dataset



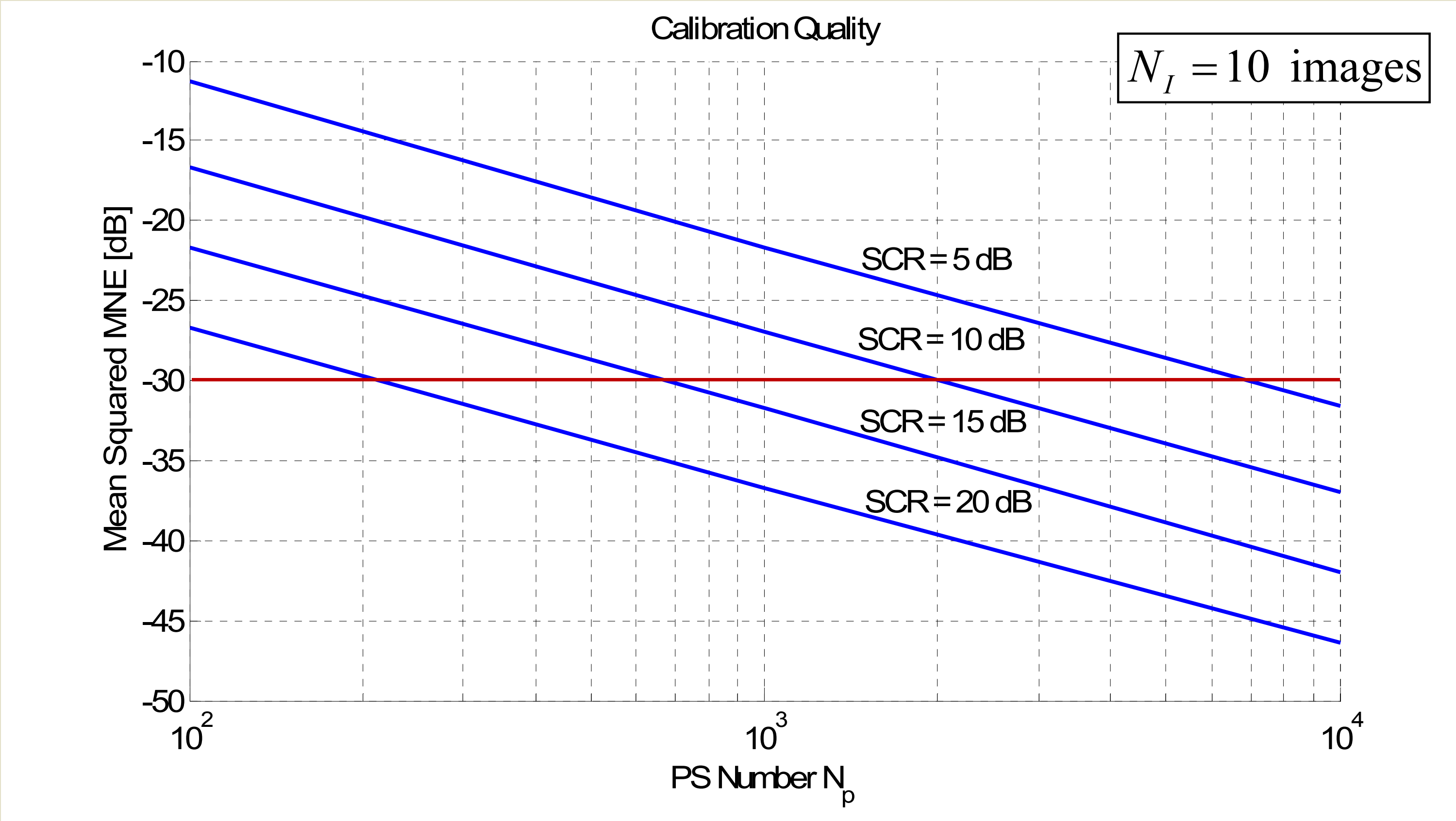


- Best 10000 PS are compared
- 51 % PS common
- Qualitative comparison shows better performance for GRLT (few PSs are found in vegetated area)

PS-Based Technique
Calibration by means of a known PDM

- Simulated performance using an image calibrated without error
- Measured through Maximum Normalized Error (Wang-Ainsworth-Lee*)
  - Distortion experienced by the most unfavourable pol

$$MNE = \max_{\mathbf{x}} \frac{\left\| \mathbf{H}^{cal} \mathbf{x} \right\|}{\left\| \mathbf{x} \right\|}$$

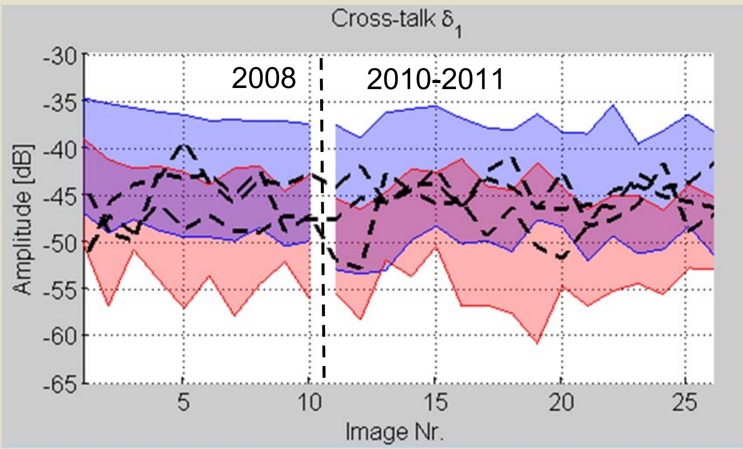


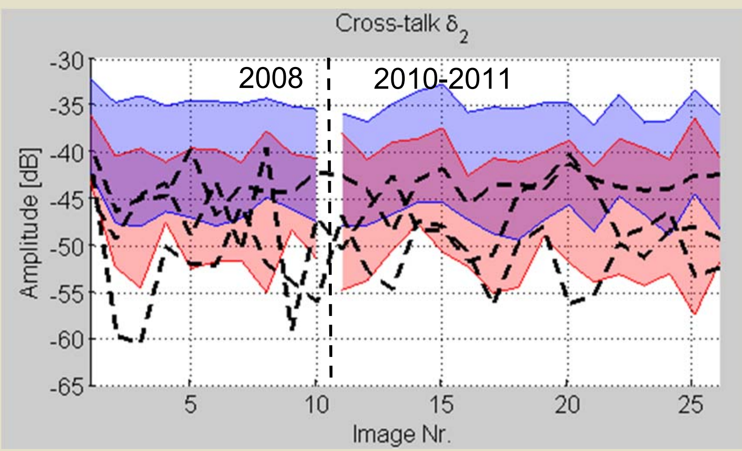
[*] Yanting Wang; Ainsworth, T.L.; Jong-Sen Lee; , "Assessment of System Polarization Quality for Polarimetric SAR Imagery and Target Decomposition," TGRS, May 2011

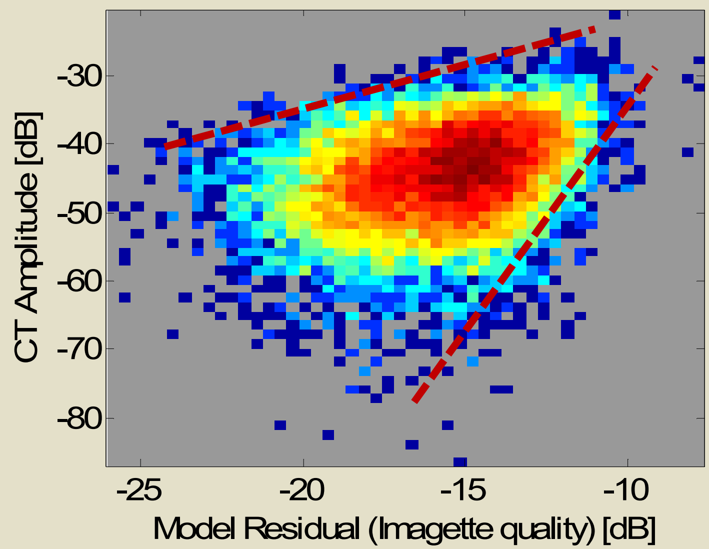
Results

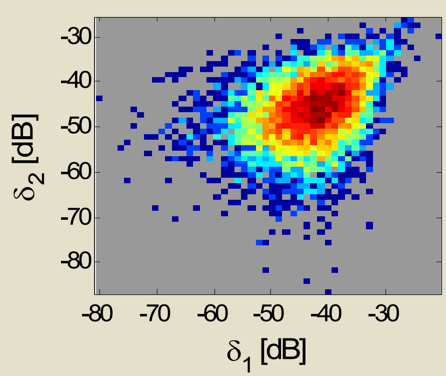
PolPSCal

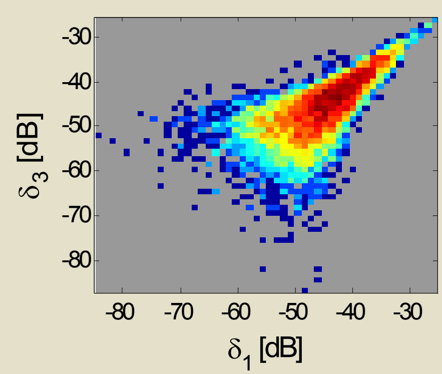
- CROSS-TALKS
  - Hardest test for PolPSCal -> The number and quality of PSs must provide higher precision than the RS2 CT level (< - 40 dB) to be effective
  - Furthermore, the nomalization step must prove to be effective enough in the removal of the ambuigity that impacts notably on the out-of-diagonal PDM elements.











PolPSCal Technique
Normalization

- **Without calibrator**
  - Exploit information provided by traditional techniques

$$\mathbf{H}_{i_0} = A_{i_0} \cdot \mathbf{H}_{i_0}^{DT} \cdot \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & f_2(i_0) & 0 \\ 0 & f_2(i_0) & 0 \\ 0 & 0 & f_2^2(i_0) \end{bmatrix}}_{\mathbf{F}_{i_0}} = \mathbf{H}_{i_0}^{PS} \cdot \mathbf{K}$$

Unknowns ($\mathbf{F}_{i_0}$, $\mathbf{K}$)

Problem is underdetermined. A residual ambiguity $\mathbf{K}_{\text{res}}$ cannot be resolved

$$\mathbf{K}_{\text{res}} = A_{res} \begin{bmatrix} 1 & & \\ & f_{res} & \\ & & f_{res}^2 \end{bmatrix}$$

It is readily verified that

$$\mathbf{F}'_{i_0} = \mathbf{F}_{i_0} \cdot \mathbf{K}_{\text{res}}$$

$$\mathbf{K}' = \mathbf{K} \cdot \mathbf{K}_{\text{res}}$$

are valid solutions of the problem

  - We then arbitrarily set: $\hat{A}(i_0) = 1$, $\hat{f}_2(i_0) = 1$

so that we yield :

$$\hat{\mathbf{H}}_i = \frac{A}{A_0} \cdot \begin{bmatrix} 1 & \delta_2 + \delta_4 & \delta_2\delta_4 \\ \delta_1 & f_1 + \delta_1\delta_4 & \delta_4 f_1 \\ \delta_3 & f_2 + \delta_2\delta_3 & \delta_2 f_2 \\ \delta_1\delta_3 & (\delta_1 + \delta_3) f_2 & f_1 f_2 \end{bmatrix} \cdot \begin{bmatrix} 1 & & \\ & f_0^{-1} & \\ & & f_0^{-2} \end{bmatrix}$$

with $A_0 = A_{i_0}$, $f_0 = f_{i_0}$

  - We can retrieve from each PDM the distortion parameters:

$$\hat{\mathbf{H}}_i \rightarrow \frac{A}{A_0}, \frac{f_1}{f_0}, \frac{f_2}{f_0}, \delta_1, \frac{\delta_2}{f_0}, \delta_3, \frac{\delta_4}{f_0}$$

Apart from a constant scale factor, we are now able to monitor the amplitude and phase trends of the imbalances

PolPSCal Technique
Theoretical Performance

- SVD provides LS optimization for $\mathbf{H}$ and $\mathbf{S}$

$$\mathbf{z}_{i,p} = e^{-j\phi(i,p)}\mathbf{y}_{i,p}$$

$$\hat{\mathbf{s}}_p, \hat{\mathbf{H}}_i = \underset{\mathbf{s}_p,\mathbf{H}_i}{\operatorname{argmin}}\left(\sum_i \sum_p \left\| \mathbf{z}_{i,p} - \mathbf{H}_i \mathbf{s}_p \right\|\right)$$

- Approximate theoretical performace can be derived
  - Unbiased for SCR $\rightarrow \infty$
  - Covariance: $\mathbf{C}_\mathbf{H}(n) = E\left[\operatorname{vec}\left(\hat{\mathbf{H}}_n - \mathbf{H}_n\right)\cdot \operatorname{vec}\left(\hat{\mathbf{H}}_n - \mathbf{H}_n\right)^H\right]$

$$= \kappa\left[\sum_p \sum_i \left(\mathbf{s}_p^* \otimes \mathbf{H}_i^H\right)\Gamma_{i,p}\left(\mathbf{s}_p^* \otimes \mathbf{H}_i^H\right)^H\right]\kappa^H +$$

$$\sum_p \left(\mathbf{S}_{\mathbf{inv}}^T \mathbf{s}_p^* \otimes \mathbf{I}_{[4\times 4]}\right)\Gamma_{n,p}\left(\mathbf{S}_{\mathbf{inv}}^T \mathbf{s}_p^* \otimes \mathbf{I}_{[4\times 4]}\right)^H +$$

$$2\operatorname{Re}\left\{\kappa \sum_p \left(\mathbf{s}_p^* \otimes \mathbf{H}_n^H\right)\Gamma_{n,p}\left(\mathbf{S}_{\mathbf{inv}}^T \mathbf{s}_p^* \otimes \mathbf{I}_{[4\times 4]}\right)^H\right\}$$

The same that would be recursively attained by the estimators

$$\hat{\mathbf{H}}_i = \sum_p \mathbf{z}_{i,p}\hat{\mathbf{s}}_p\left(\sum_p \hat{\mathbf{s}}_p \hat{\mathbf{s}}_p^H\right)^{-1}$$

$$\hat{\mathbf{s}}_p = \left(\sum_i \hat{\mathbf{H}}_i^H \hat{\mathbf{H}}_i\right)^{-1} \sum_i \hat{\mathbf{H}}_i^H \mathbf{z}_{i,p}$$

LEGEND

$$\kappa = \mathbf{S}_{\mathbf{inv}}^T \otimes \mathbf{H}_n \mathbf{H}_{\mathbf{inv}}$$

$$\mathbf{S}_{\mathbf{inv}} = \left(\sum_p \mathbf{s}_p \mathbf{s}_p^H\right)^{-1}$$

$$\mathbf{H}_{\mathbf{inv}} = \left(\sum_i \mathbf{H}_i^H \mathbf{H}_i\right)^{-1}$$

PolPSCal Technique

Theoretical Performance

- SVD provides LS optimization for **H** and **S**

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$$\hat{\mathbf{s}}_p, \hat{\mathbf{H}}_i = \underset{\mathbf{s}_p, \mathbf{H}_i}{\operatorname{argmin}} \left(\sum_i \sum_p \left\| \mathbf{z}_{i,p} - \mathbf{H}_i \mathbf{s}_p \right\| \right)$$

The same that would be recursively attained by the estimators

$$\hat{\mathbf{H}}_i = \sum_p \mathbf{z}_{i,p} \hat{\mathbf{s}}_p \left(\sum_p \hat{\mathbf{s}}_p \hat{\mathbf{s}}_p^H \right)^{-1}$$

$$\hat{\mathbf{s}}_p = \left(\sum_i \hat{\mathbf{H}}_i^H \hat{\mathbf{H}}_i \right)^{-1} \sum_i \hat{\mathbf{H}}_i^H \mathbf{z}_{i,p}$$

- Approximate theoretical performace can be derived
  - Unbiased for SCR $\rightarrow \infty$
  - Covariance: $\mathbf{C}_{\mathbf{H}}(n) = E\left[\operatorname{vec}\left(\hat{\mathbf{H}}_n - \mathbf{H}_n\right) \cdot \operatorname{vec}\left(\hat{\mathbf{H}}_n - \mathbf{H}_n\right)^H \right]$

LEGEND

$$\kappa = \mathbf{S}_{\mathbf{inv}}^T \otimes \mathbf{H}_n \mathbf{H}_{\mathbf{inv}}$$

$$\mathbf{S}_{\mathbf{inv}} = \left(\sum_p \mathbf{s}_p \mathbf{s}_p^H \right)^{-1}$$

$$\mathbf{H}_{\mathbf{inv}} = \left(\sum_i \mathbf{H}_i^H \mathbf{H}_i \right)^{-1}$$

Not straightforward

$\operatorname{Tr}\{.\} \approx$ Total MSE

Simplification: $\mathbf{H} = \mathbf{I}, \quad \Gamma_{i,p} = \Gamma_W$

$$\operatorname{Tr}\{\mathbf{C}_{\mathbf{H}}\} = \left(\frac{1}{N_I} + 2 \right) \cdot \operatorname{Tr}\{\mathbf{S}_{\mathbf{inv}}\} \left(\operatorname{Tr}\{\Gamma_W\} - \sigma_n \right) + \operatorname{Tr}\{\mathbf{S}_{\mathbf{inv}}\} \operatorname{Tr}\{\Gamma_W\}$$

- N_I is not influent for estimation (it is only important for detection)
- PS density and quality determine:

$$\Delta_{[\mathrm{dB}]} MSE \cong \Delta_{[\mathrm{dB}]} SCR$$

$$\Delta_{[\mathrm{dB}]} MSE \cong -\Delta_{[\mathrm{dB}]} N_p$$

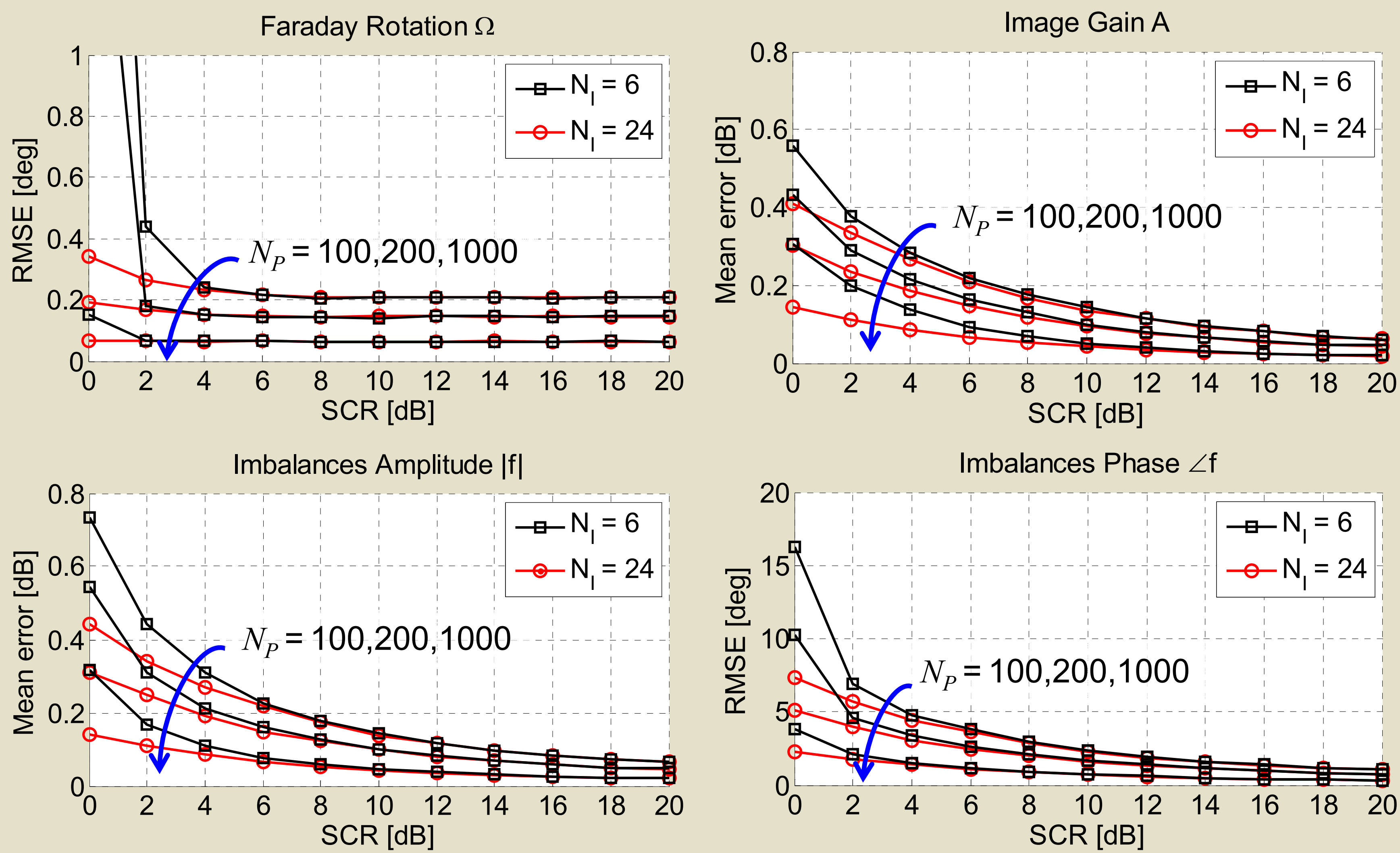
Impact of the number of images N_I and of the clutter to thermal noise ratio

Dependence on the number of targets N_p and on the SCR

Estimation Step

Theoric performance

PS			Clutter (temporally uncorralated)		Thermal et al.
P_{hh} [dB]	P_{hv} [dB]	P_{vv} [dB]	coher. $c_{HH,VV}$	$c_{HV,HH} = c_{HV,VV}$	NESZ [dB]
0	-7	1.5	0.2·exp(j·20°)	0	-15



Estimation Step

Theoric performance – C-band (no Faraday)

Images	P_{hh} [dB]	P_{hv} [dB]	P_{hv} [dB]	SCR [dB]	NESZ [dB]
8	0	-12	0	from 5 to 20	-25